Perfect Security

601.442/642 Modern Cryptography

22nd January 2026

Agenda

- Private communication and encryption schemes
- Defining an encryption scheme
  - First crypto definition!
- One-time pads
  - First crypto scheme!

A Few Remarks

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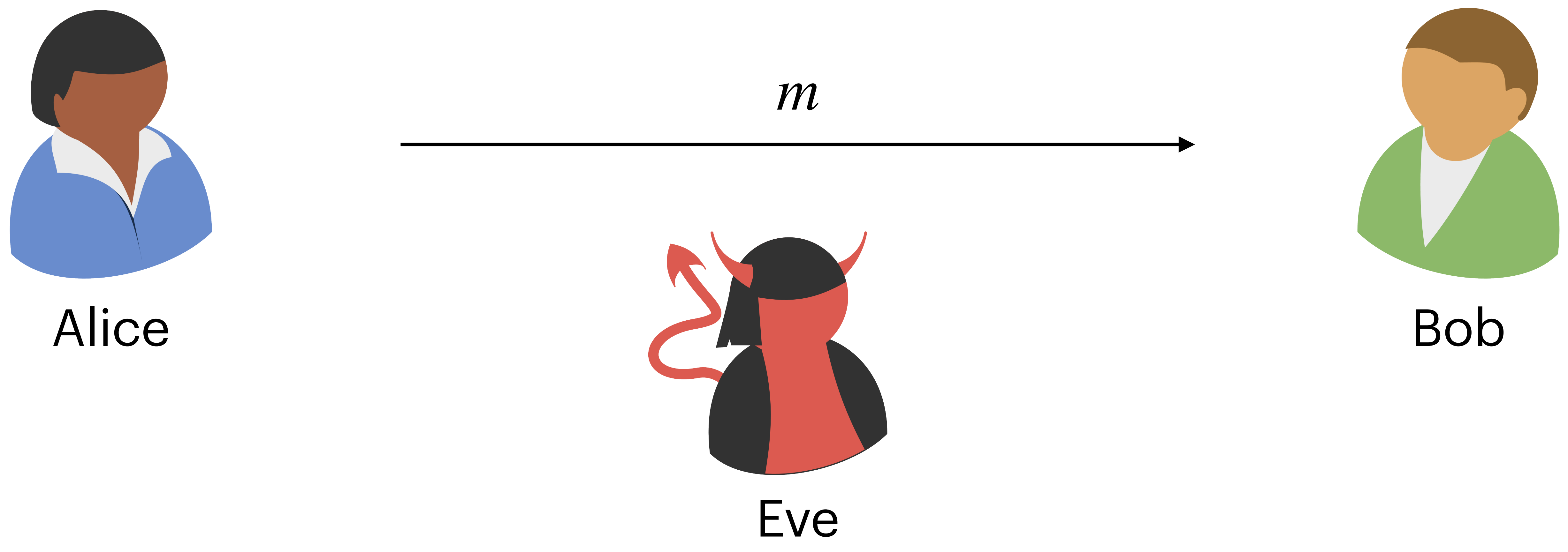
“It is by logic that we prove, but by intuition that we discover.”

- Henri Poincaré

Private Communication

Private Communication

The Private Communication Problem



Alice wants to send a message m to Bob, while keeping the message hidden from an eavesdropper Eve.

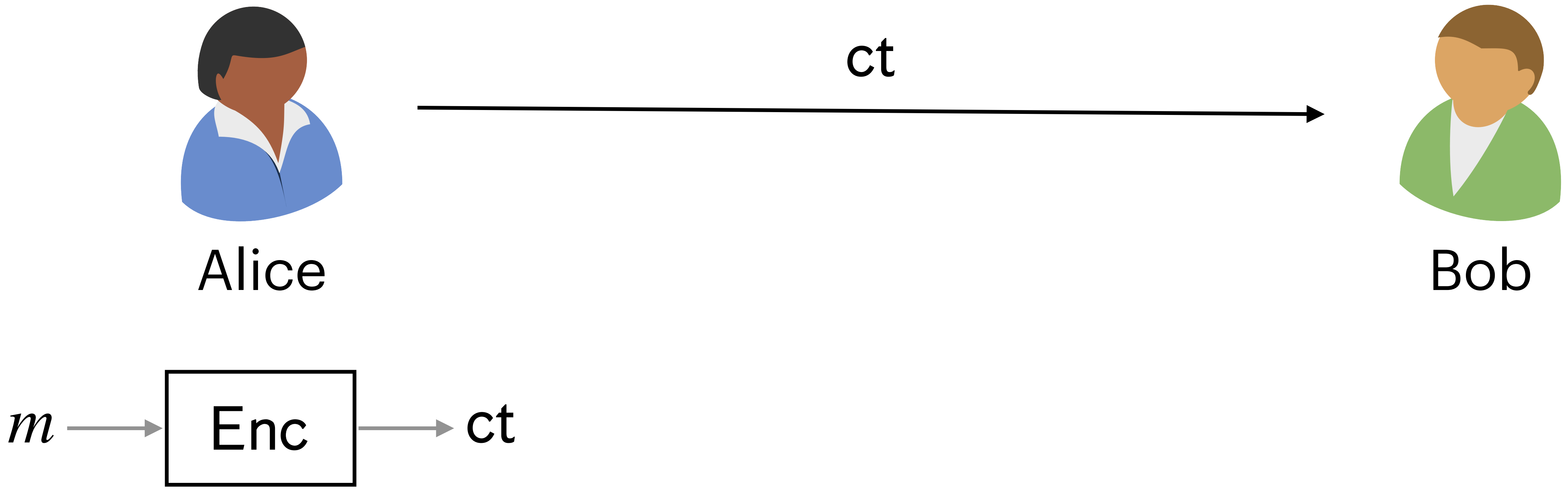
Encryption

Alice

Bob

Encryption

Encryption



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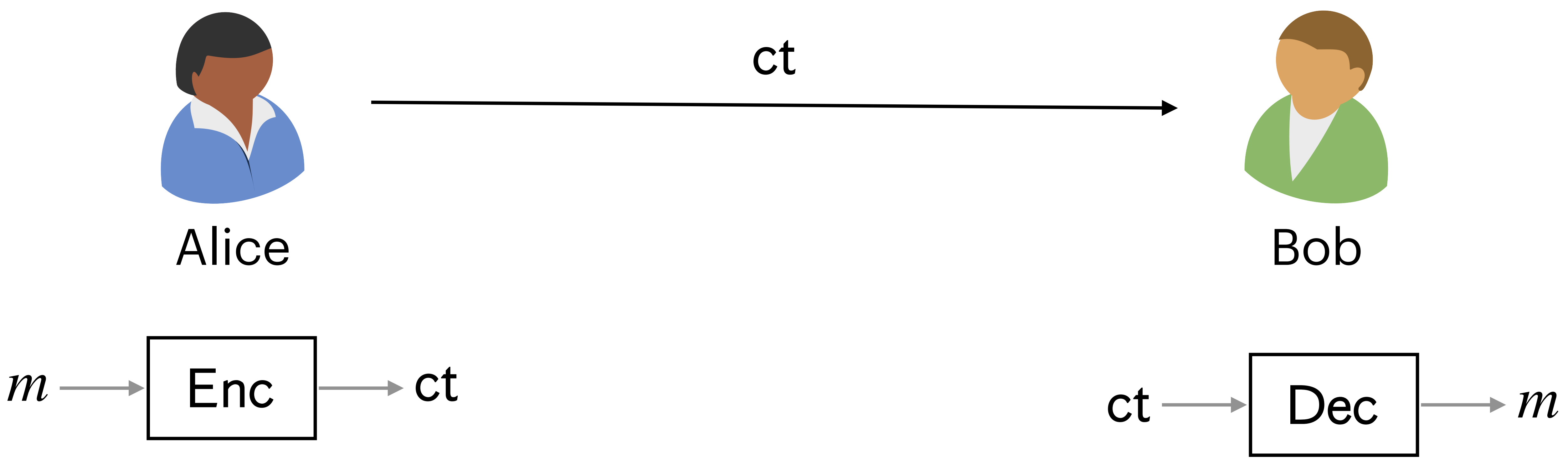

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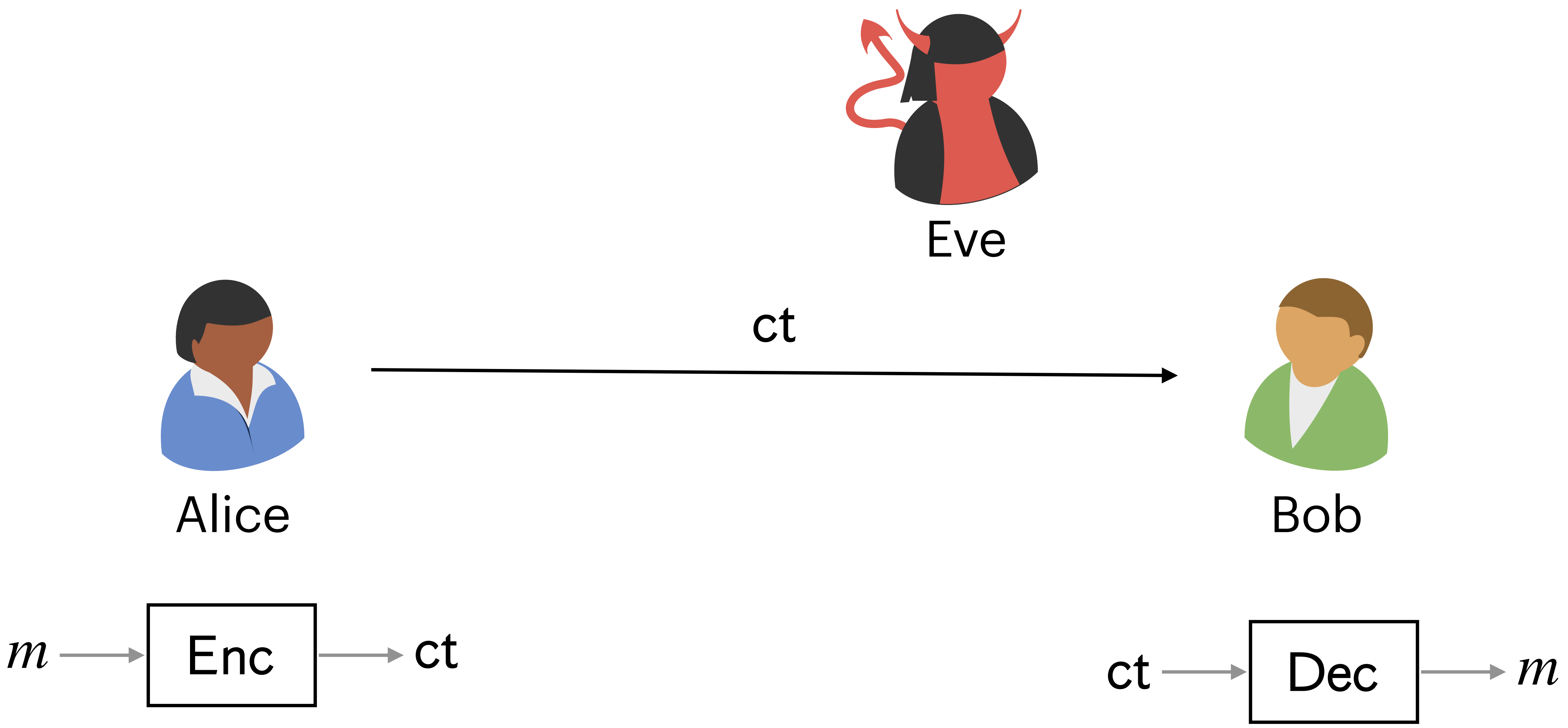

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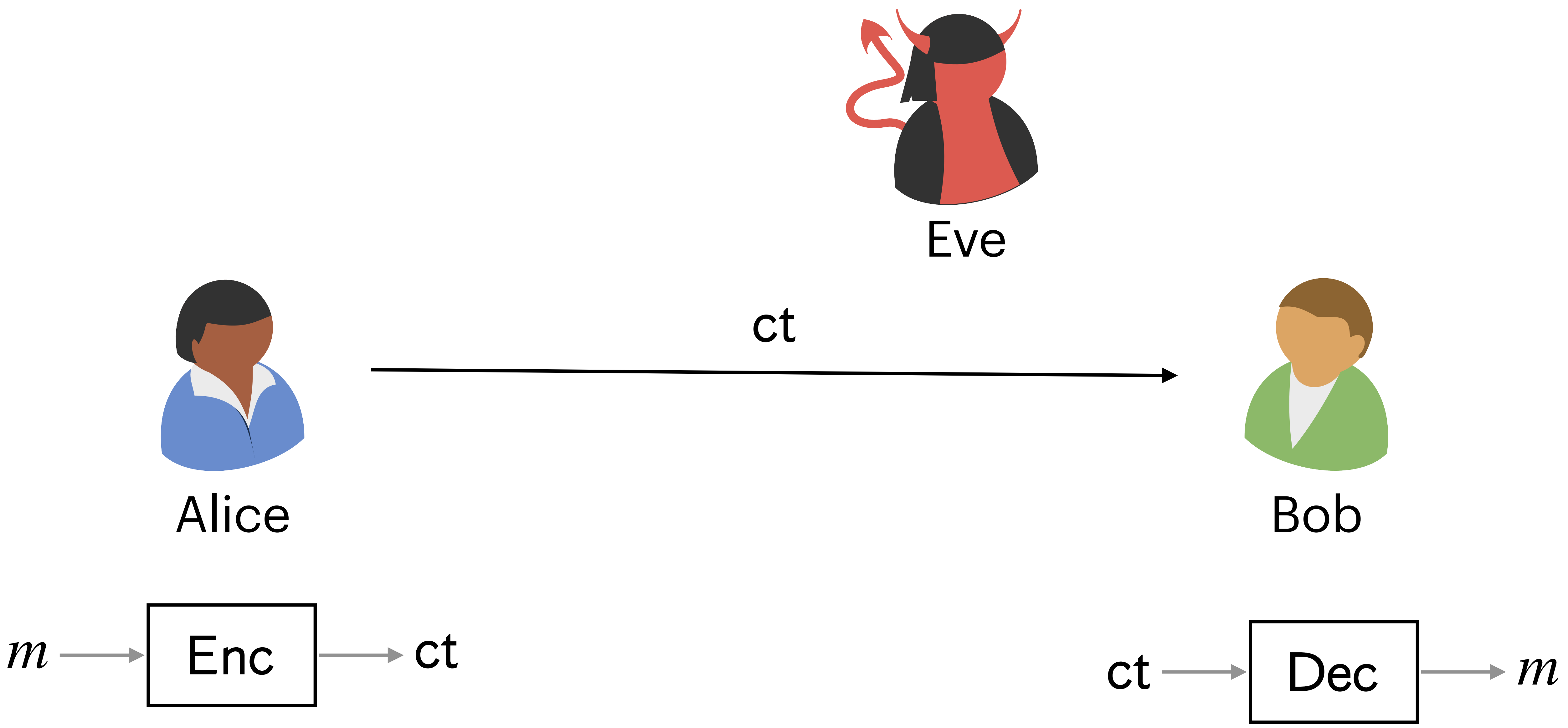

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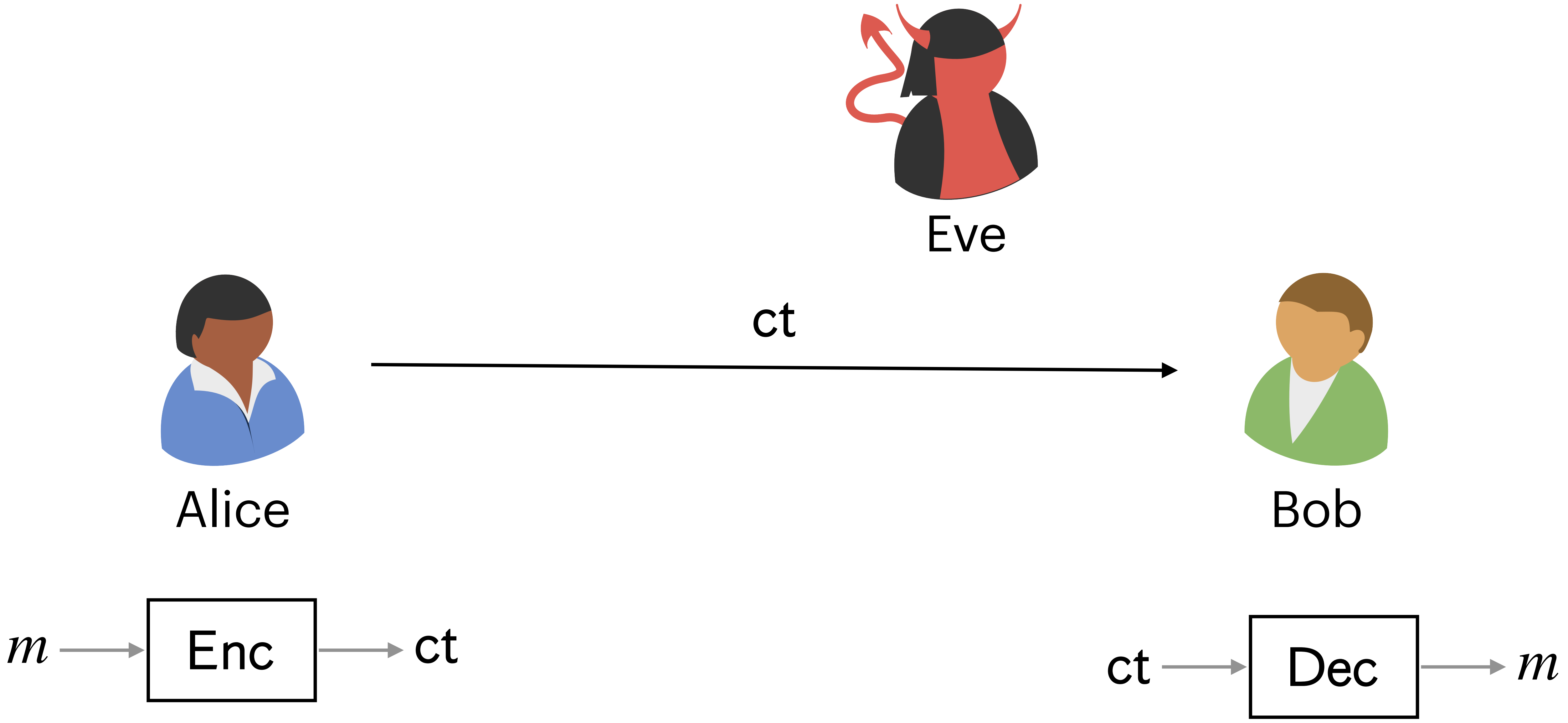

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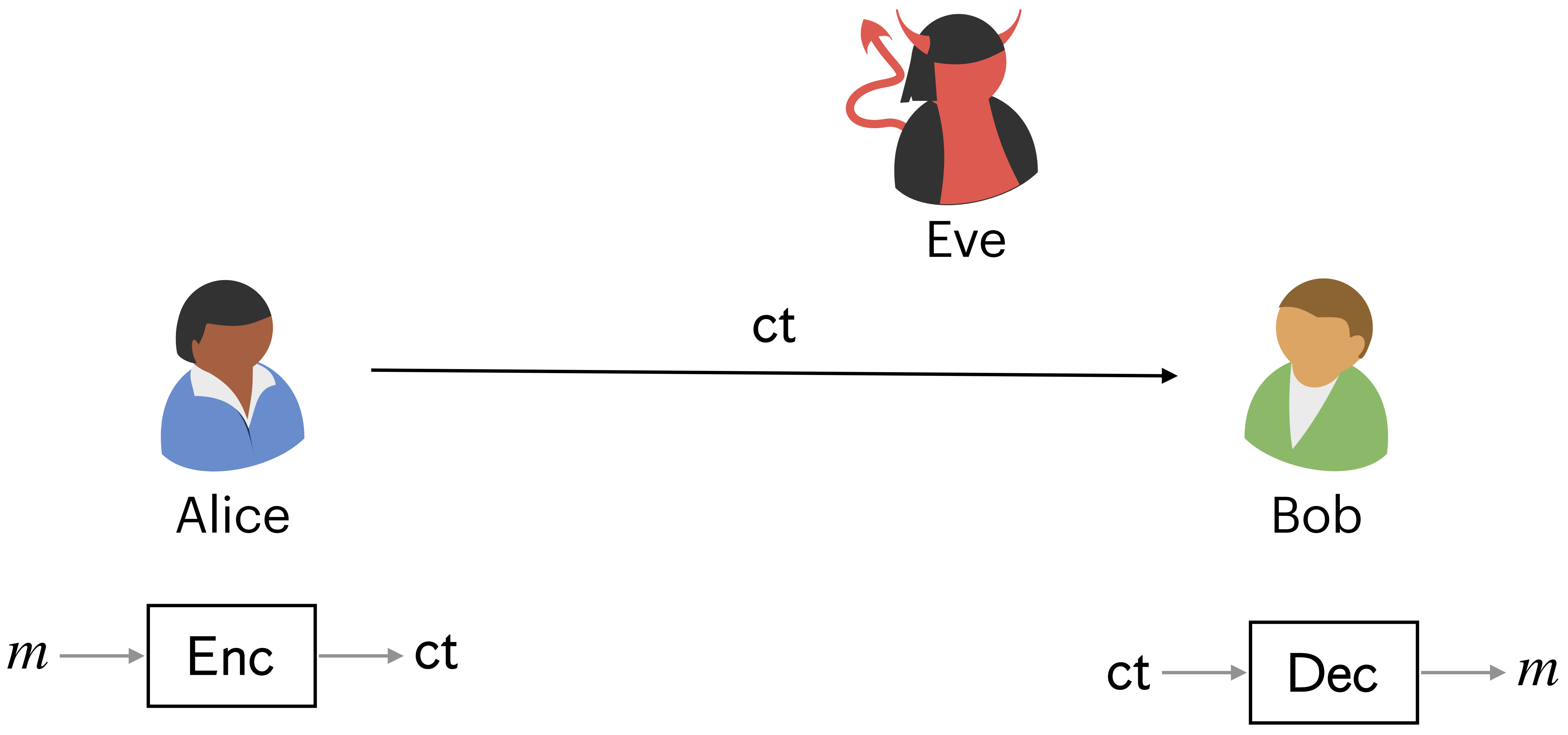

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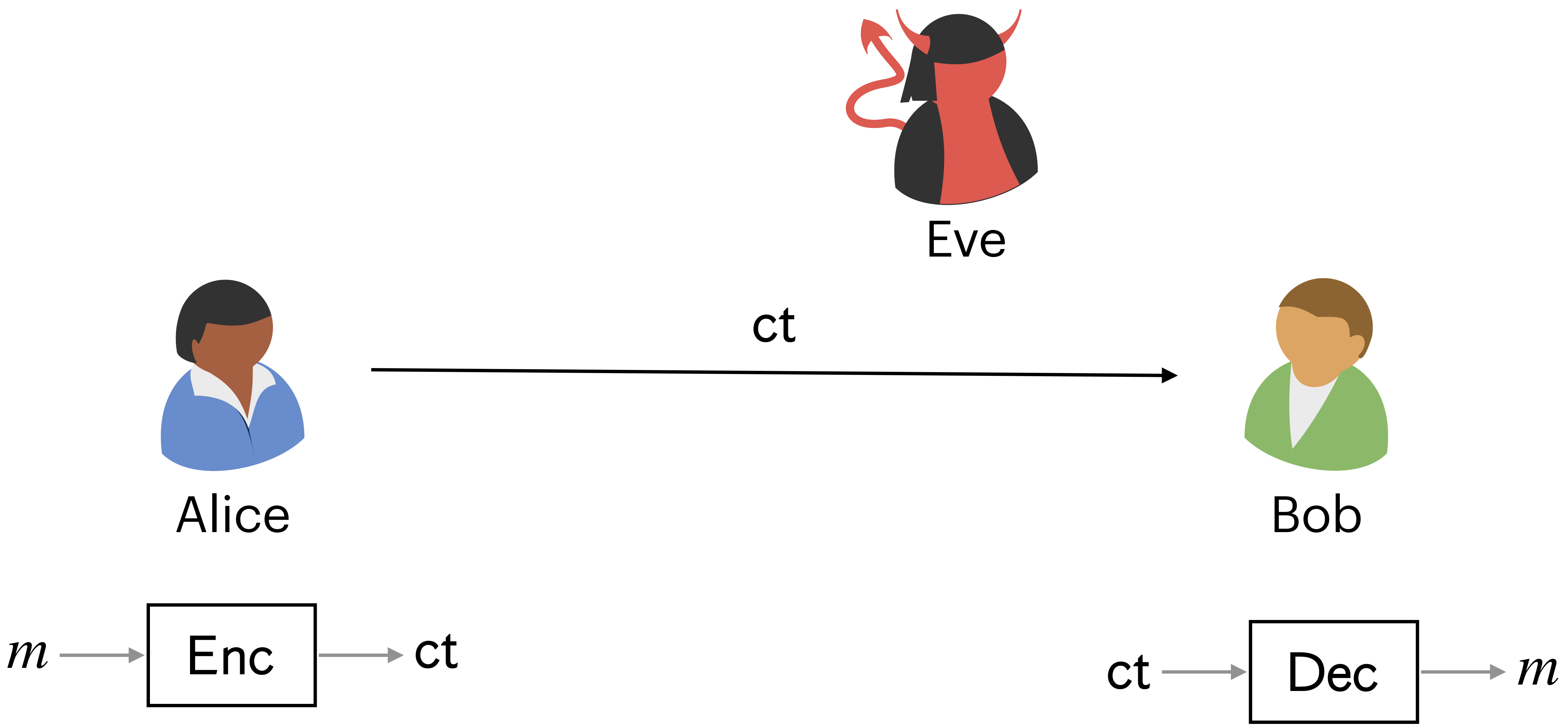

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- Should we rely on keeping the **details** of the Enc and Dec algorithms secret from Eve?
  - No! If Eve eventually learns the details of Enc and Dec, we will have to invent new algorithms.
  - Security through obscurity is fragile and unsustainable.

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Design your system to be secure even if the adversary has complete knowledge of all its algorithms.

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  - It is easier to ensure the secrecy of a key than that of an algorithm.
  - Algorithms can be made public, analyzed and standardized. Crucial for large-scale deployments.

Encryption: Syntax

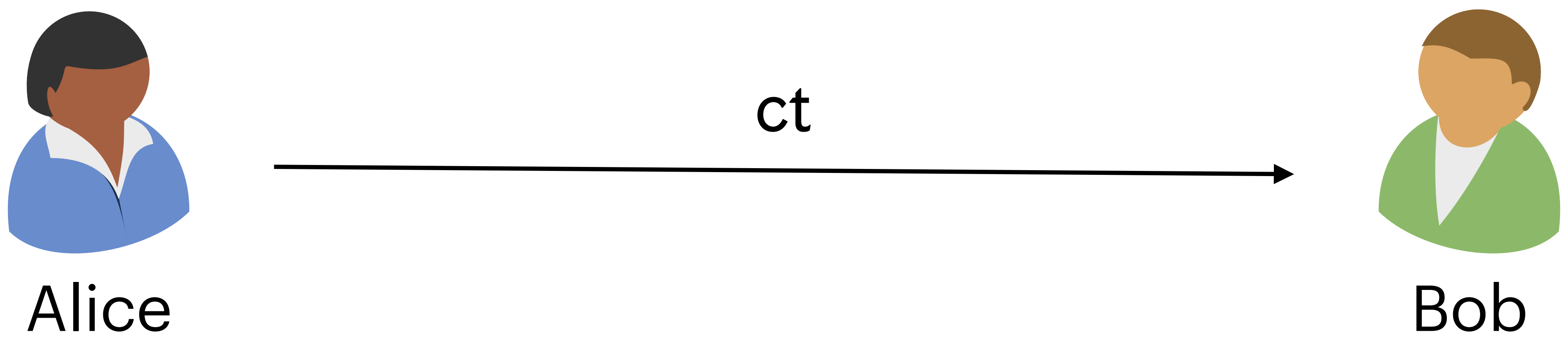
Alice

k

Bob

k

Encryption: Syntax



Alice	Bob
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$\mathsf{ct} \leftarrow \mathsf{Enc}(k, m)$	$m \leftarrow \mathsf{Dec}(k, \mathsf{ct})$

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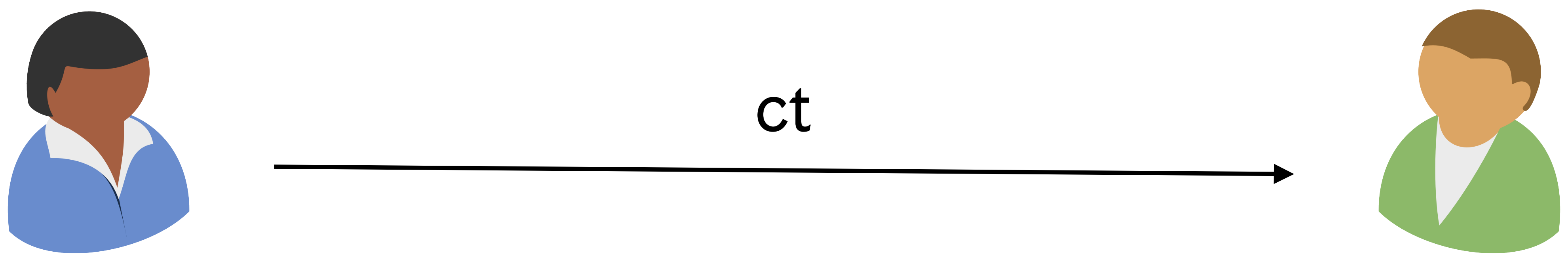

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Encryption Scheme Syntax

An encryption scheme consists of three (possibly probabilistic) algorithms:

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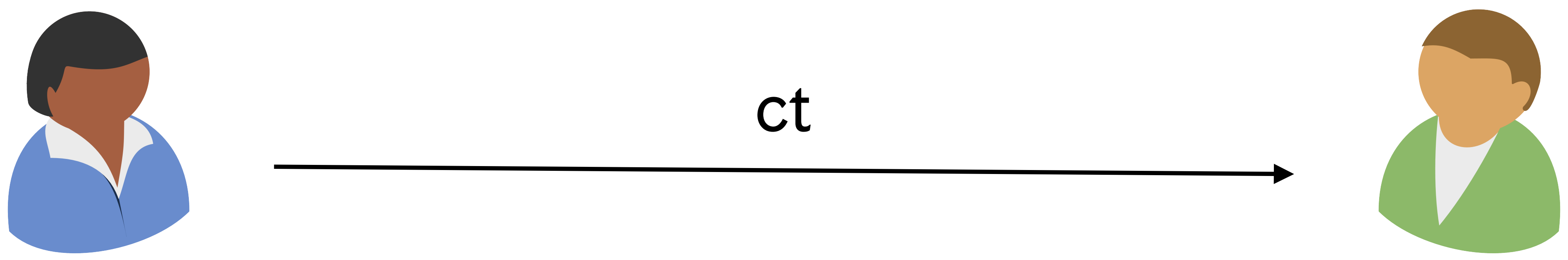

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ct →

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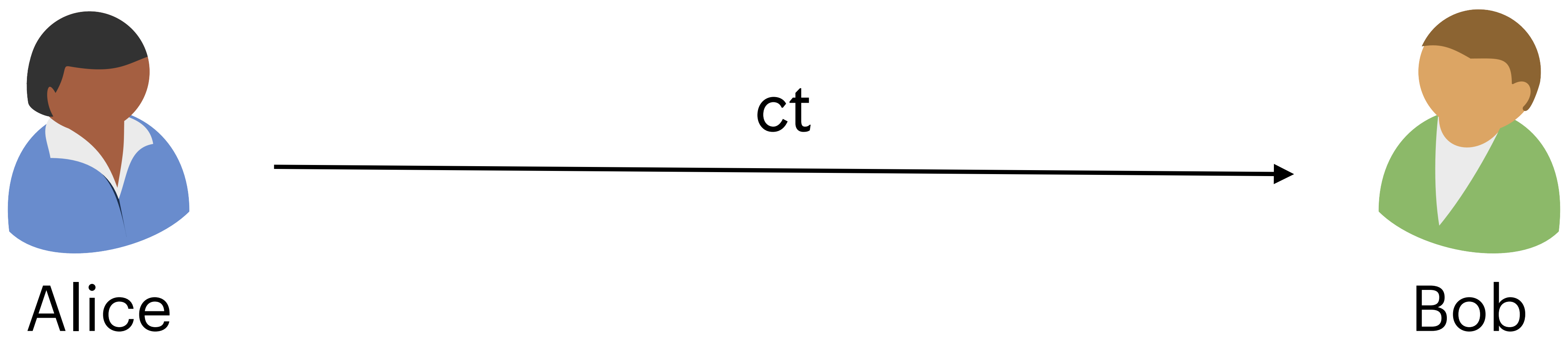

k $\qquad$ k

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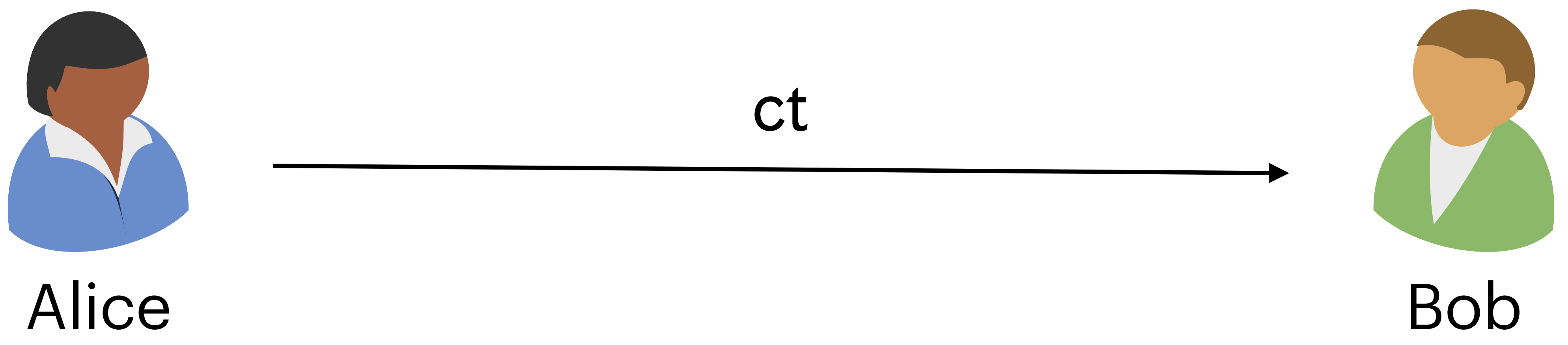

Alice: k — $\mathsf{ct} \leftarrow \mathsf{Enc}(k, m)$

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<u>Encryption Scheme Syntax</u>

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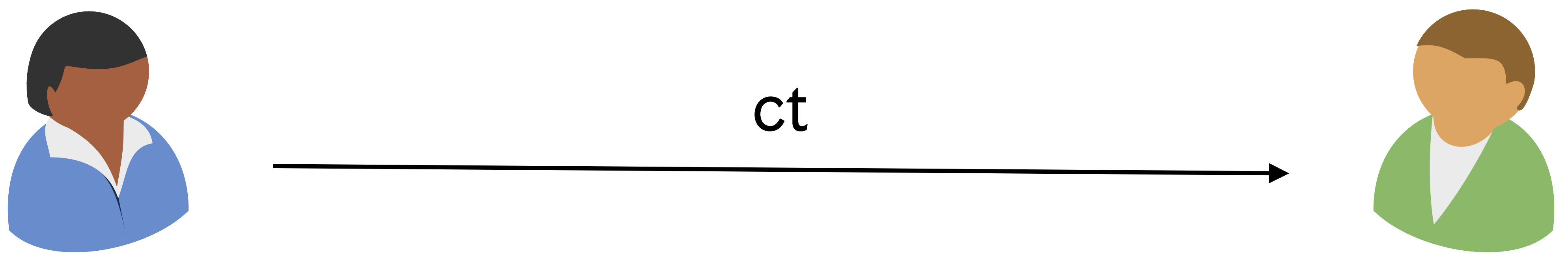
- $\mathsf{KeyGen}() \to k$ outputs a key $k \in \mathscr{K}$.
- $\mathsf{Enc}(k, m) \to \mathsf{ct}$ takes key k and message $m \in \mathscr{M}$ and outputs ciphertext $\mathsf{ct} \in \mathscr{C}$.

Message space

Ciphertext space

Encryption: Syntax



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- $\mathsf{Dec}(k, \mathsf{ct}) \rightarrow m$ takes key k and ciphertext ct and outputs message m.

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Adversarial Model: Eve is “passive”. She reads ciphertexts but does not interfere.

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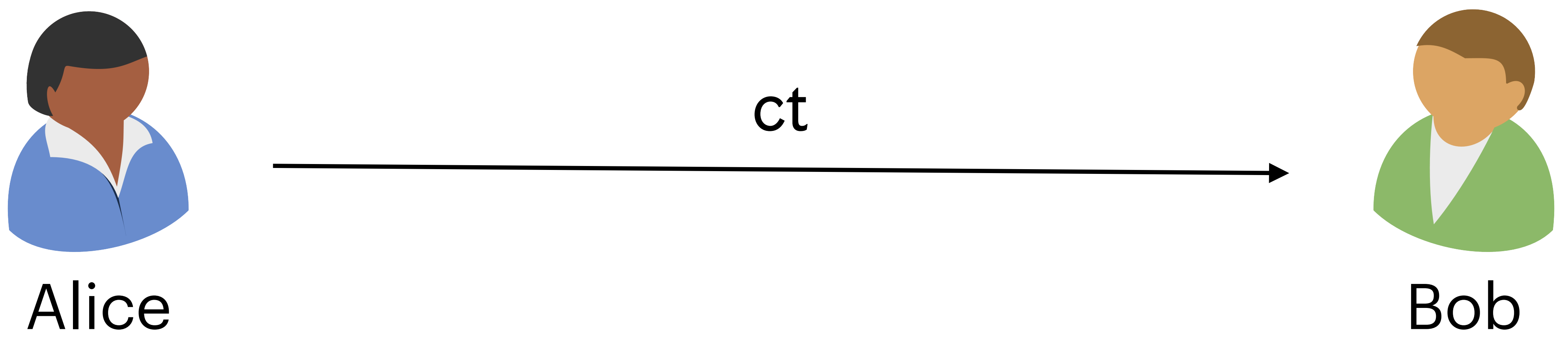

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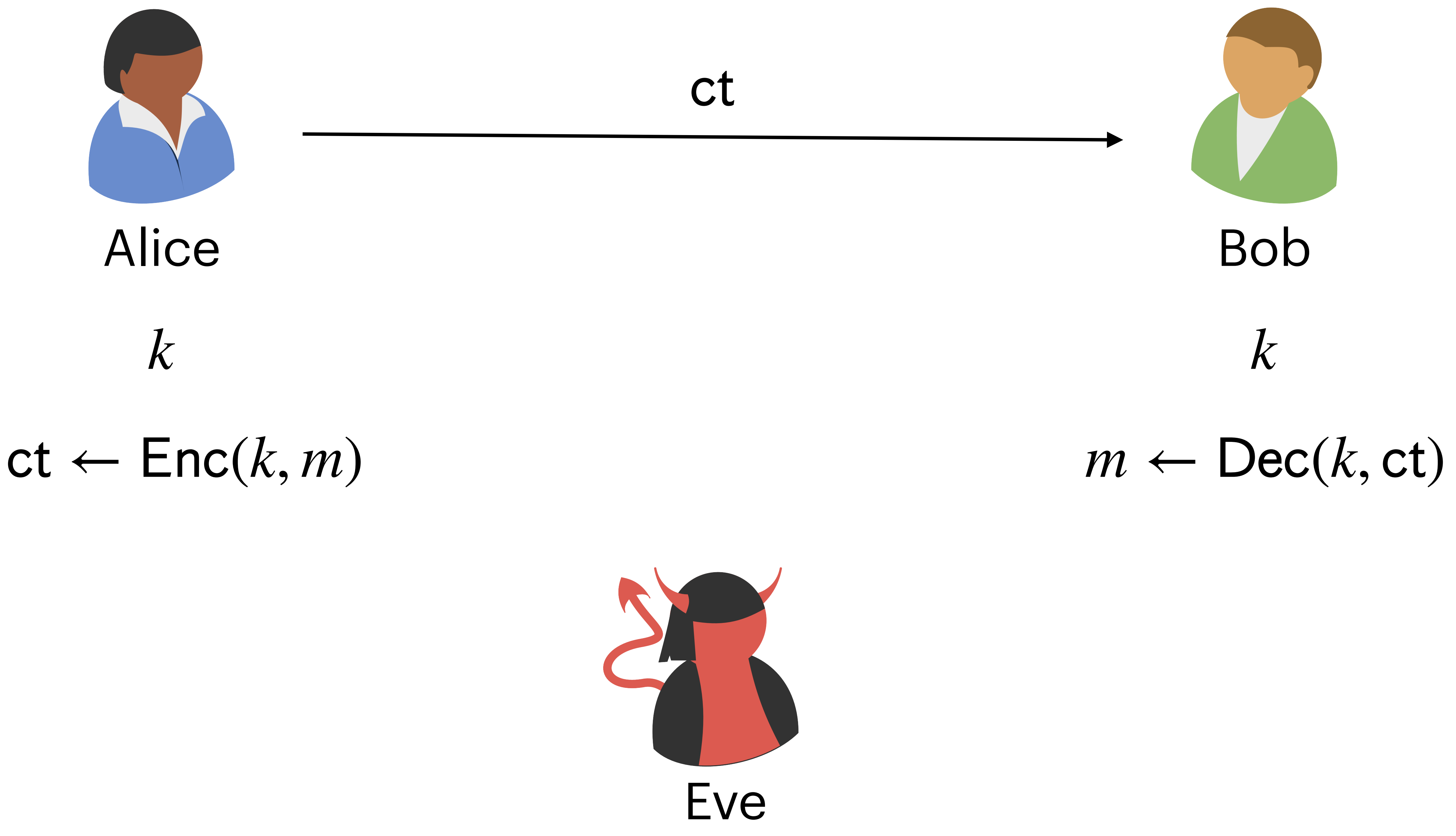

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- **Simplification:** We will focus on the case of encrypting a single message. We will consider multi-message security later in the course.

One-Time Pad

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Let λ be a positive integer and let $\mathcal{K} = \mathcal{M} = \mathcal{C} = \{0,1\}^\lambda$.

- KeyGen(): $k \xleftarrow{\$} \{0,1\}^\lambda$.
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Proof: Fix arbitrary $k \in \mathcal{K}$ and $m \in \mathcal{M}$. We have

$$\begin{aligned}\text{Dec}(k, \text{Enc}(k, m)) &= \text{Dec}(k, k \oplus m) \\ &= k \oplus k \oplus m \\ &= m\,.\end{aligned}$$

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Why is one-time pad secure?

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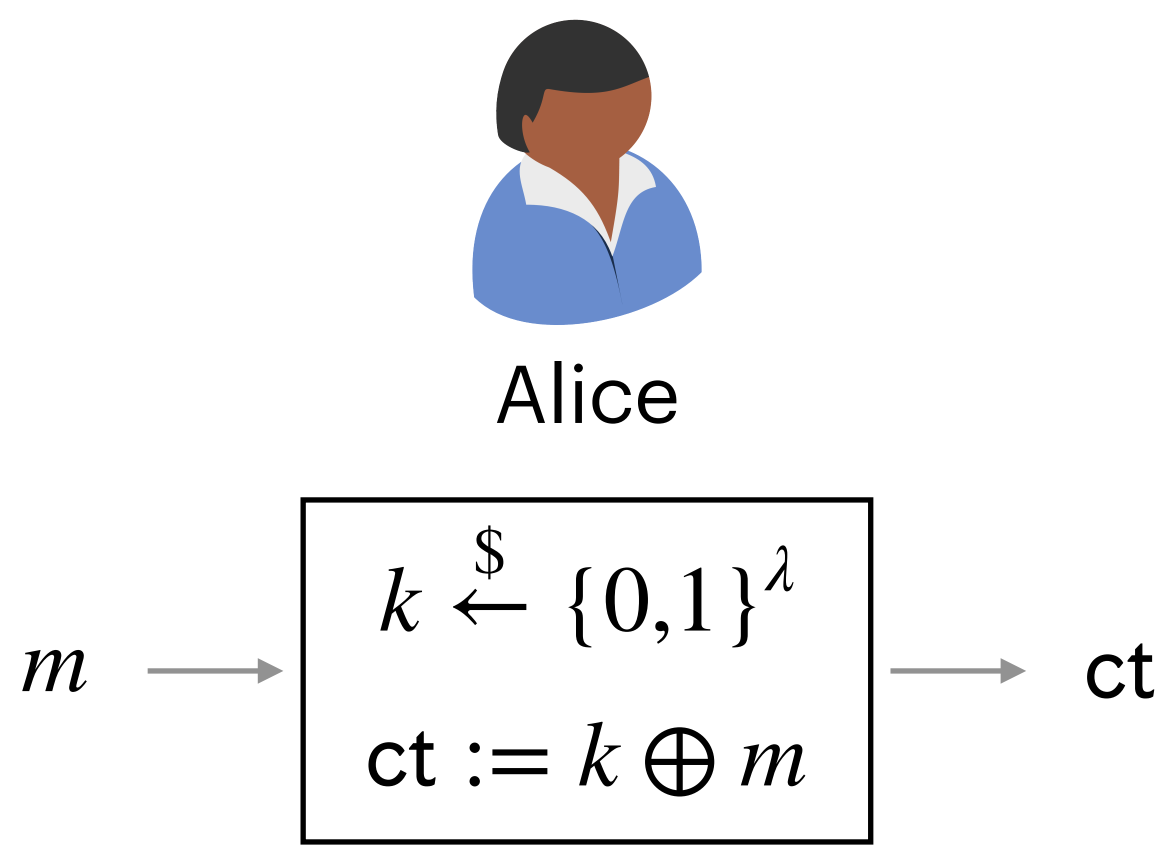

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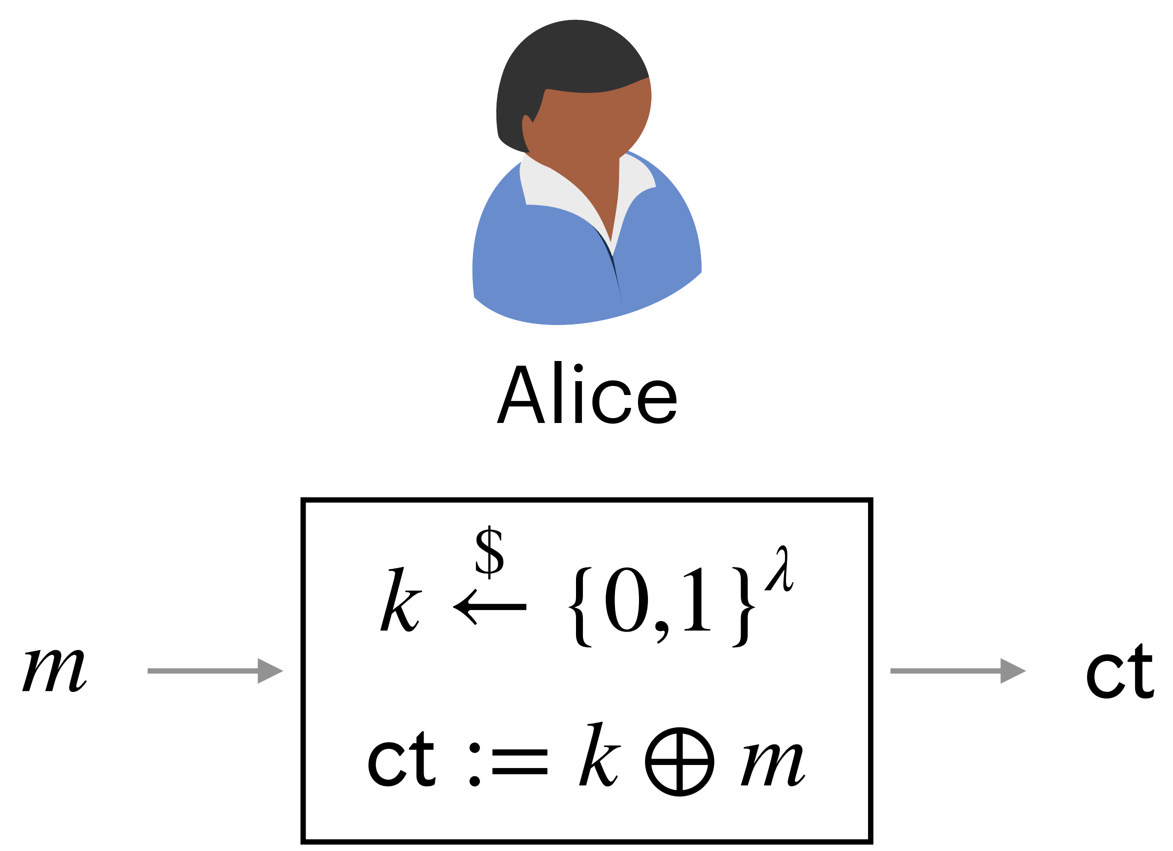

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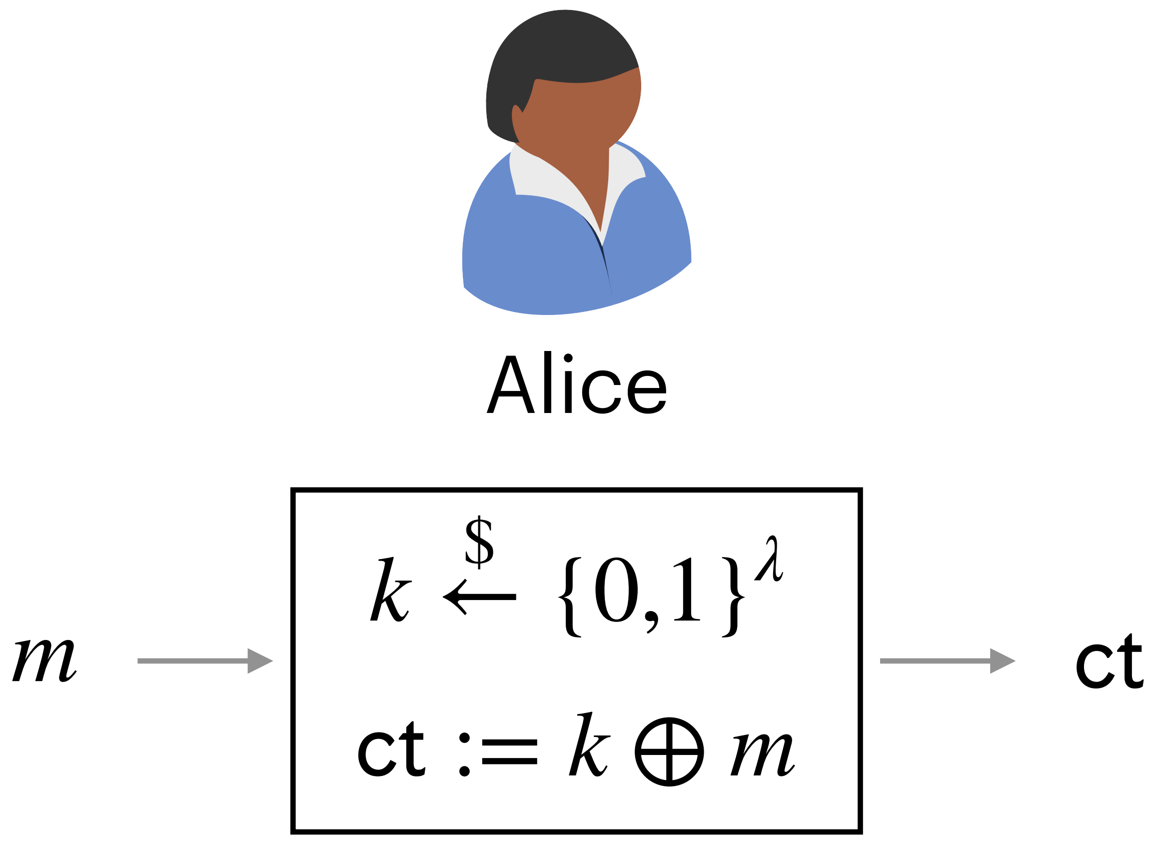

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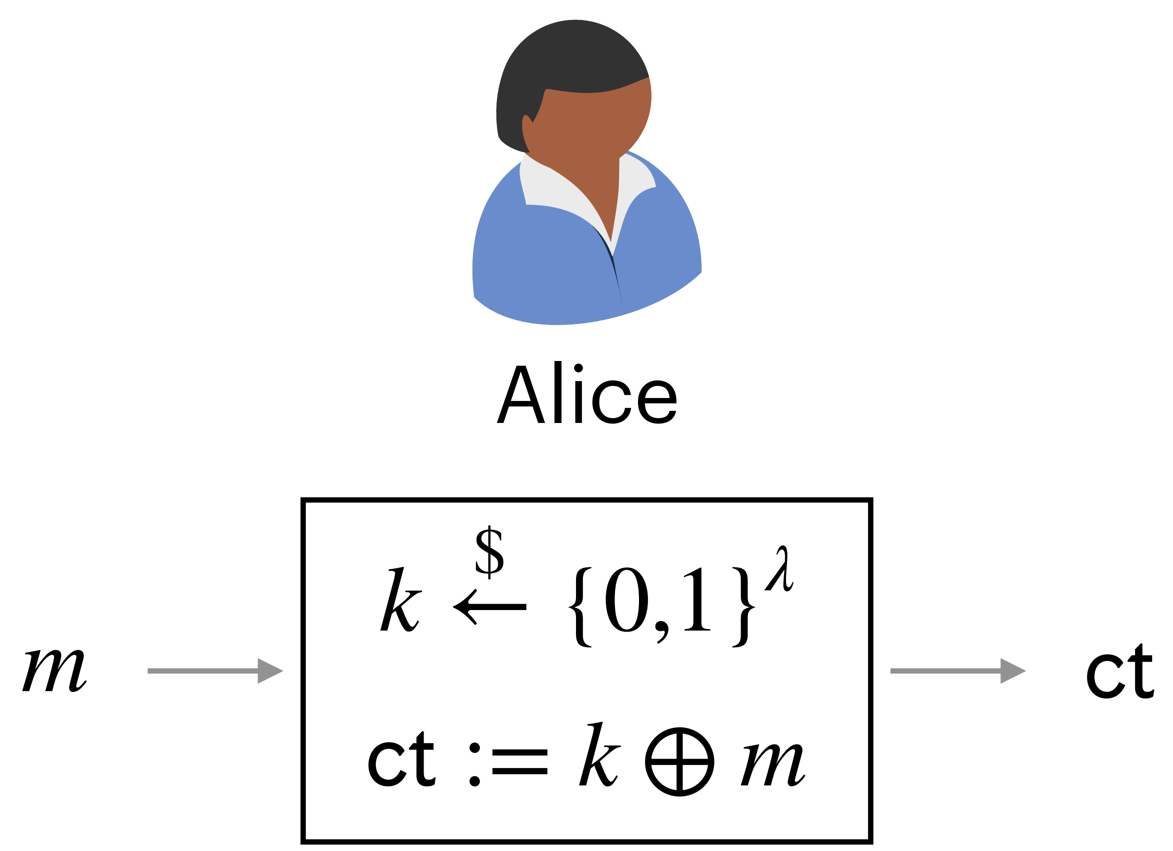

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Pr	k	ct $= k \oplus 010$
1/8	000	010
1/8	001	011
1/8	010	000
1/8	011	001
1/8	100	110
1/8	101	111
1/8	110	100
1/8	111	101

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Alice

$m \longrightarrow$ [$k \xleftarrow{\$} \{0,1\}^\lambda$; ct $:= k \oplus m$] $\longrightarrow$ ct

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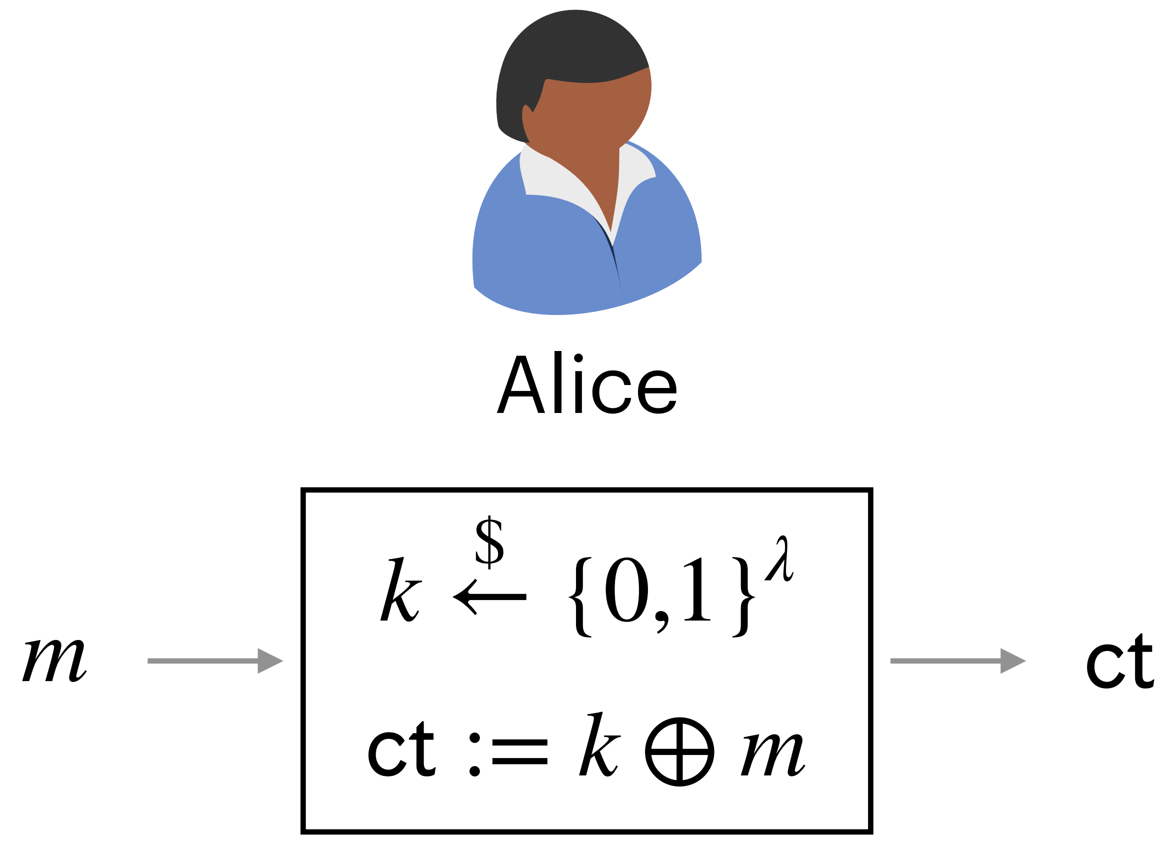

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  - Every string in $\{0,1\}^3$ occurs exactly once as a ciphertext.
  - Since the key is sampled uniformly at random, for any $s \in \{0,1\}^3$, the probability that **ct** $= s$ is $1/8$ i.e., **the ciphertext is uniformly random** over $\{0,1\}^3$.

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$1/8$	001	011
$1/8$	010	000
$1/8$	011	001
$1/8$	100	110
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$1/8$	110	100
$1/8$	111	101

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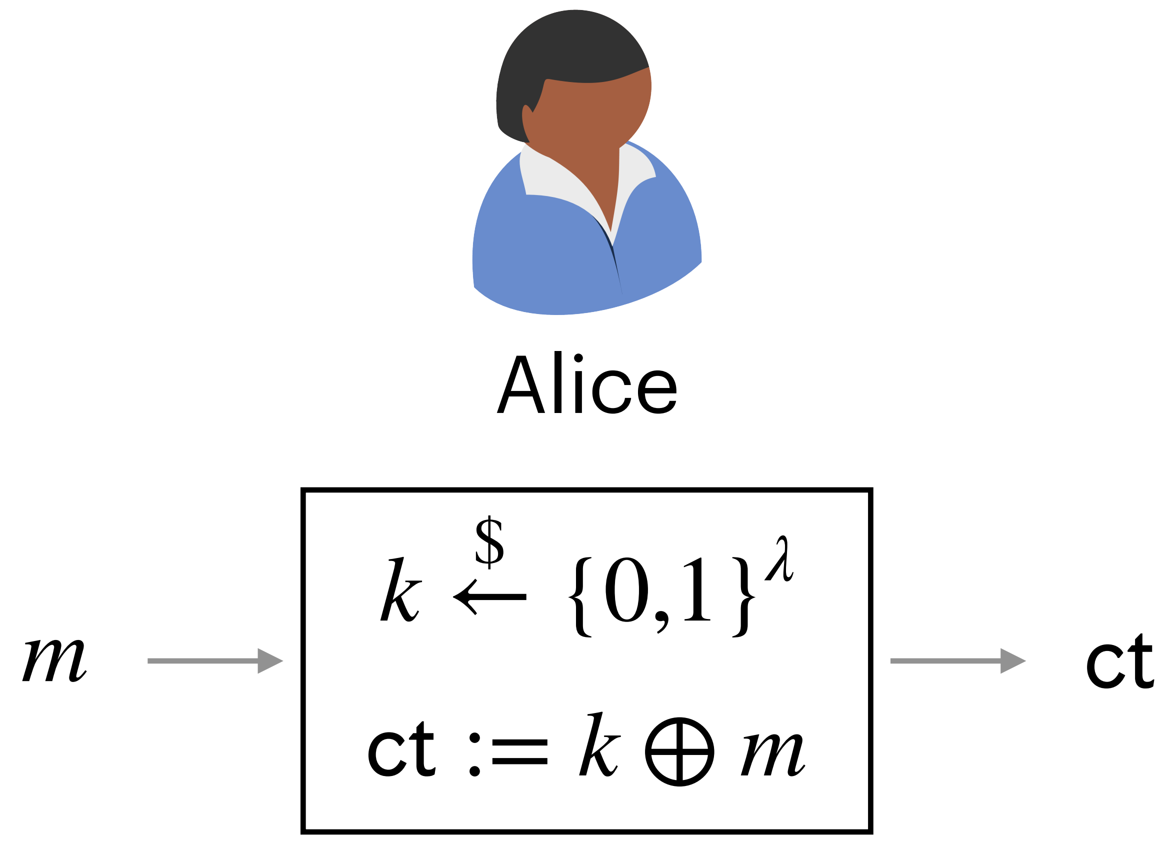

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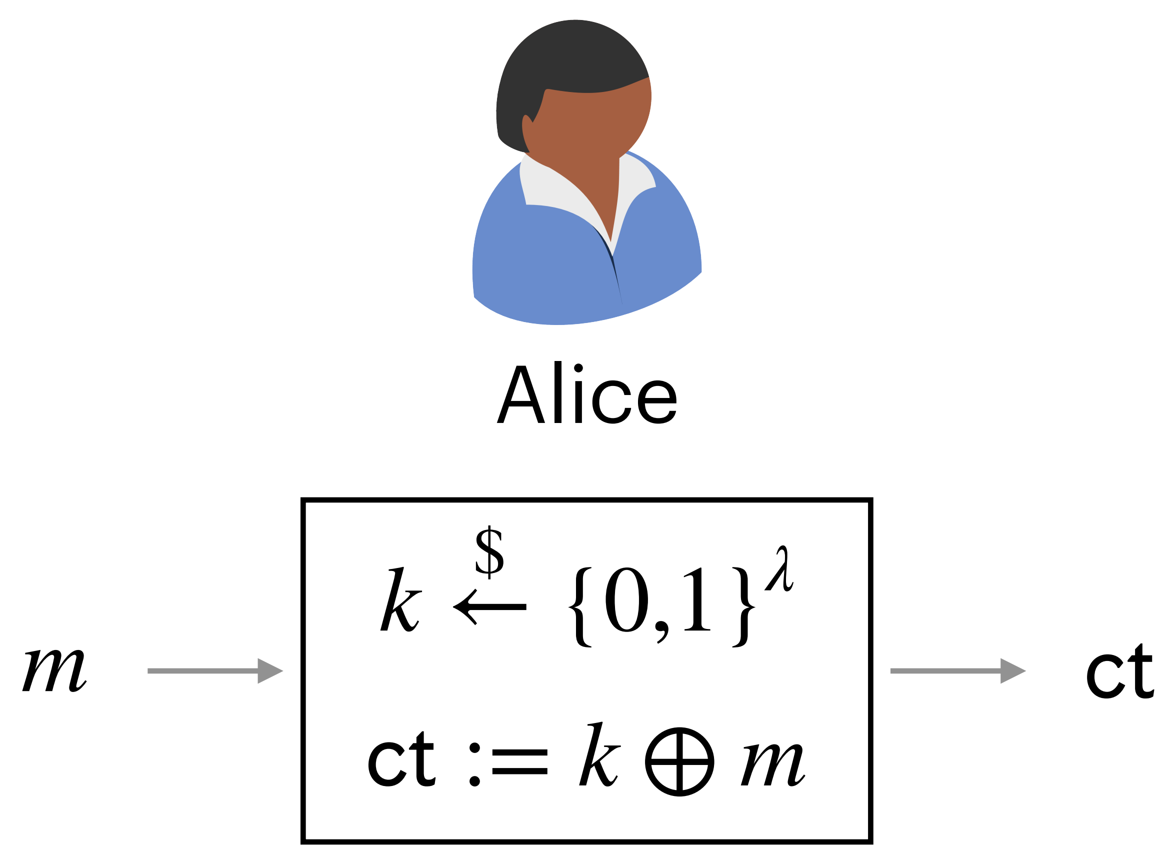

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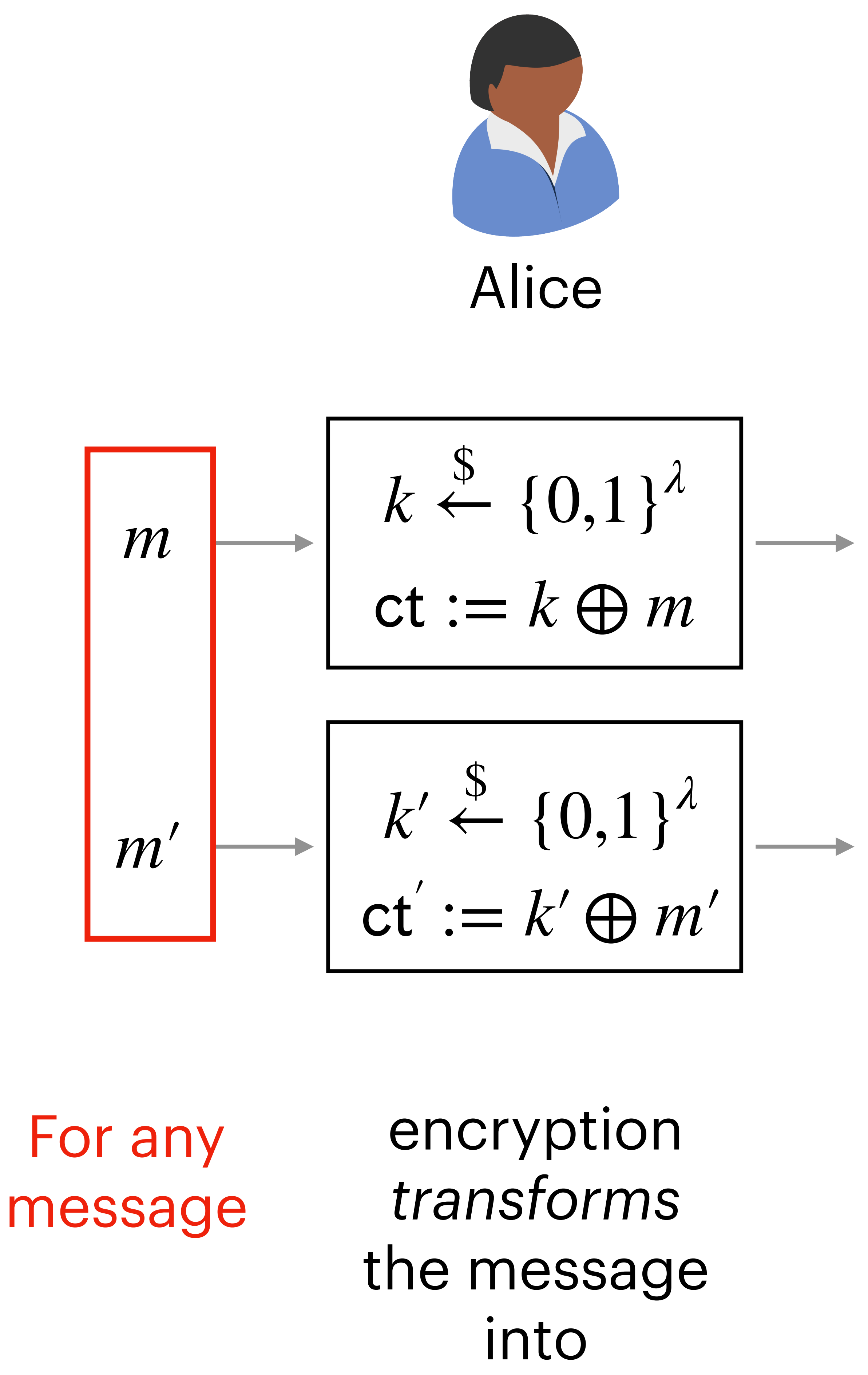

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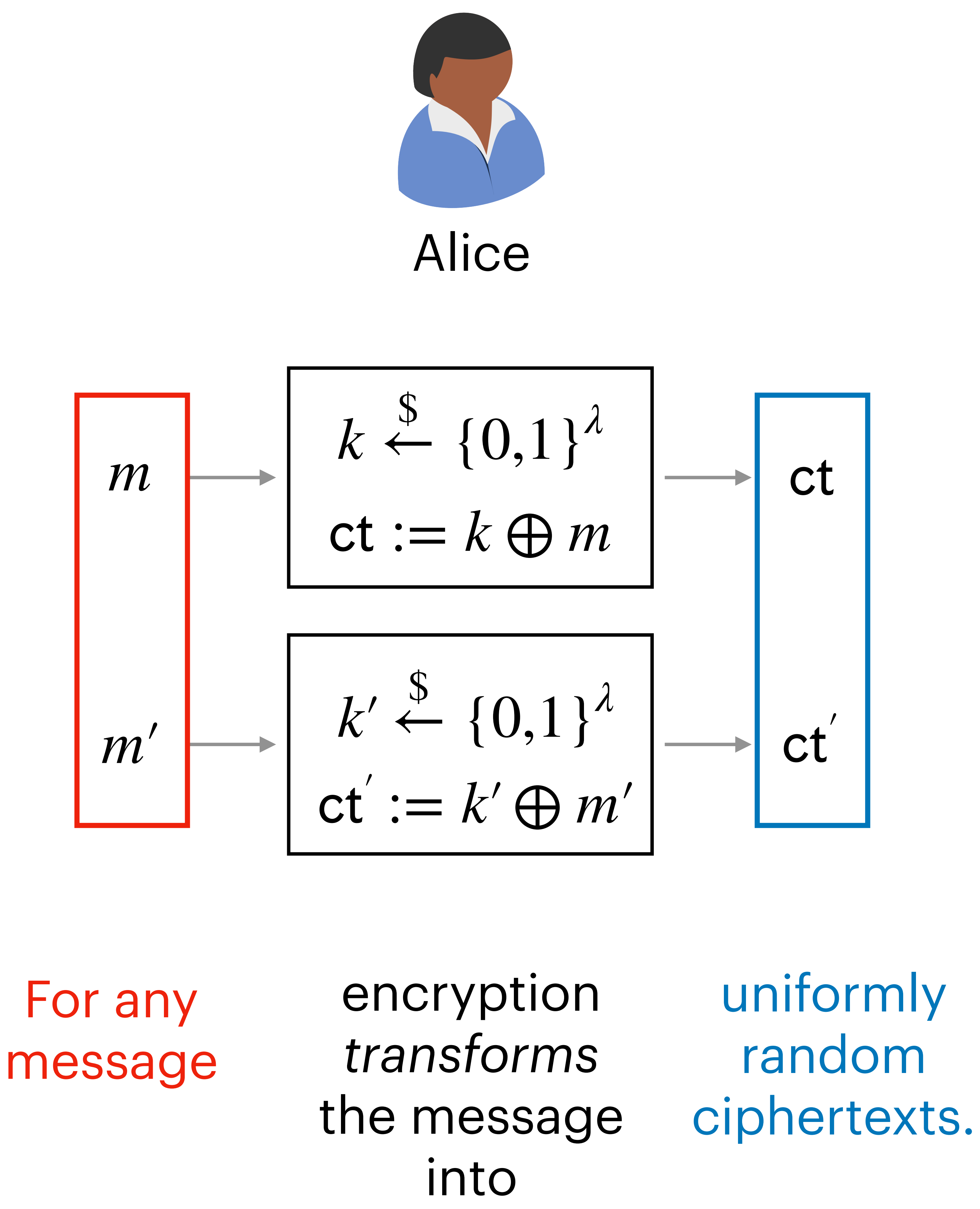

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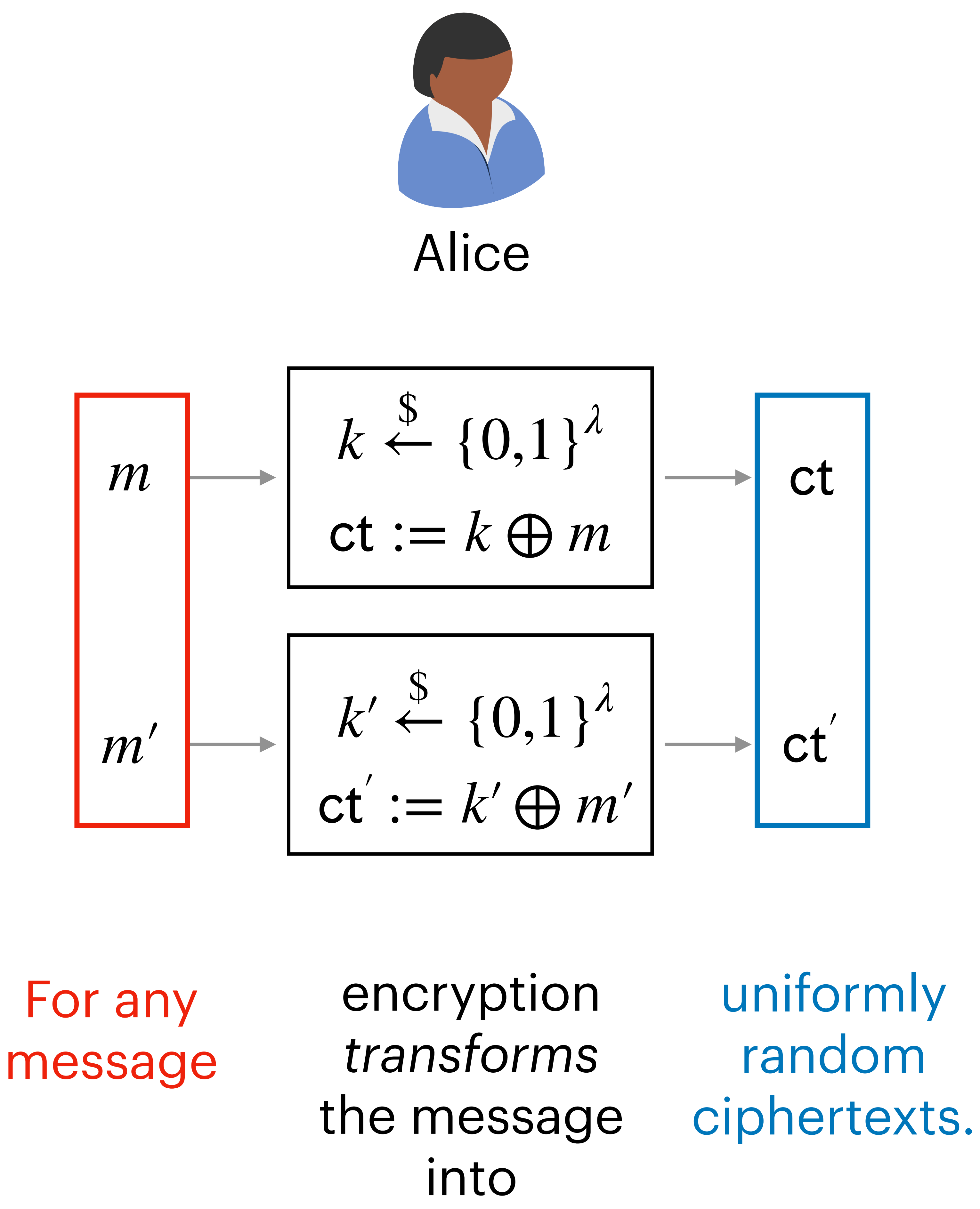

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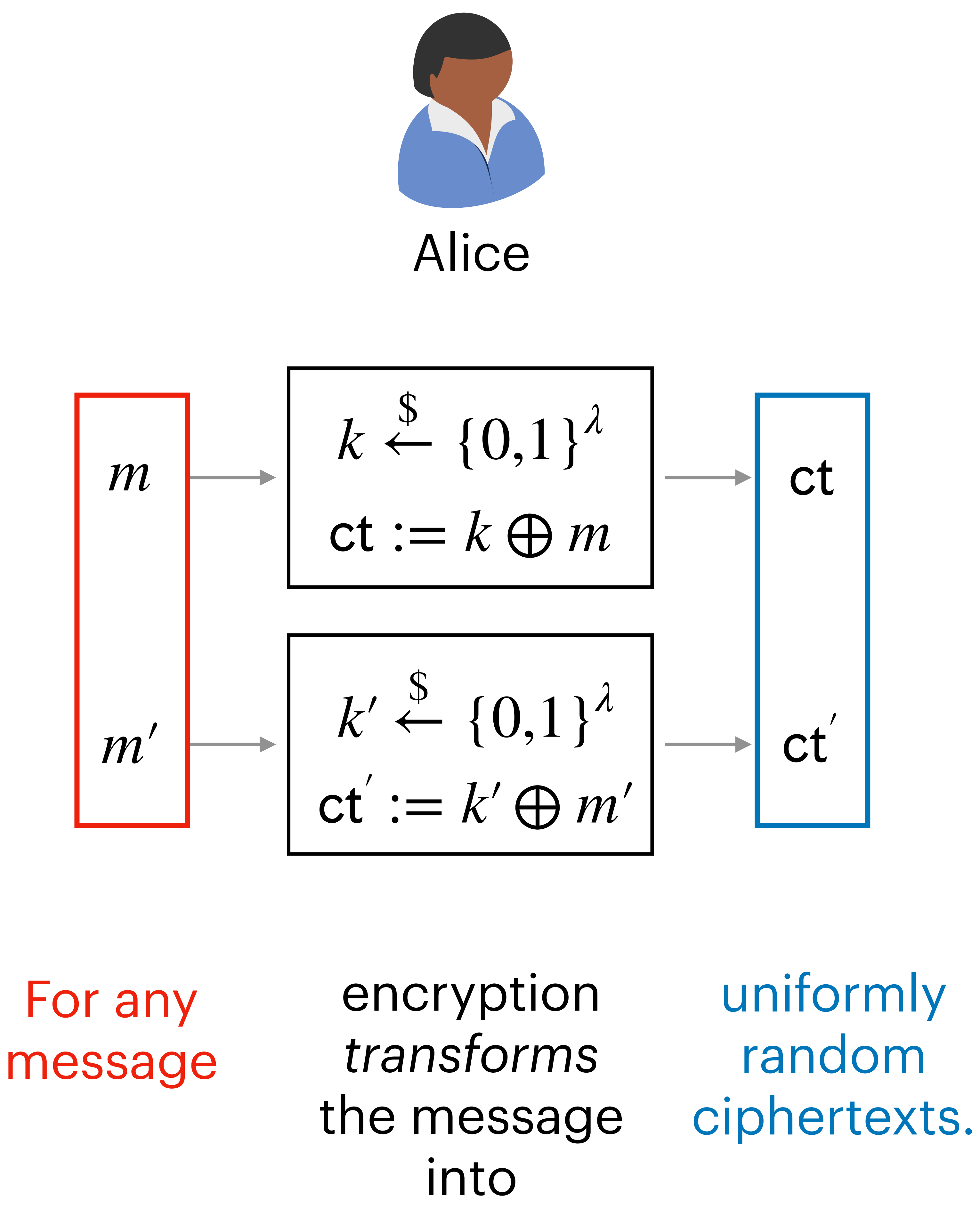

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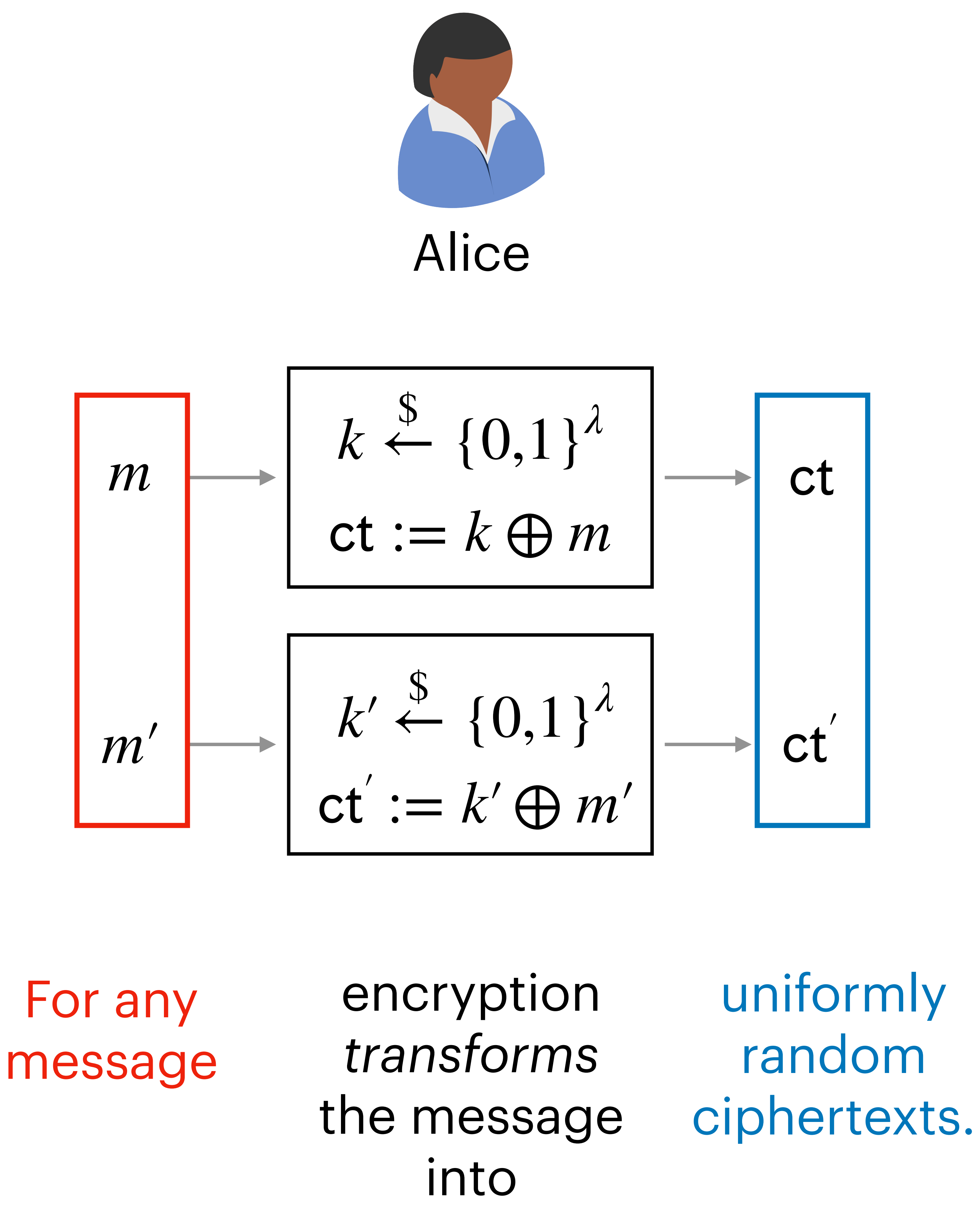

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  - Ones that should hold in the absence of an attacker e.g., correctness
  - Ones that specify what can happen to a system in the presence of an attacker e.g., security.
- **Eventual Goal:** Write formal definitions to capture all required properties from any given system.

Encryption: Correctness

<u>Encryption Scheme Syntax</u>

An encryption scheme consists of three (possibly probabilistic) algorithms:

- $\mathsf{KeyGen}() \to k$ outputs a key $k \in \mathscr{K}$.
- $\mathsf{Enc}(k, m) \to \mathsf{ct}$ takes key k and message $m \in \mathscr{M}$ and outputs ciphertext $\mathsf{ct} \in \mathscr{C}$.
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Encryption Scheme Correctness

An encryption scheme satisfies correctness if $\forall k \in \mathcal{K}$, $\forall m \in \mathcal{M}$, we have

$$\Pr[\mathsf{Dec}(k, \mathsf{Enc}(k, m)) = m] = 1,$$

where the probability is over the randomness used in encryption and decryption.

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Identical distributions

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From Eve's view, the ciphertext carries no information about the plaintext.

Comparing Both Security Notions

Claim: If an encryption scheme is one-time uniform ciphertext secure, then it is also perfectly secure.

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We want to show that $\forall m_0, m_1 \in \mathcal{M}$, $$D_0' = \left\{ \mathsf{ct} : \begin{array}{c} k \leftarrow \mathsf{KeyGen}() \\ \mathsf{ct} \leftarrow \mathsf{Enc}(k, m_0) \end{array} \right\} \equiv D_1' = \left\{ \mathsf{ct} : \begin{array}{c} k \leftarrow \mathsf{KeyGen}() \\ \mathsf{ct} \leftarrow \mathsf{Enc}(k, m_1) \end{array} \right\}.$$

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We will consider the following sequence of distributions, called hybrids.

$$H_0 = \left\{ \mathsf{ct} : \begin{array}{c} k \leftarrow \mathsf{KeyGen}() \\ \mathsf{ct} \leftarrow \mathsf{Enc}(k, m_0) \end{array} \right\}$$

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Our goal is to show that $H_0 \equiv H_2$. We will do this in two steps using the "intermediate" hybrid H_1.

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$$H_0 = \left\{ \text{ct} : \begin{array}{c} k \leftarrow \text{KeyGen}() \\ \text{ct} \leftarrow \text{Enc}(k, m_0) \end{array} \right\}$$

$H_0 \equiv H_1$ because of one-time uniform ciphertext security.

$$H_1 = \left\{ \text{ct} : \text{ct} \xleftarrow{\$} \mathscr{C} \right\}$$

$$H_2 = \left\{ \text{ct} : \begin{array}{c} k \leftarrow \text{KeyGen}() \\ \text{ct} \leftarrow \text{Enc}(k, m_1) \end{array} \right\}$$

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Claim: If an encryption scheme is one-time uniform ciphertext secure, then it is also perfectly secure.

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$H_0 \equiv H_1$ because of one-time uniform ciphertext security.

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$H_0 \equiv H_1$ because of one-time uniform ciphertext security.

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By transitivity, $H_0 \equiv H_2$.

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Proof:

We are given that $\forall m \in \mathcal{M}$, $\quad D_0 = \left\{ \mathsf{ct} : \begin{array}{c} k \leftarrow \mathsf{KeyGen}() \\ \mathsf{ct} \leftarrow \mathsf{Enc}(k, m) \end{array} \right\} \equiv D_1 = \left\{ \mathsf{ct} : \mathsf{ct} \xleftarrow{\$} \mathcal{C} \right\}$.

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$H_0 \equiv H_1$ because of one-time uniform ciphertext security.

$H_1 \equiv H_2$ because of one-time uniform ciphertext security.

By transitivity, $H_0 \equiv H_2$.

The hybrid technique is very common in cryptographic proofs.
We will use it repeatedly throughout the course.

Comparing Both Security Notions

Claim: If an encryption scheme is one-time uniform ciphertext secure, then it is also perfectly secure.

Corollary: One-time pad is perfectly secure.