Introduction

601.442/642 Modern Cryptography

20th January 2026

Course Staff

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What is Cryptography?

A Brief History of Cryptography

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Classical Cryptography: The art of secret writing

Pre-1950

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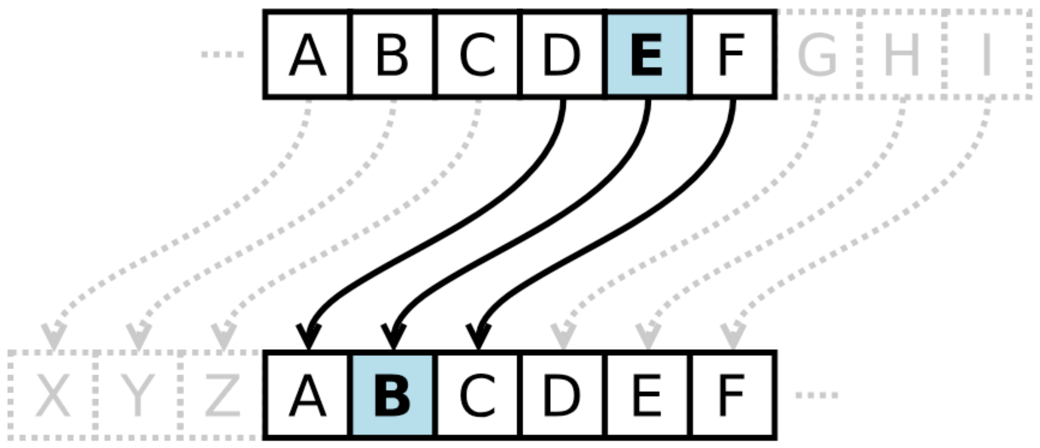
Pre-1950



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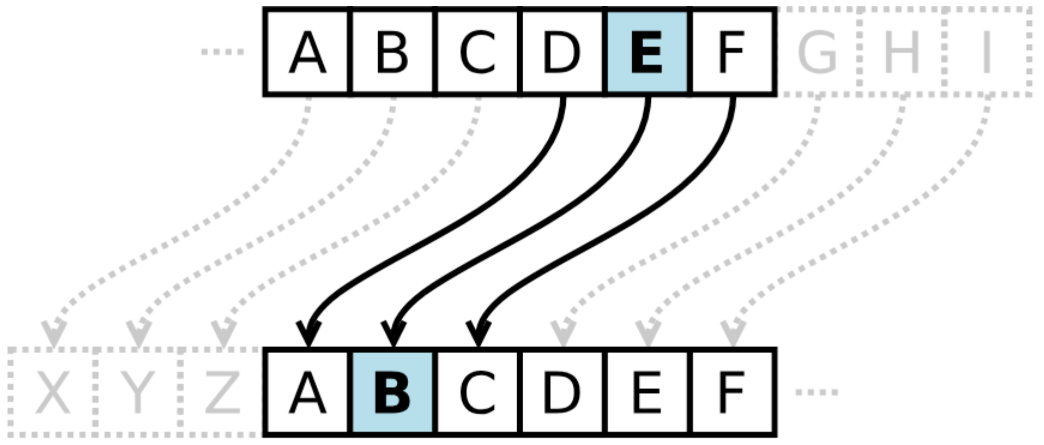

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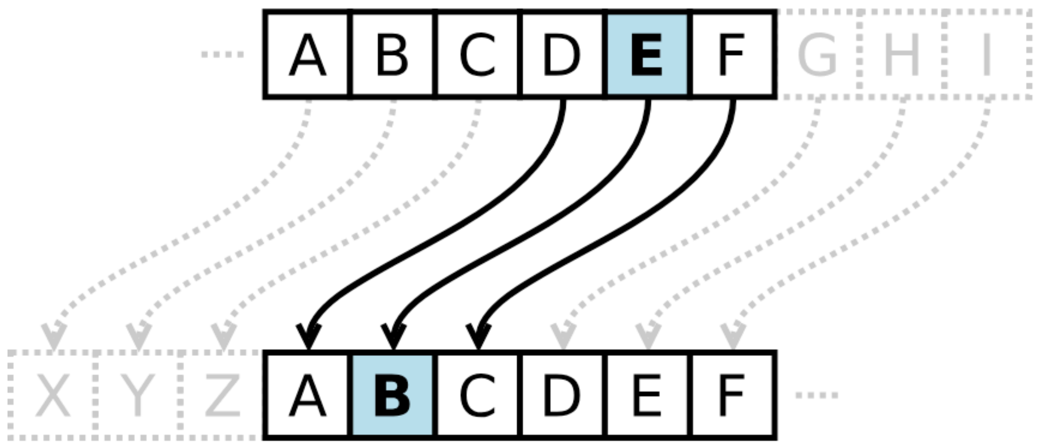

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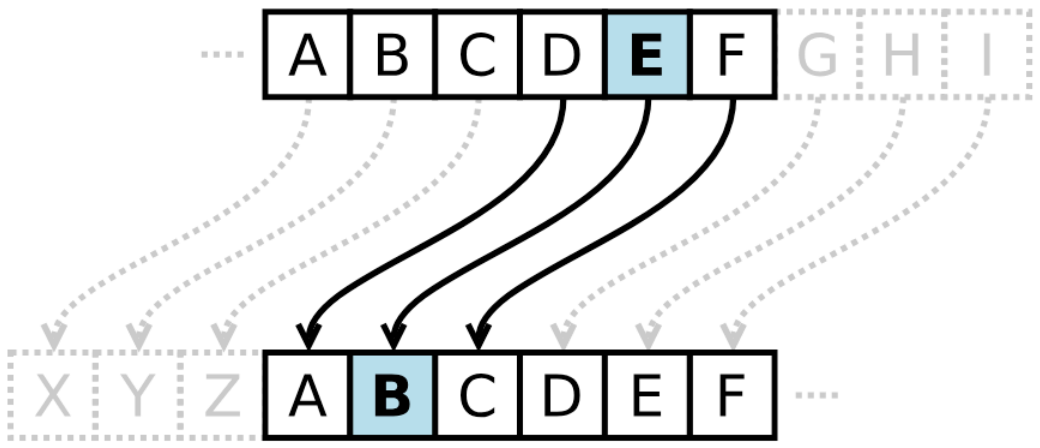

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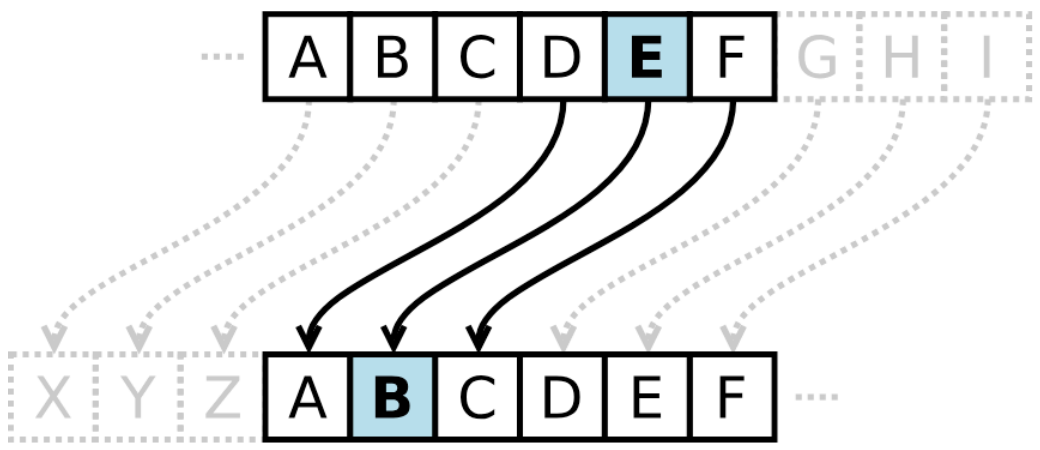

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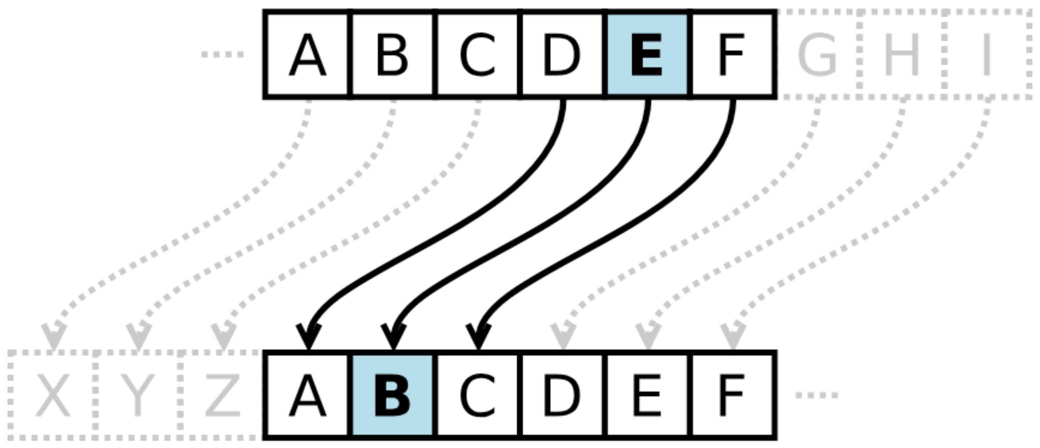

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One of the first applications of computing

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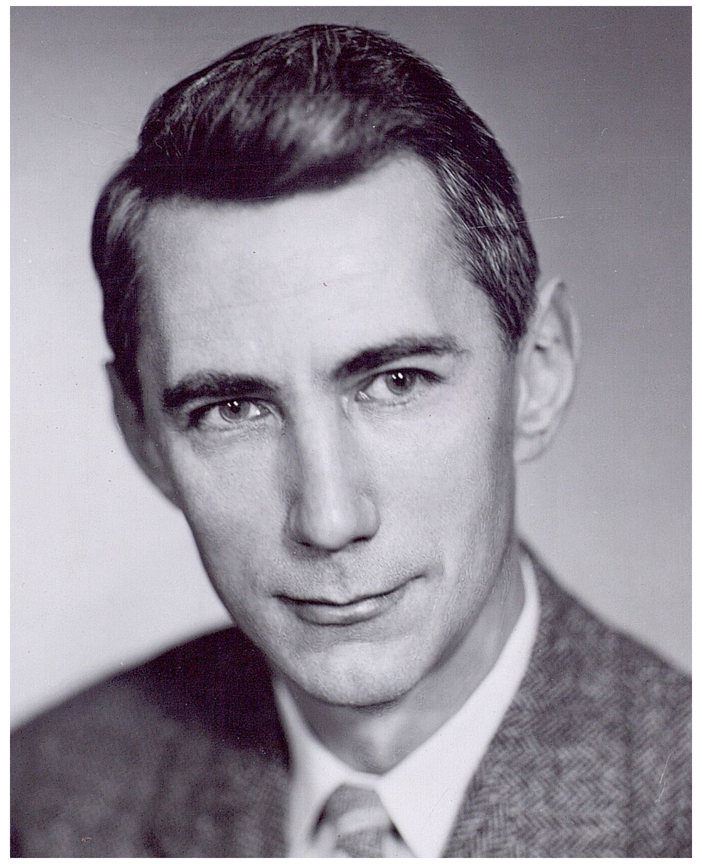

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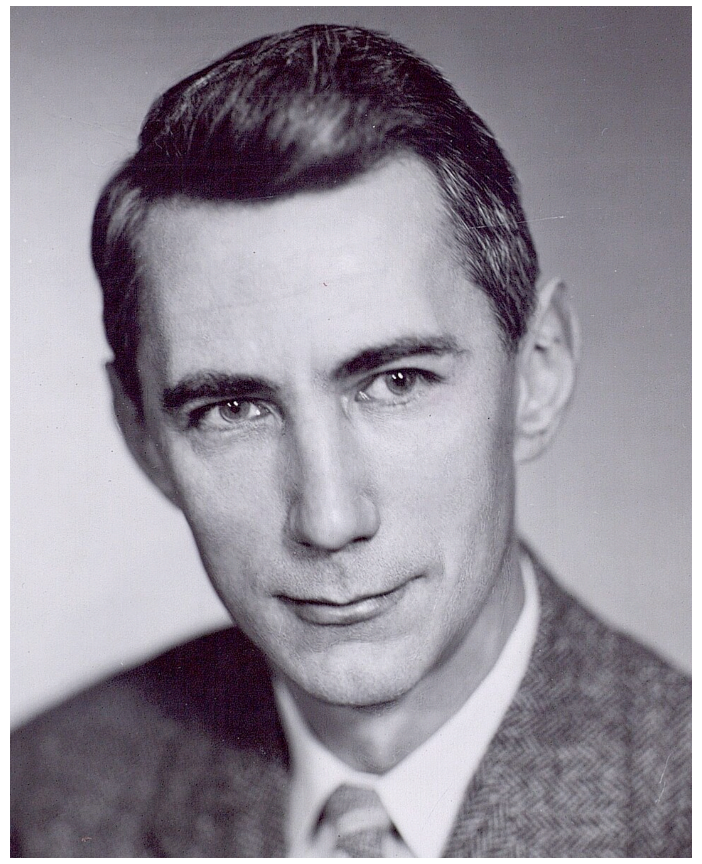

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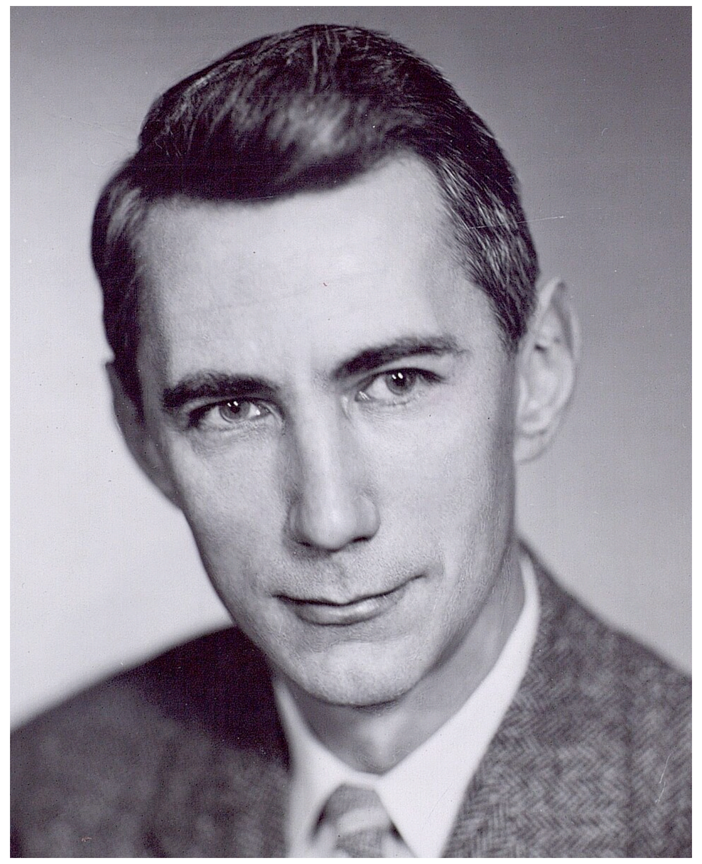

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"Communication Theory of Secrecy Systems" (1949)

Mathematical foundations of secrecy

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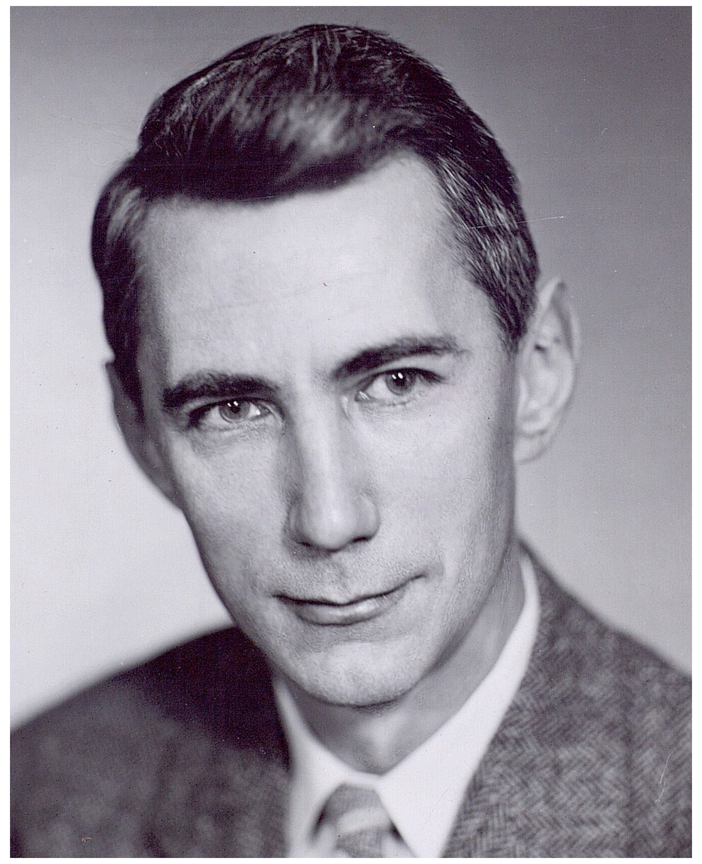
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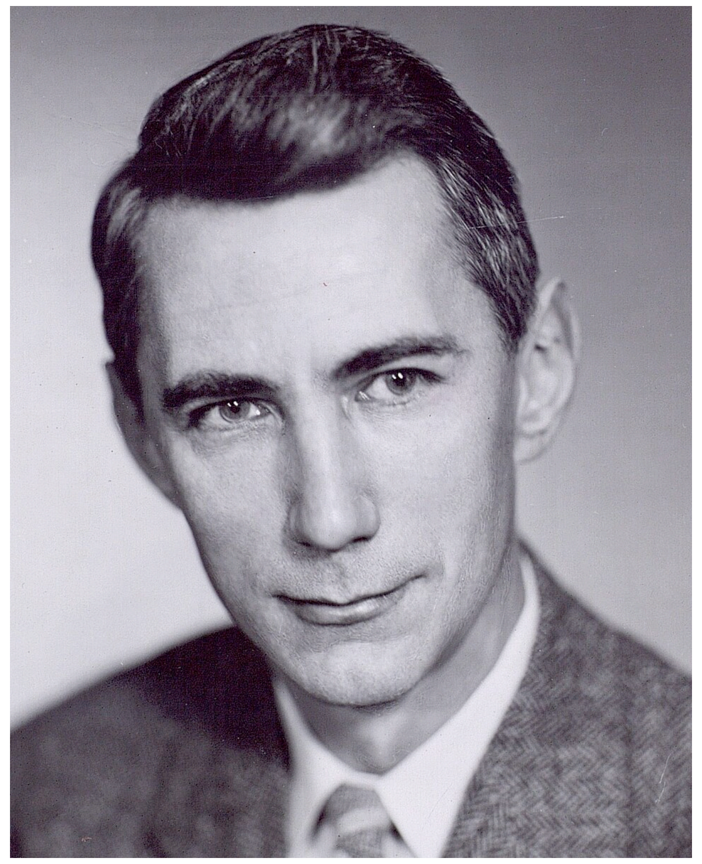

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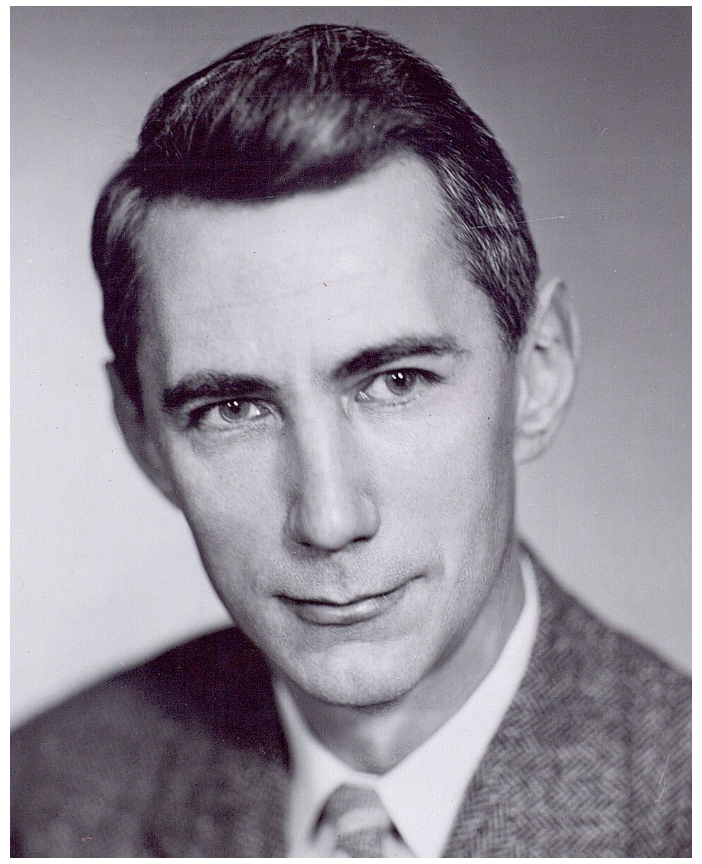

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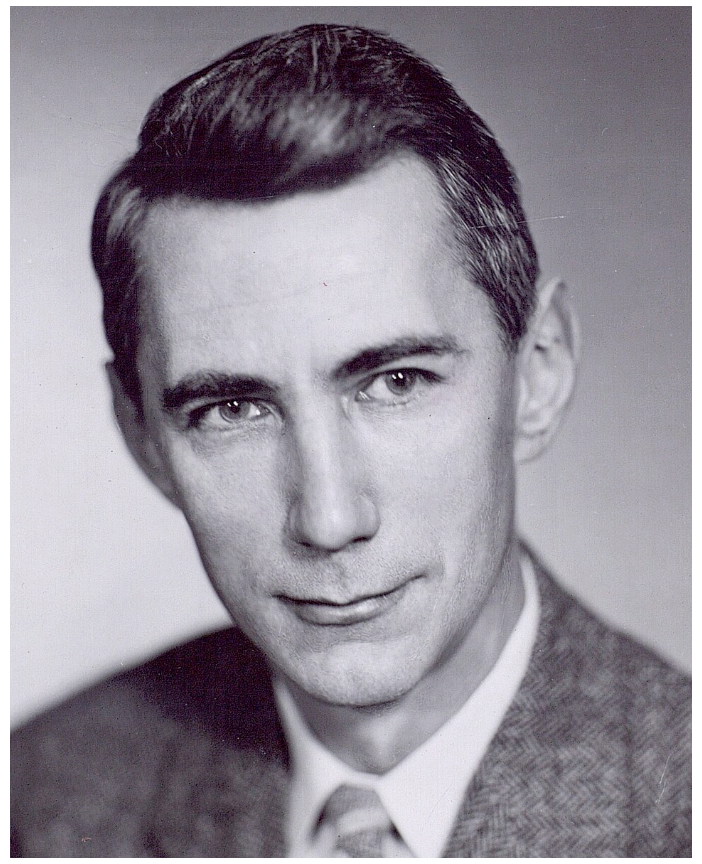

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8 of them have since won the Turing Award (10 in total so far).

Modern Cryptography

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Design and analysis of systems that need to withstand malicious attempts to abuse it.

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Another Definition:

Algorithmic and mathematical foundations of secure communication and computation.

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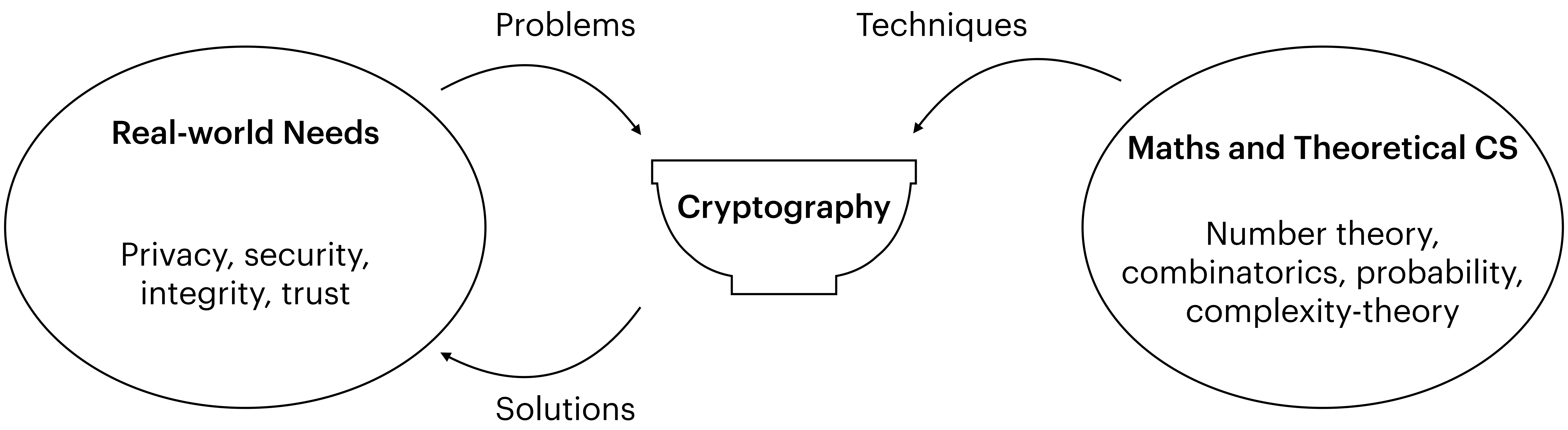

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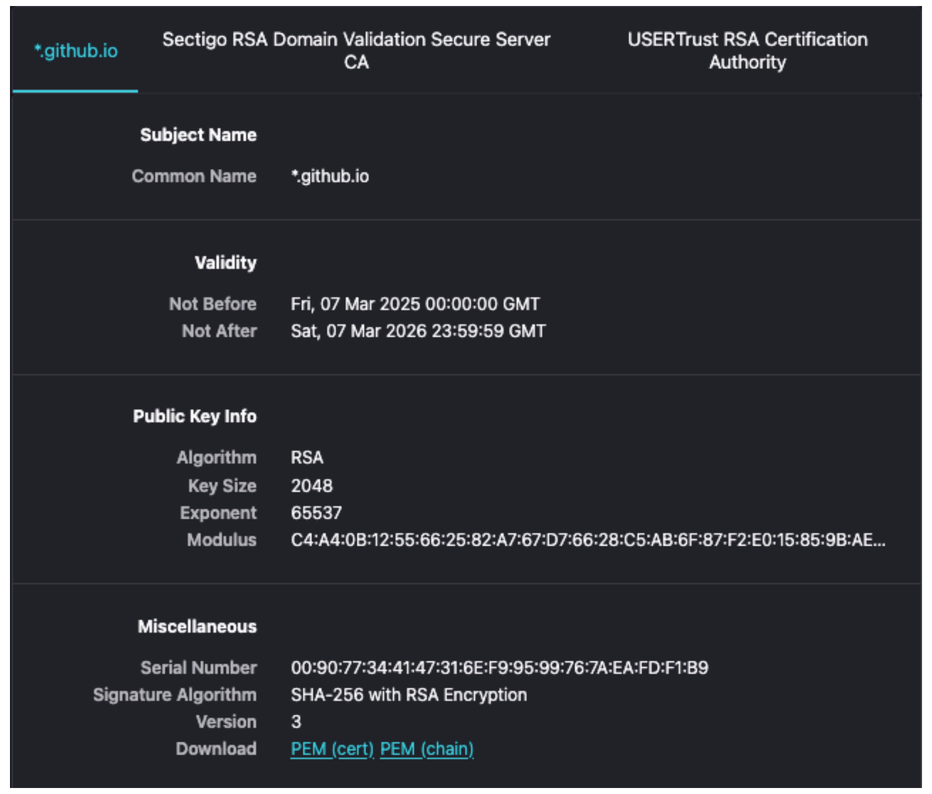

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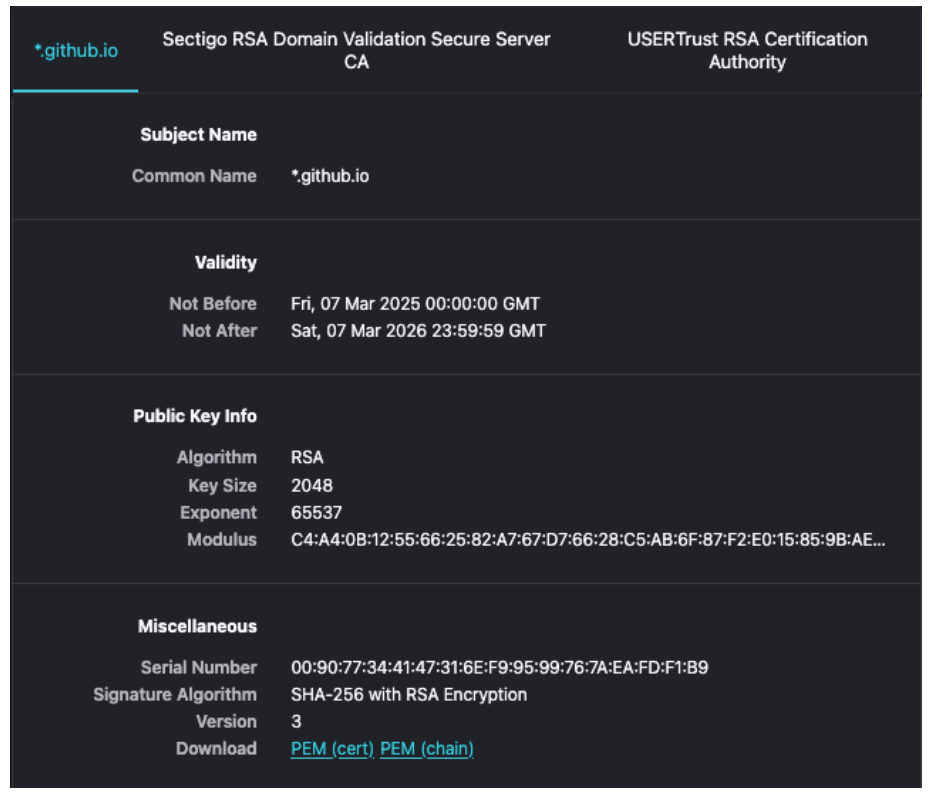

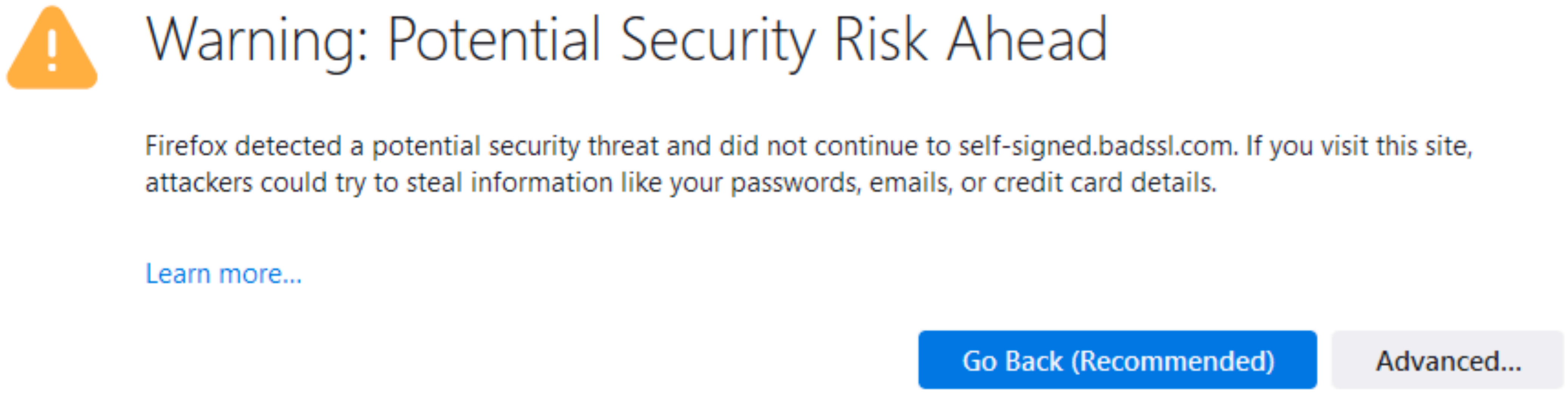

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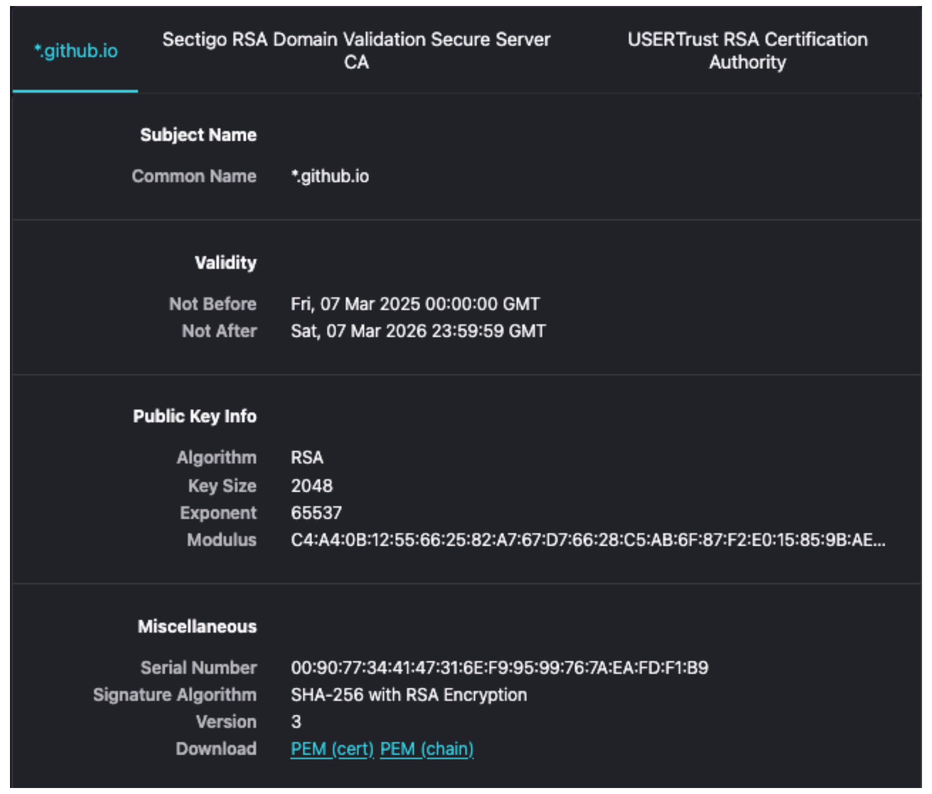

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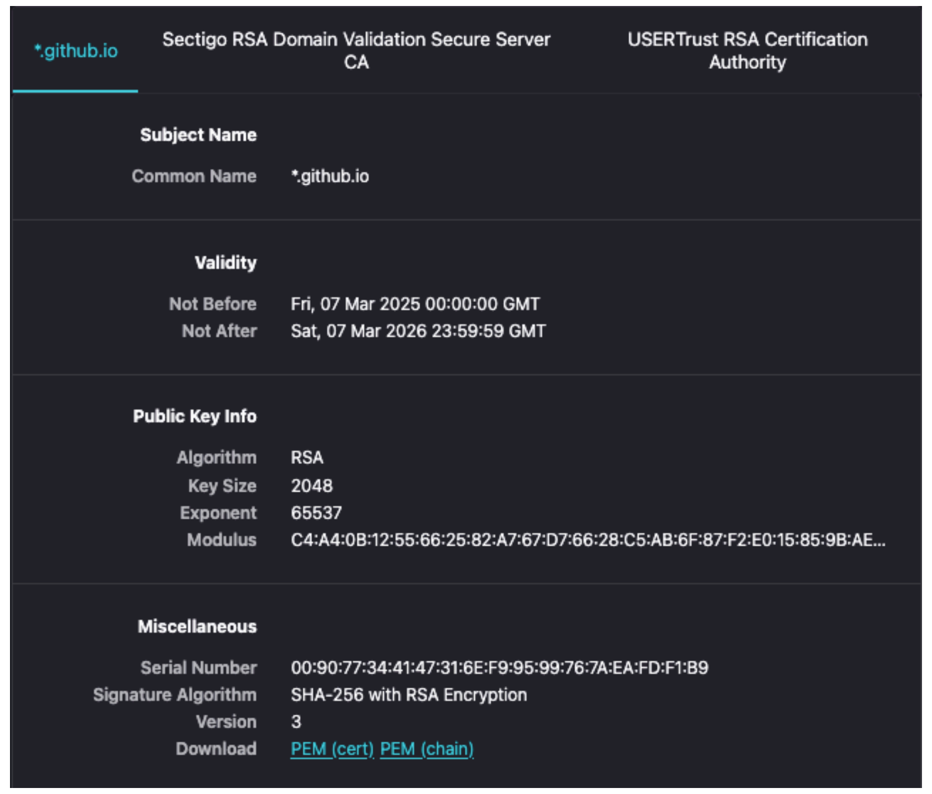

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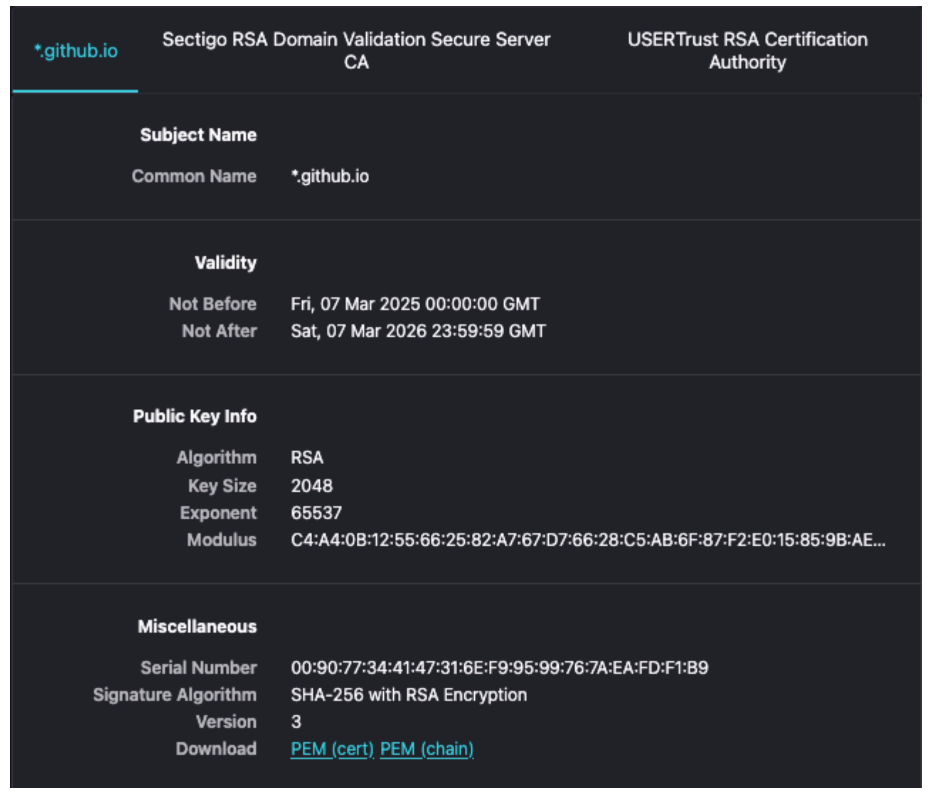

This course is about the foundations of Cryptography.

- Pseudorandomness
- Public-key Encryption
- Digital Signatures
- Hash Functions
- Zero-knowledge Proofs
- Secure Computation

The Pillars of Modern Cryptography

Definitions

Hardness Assumptions

Proofs

The Pillars of Modern Cryptography

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Hardness Assumptions

Proofs

If you cannot **define** something, you cannot achieve it.

The Pillars of Modern Cryptography

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Hardness Assumptions

Proofs

If you cannot **define** something, you cannot achieve it.

Model Worst-case Adversary:
- What they know
- What they can do
- What are their goals

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Hardness Assumptions

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Use hard problems to **constrain** the adversary.

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Source of hard problems: number theory,

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Cryptography is the science of useful hardness.

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Hardness Assumptions

Proofs

Formally argue why a system satisfies the definition.

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Reductions: If an adversary breaks system S w.r.t. definition D

then

there is an adversary that breaks the hardness assumption.

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Hardness Assumptions

Proofs

Formally argue why a system satisfies the definition.

Reductions: If an adversary breaks system S w.r.t. definition D

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there is an adversary that breaks the hardness assumption.

Either ensure security or solve a hard problem!

Course Objectives

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- Learn the modern, provable security-based approach to cryptography.

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  - Framework to reason about security guarantees
  - Understand what goes on “under the hood”
- Develop “crypto mindset”

Topics

- Perfect Security
- Computational Security
- One-way Functions
- Pseudorandomness
- Symmetric-key Encryption
- Key Agreement
- Public-key Encryption
- Message Authentication Codes
- Hash Functions
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Foundations of provable security

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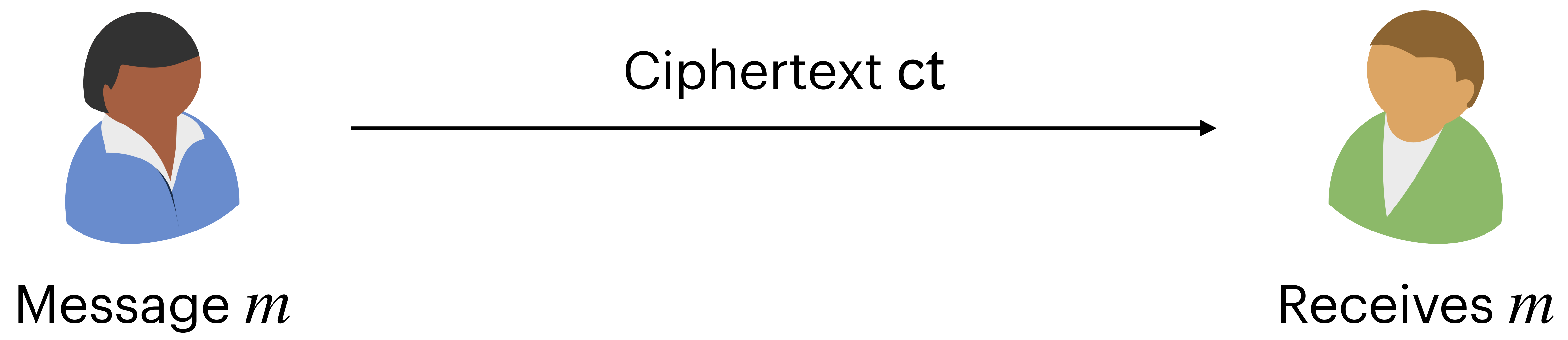

Encryption Schemes

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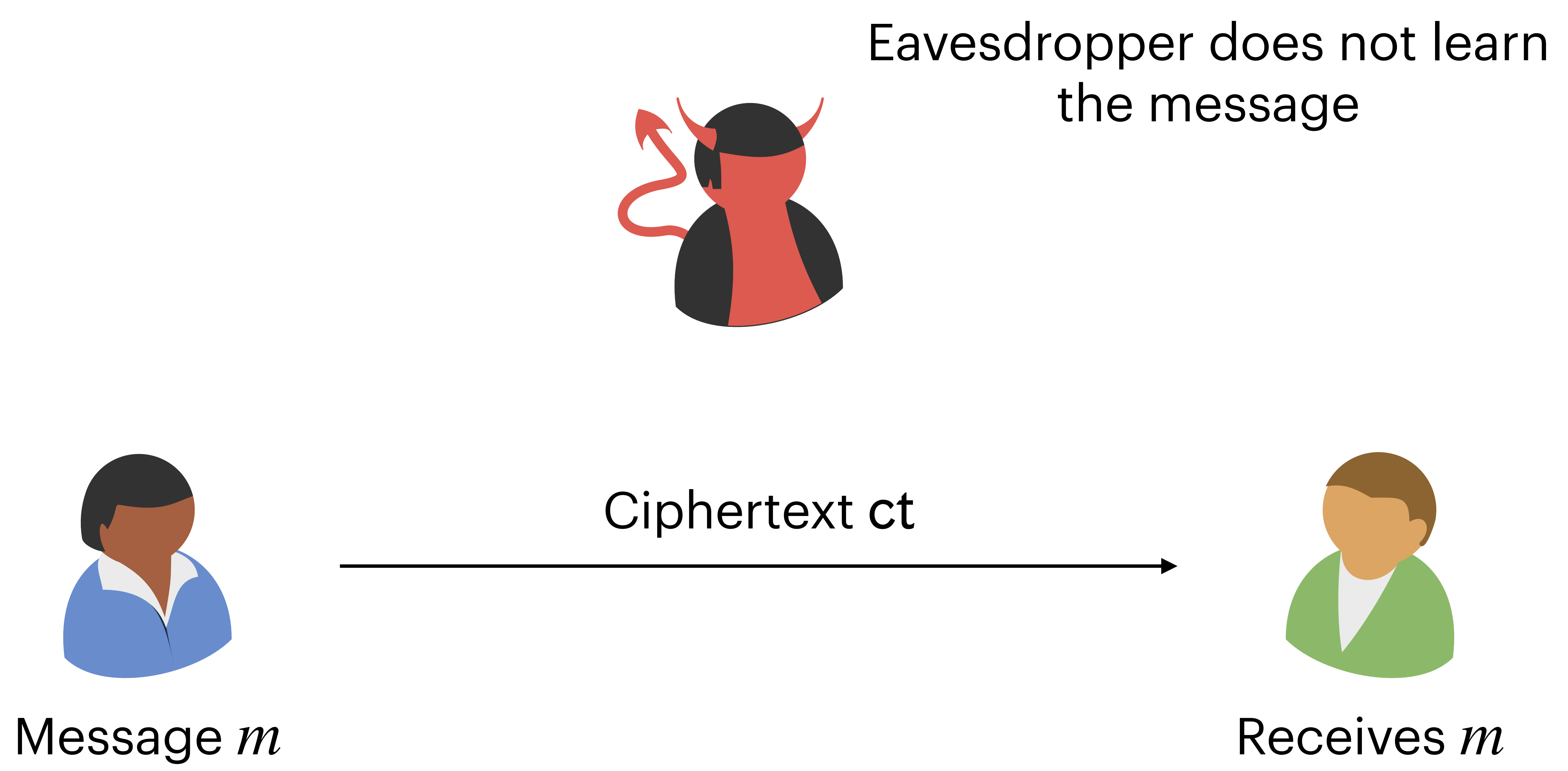

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Downloads software update

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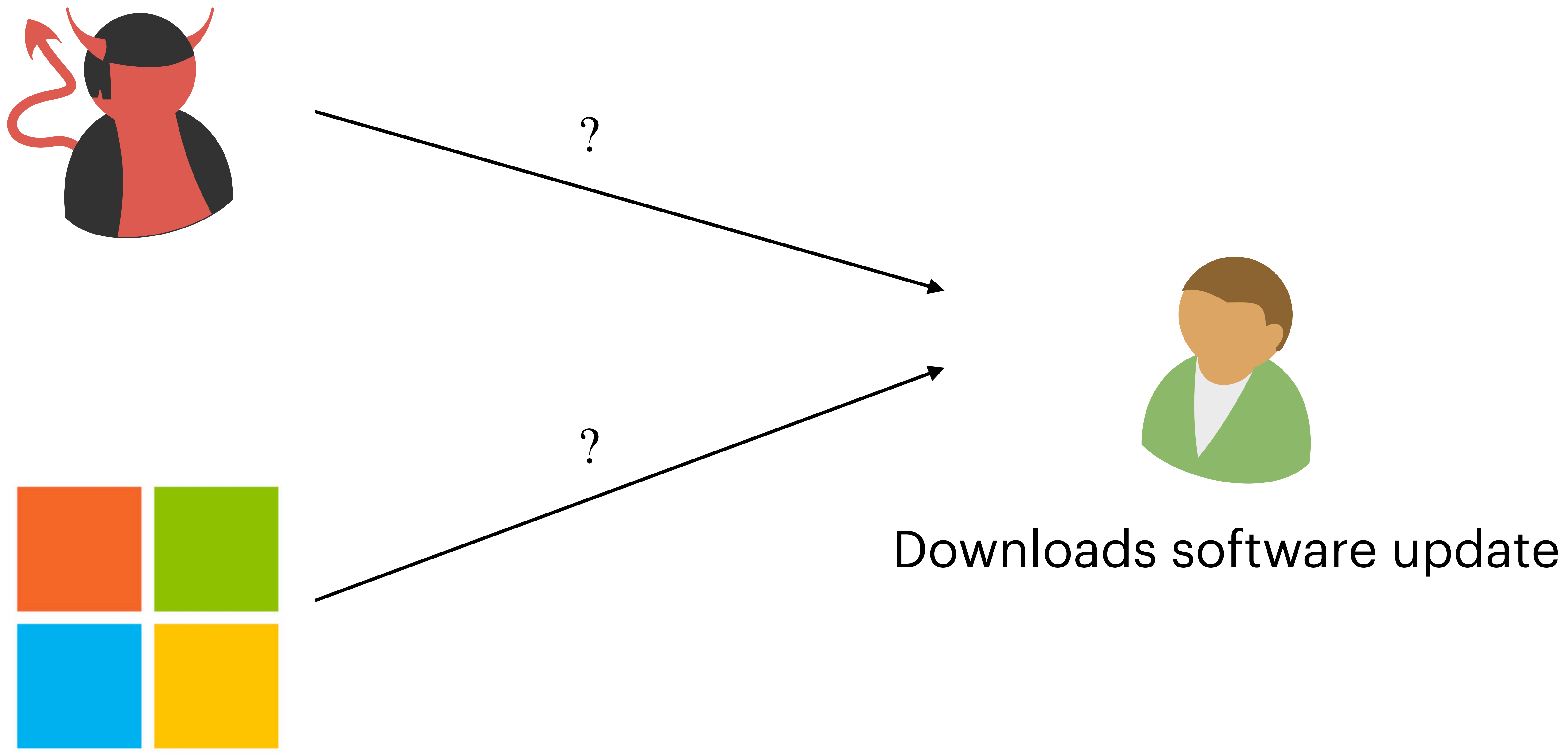

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Prove that a statement is true without conveying any additional knowledge.

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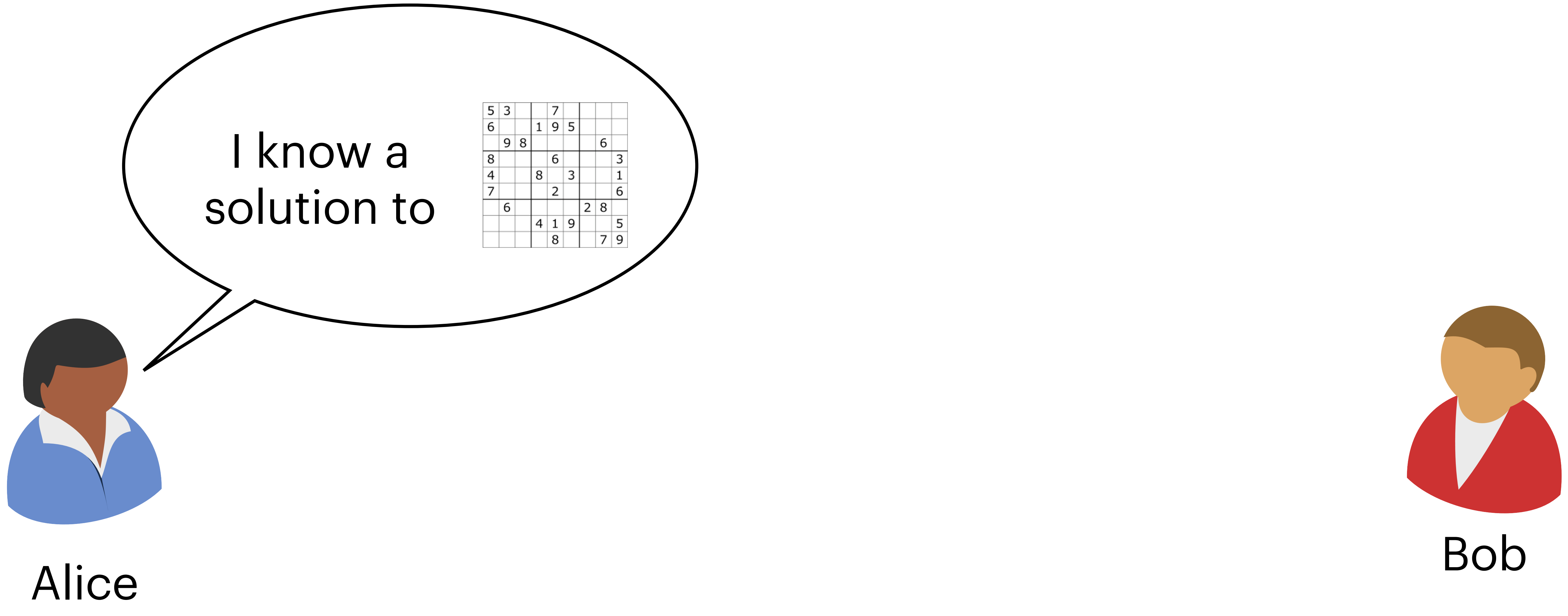

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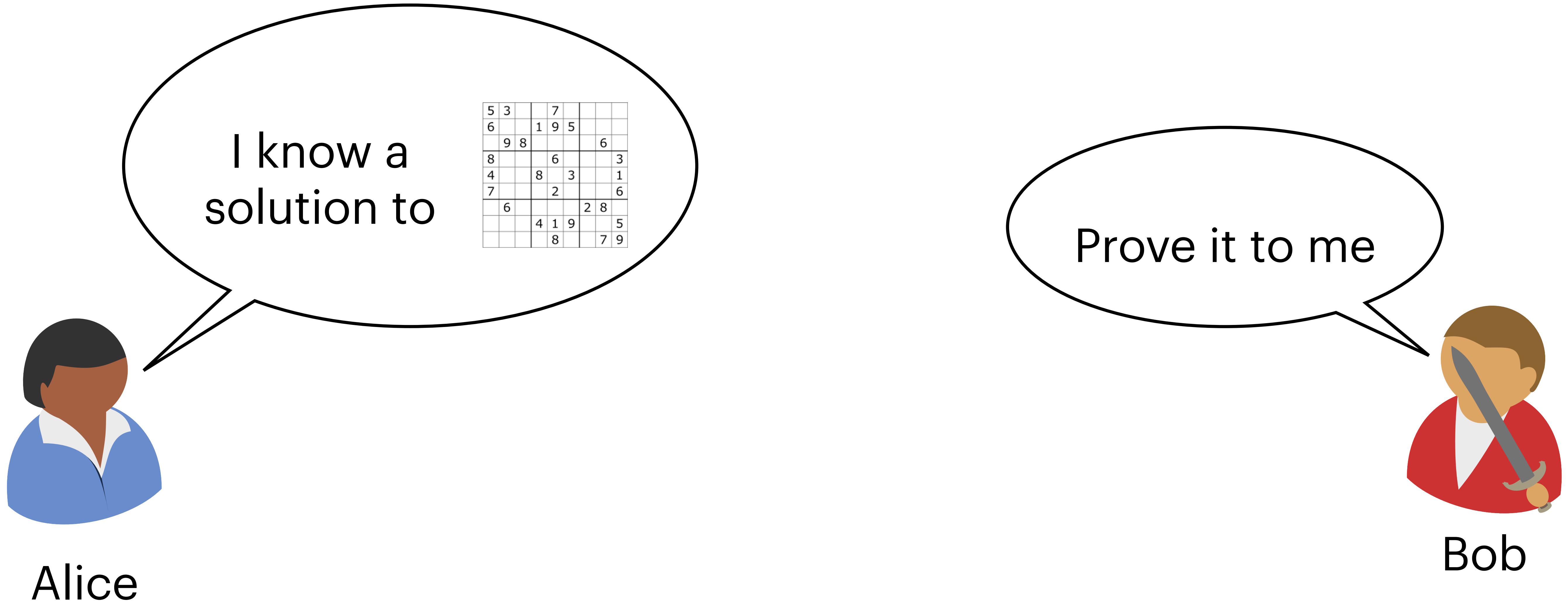

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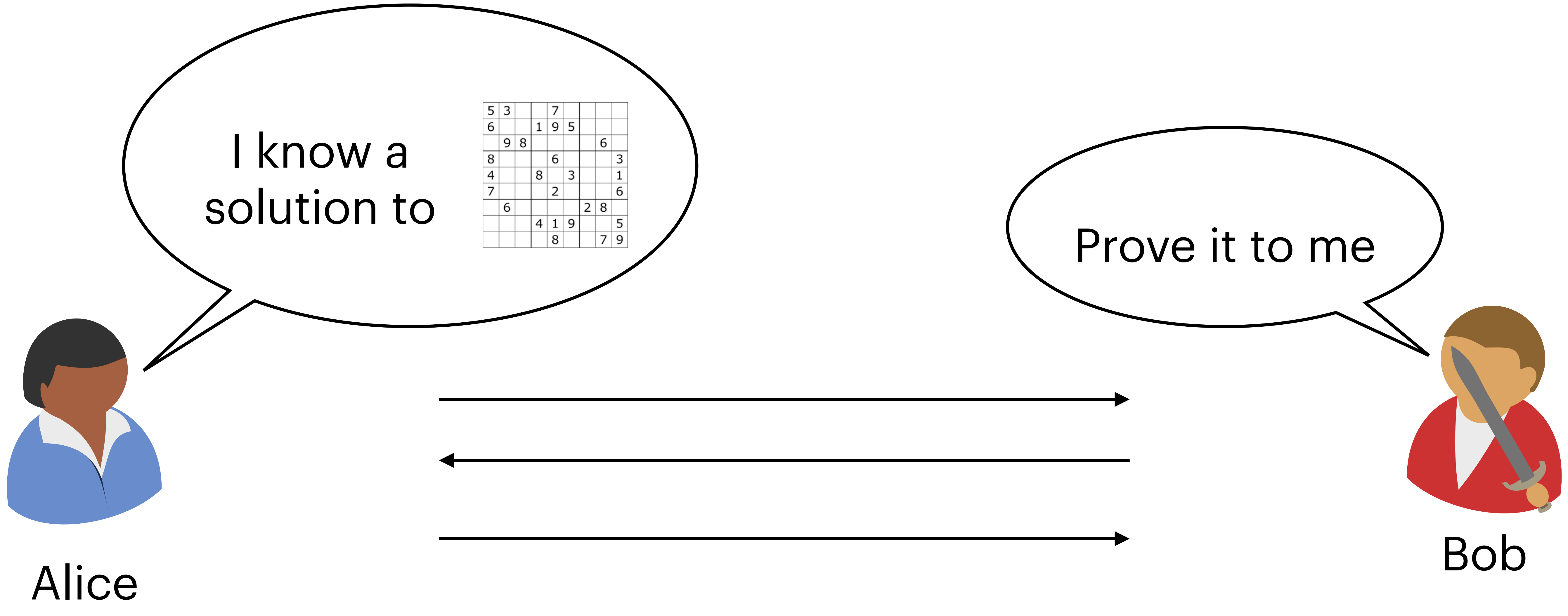

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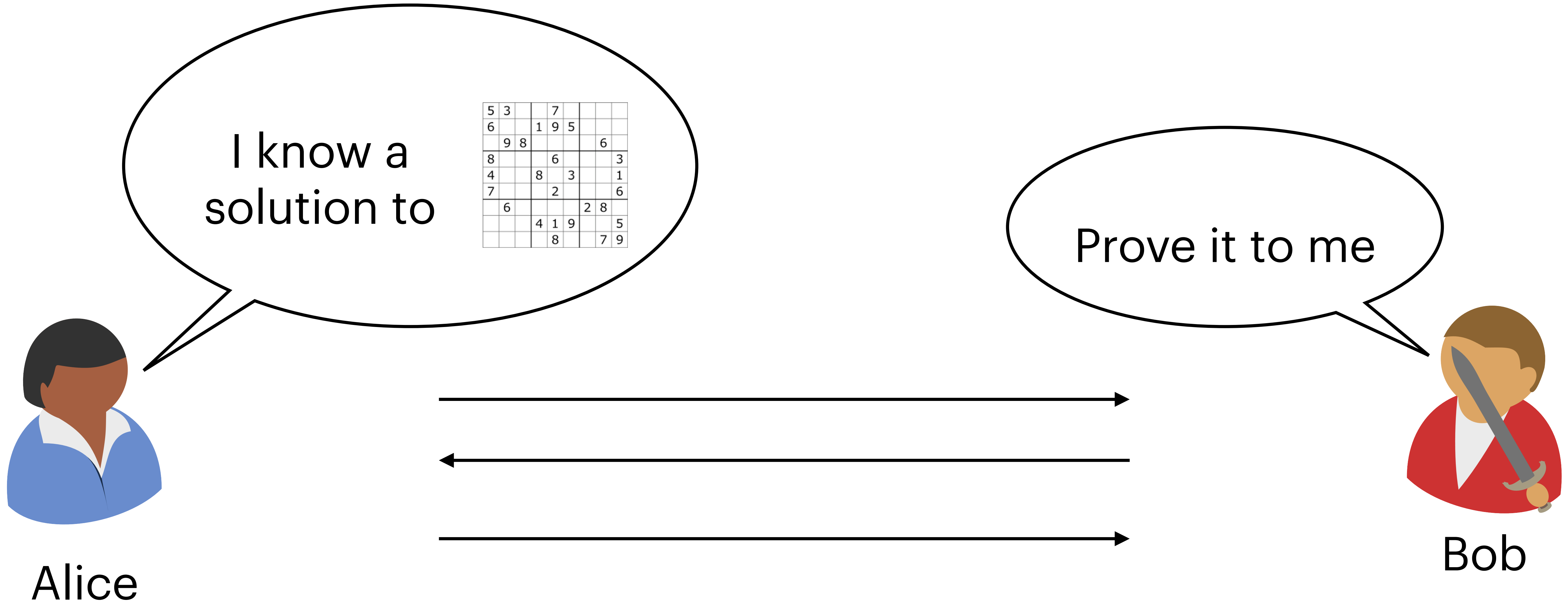

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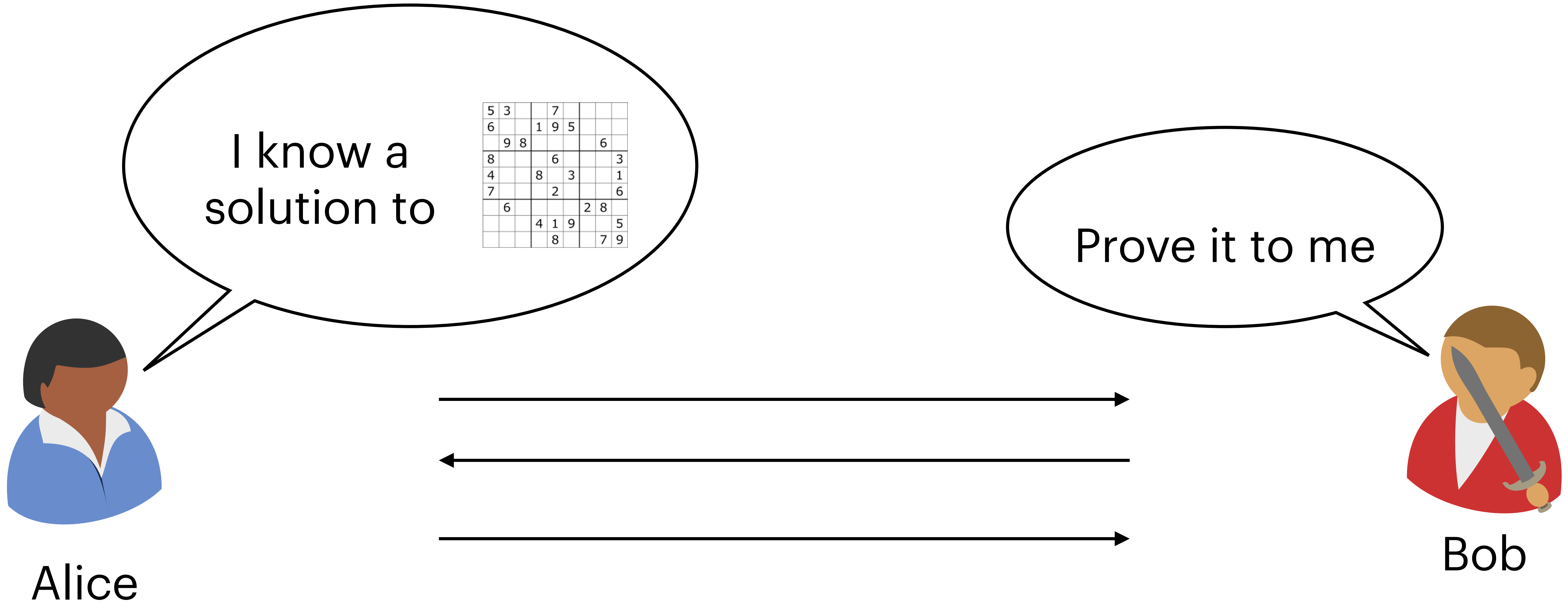

Bob is convinced that Alice has a solution

Bob learns nothing about the solution

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Compute on private inputs to only learn the output.

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Net worth x

Net worth y

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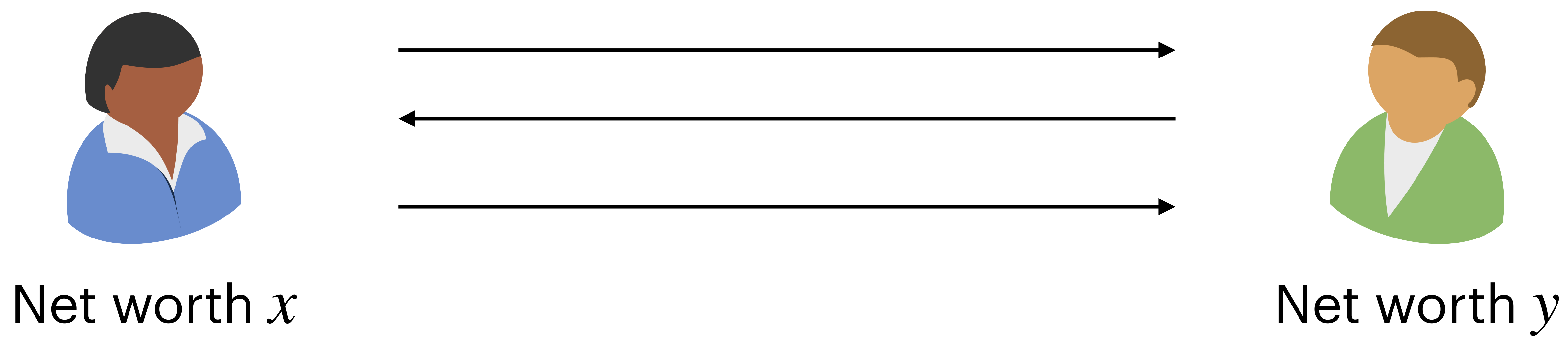

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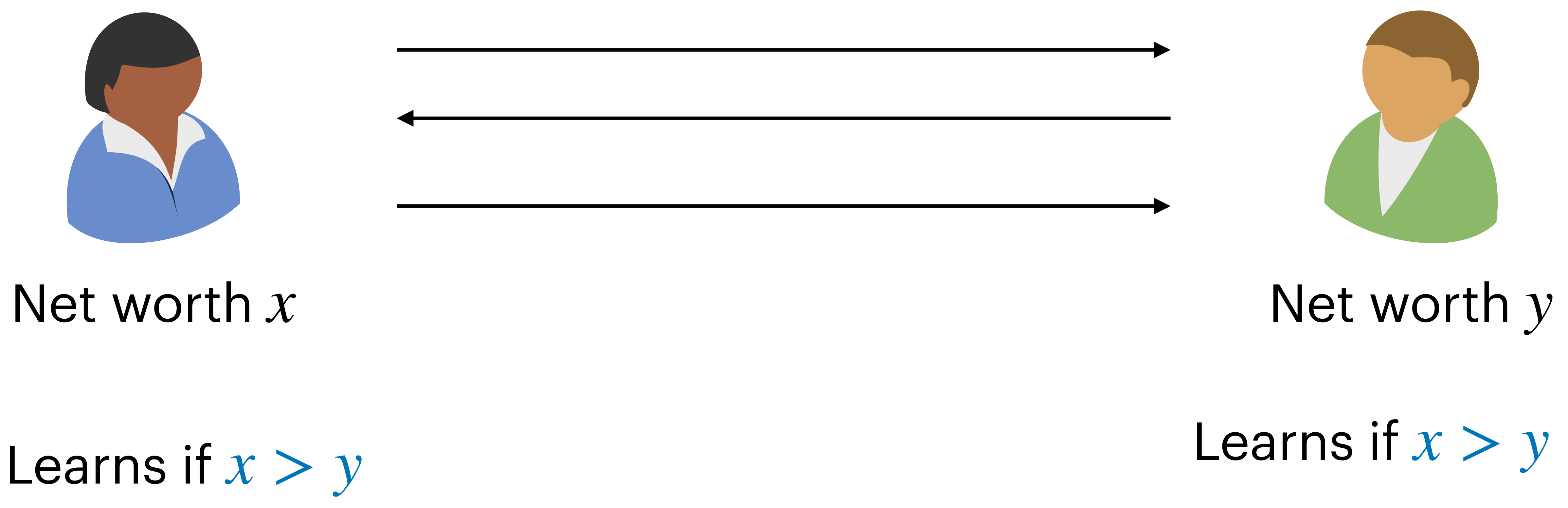

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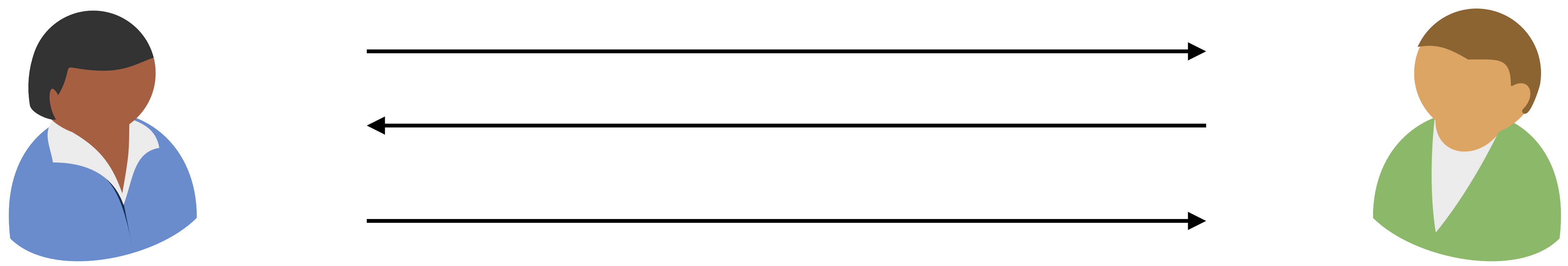

Net worth x

Net worth y

Learns if $x > y$

Learns nothing about y

Learns if $x > y$

Learns nothing about x

Compute on private inputs to only learn the output.

Logistics

Course Logistics

- Course website: https://adishegde.github.io/modern_crypto_sp26/
- In person classes, no Zoom or recordings
- Use Canvas for homework submission, discussion board, and announcements
- Grading:
  - 25% Homework
  - 15% Midterm 1
  - 25% Midterm 2
  - 30% Final
  - 5% Class participation

Homework

- Weekly assignments
- Submit via Canvas
- Must be typeset (use LaTeX or Typst)
- 48 “late hours”
- Okay to collaborate, list your collaborators
- No using AI on homeworks

Textbook and References

- No official textbook
- Free textbook *A Graduate Course in Applied Cryptography* is a great reference: https://toc.cryptobook.us/
- Syllabus, lecture notes, and slides will be available on the course website

Prerequisite / Background

Required reading before next class: pre-req lecture notes

https://adishegde.github.io/modern_crypto_sp26/notes/prerequisite_notes.pdf

Logic

Logic

x	y	x AND y
0	0	0
0	1	0
1	0	0
1	1	1

Logic

x	y	x AND y
0	0	0
0	1	0
1	0	0
1	1	1

x	y	x OR y
0	0	0
0	1	1
1	0	1
1	1	1

Logic

x	y	x AND y
0	0	0
0	1	0
1	0	0
1	1	1

x	y	x OR y
0	0	0
0	1	1
1	0	1
1	1	1

x	y	x XOR y
0	0	0
0	1	1
1	0	1
1	1	0

Logic

x	y	x AND y
0	0	0
0	1	0
1	0	0
1	1	1

x	y	x OR y
0	0	0
0	1	1
1	0	1
1	1	1

x	y	x XOR y
0	0	0
0	1	1
1	0	1
1	1	0

$$0 \oplus 1 = 1$$

Logic

x	y	x AND y
0	0	0
0	1	0
1	0	0
1	1	1

x	y	x OR y
0	0	0
0	1	1
1	0	1
1	1	1

x	y	x XOR y
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0	1	1
1	0	1
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$$0 \oplus 1 = 1$$

$$\begin{array}{r} 01010 \\ \oplus\ 11011 \\ \hline 10001 \end{array}$$

Logic

$x \wedge y$

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Logic: Implication

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$$P \Rightarrow Q$$

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$P \Rightarrow Q$ “If x = 19, then x is prime”

Logic: Implication

$$P \Rightarrow Q$$

"If x = 19, then x is prime"

Contrapositive:

$$\neg Q \Rightarrow \neg P$$

Logic: Implication

$P \Rightarrow Q$ “If *x* = 19, then *x* is prime”

Contrapositive:

$\neg Q \Rightarrow \neg P$ “If *x* is *not* prime, then *x* != 19”

Logic: Implication

$P \Rightarrow Q$ "If x = 19, then x is prime"

Contrapositive:

$\neg Q \Rightarrow \neg P$ "If x is *not* prime, then x != 19"

A statement and its contrapositive are *logically equivalent*. Often when we want to prove a statement we will prove its contrapositive.

Logic: Quantifiers

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Universal Quantifier

$$\forall x \in A, P(x)$$

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$\forall x \in A$

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"For all integers x

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$\forall x \in A$ $P(x)$

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Logic: Nesting Quantifiers

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$\forall x \in A, \exists y \in A, P(x, y)$

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"For all integers *x*

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"For all integers x , there exists an integer y such that

Logic: Nesting Quantifiers

$\forall x \in A$ $\exists y \in A$

$\forall x \in A, \exists y \in A, P(x, y)$ "For all integers x, there exists an integer y such that $x + y = 0$" $P(x, y)$

Logic: Nesting Quantifiers

$\forall x \in A$

$\exists y \in A$

$$\forall x \in A, \exists y \in A, P(x, y)$$

"For all integers *x*, there exists an integer *y* such that *x* + *y* = 0"

$P(x, y)$

$$\exists y \in A, \forall x \in A, P(x, y)$$

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“For all integers *x*, there exists an integer *y* such that *x* + *y* = 0” $P(x, y)$

Order of quantifiers really matters!

$\exists y \in A$ $\forall x \in A$

$$\exists y \in A, \forall x \in A, P(x, y)$$

“There exists an integer *y* such that for all integers *x* *x* + *y* = 0” $P(x, y)$

Logic: Negating Quantifiers

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$\forall x \in A$	$P(x)$
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$$\forall x \in A, P(x)$$

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$$\exists x \in A, \neg P(x)$$

"There exists an integer x such that $x < 0$

Logic: Negating Nested Quantifiers

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$$\forall x \in A, \exists y \in B, \exists z \in C, P(x)$$

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Negate each quantifier in turn

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Negate each quantifier in turn

$$\exists x \in A, \forall y \in B, \forall z \in C, \neg P(x)$$

Logic: Putting it all Together

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$$\forall x, P(x) \wedge \forall y, P(y) \Rightarrow \forall z, Q(Z)$$

Logic: Putting it all Together

$$\forall x, P(x) \land \forall y, P(y) \Rightarrow \forall z, Q(Z)$$

$$\exists z, \neg Q(z) \Rightarrow \exists x, \neg P(X) \lor \exists y, \neg P(y)$$

Probability: Distributions

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Sample Space: the possible outcomes of a probabilistic experiment

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Example: All binary strings of length 3 (000, 001, 010,…)

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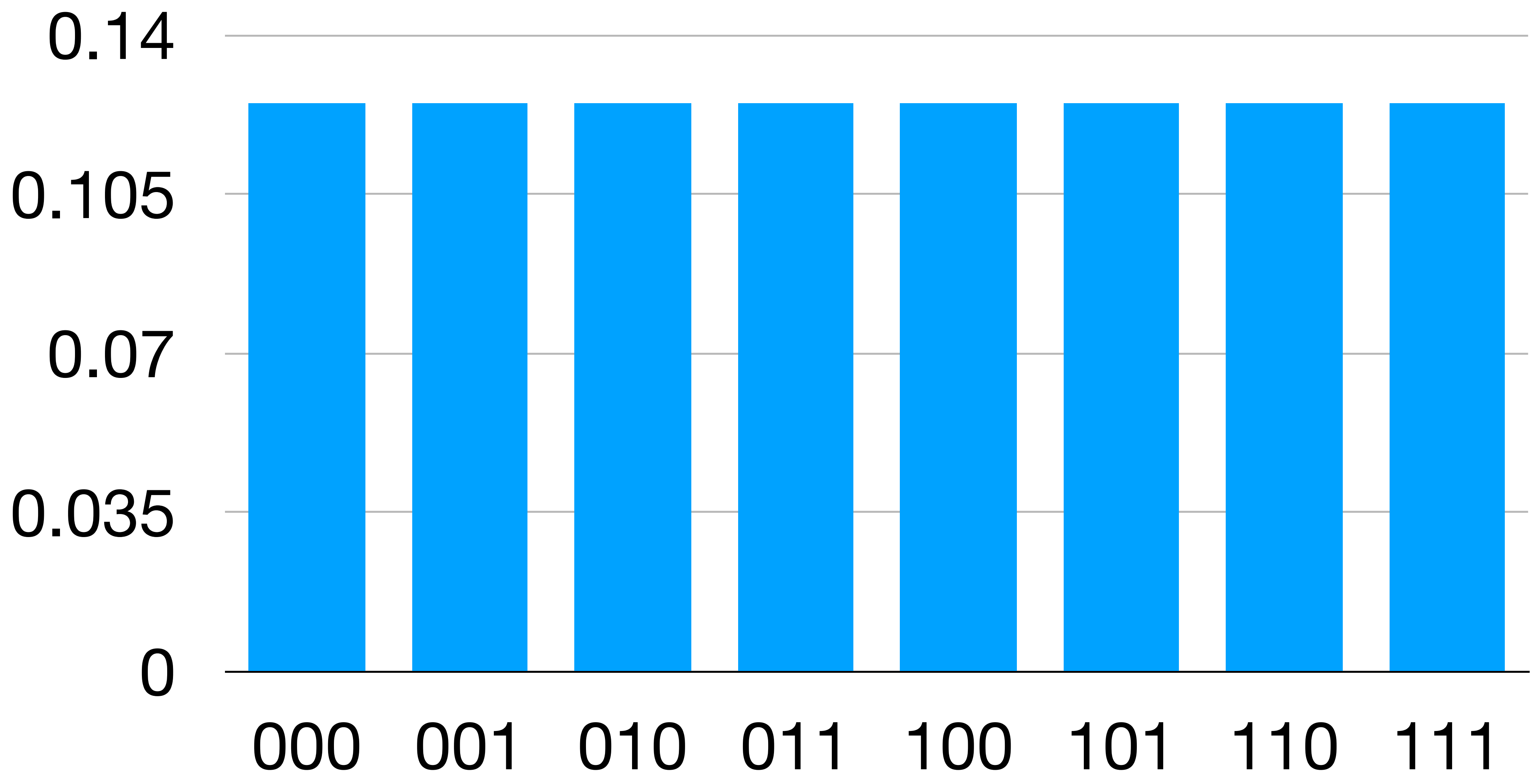

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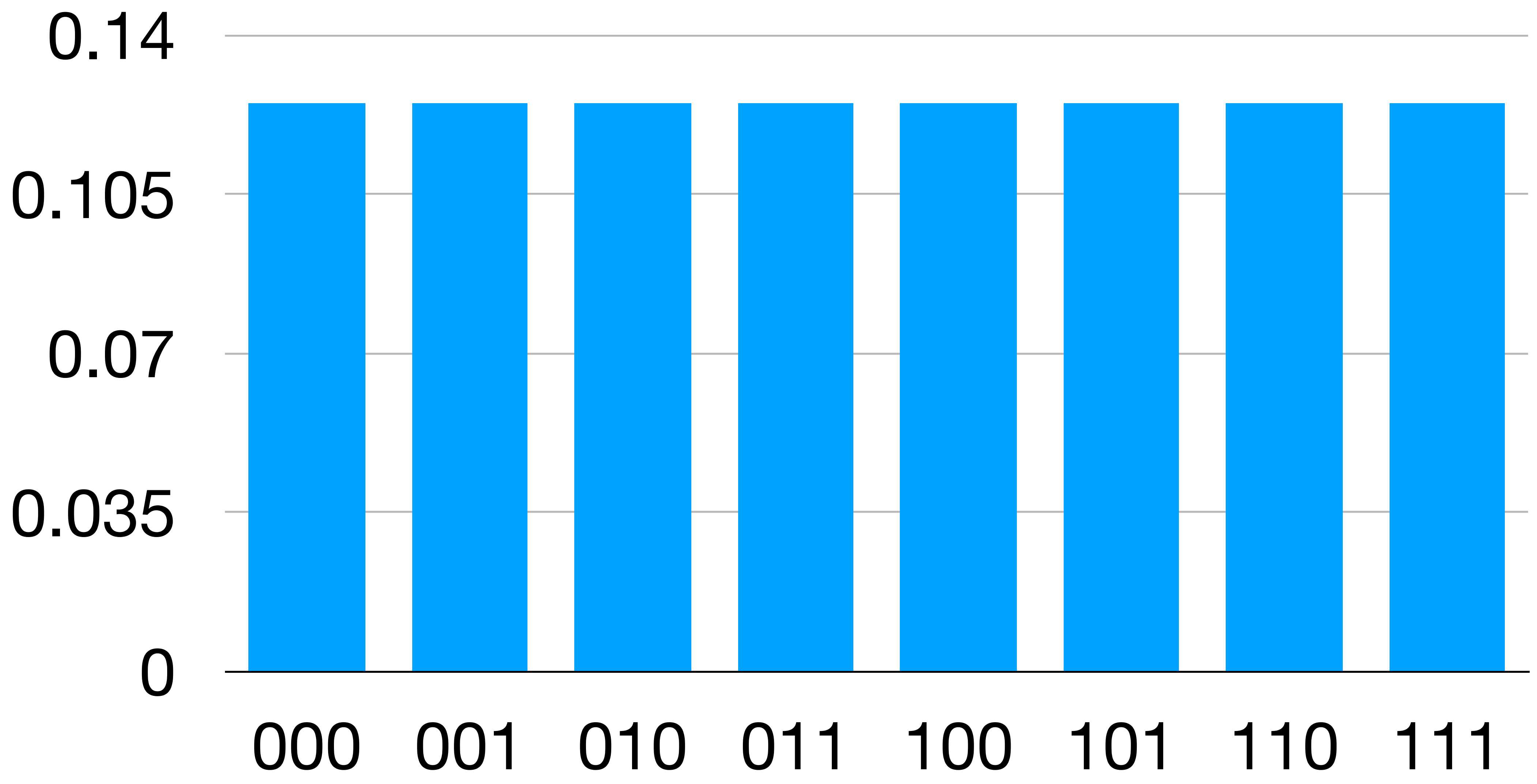

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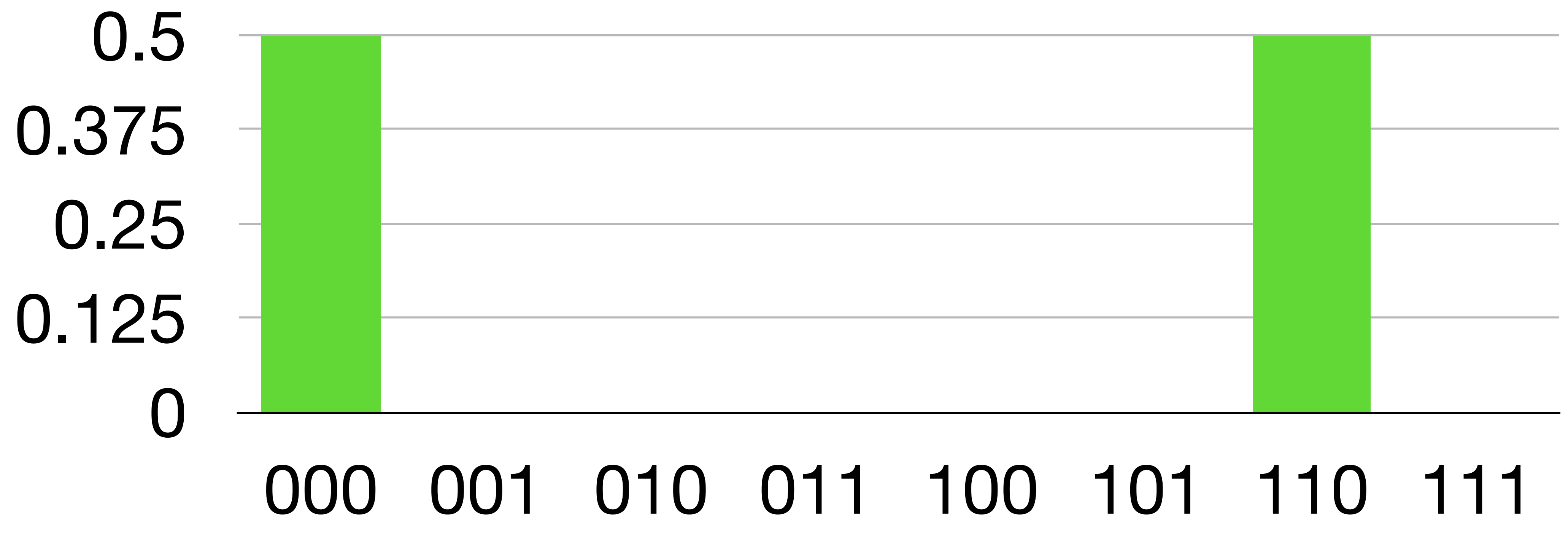

Probability: Distributions

Sampling from a distribution means selecting an element in accordance with the assigned probabilities

Sample Space: the possible outcomes of a probabilistic experiment

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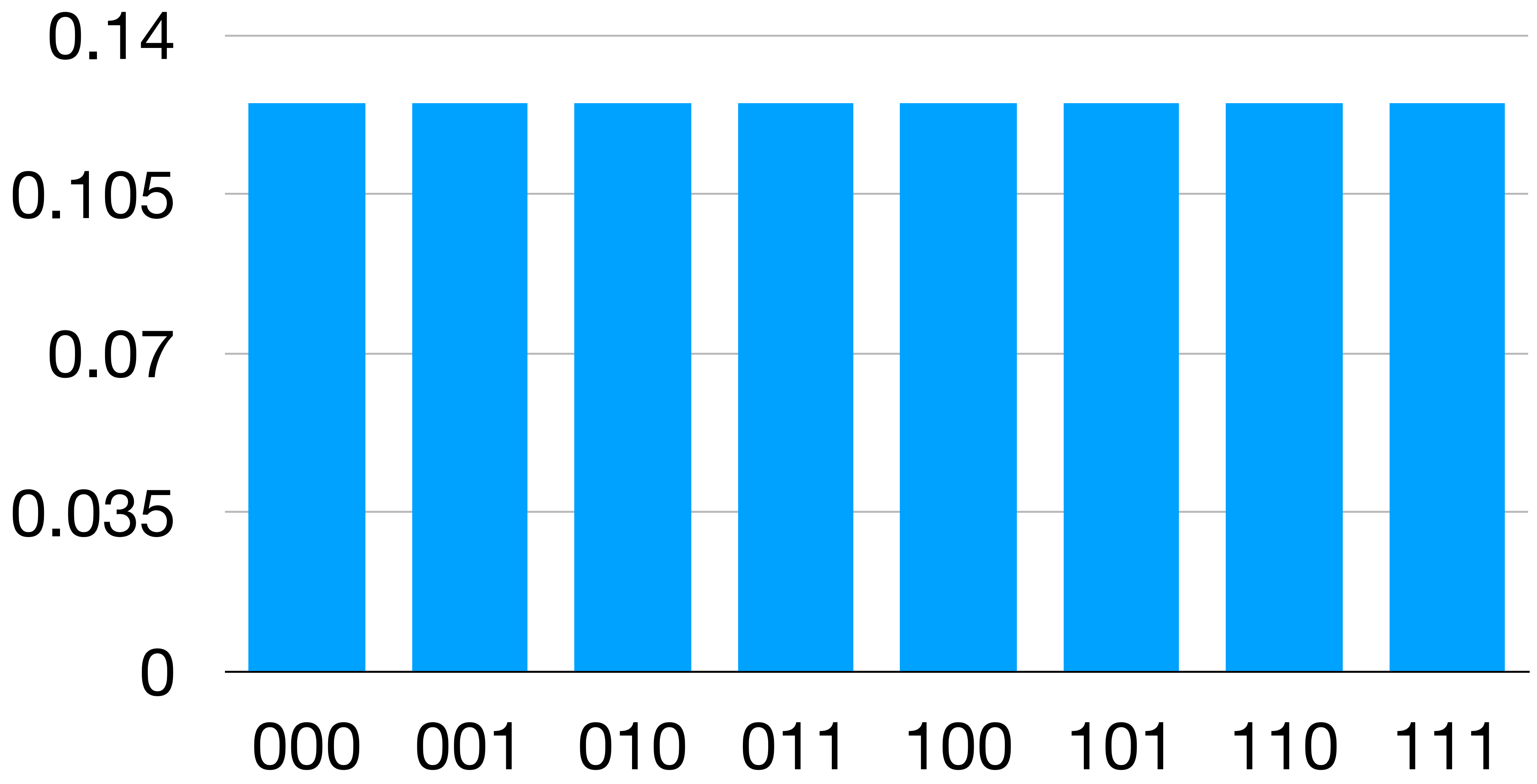

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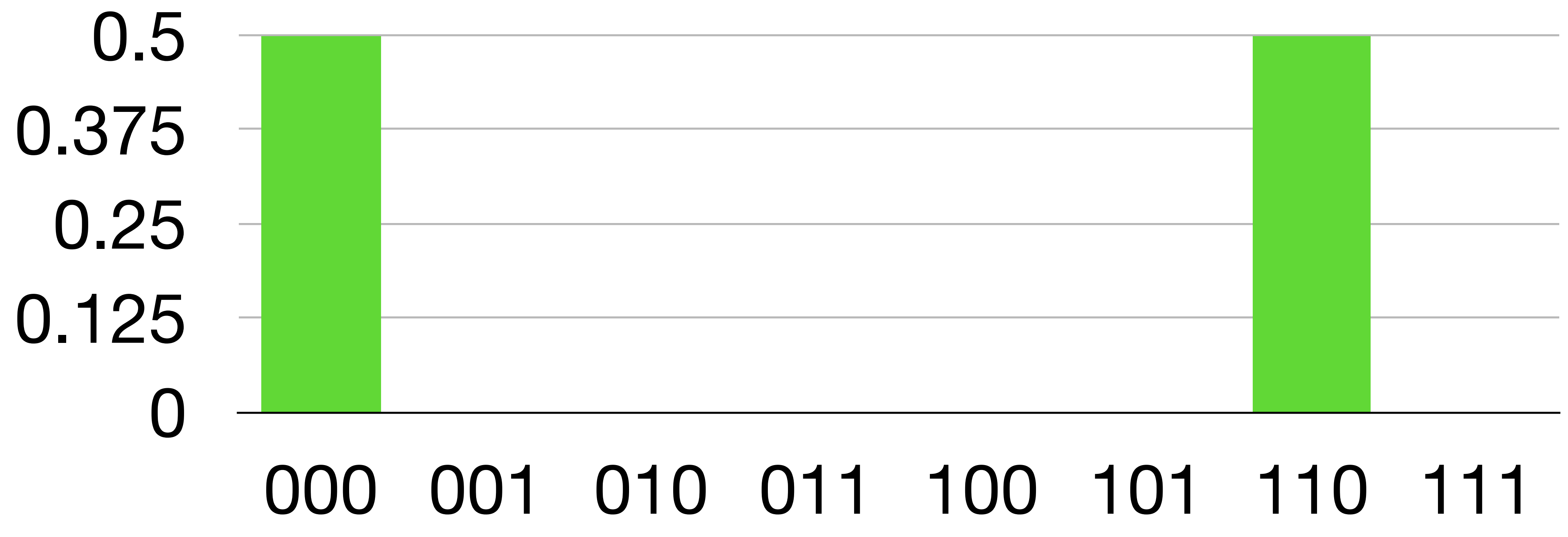

Turing Machines

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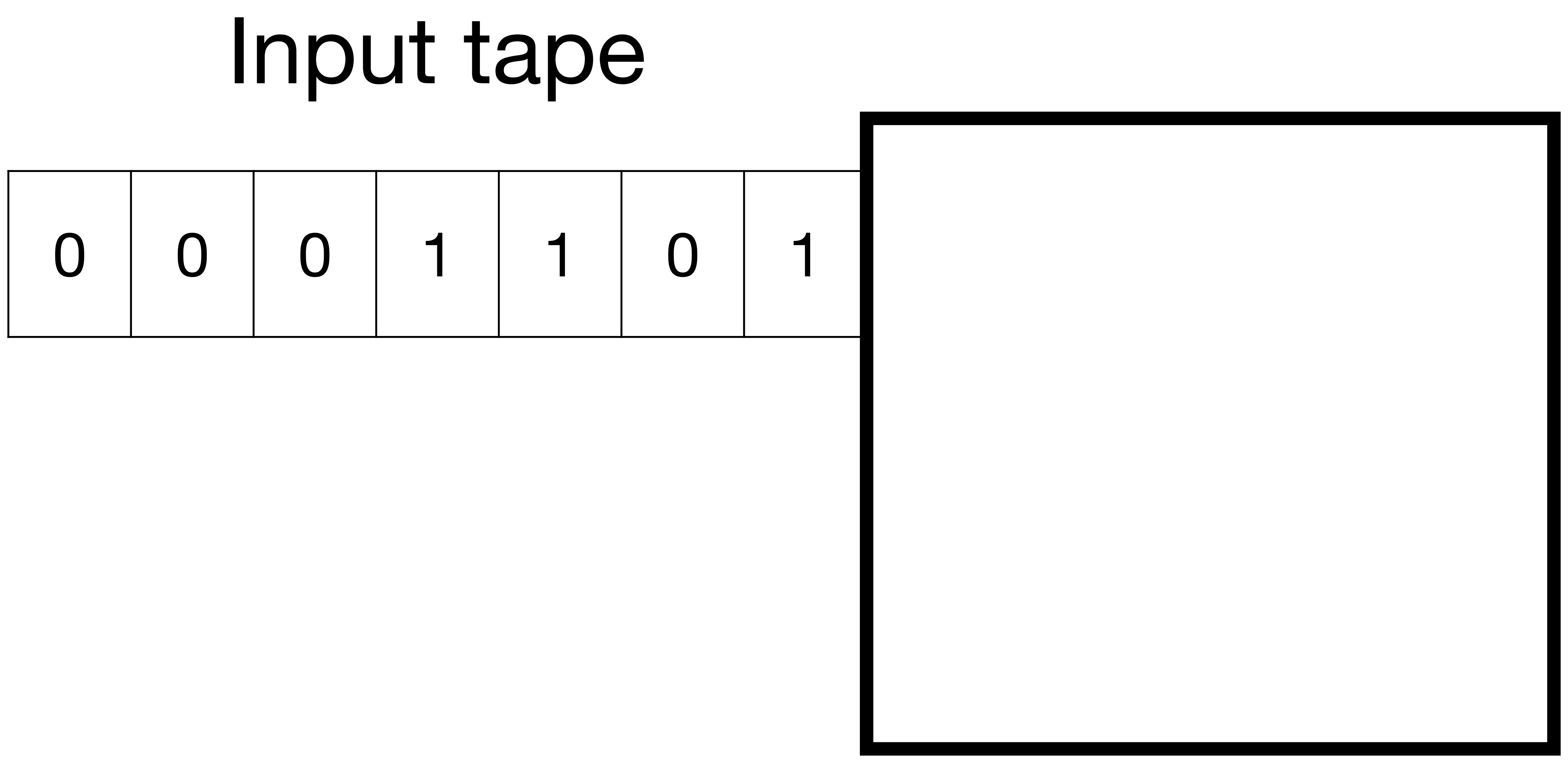

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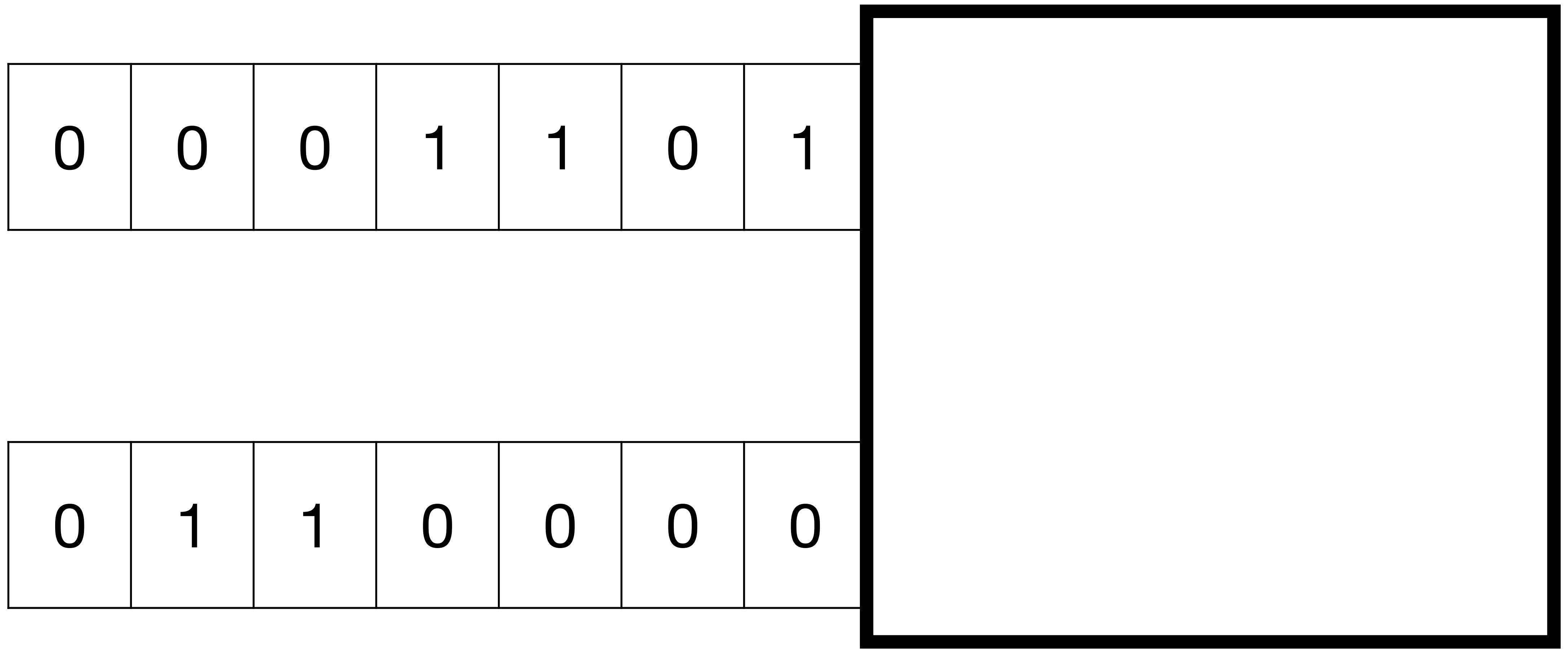
Input tape



Randomness tape (uniform 0s and 1s)

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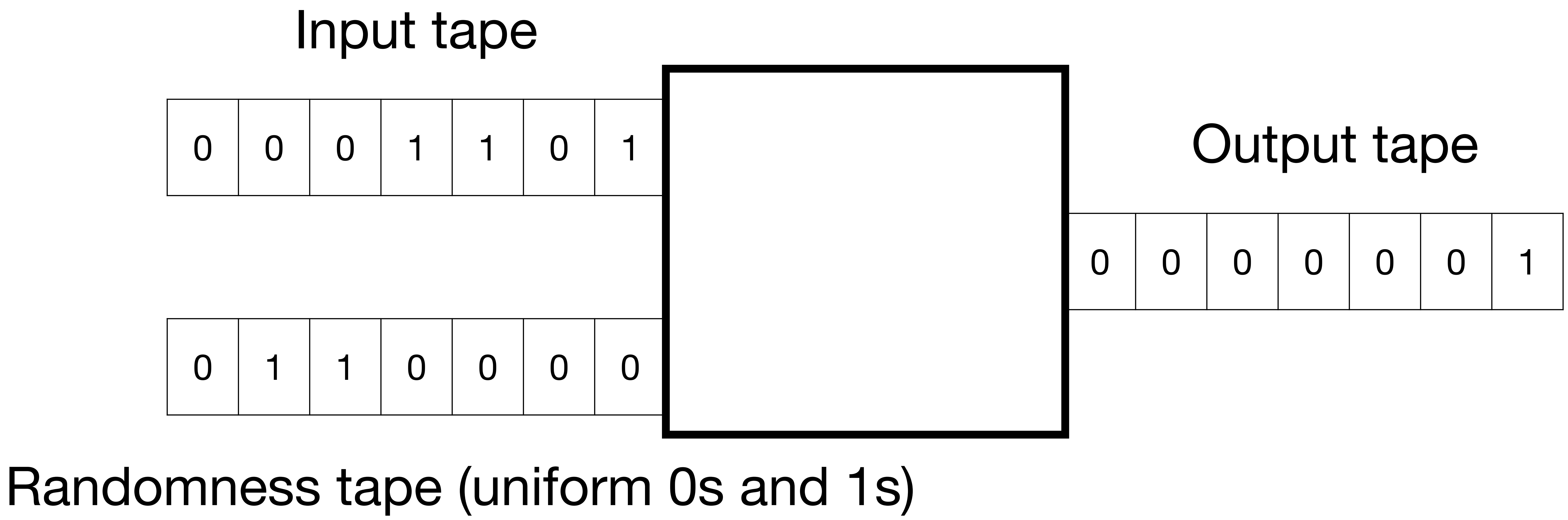

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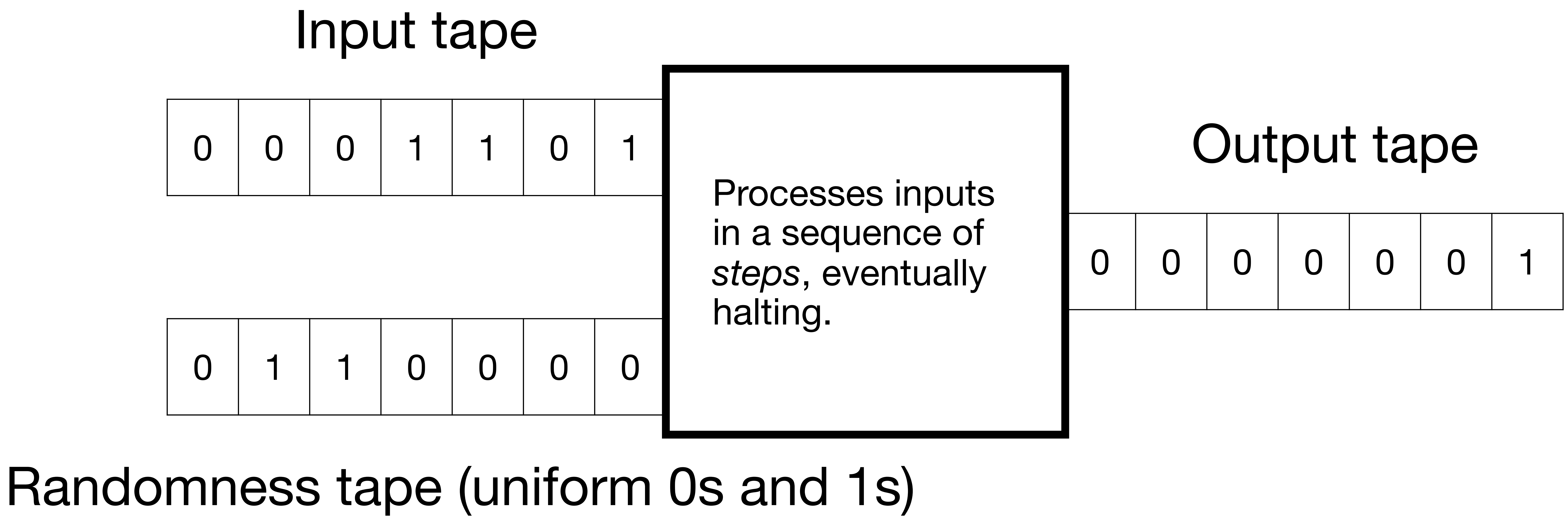

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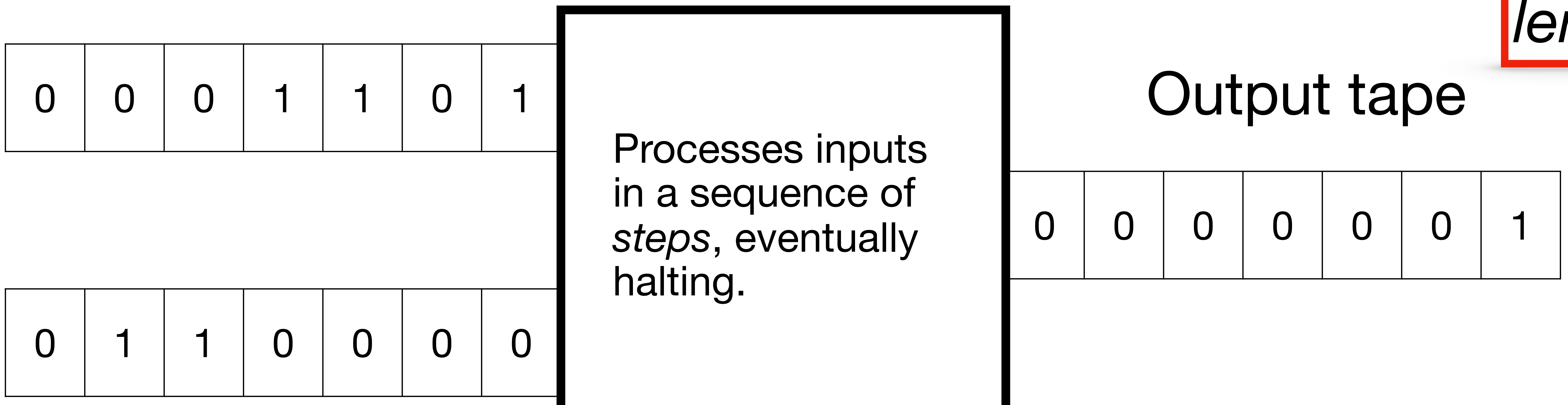

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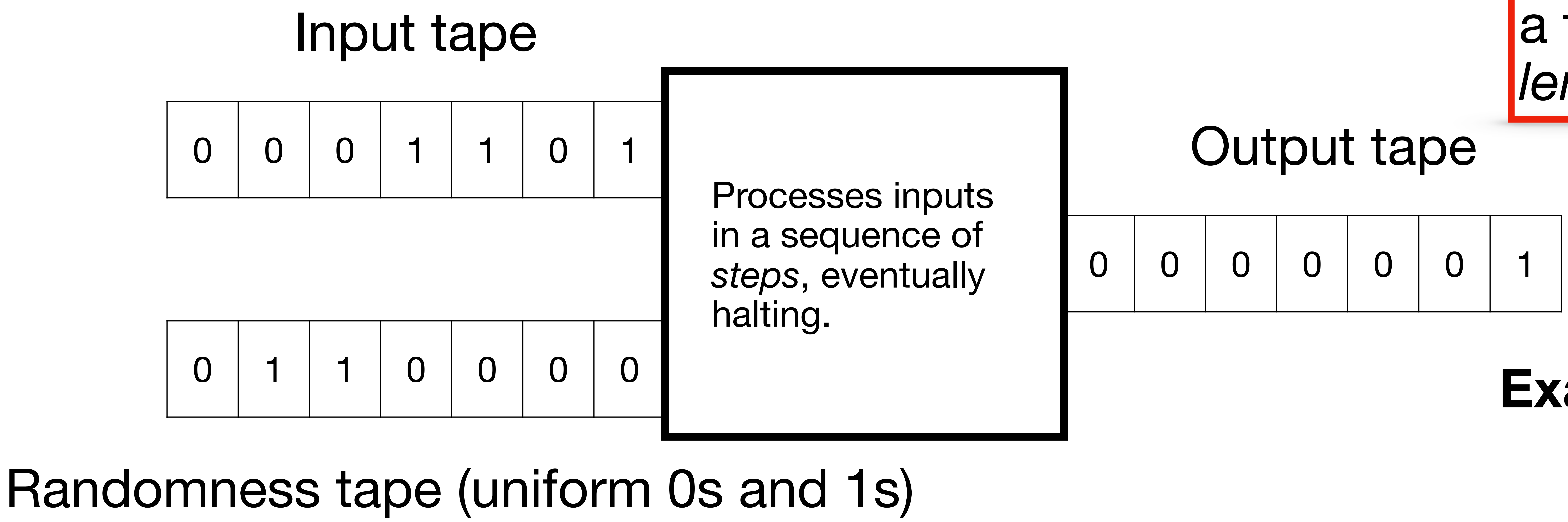

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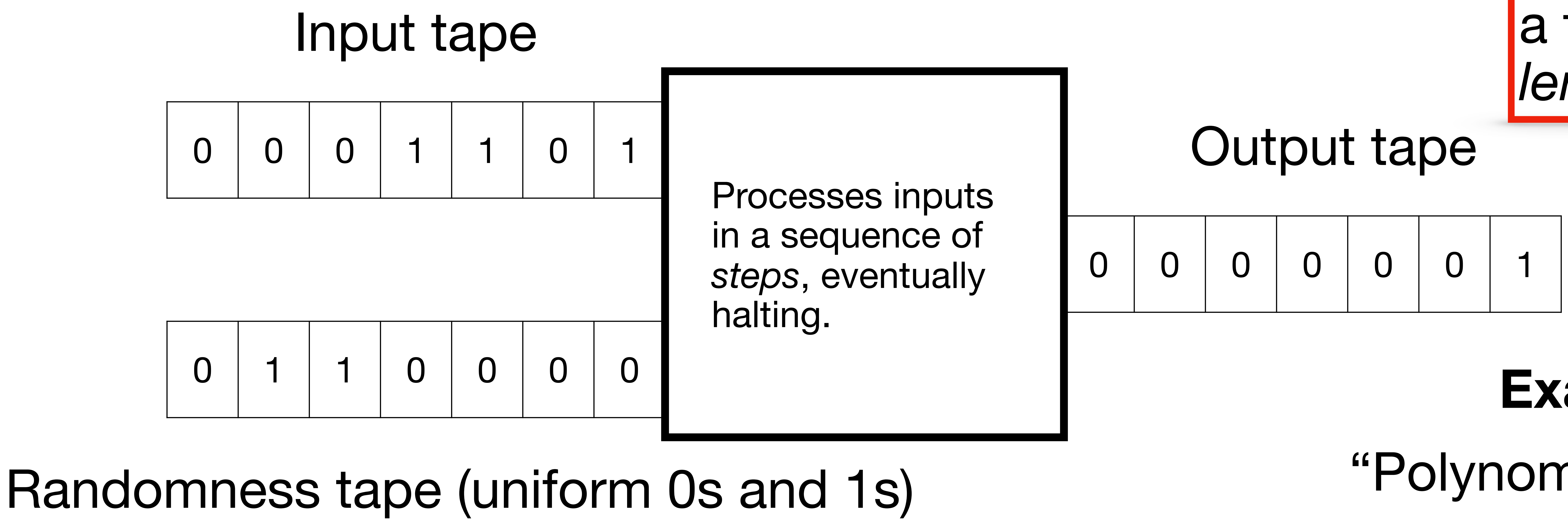

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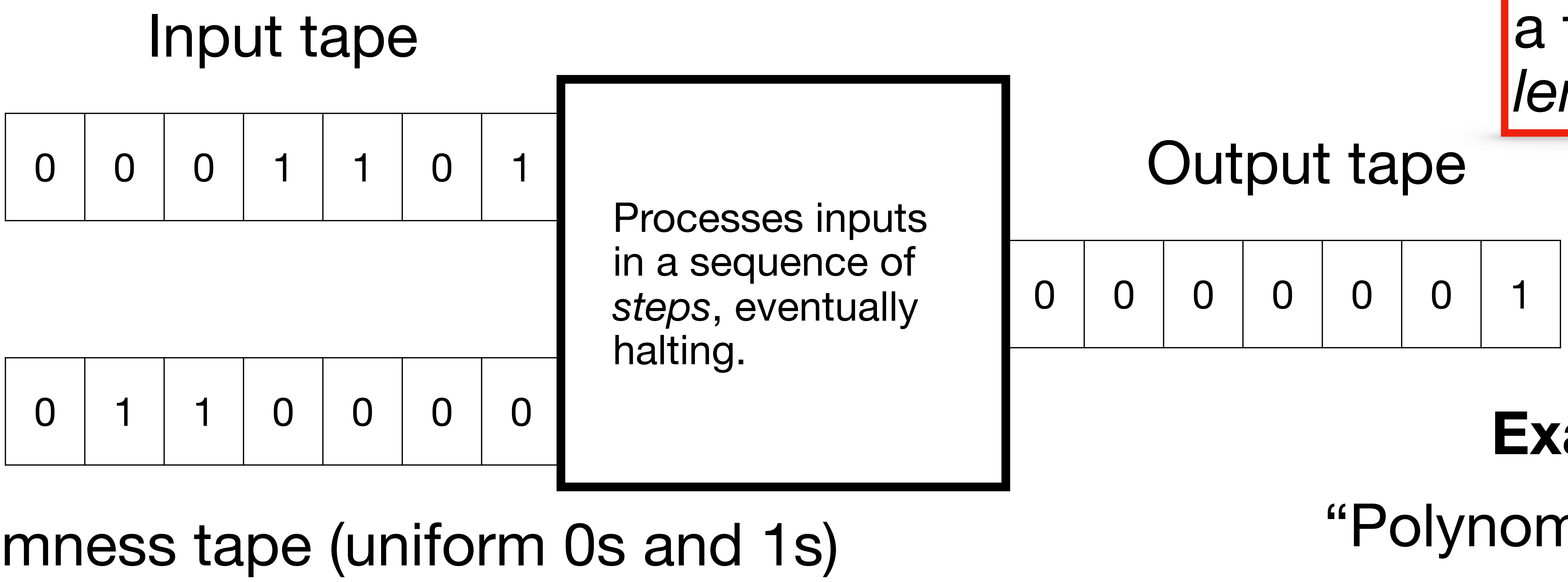

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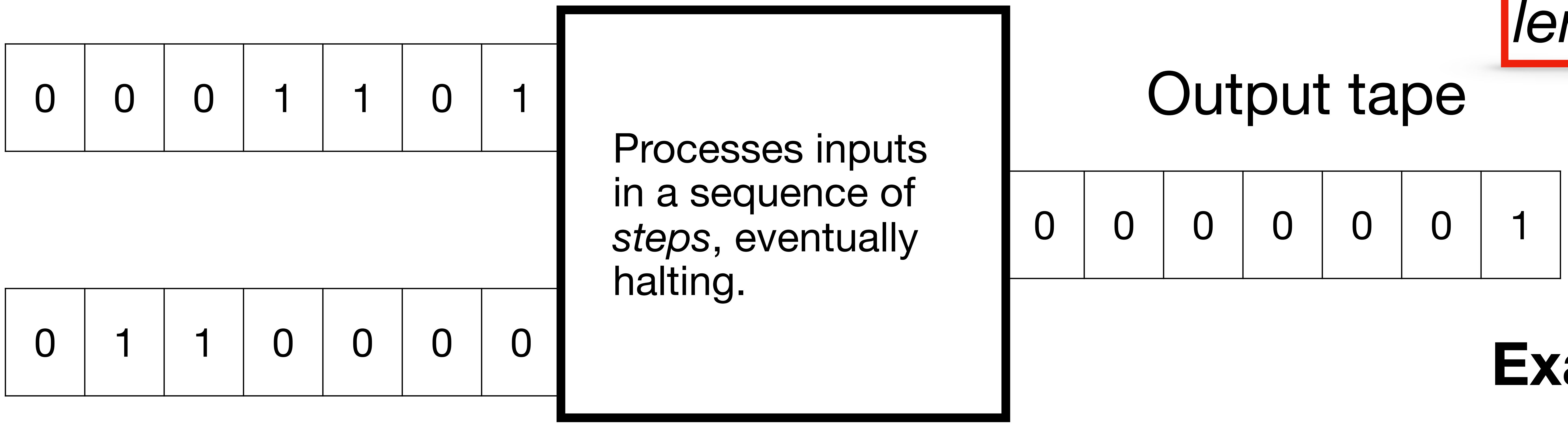

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PPT = “Probabilistic Polynomial Time”

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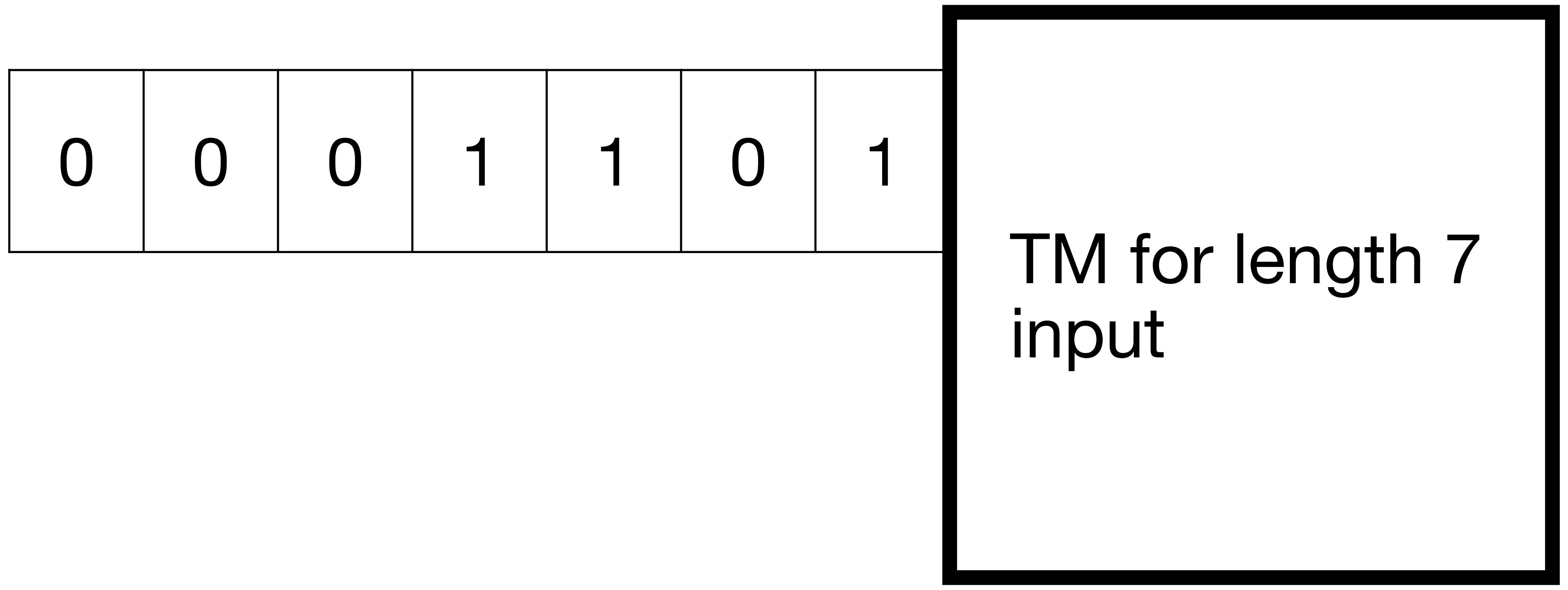

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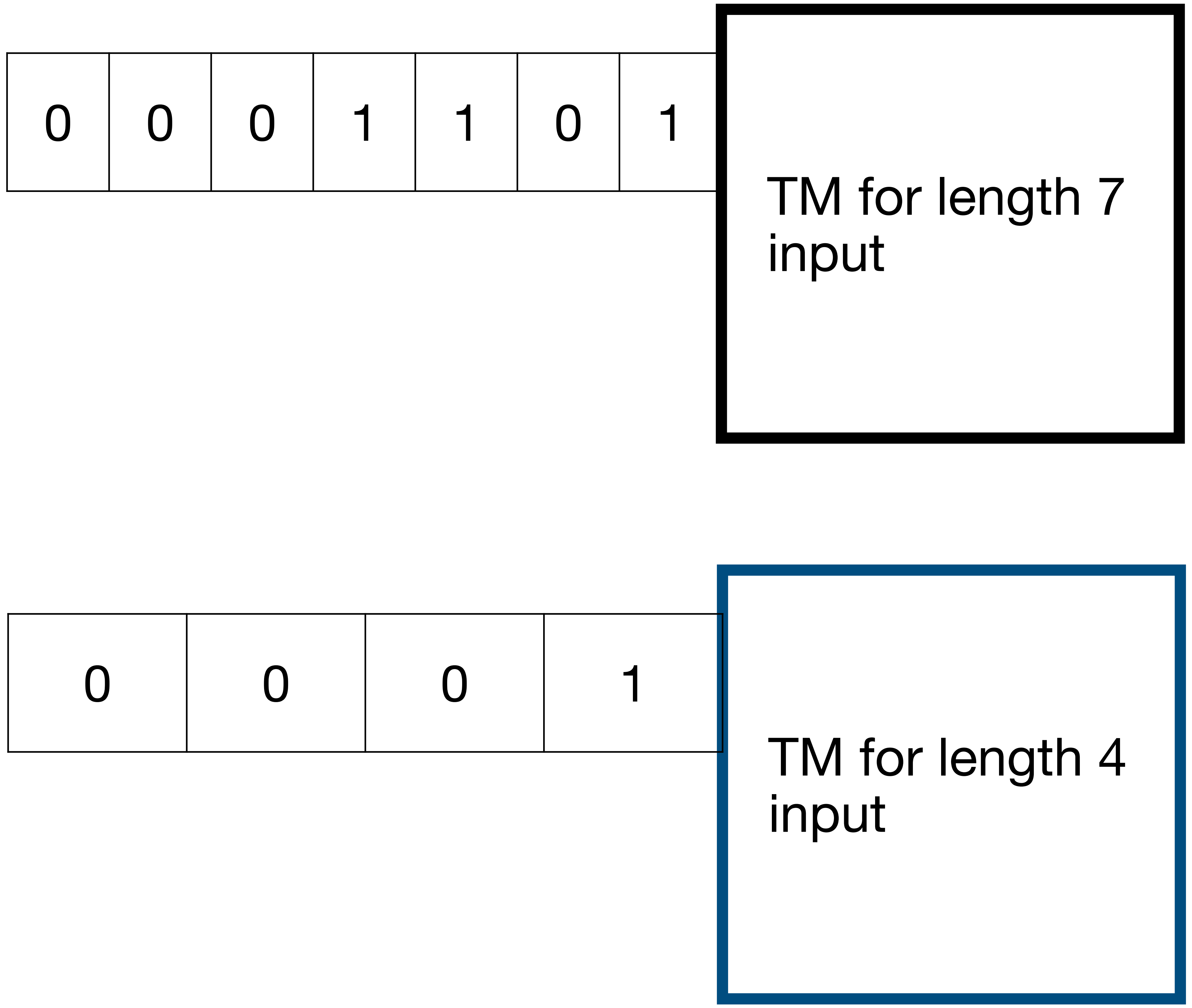

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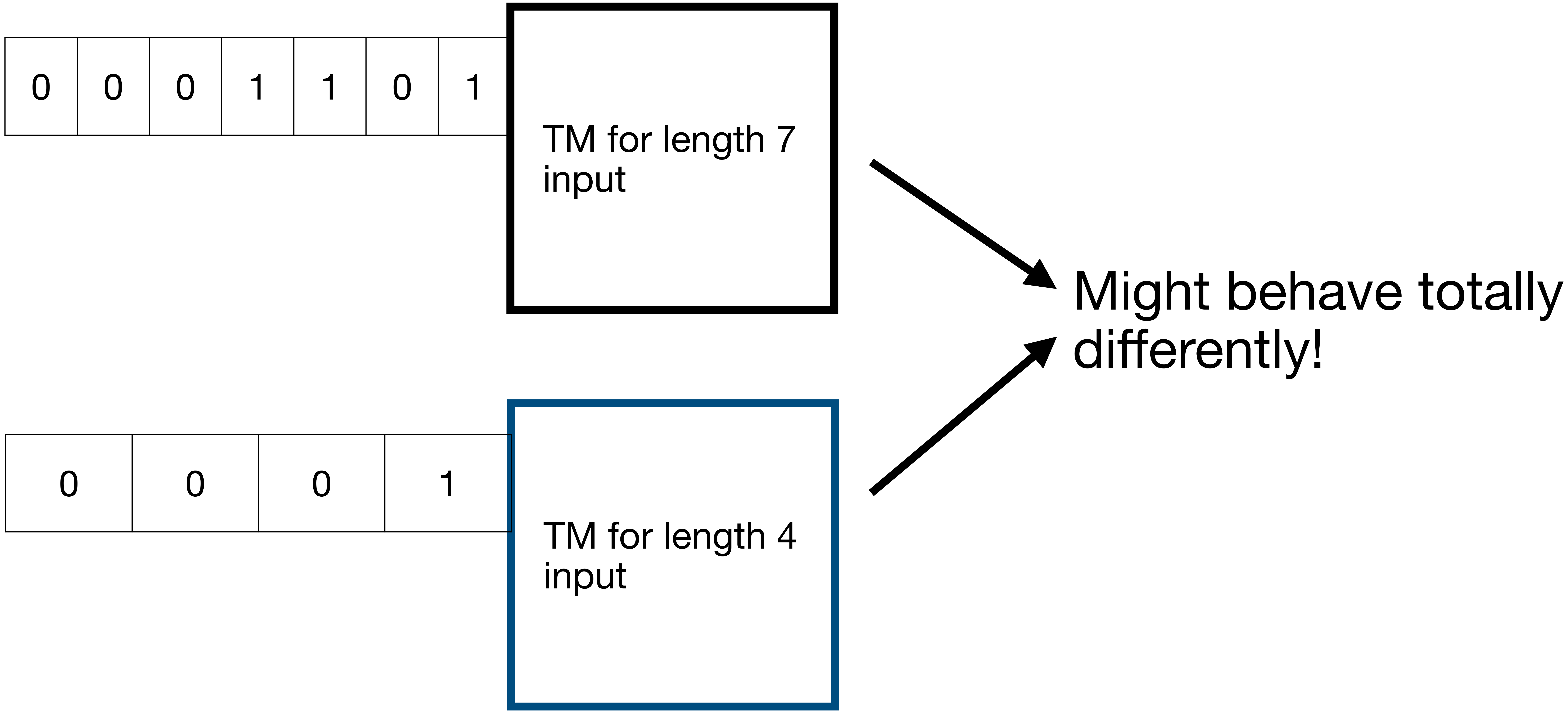

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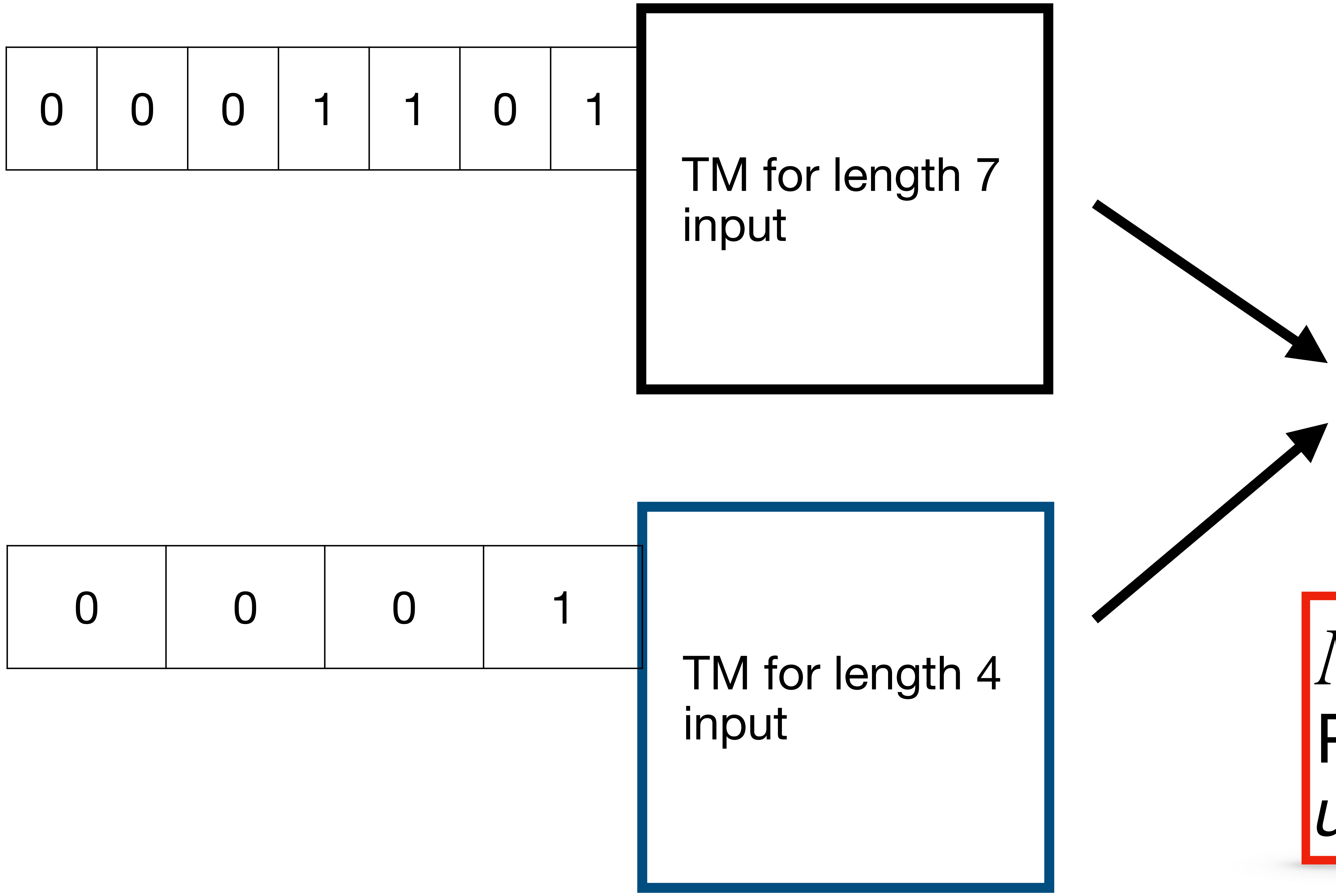

Might behave totally differently!

$M = \{M_1, M_2, \dots\}$ where each M_1 is a PPT Turing Machine is called a *Non-uniform PPT Turing Machine* (NUPPT)

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- Specifically, we care about the *limit* of this function: what does it trend towards?

Asymptotic Notation

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Big O: $f(x) \in O(g(x))$ if

$$\exists c, n_0 \in \{1,2,3,\ldots\} \text{ such that } \forall n \geq n_0,\ 0 \leq f(n) \leq c \cdot g(n)$$

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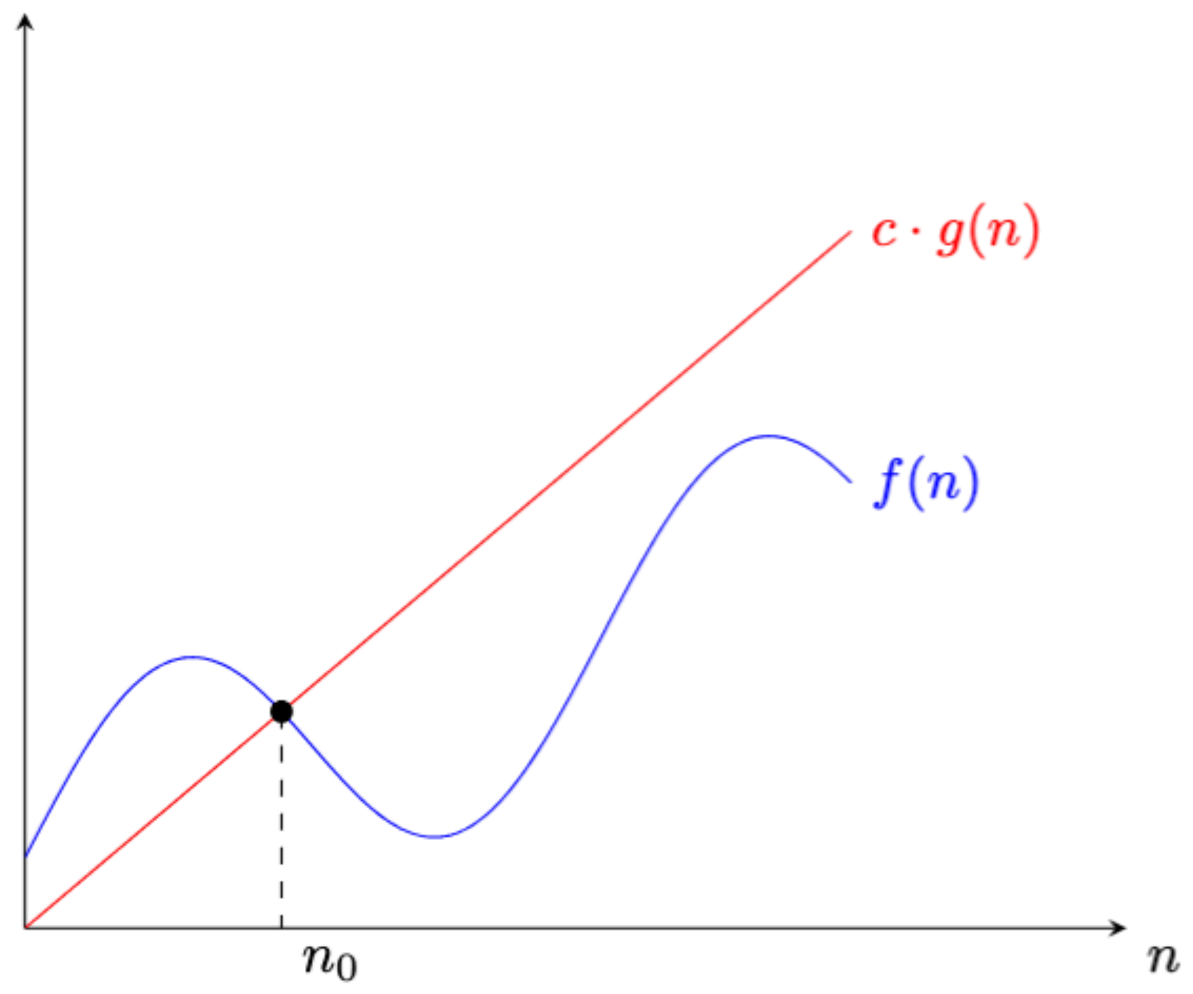

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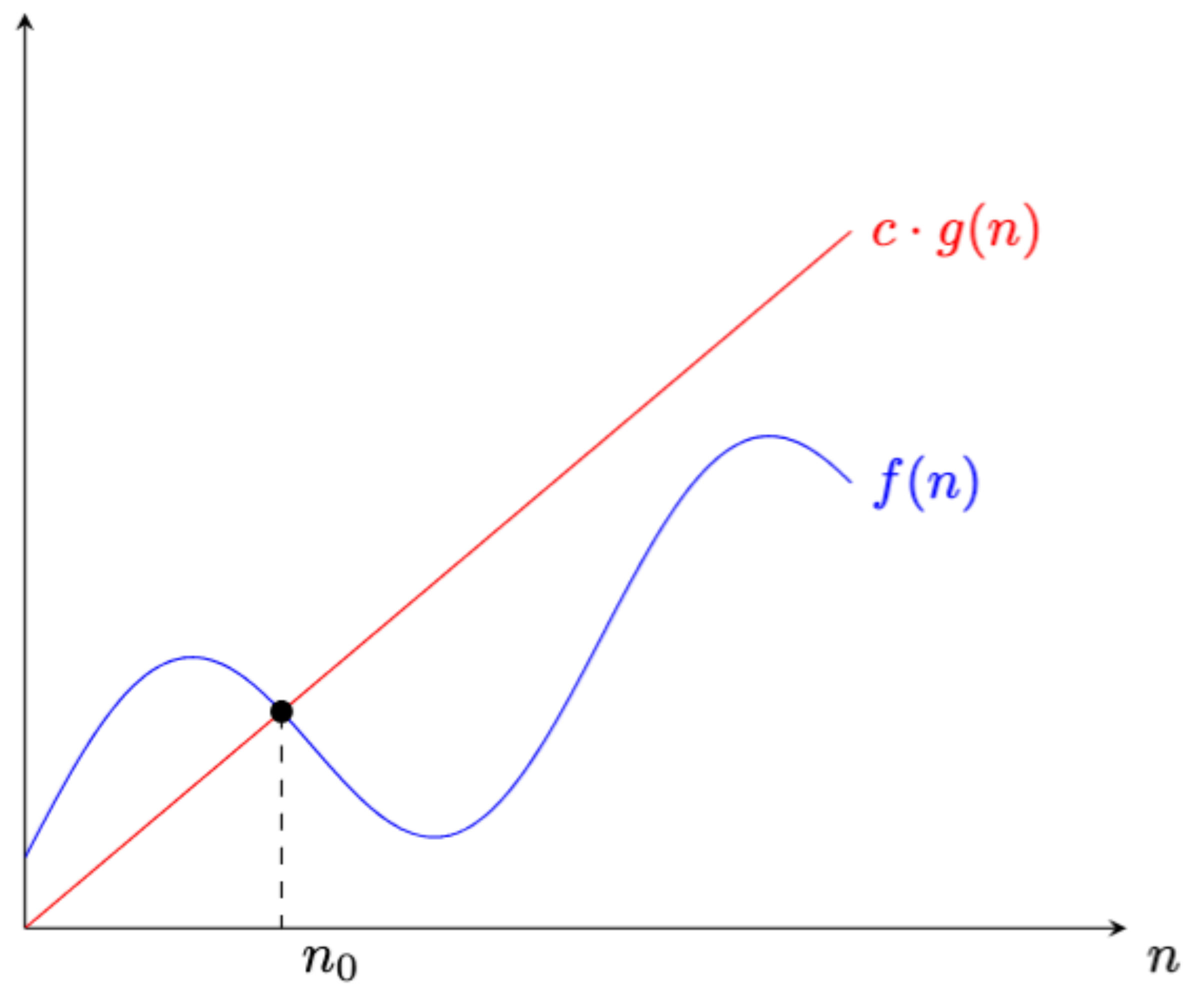

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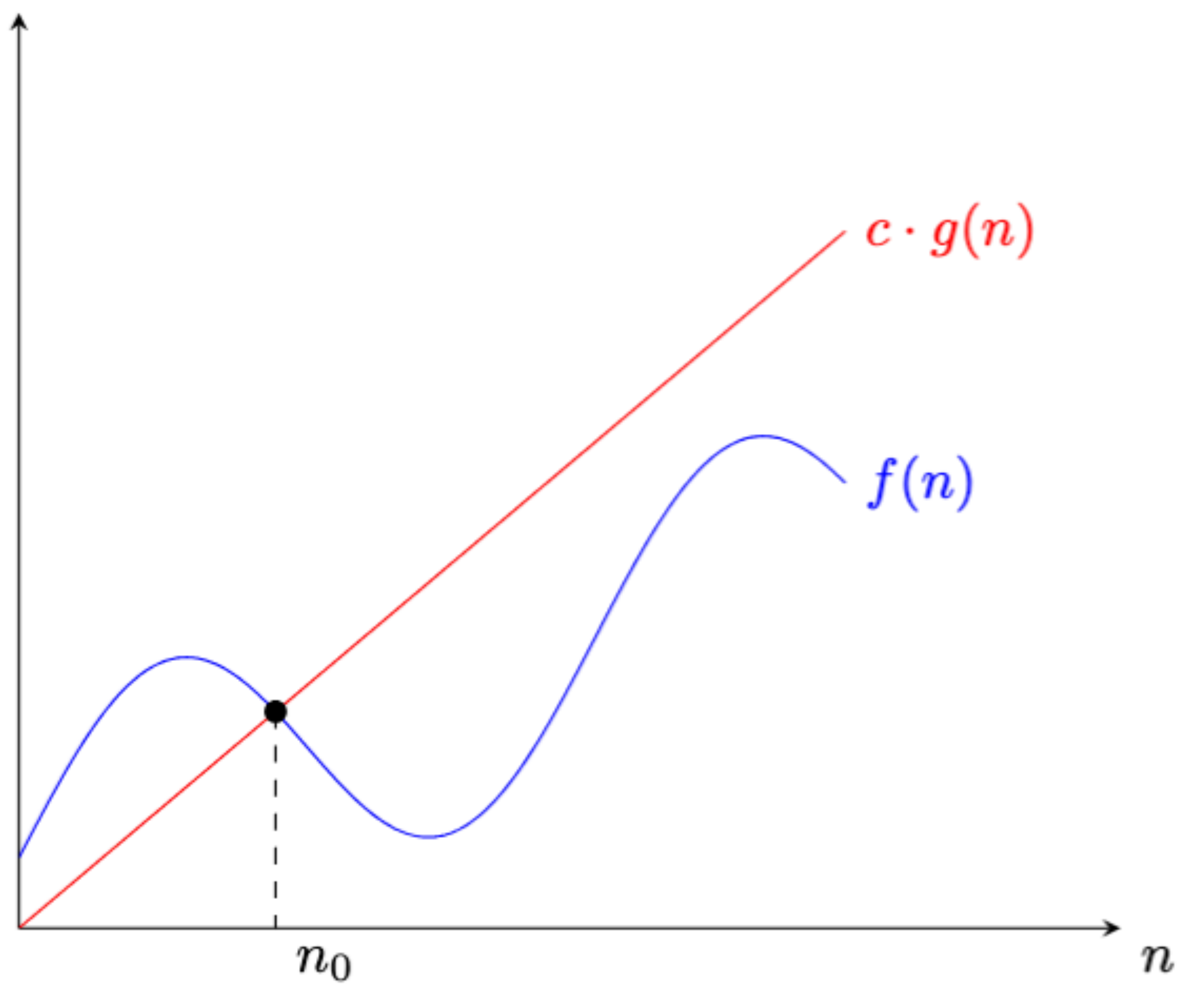

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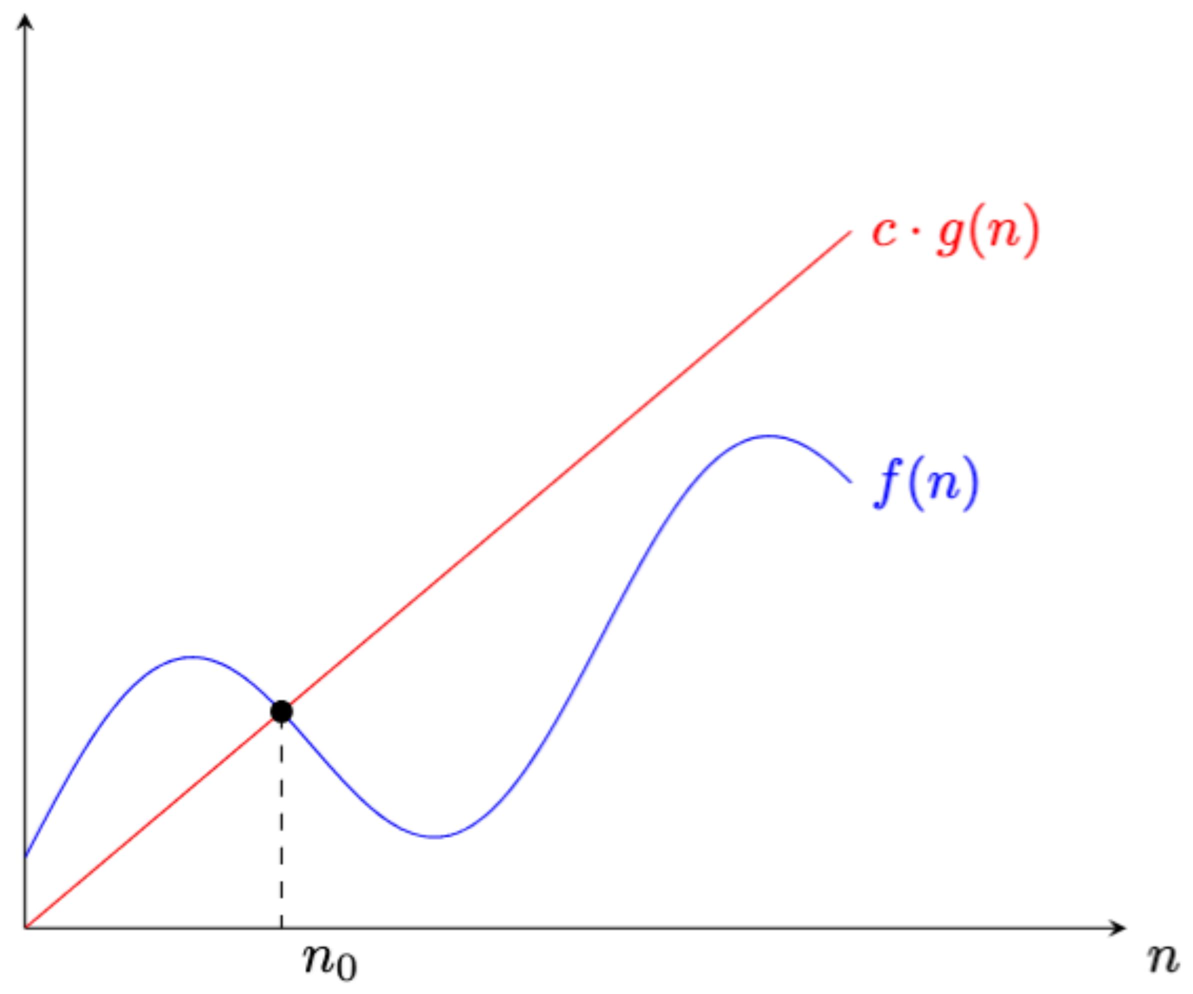

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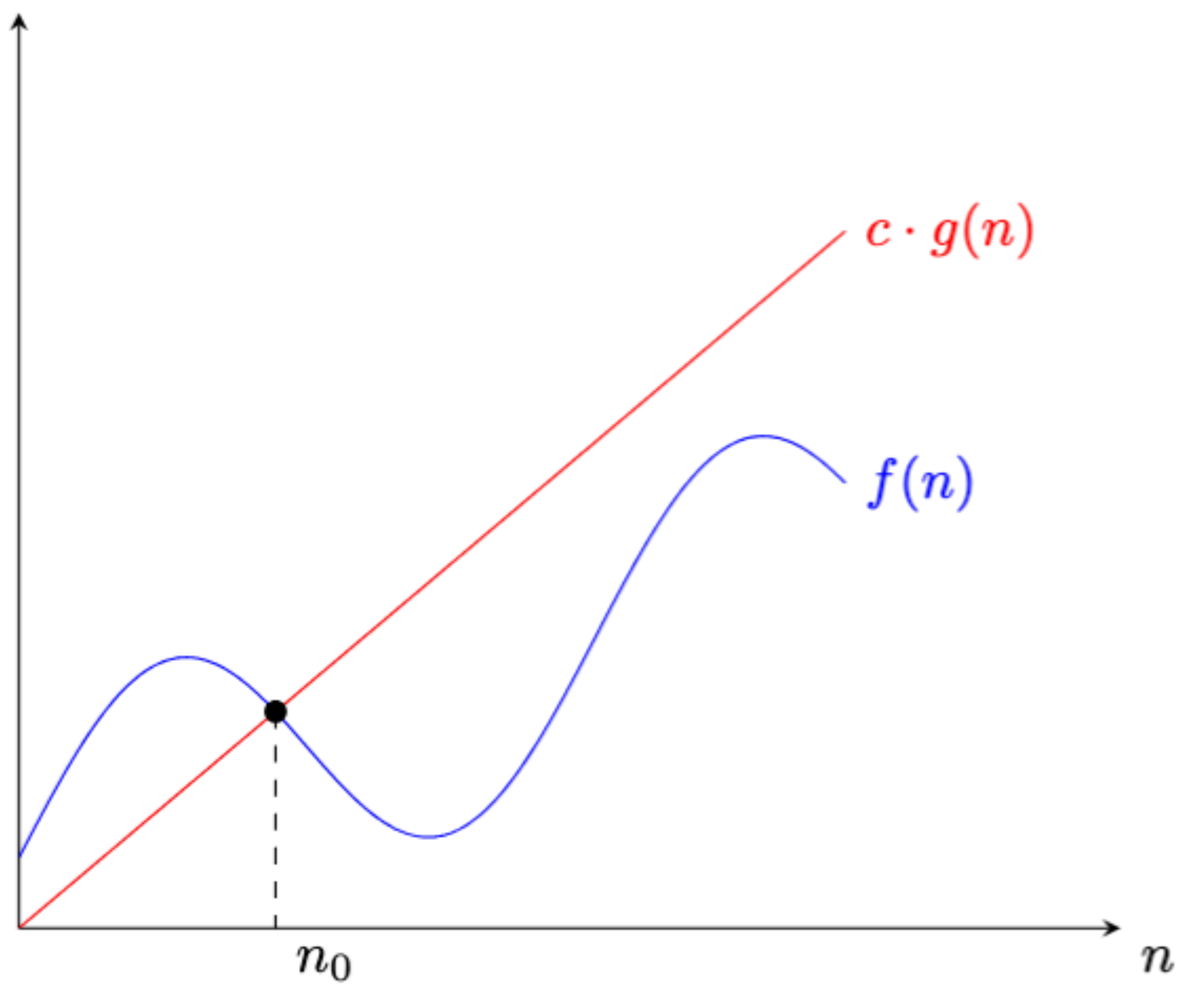

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We may say that a $f(x)$ is "super-polynomial" to mean that $f(x) \in \omega(x^d)$ for any constant d