Authentication I

601.442/642 Modern Cryptography

10th March 2026

Logistics

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  - Search: given $y = f(x)$, find x
  - Decision: given $y = f(x)$, *predict* $\mathsf{hc}(x)$

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$$\Pr\left[f(x') = y \;:\; \begin{array}{r} x \xleftarrow{\$} \{0,1\}^\lambda \\ y := f(x) \\ x' \xleftarrow{\$} \mathscr{A}(1^\lambda, y) \end{array} \right] \leq \mathsf{negl}(\lambda)$$

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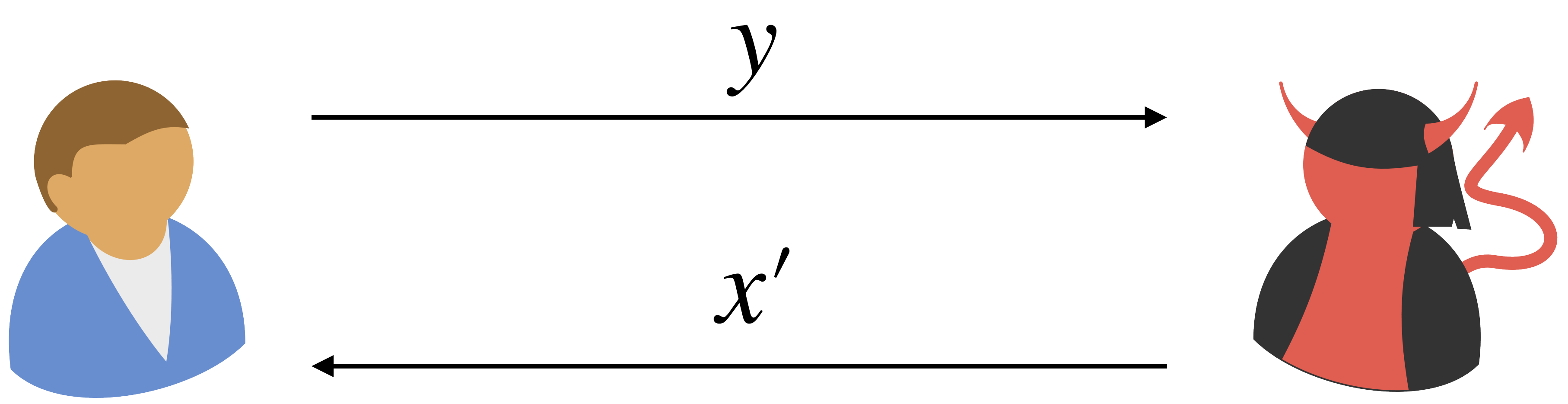
$$\Pr\left[\mathscr{A} \text{ wins OWFGame}\right] \leq \mathsf{negl}(\lambda)$$

$$x \xleftarrow{\$} \{0,1\}^\lambda$$
$$y := f(x)$$



Wins if $f(x') = y$

Authentication

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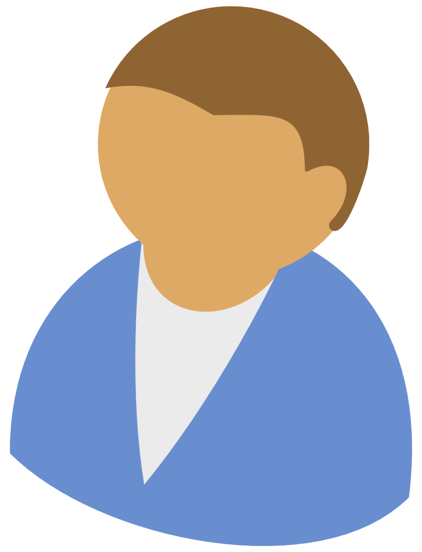

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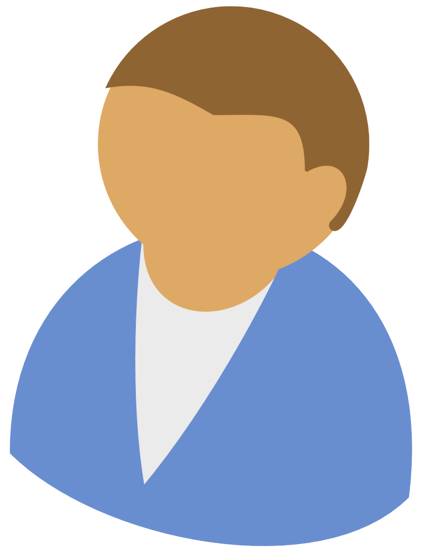

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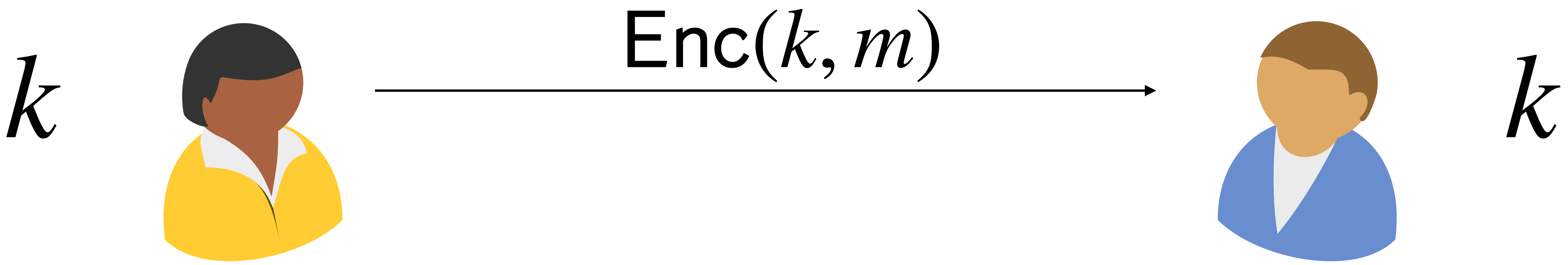

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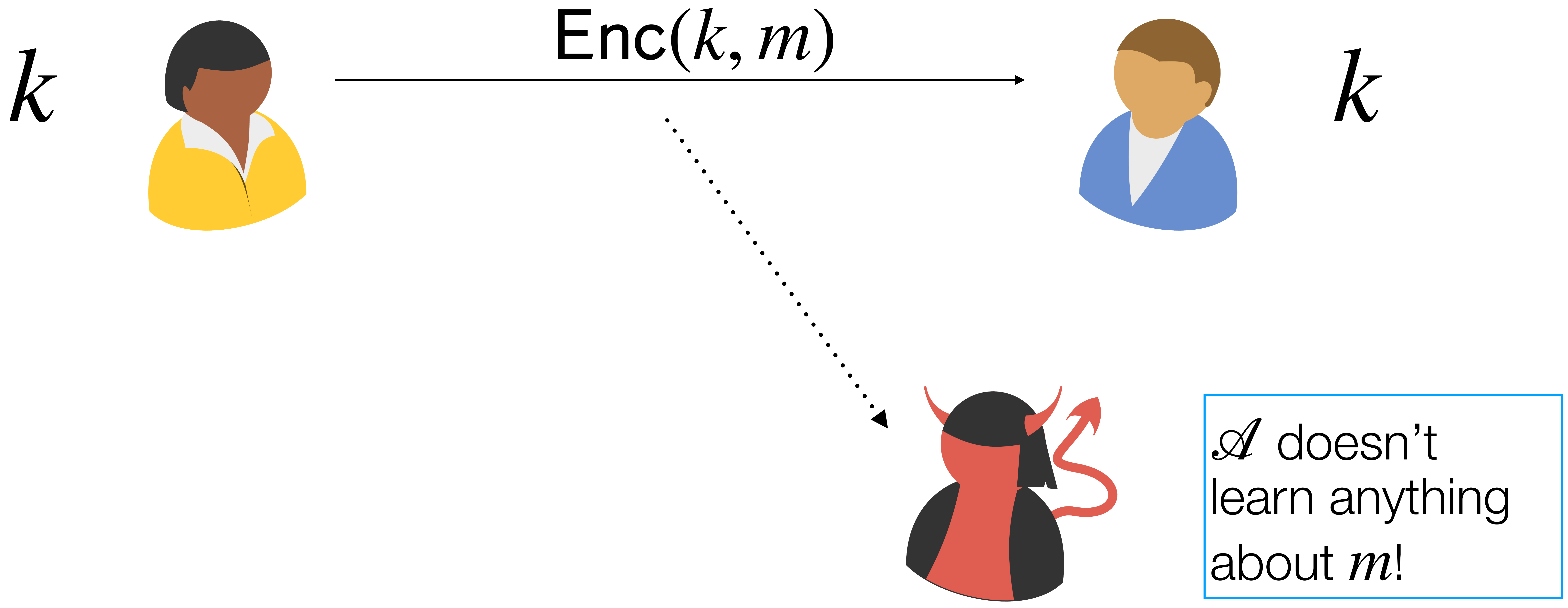

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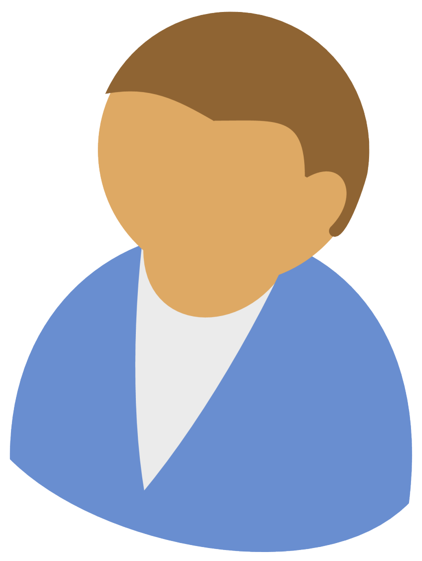 k

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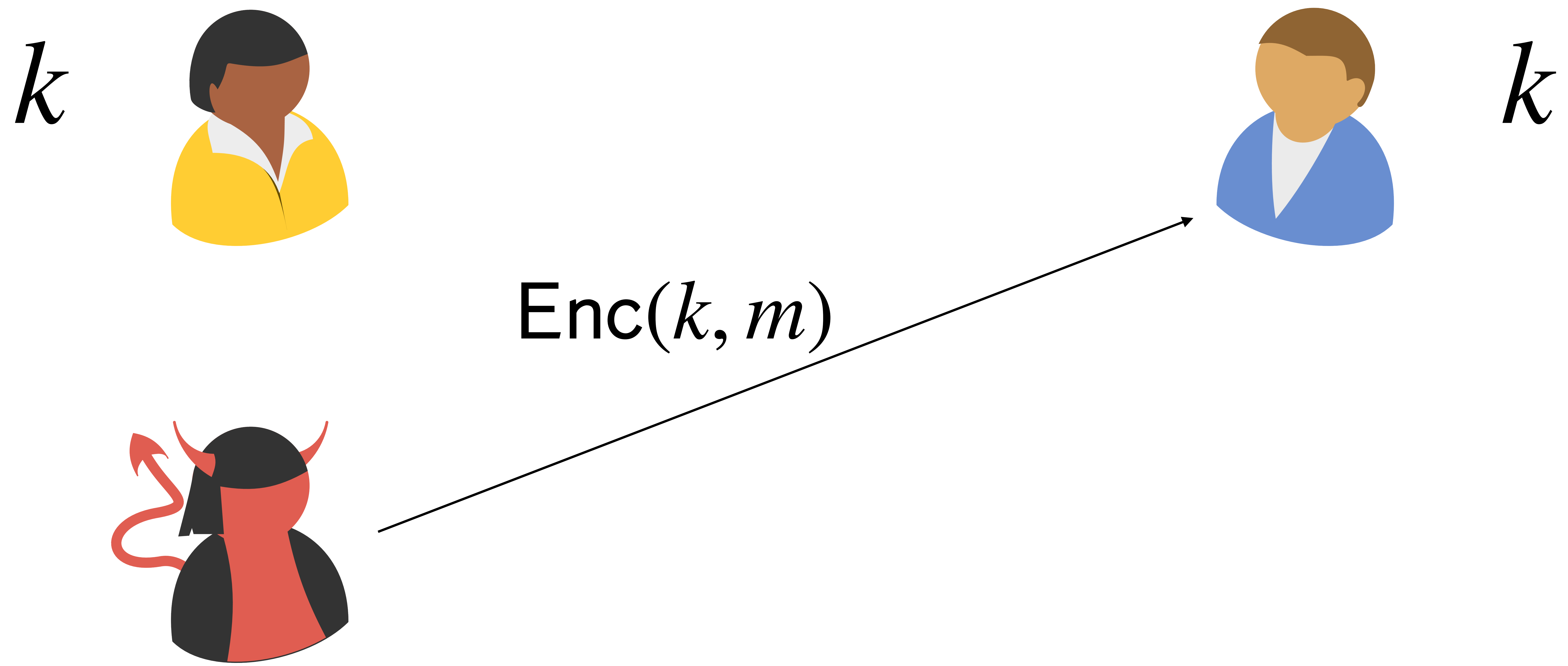

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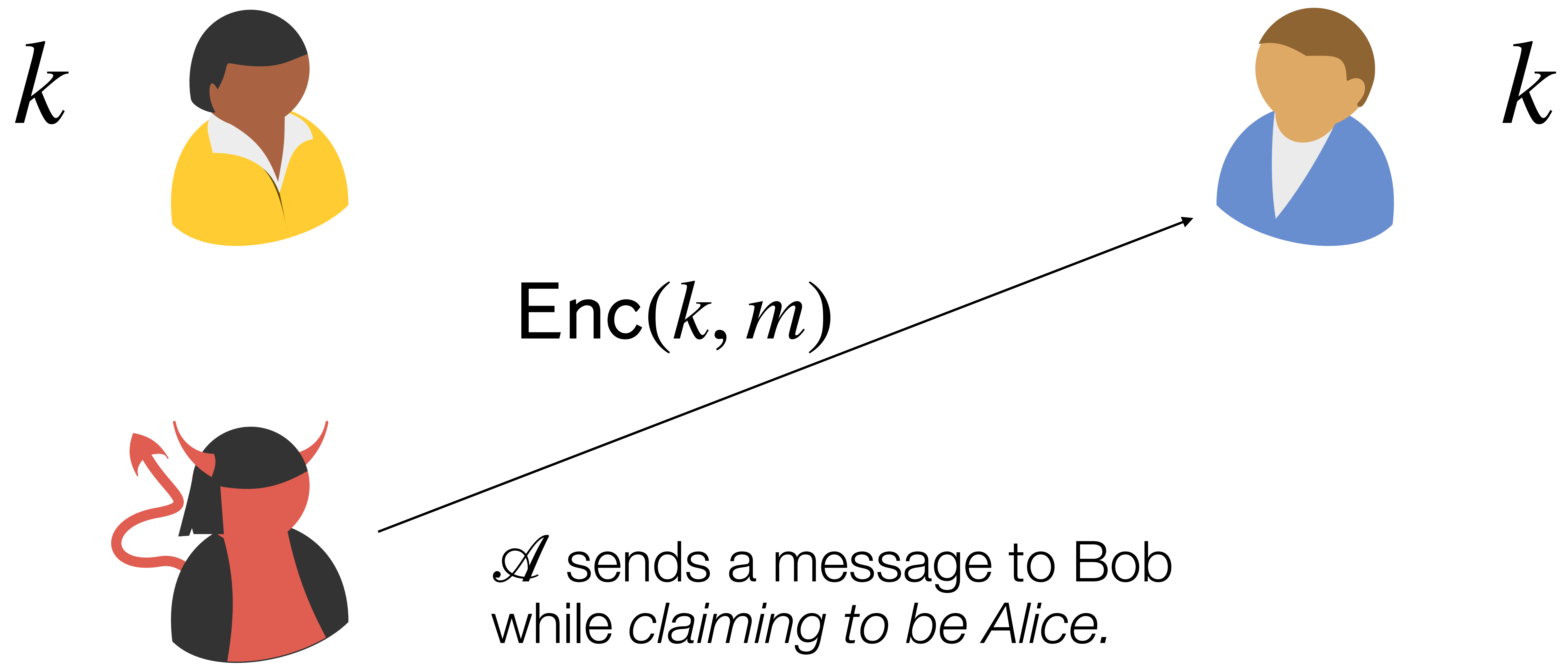

$\mathscr{A}$ sends a message to Bob
while *claiming to be Alice.*

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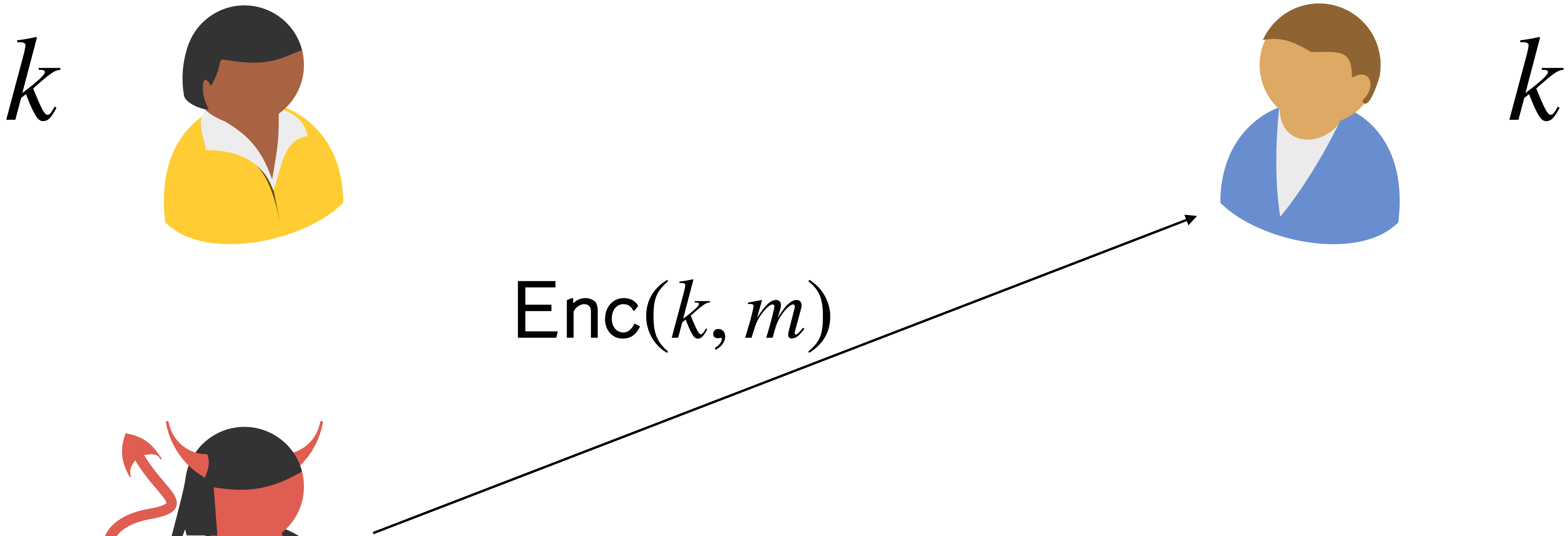

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How can Bob tell that a message *really did* come from Alice?

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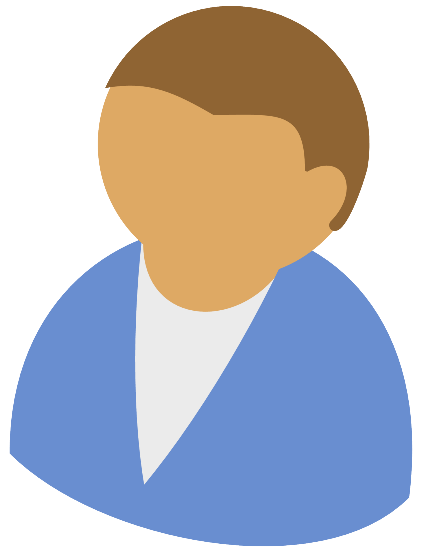

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$\mathsf{Enc}(k, m)$

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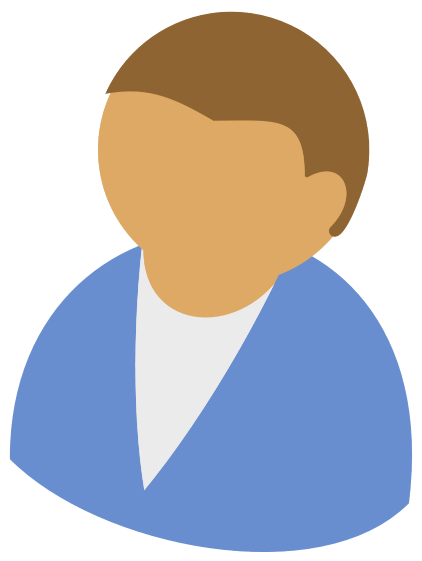

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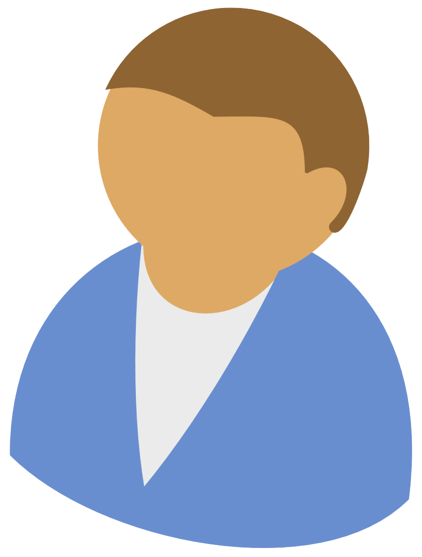

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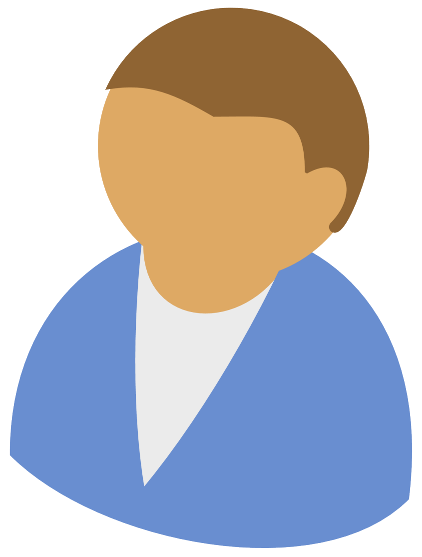

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How can Bob tell that a message *really did* come from Alice?

Goal: something like a *signature*

- Alice can "sign" a message m to produce a signature σ
- Bob can *verify* that σ is correct for m
- $\mathscr{A}$ cannot *forge* a signature

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- Public Key: Digital Signatures

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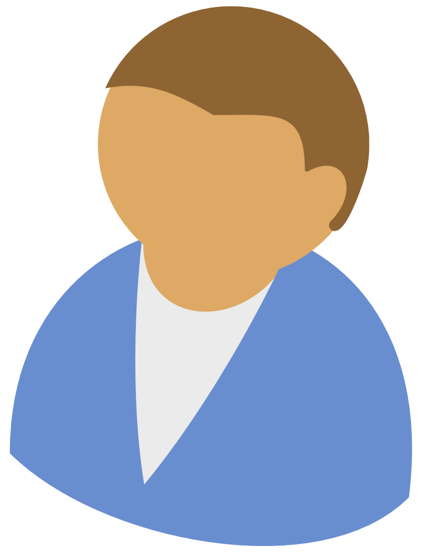

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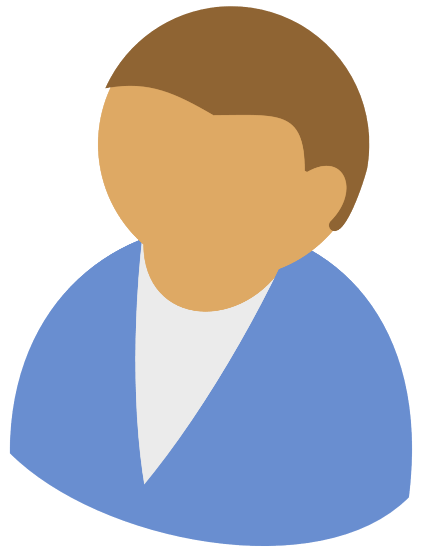

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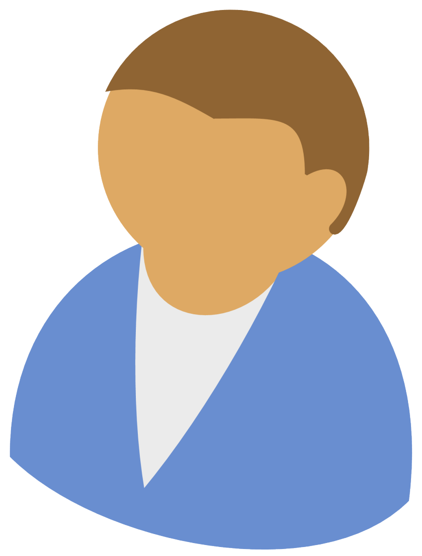
$\sigma \leftarrow \mathsf{Tag}(k, m)$



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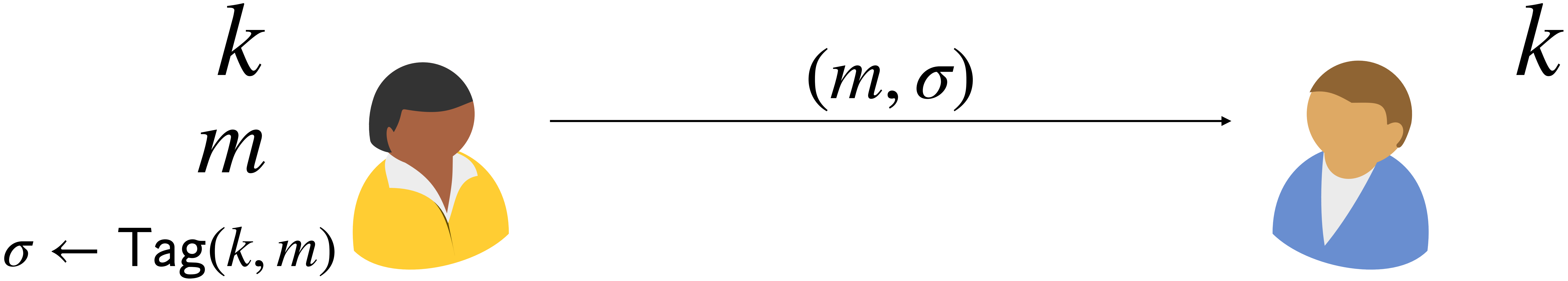

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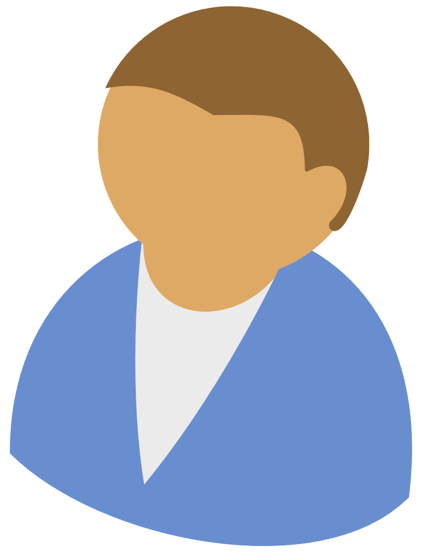

k

If $\mathsf{Ver}(k, m, \sigma) = 1$, accept

else, reject

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Goal: $\mathscr{A}$ can't produce a (m', σ') pair that verifies: even after seeing a bunch of valid signatures (m, σ)

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Correctness: $$\Pr\left[\mathsf{Ver}(k, m, \sigma) = 1 \;:\; \begin{matrix} k \leftarrow \mathsf{KeyGen}(1^\lambda) \\ \sigma \leftarrow \mathsf{Tag}(k, m) \end{matrix}\right] = 1$$

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A MAC scheme $(\mathsf{KeyGen}, \mathsf{Tag}, \mathsf{Ver})$ satisfies *unforgeability under chosen message attack* (UF-CMA) if for all NUPPT $\mathscr{A}$, there exists a negligible function $\mathsf{negl}(\cdot)$. such that $\forall \lambda \in \mathbb{N}$:

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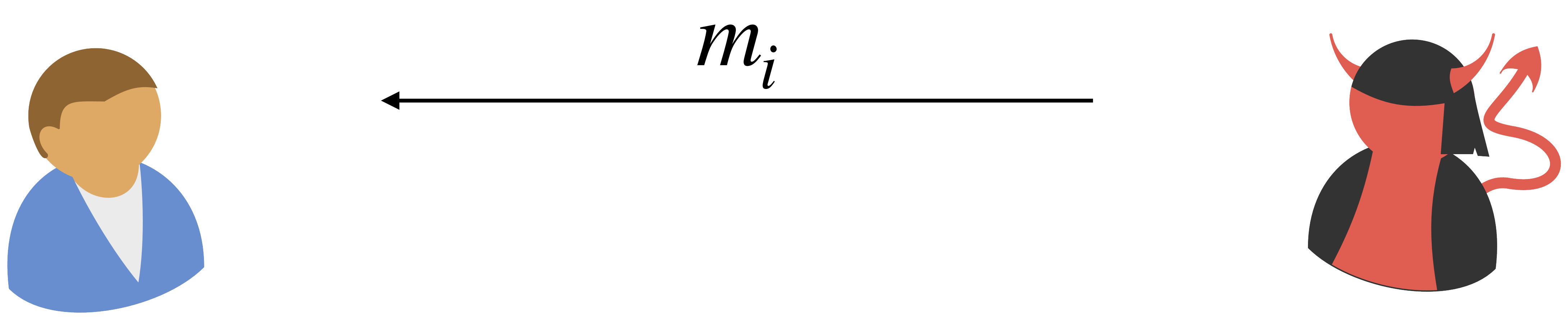

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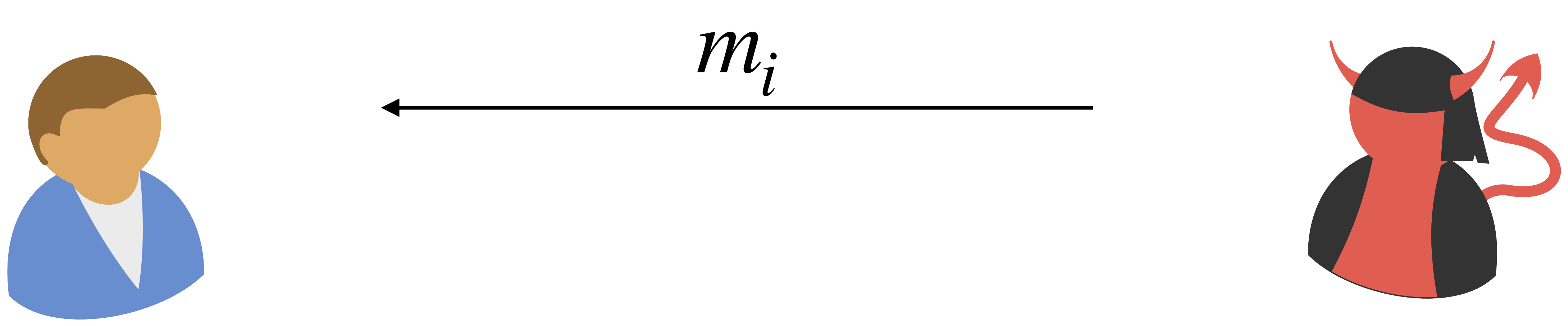

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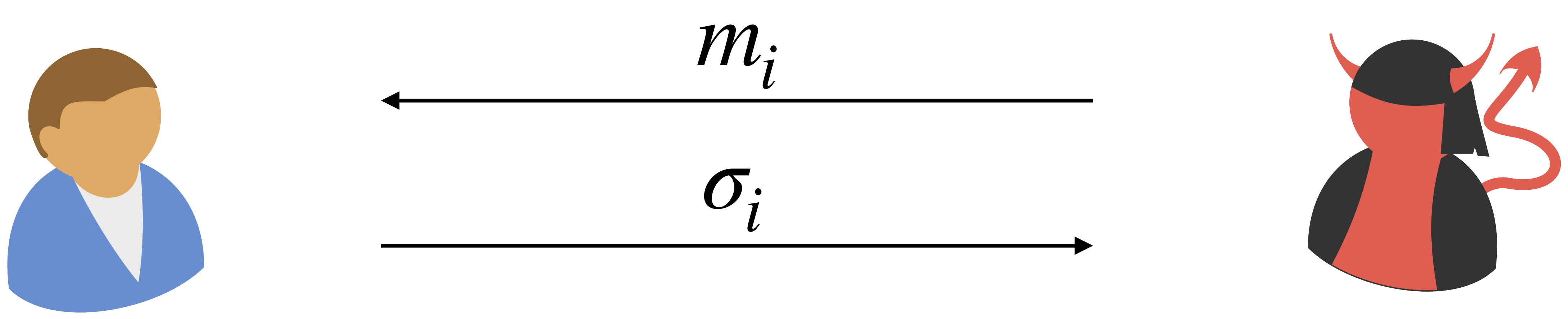

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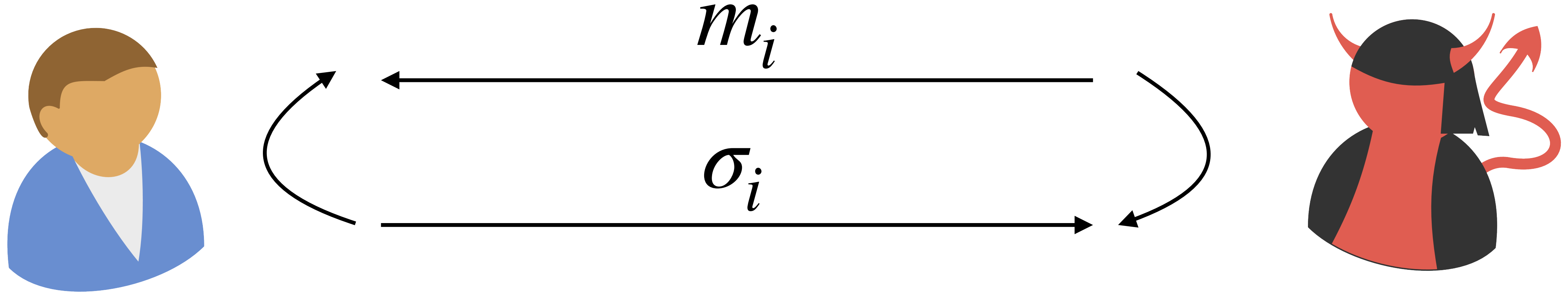

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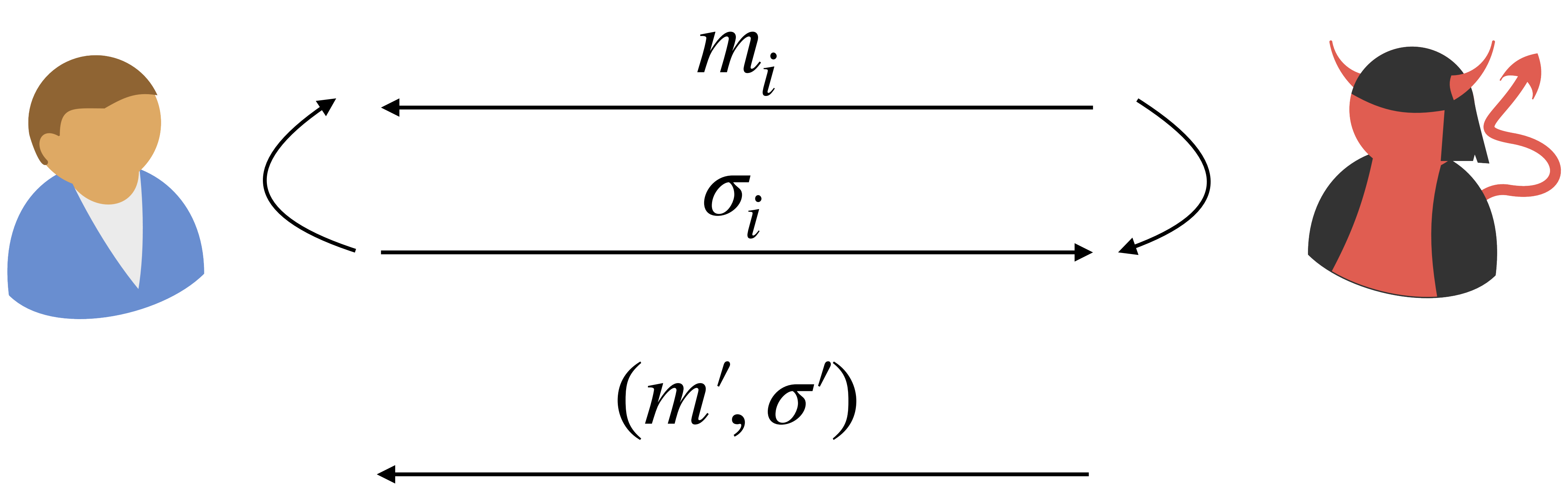

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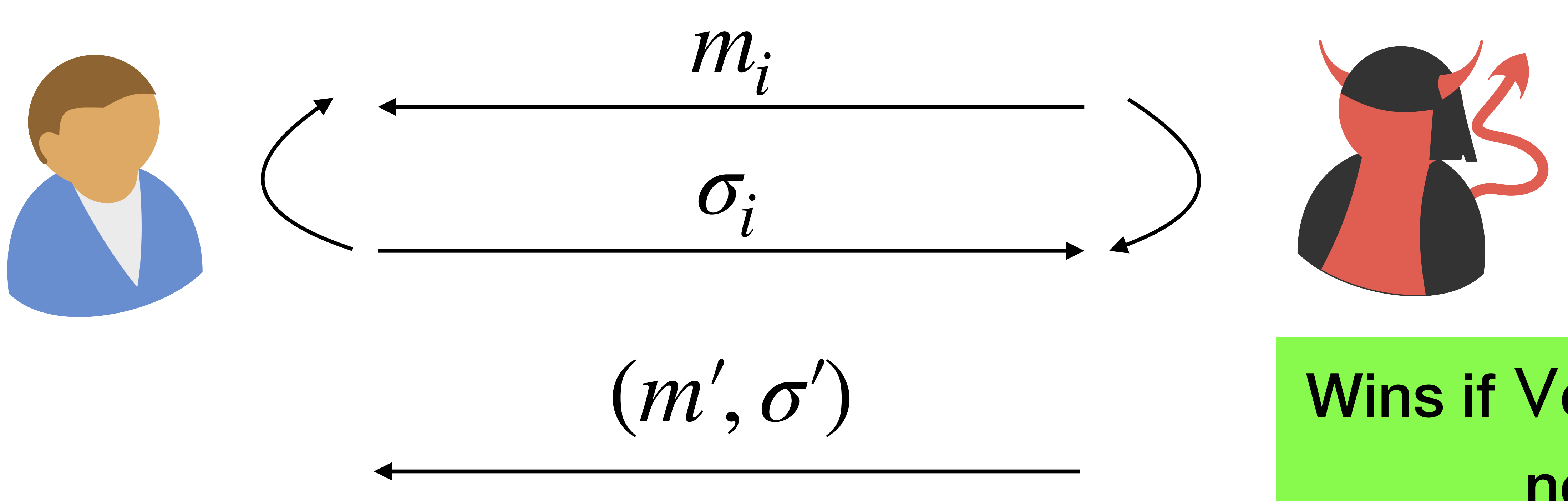

Wins if $\text{Ver}(k, m', \sigma') = 1$ **and** $\mathscr{A}$ **never queried** m'

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This is a *search* problem!

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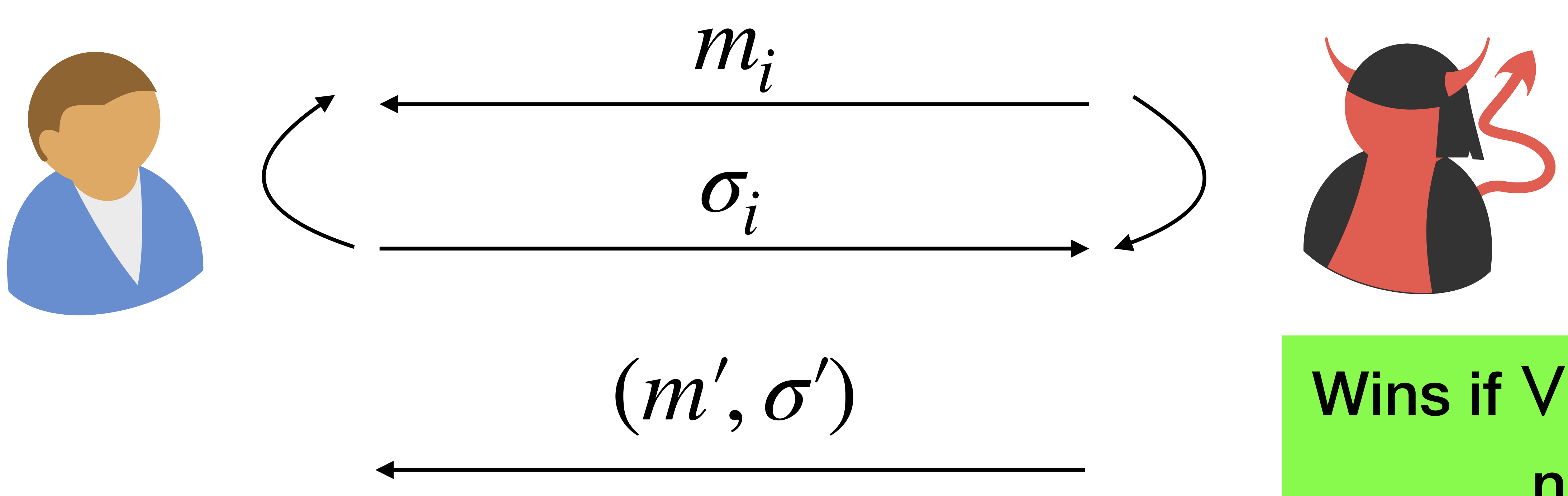

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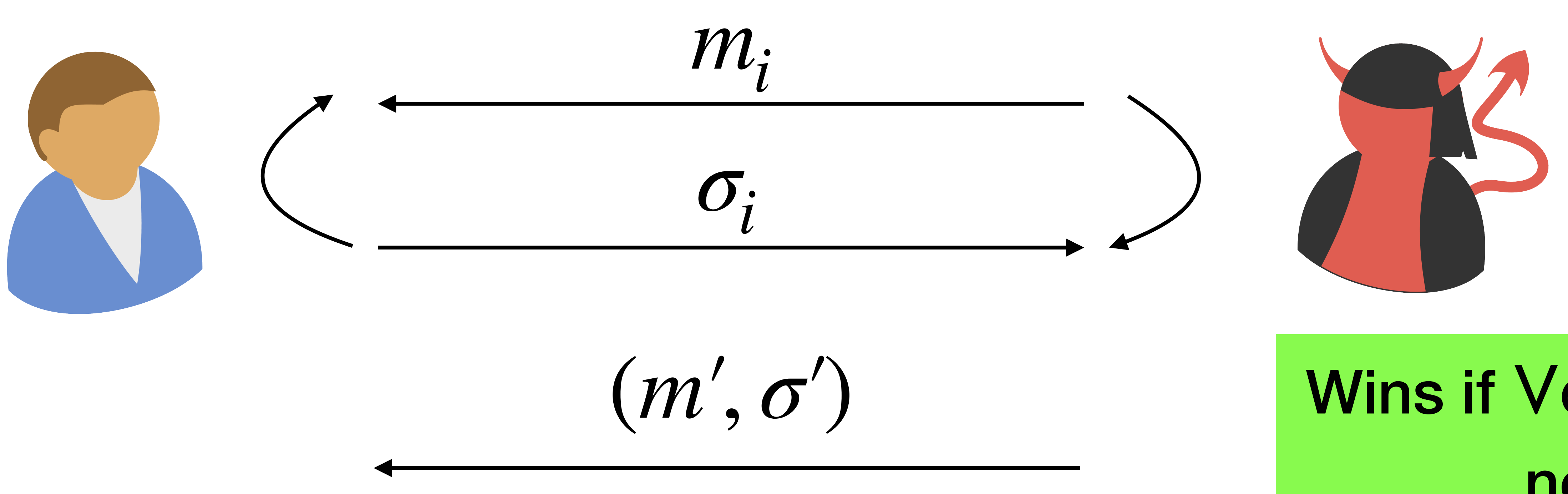

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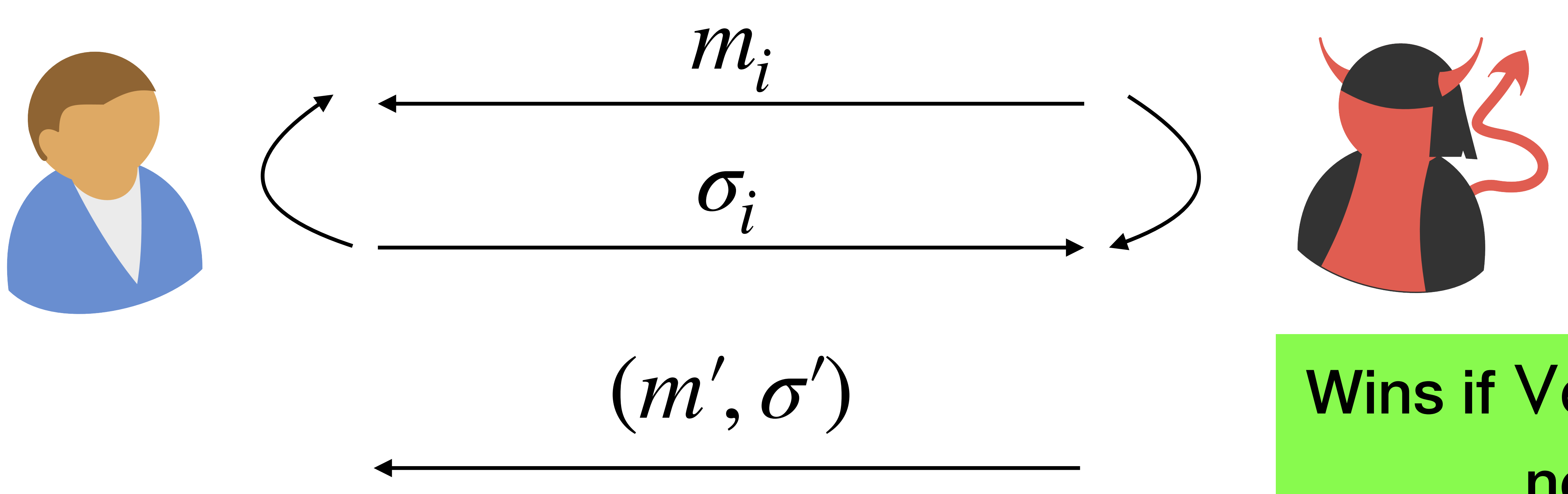

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Let λ be the security parameter and, Let $\{F_k\}_{k \in \{0,1\}^\lambda}$ be a secure family of PRFs, where $F_k : \{0,1\}^\lambda \rightarrow \{0,1\}^\lambda$.

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Intuition: to forge a signature $\mathscr{A}$ would need to *predict* the output of a PRF. This should be impossible!

Recap: PRF Security

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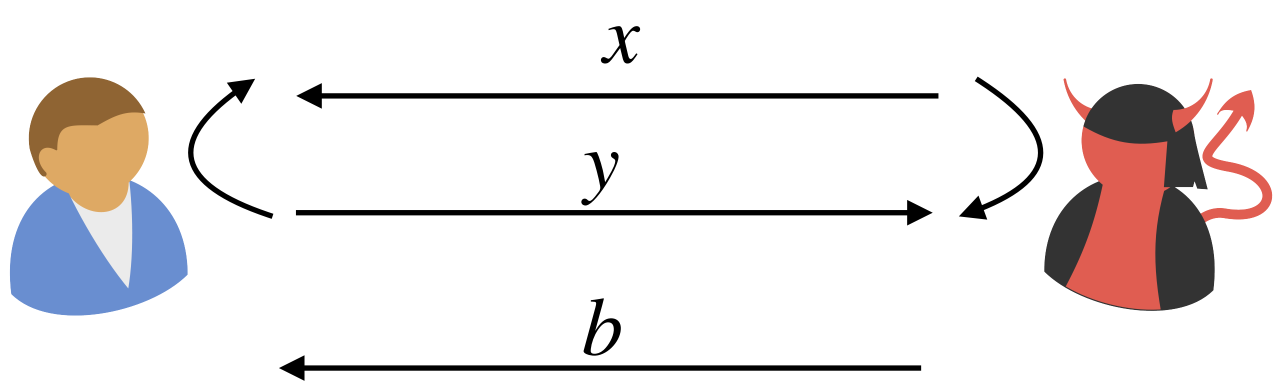
Game$_0$

$k \xleftarrow{\$} \{0,1\}^\lambda \qquad y = F_k(x)$



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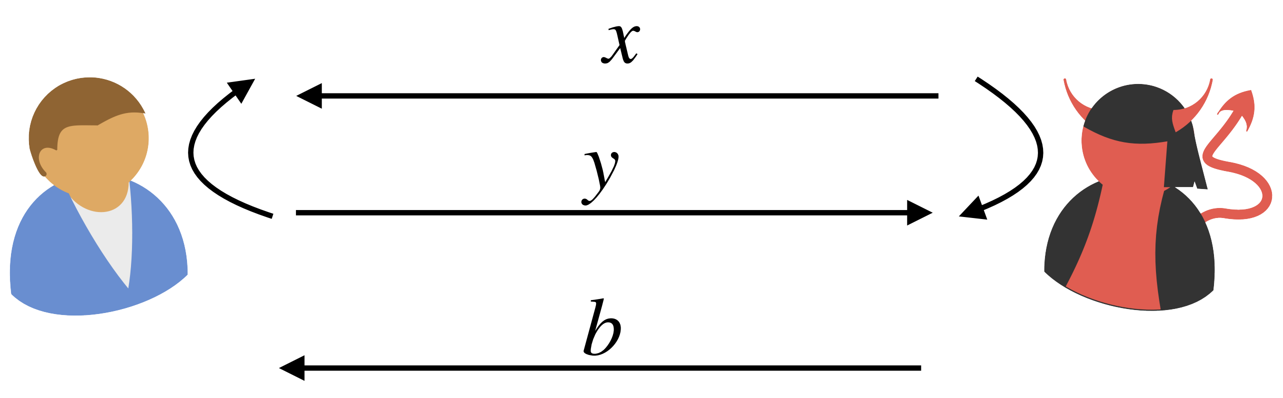

Game_1

$T := \{\}$

if $x \notin T$

$r \xleftarrow{\$} \{0,1\}^\lambda$

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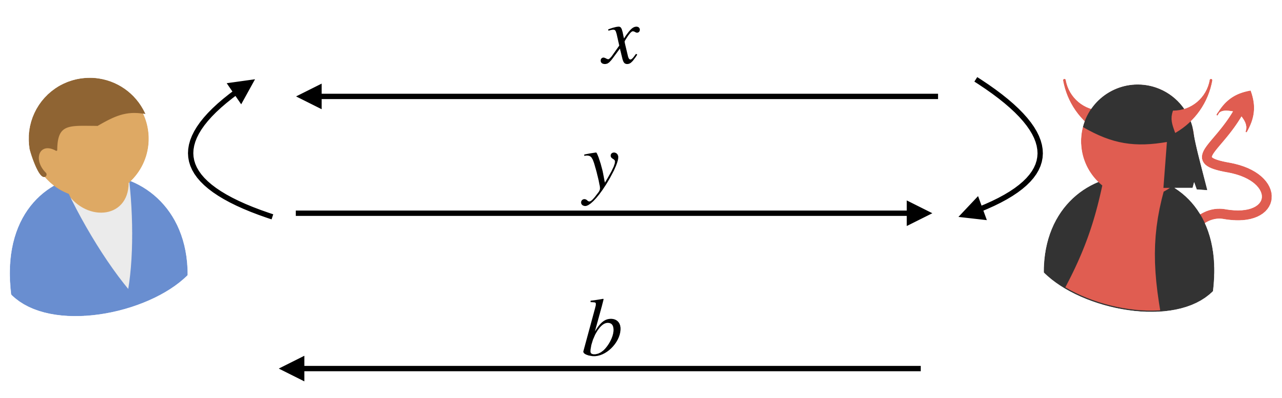

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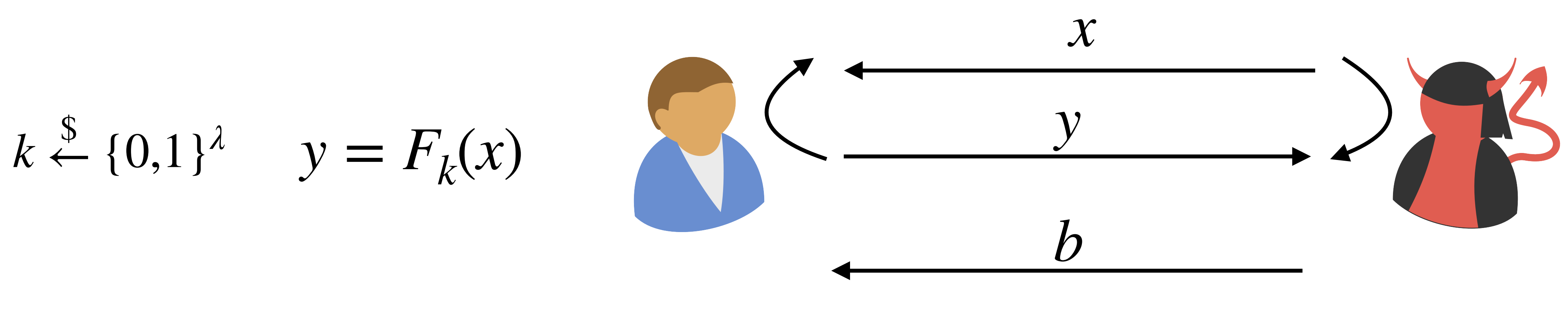

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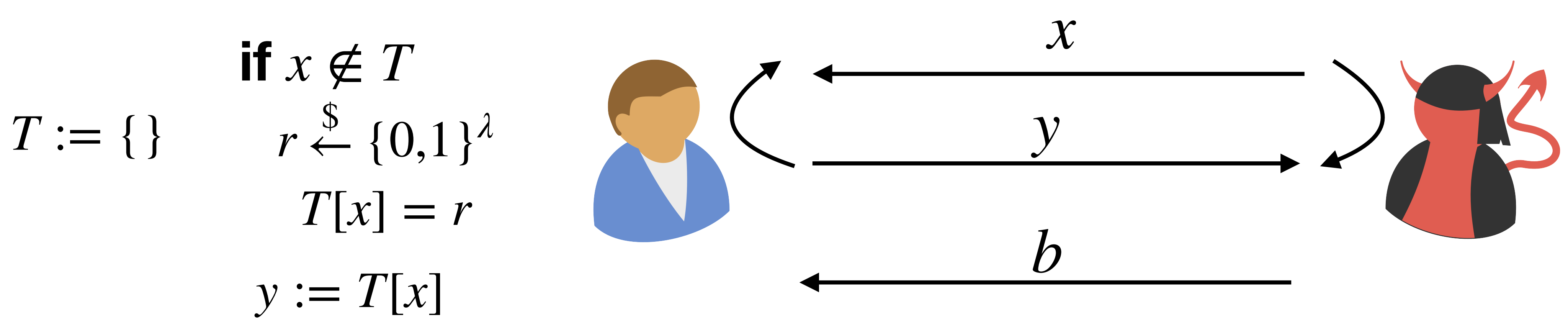

Let W_b be the event that $\mathscr{A}$ outputs 0 in Game$_b$

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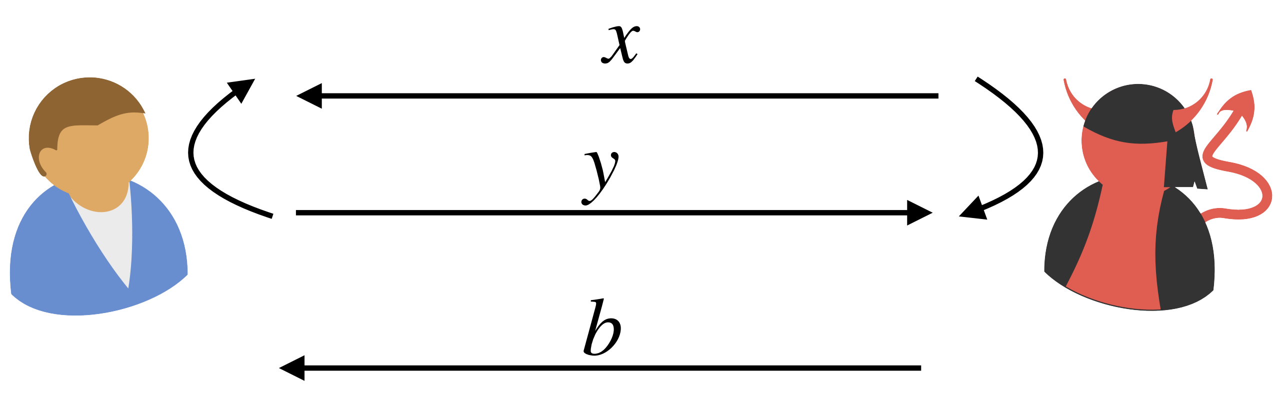

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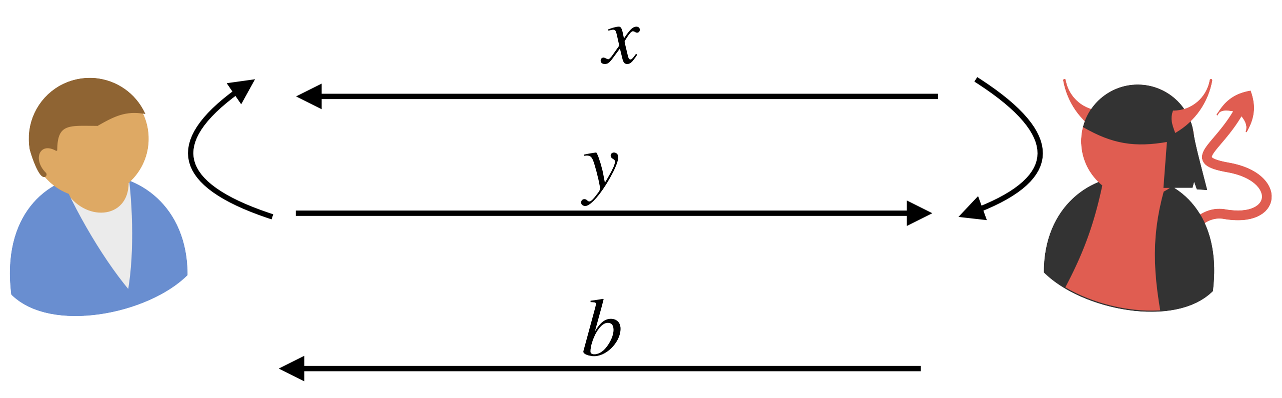
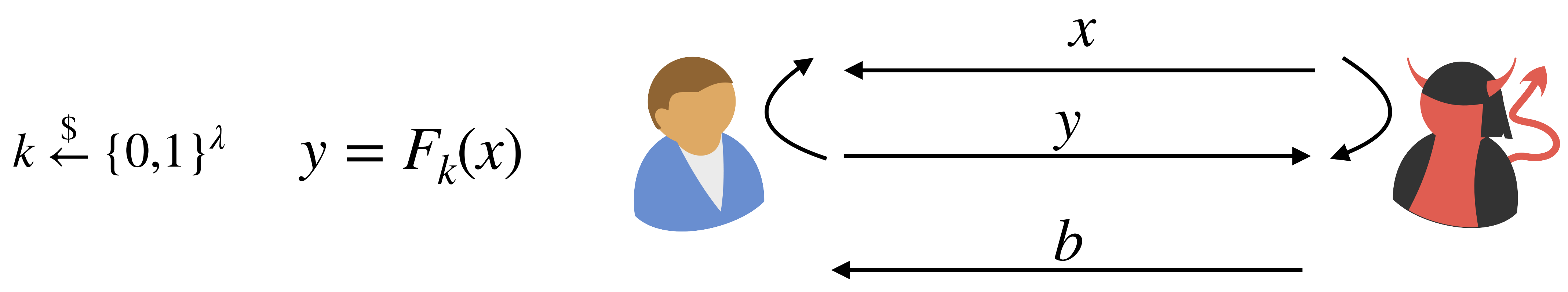


Let W_b be the event that $\mathscr{A}$ outputs 0 in Game_b

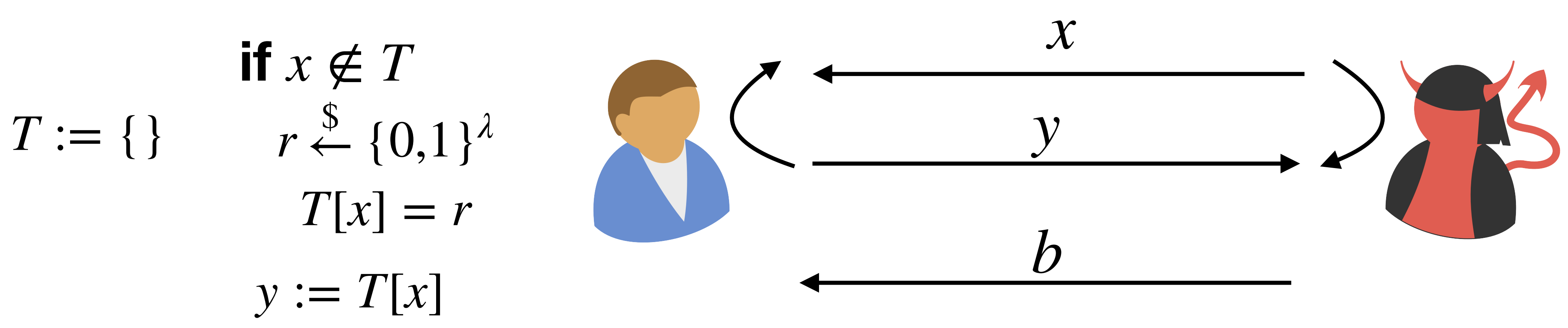
$$\left|\Pr[W_0] - \Pr[W_1]\right| \leq \text{negl}(\lambda)$$

Recap: PRF Security

Game_0



Game_1



Let W_b be the event that $\mathscr{A}$ outputs 0 in Game_b

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This is a *decision* problem!
Need to be careful in the proof as we relay on the *decisional* security to prove that a *search* problem is hard

Proof of Security

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

$\mathsf{Tag}(k, m) : \sigma := F_k(m)$

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H_0

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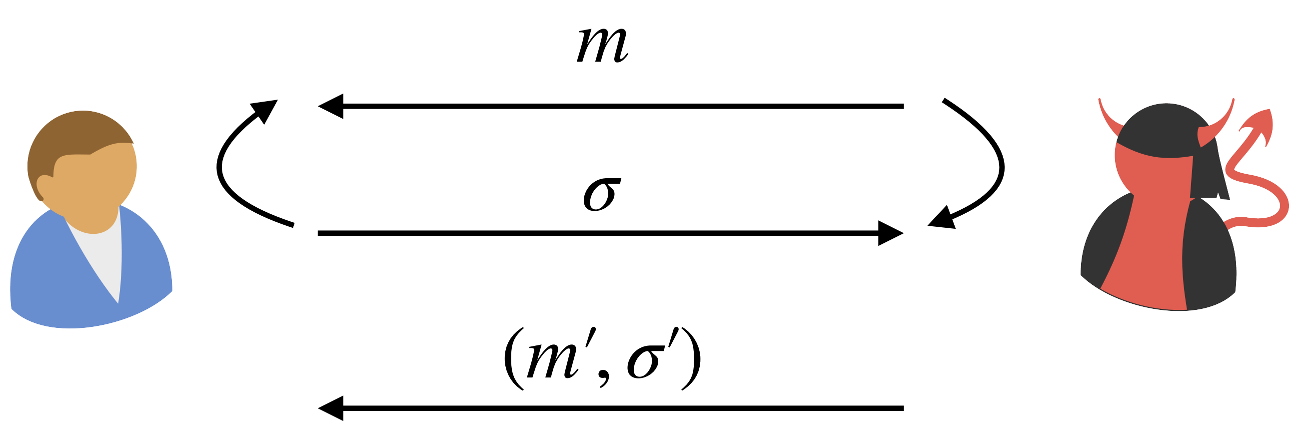

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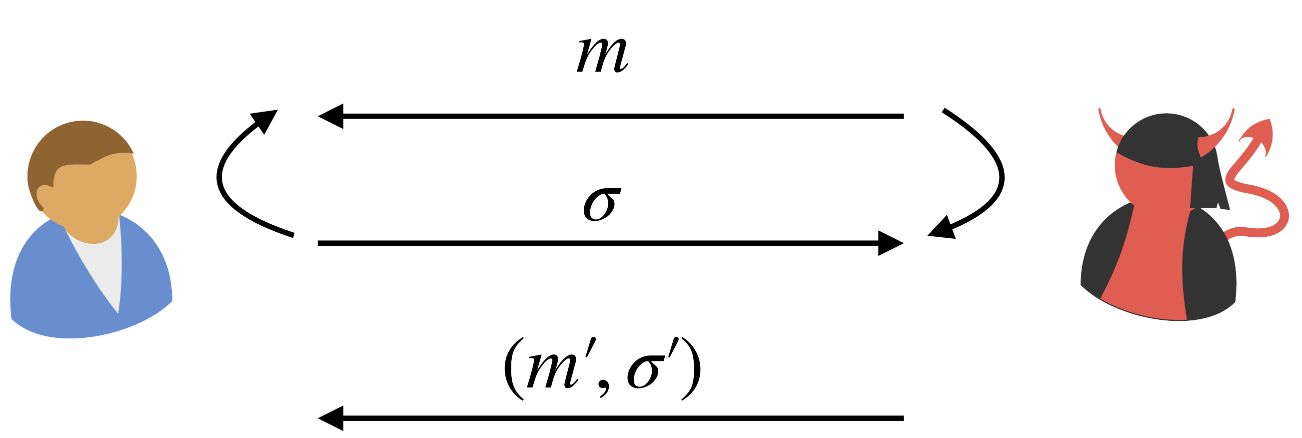

Claim: If F is a secure family of PRFs, then PRF-MAC is a secure MAC

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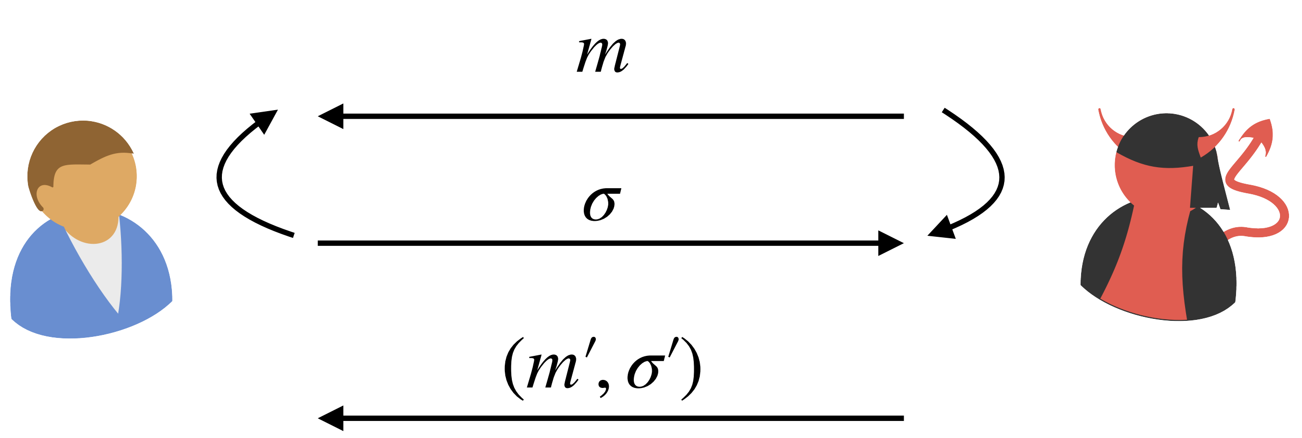

Claim: If F is a secure family of PRFs, then **Pr**$[\mathscr{A}$ wins in $H_0] \leq$ **negl**(λ)

Proof of Security

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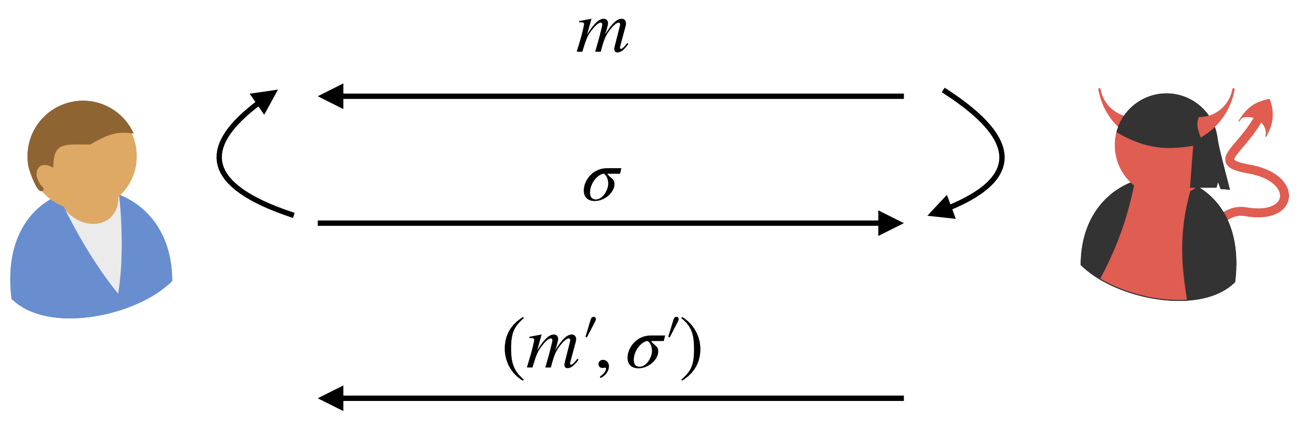

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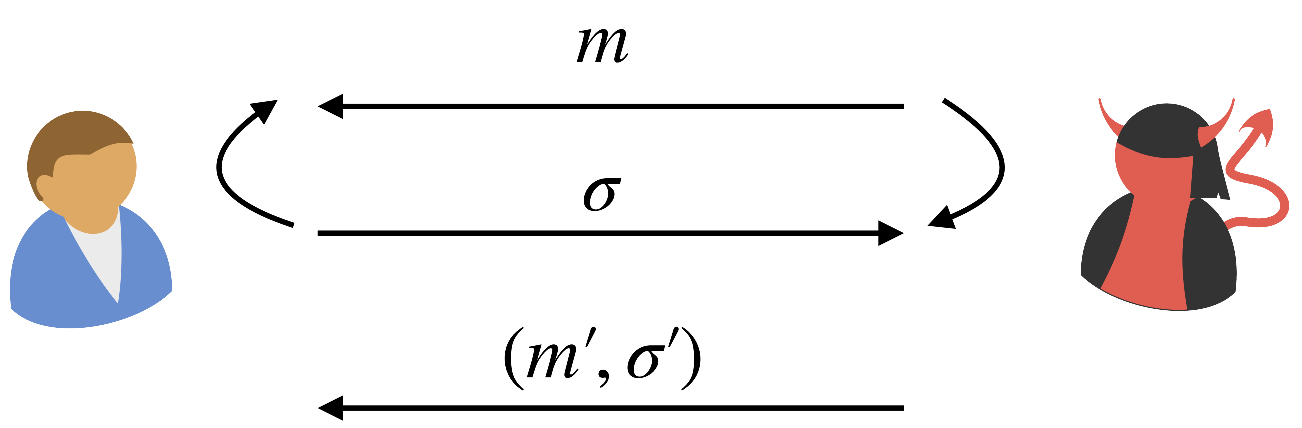

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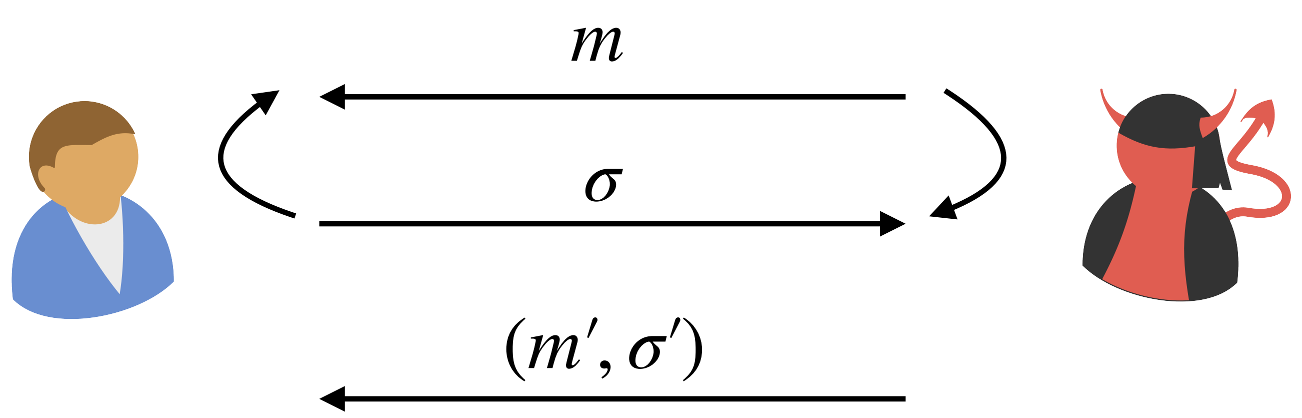

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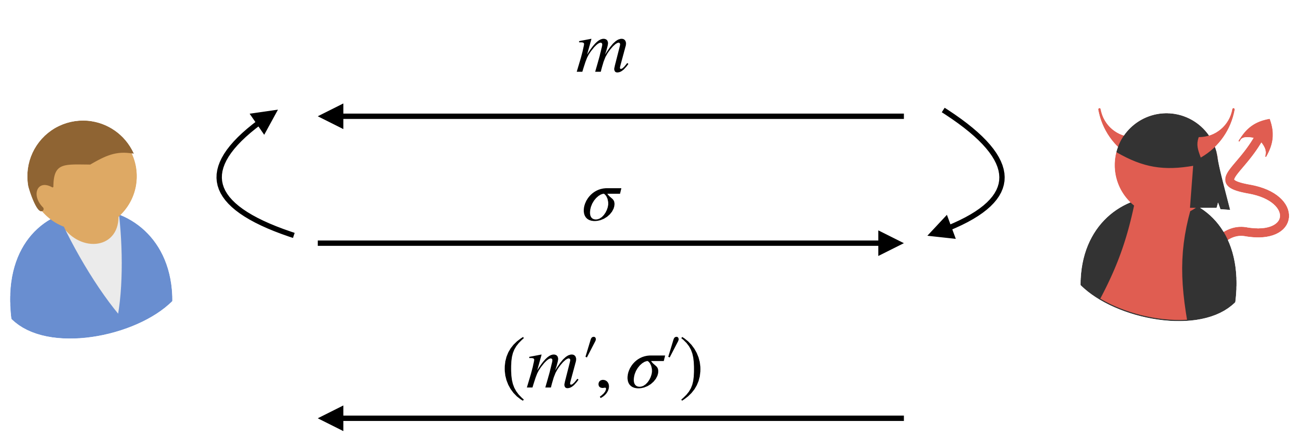

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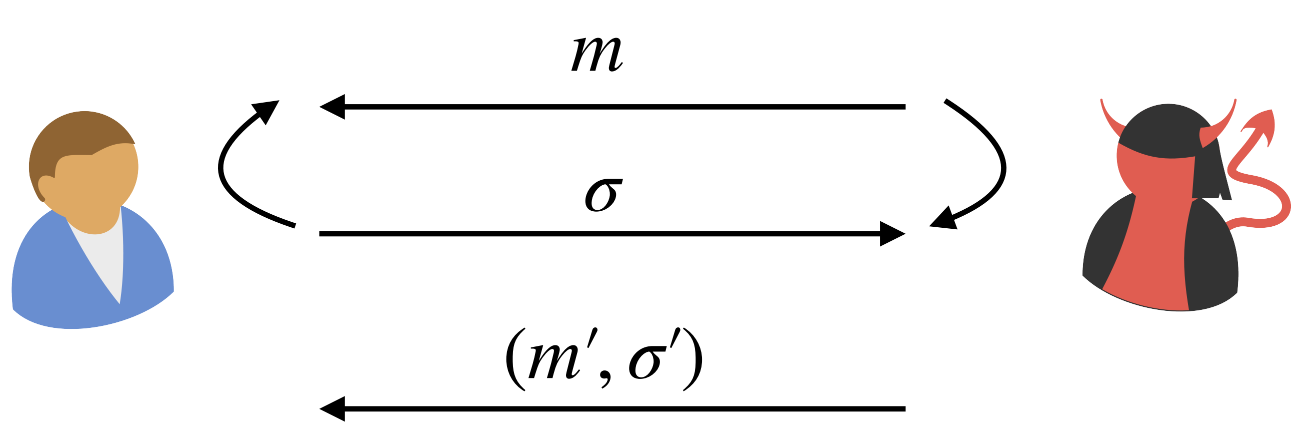

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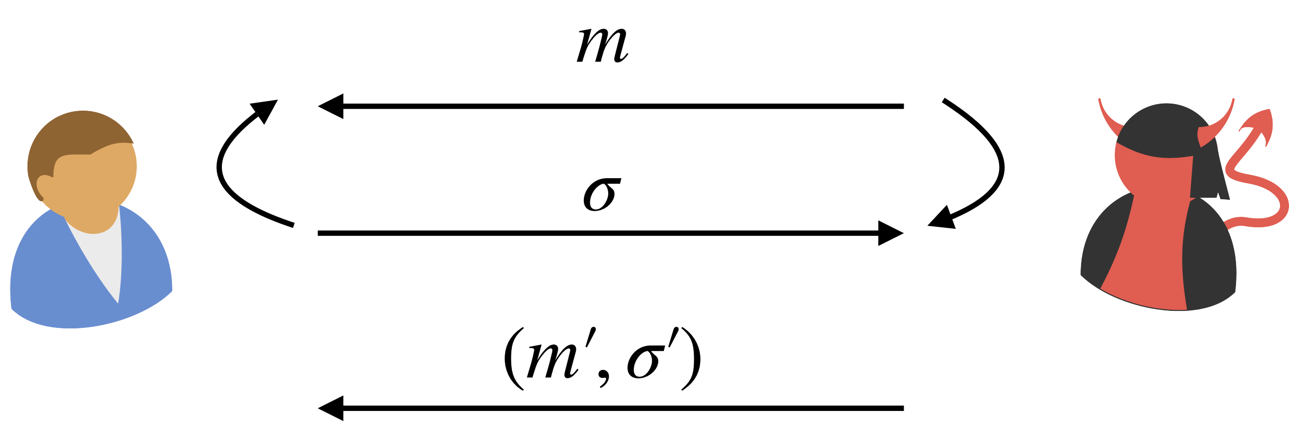

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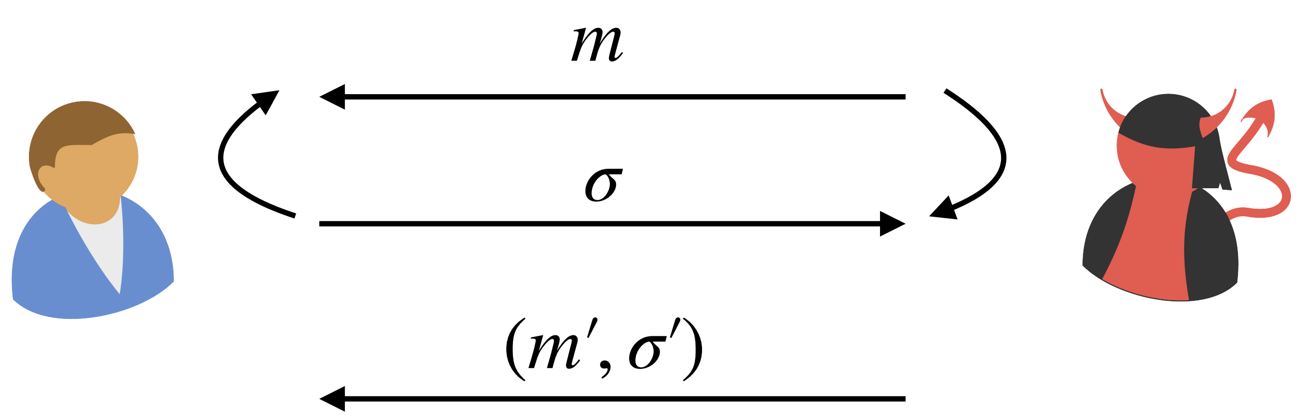

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Why? PRF Security!

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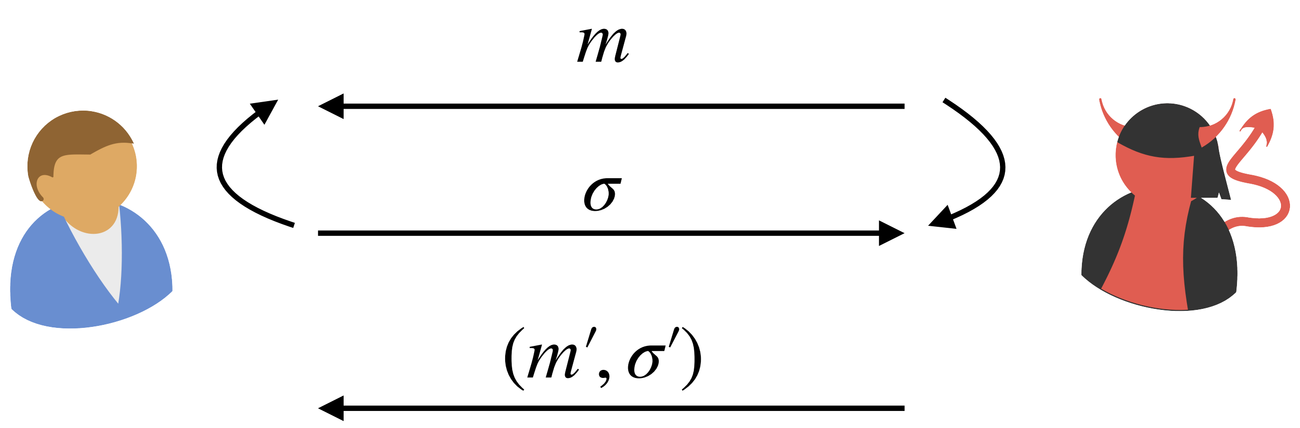

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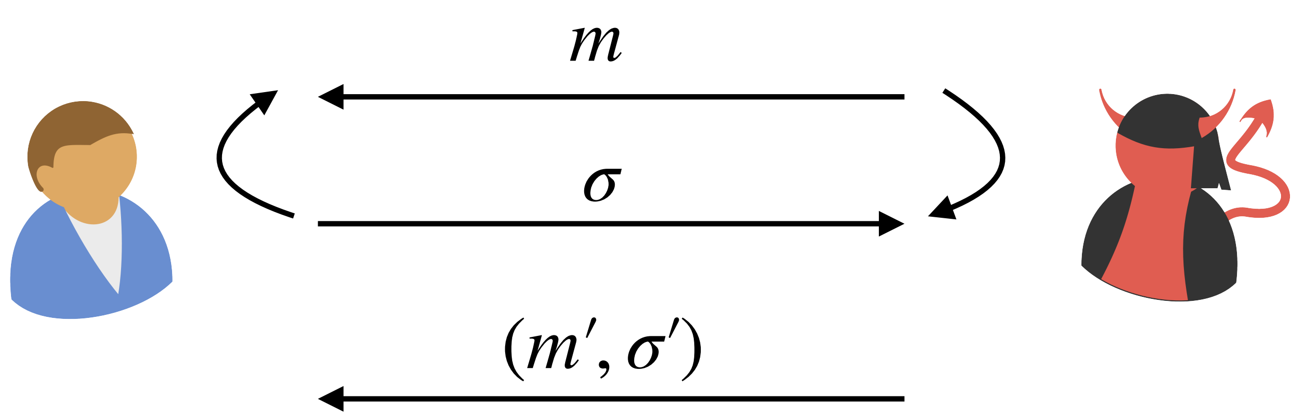

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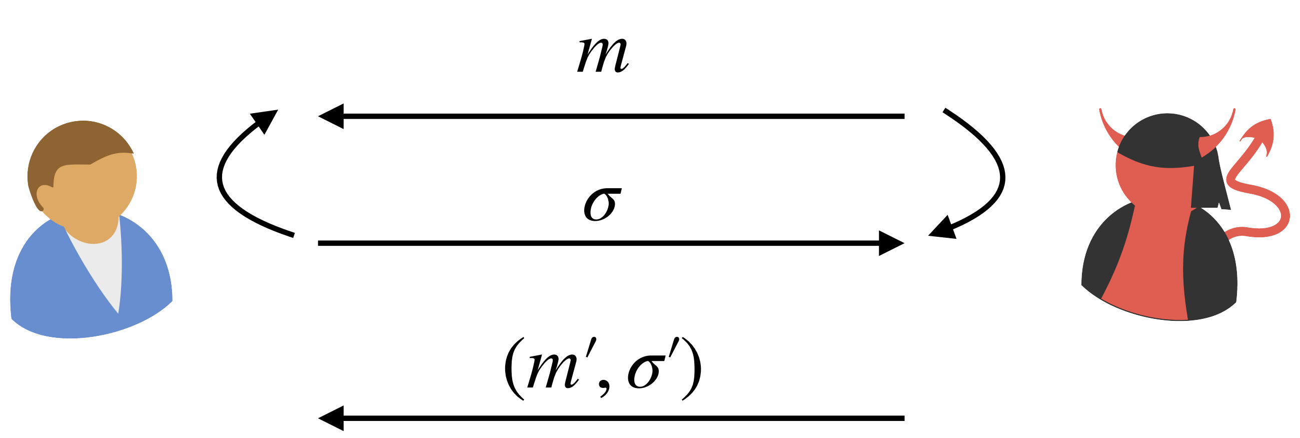

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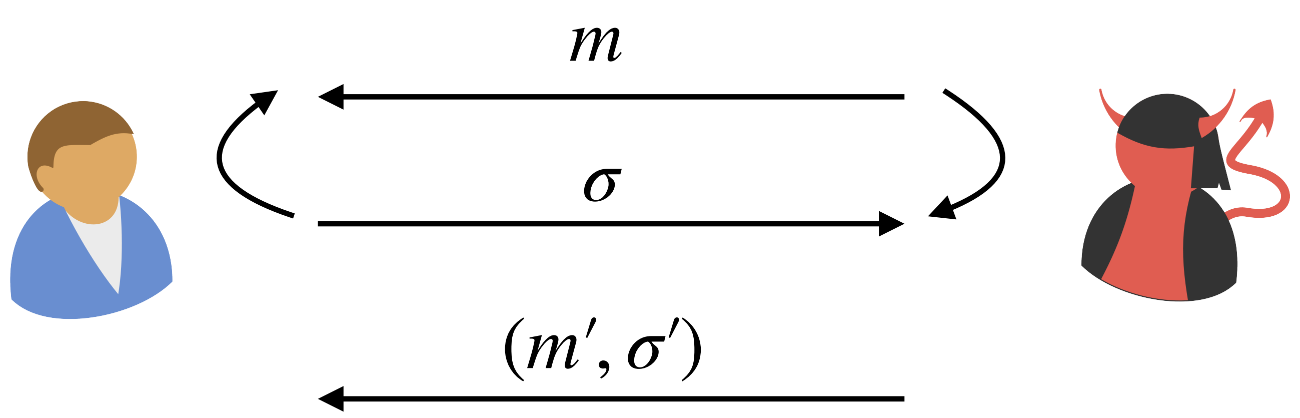

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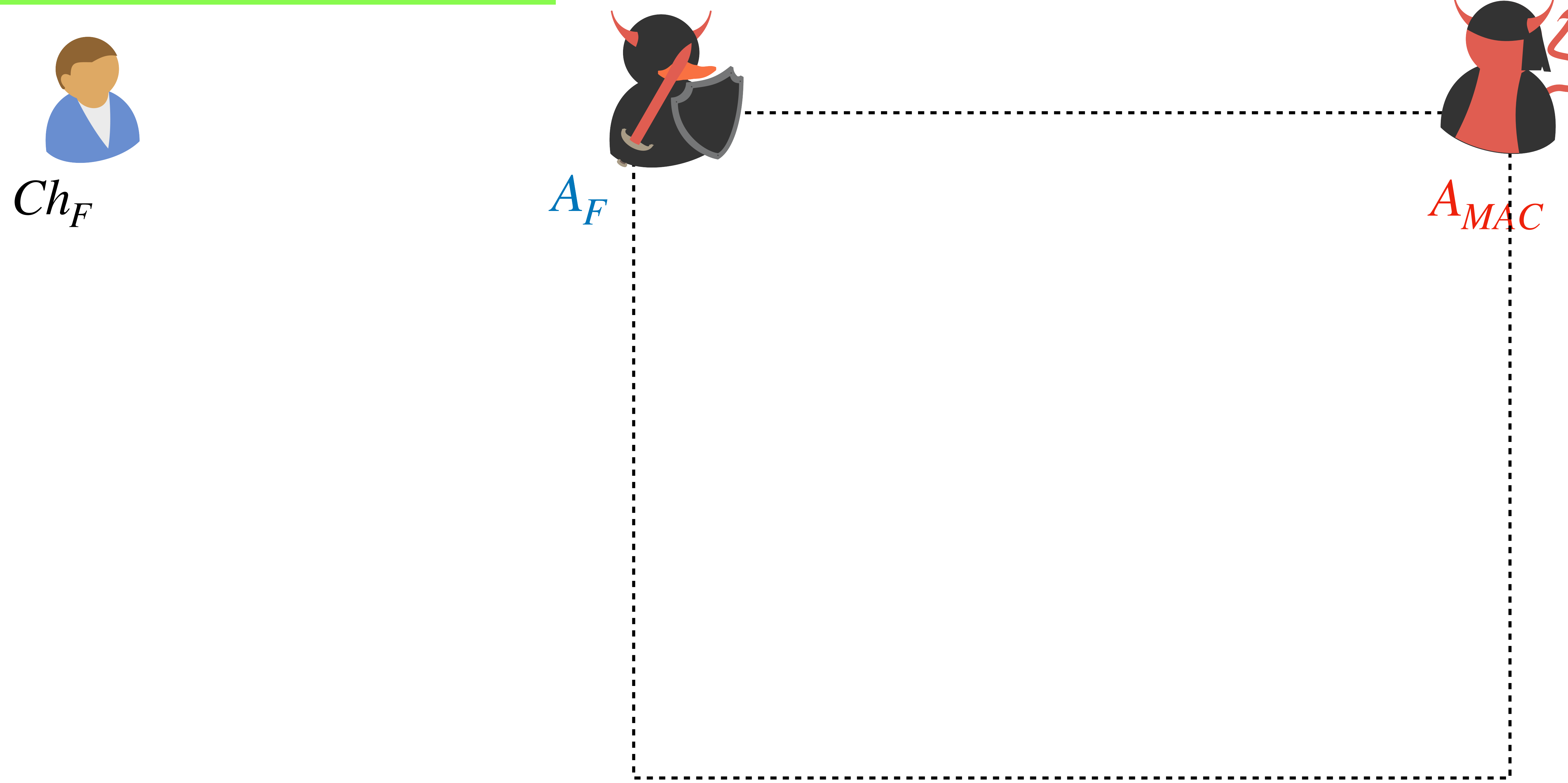

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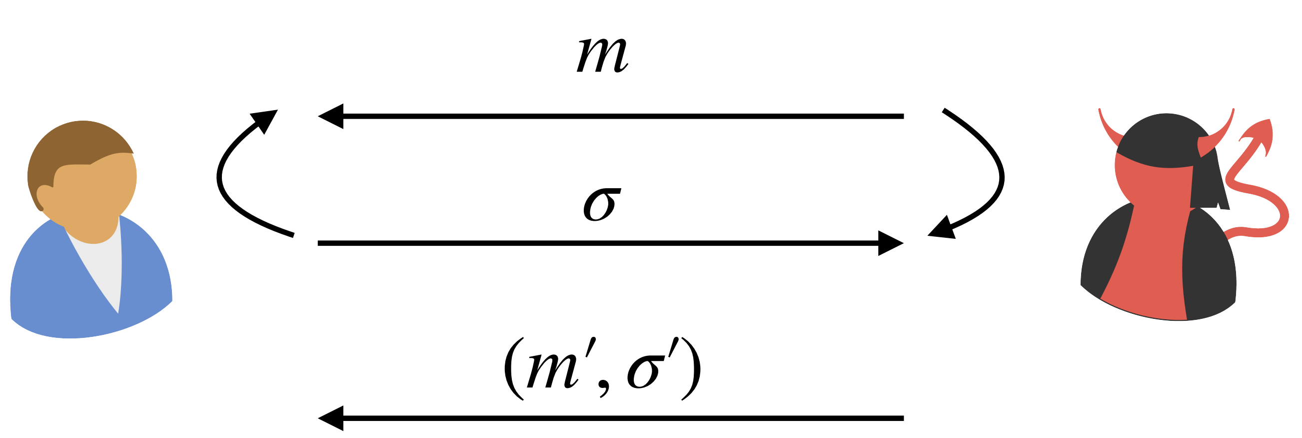

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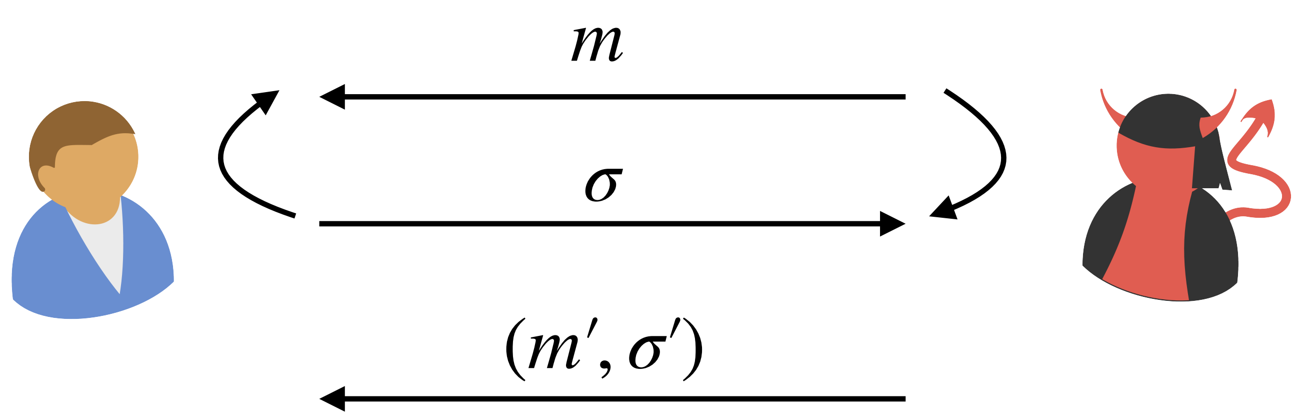

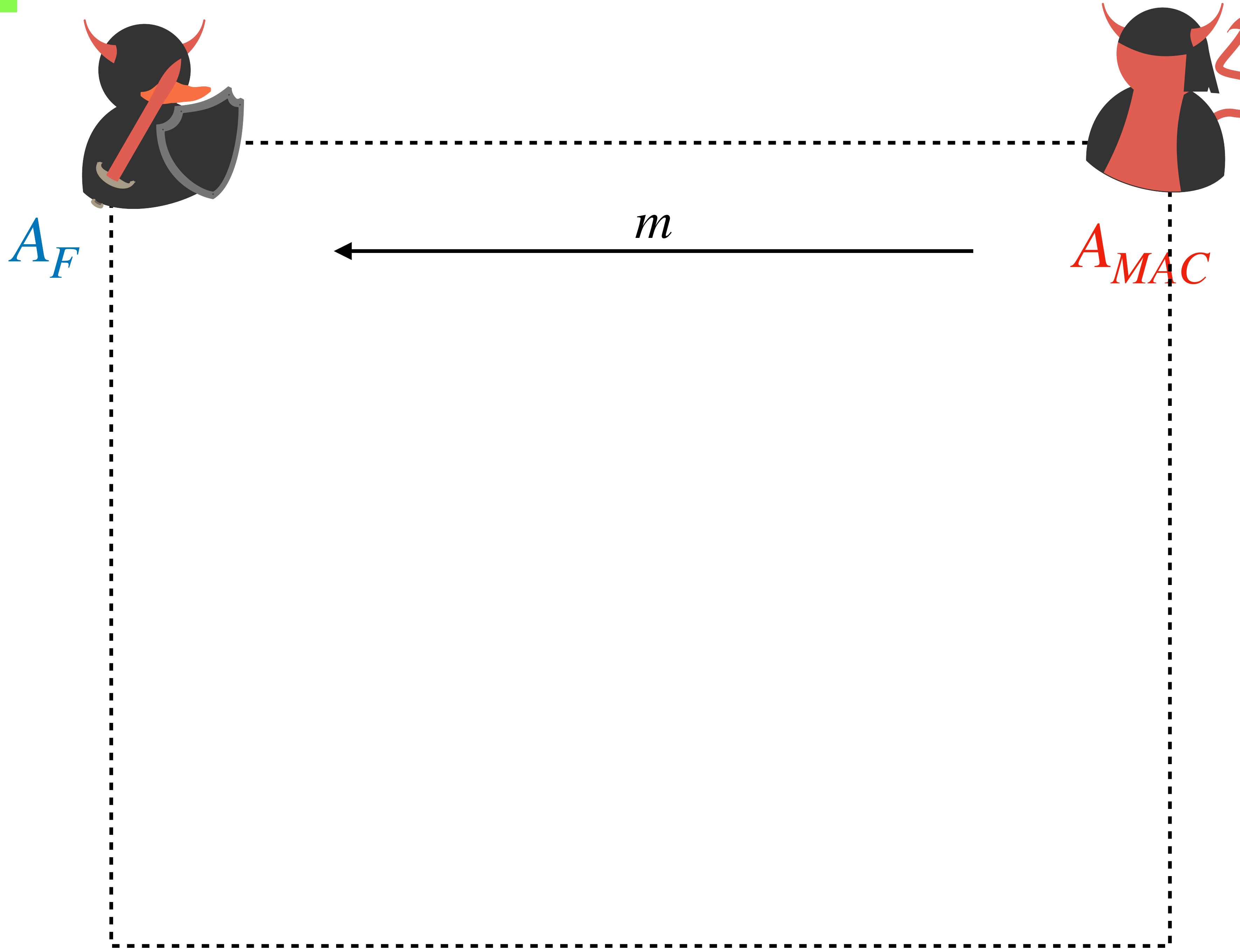

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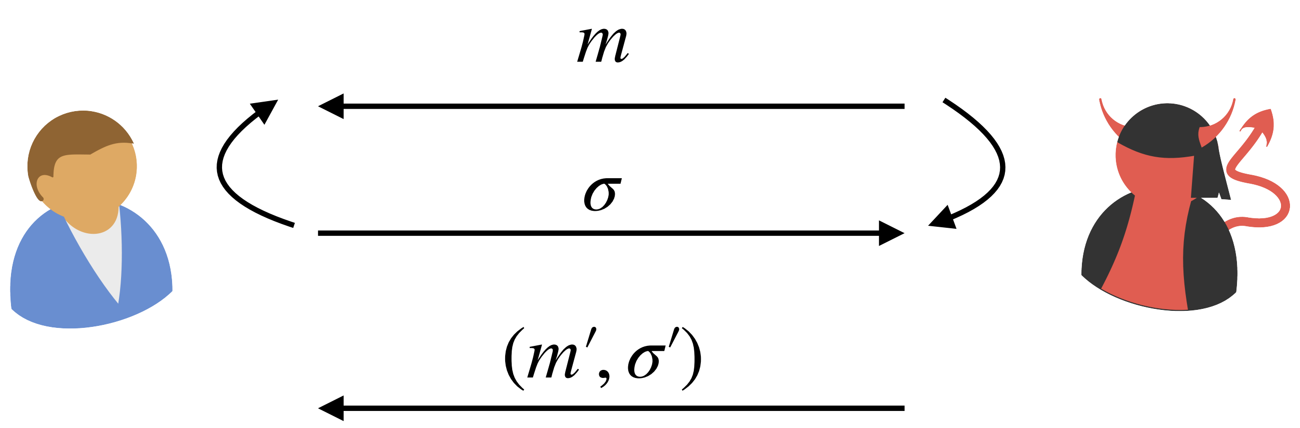

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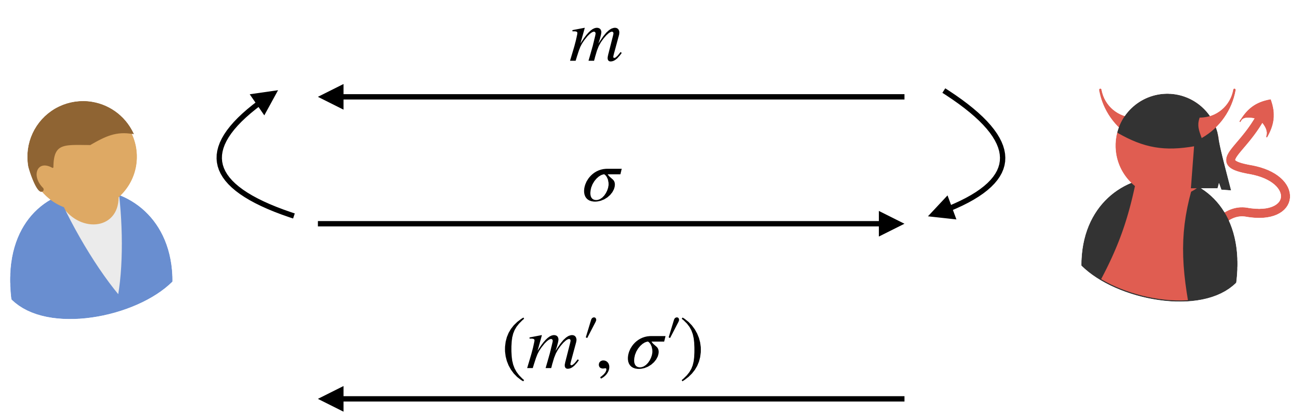

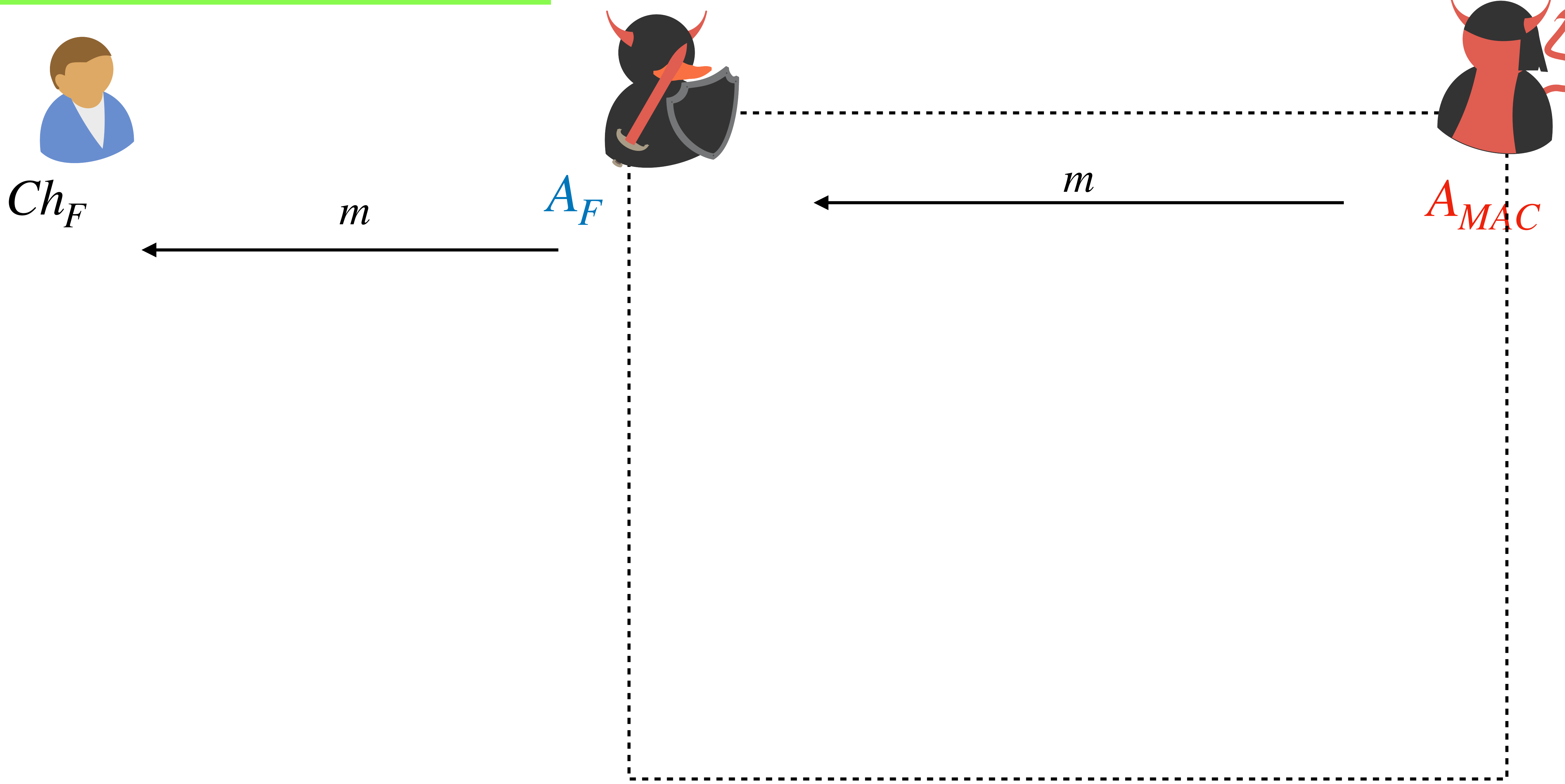

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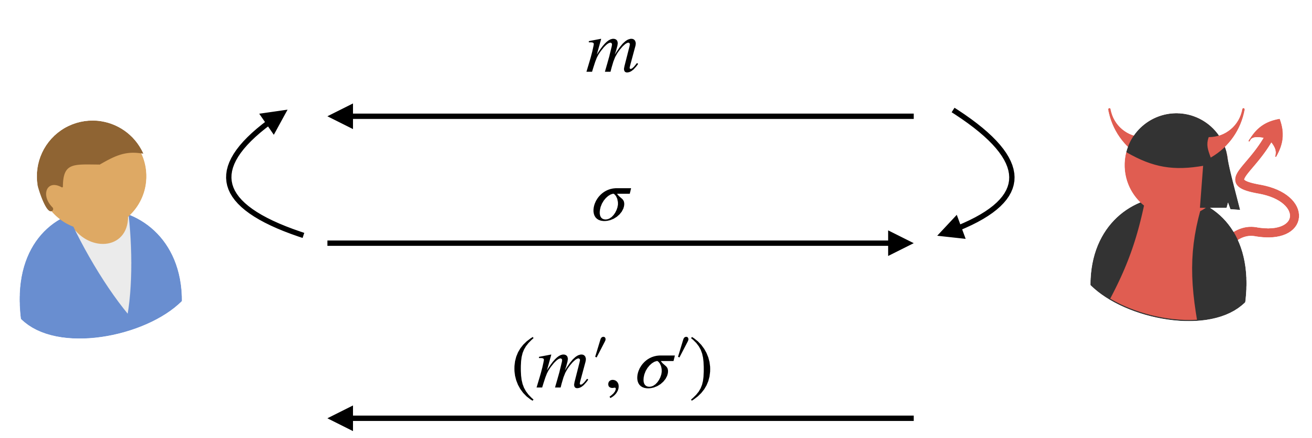

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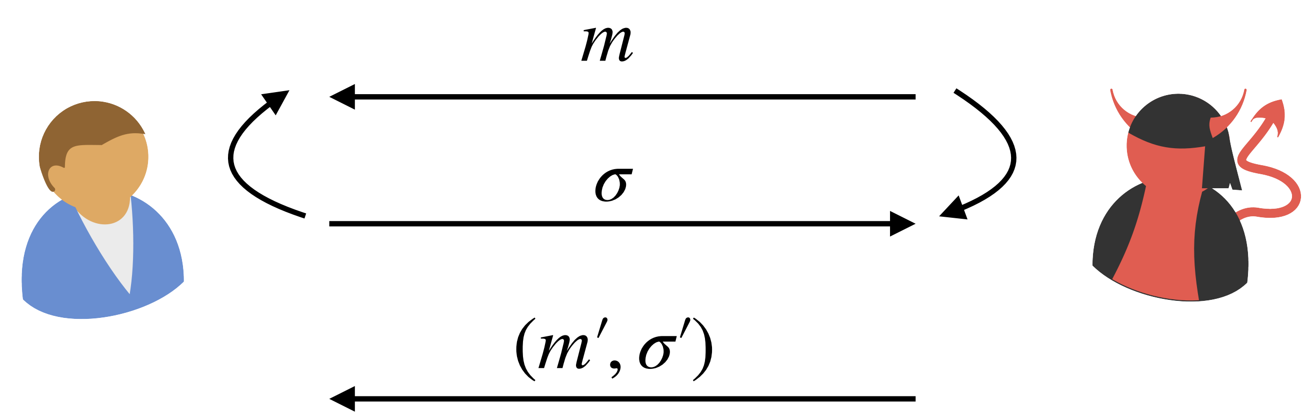

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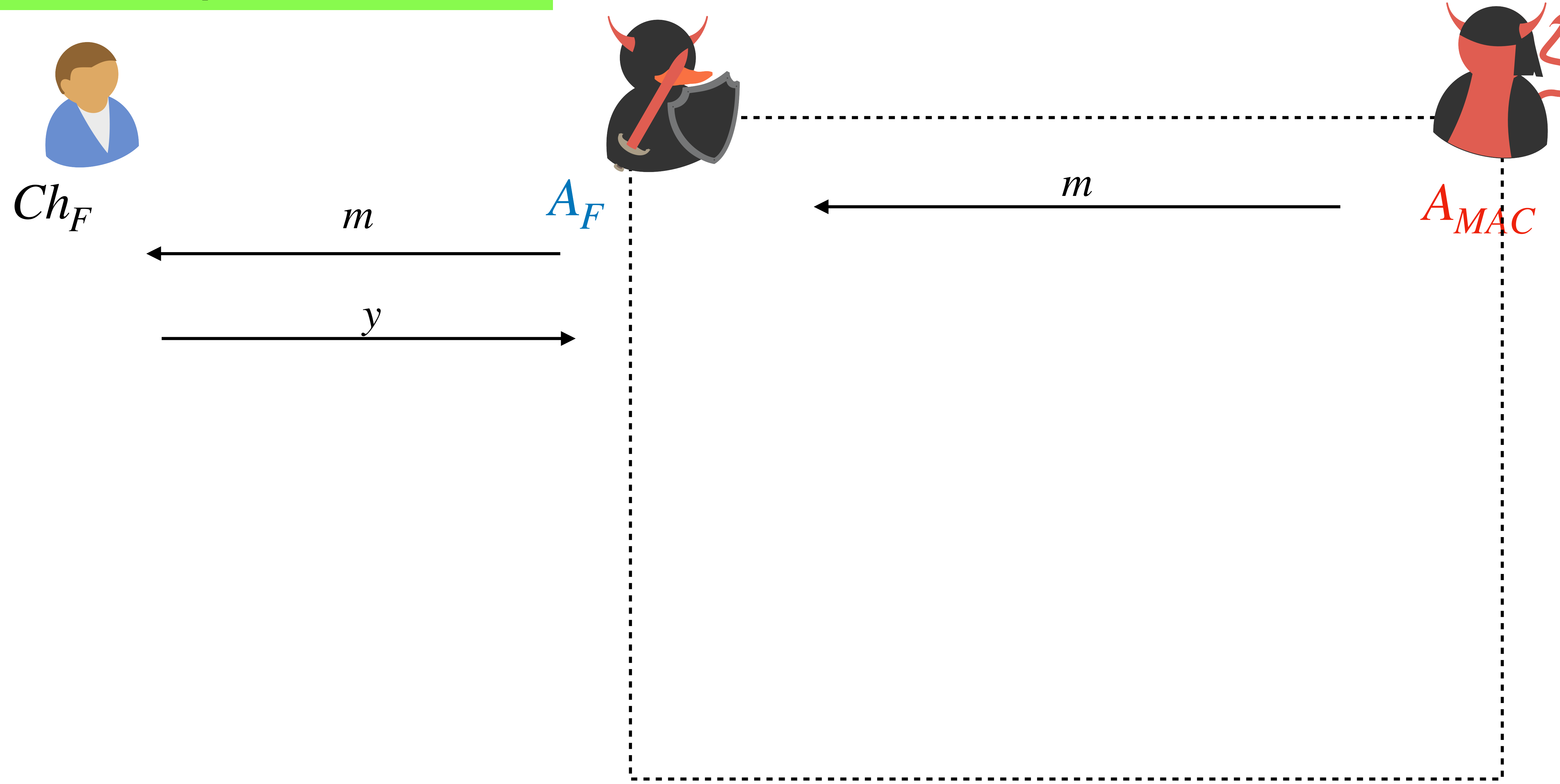

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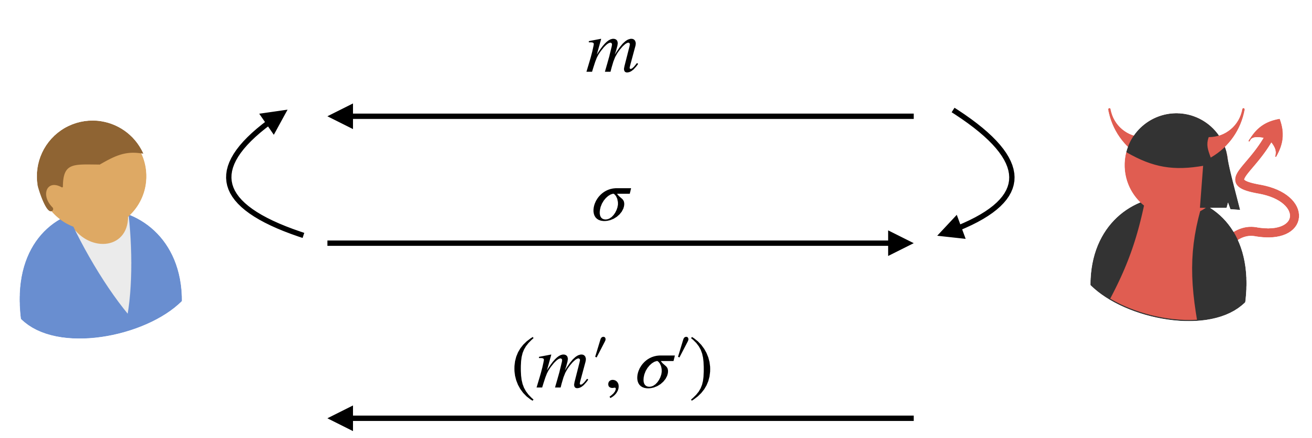

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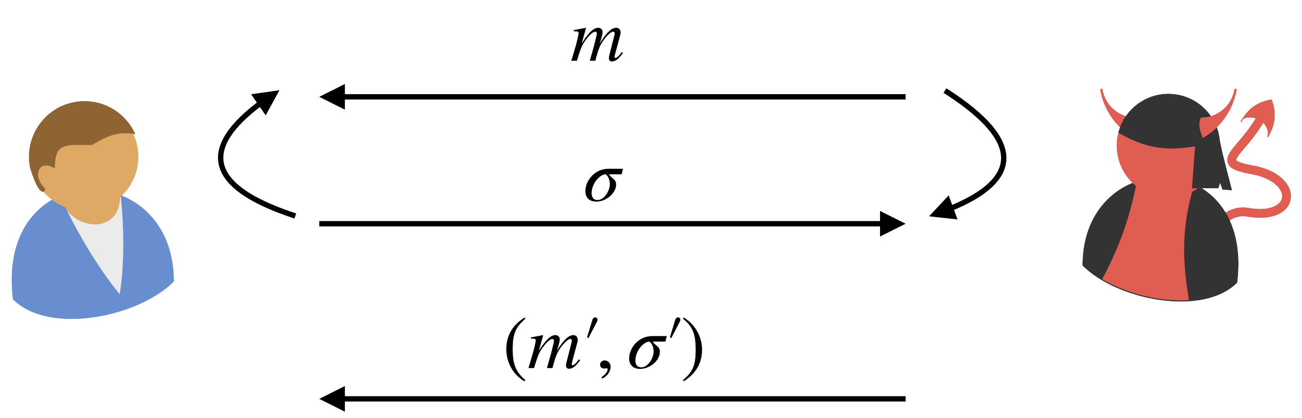

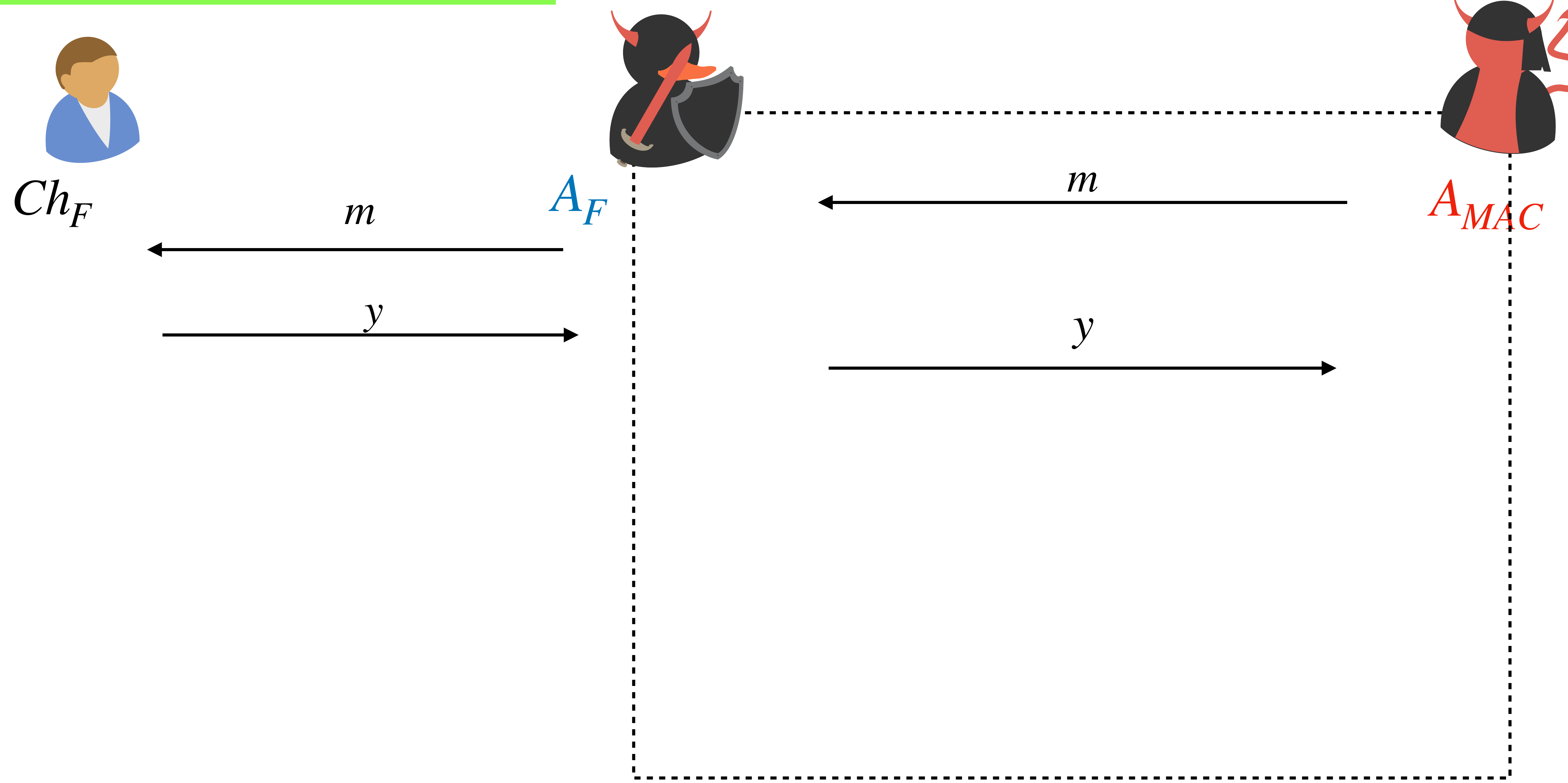

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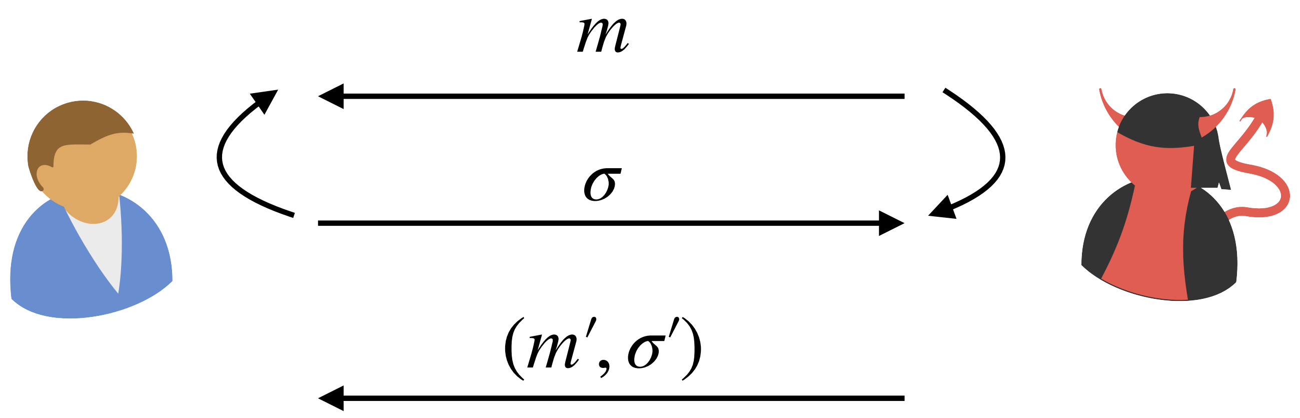

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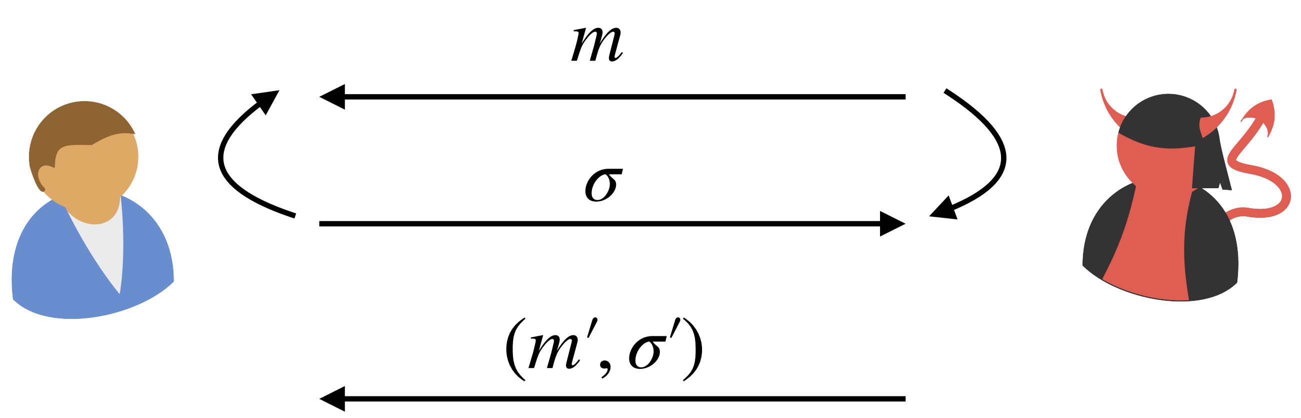

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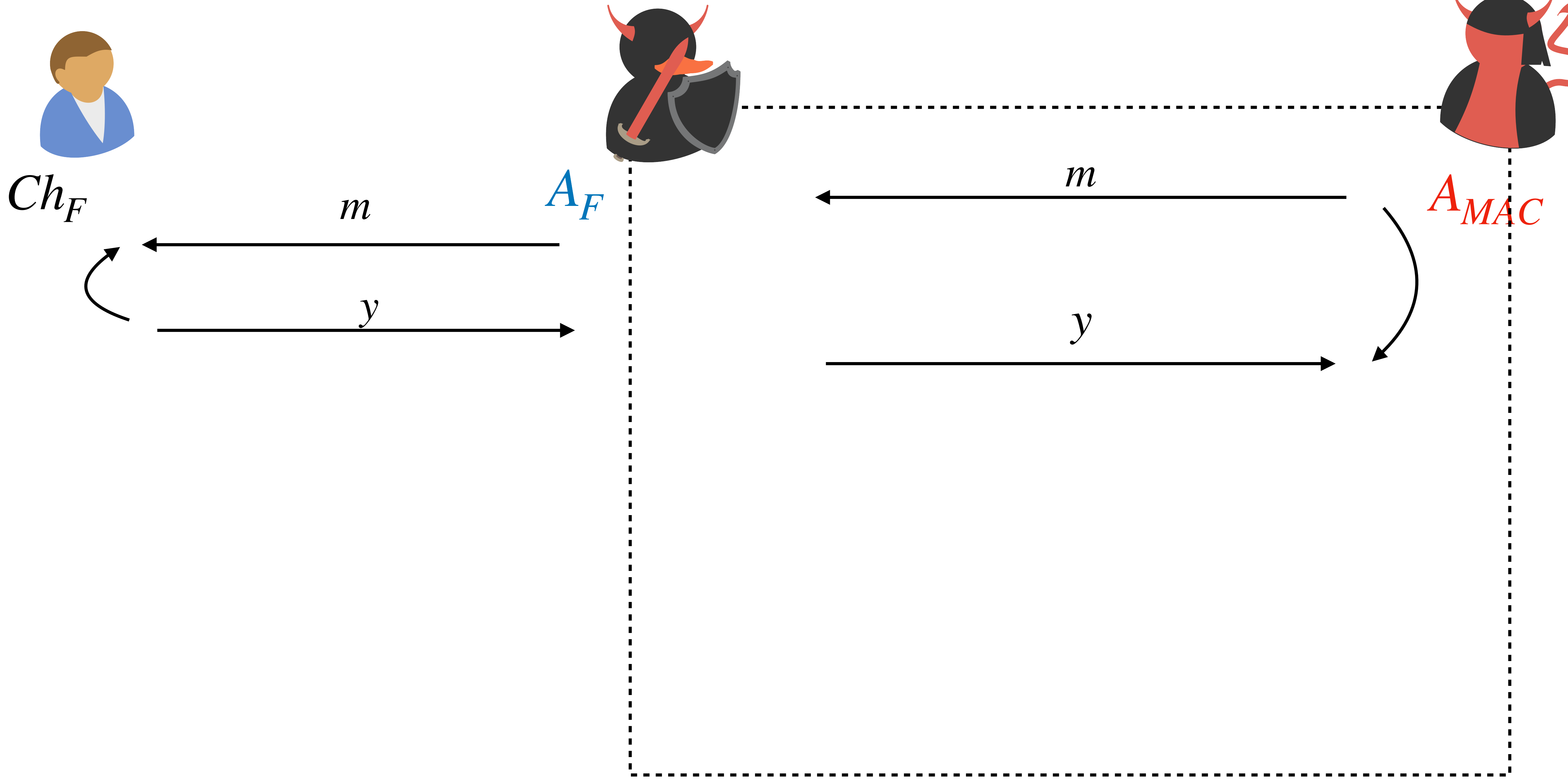

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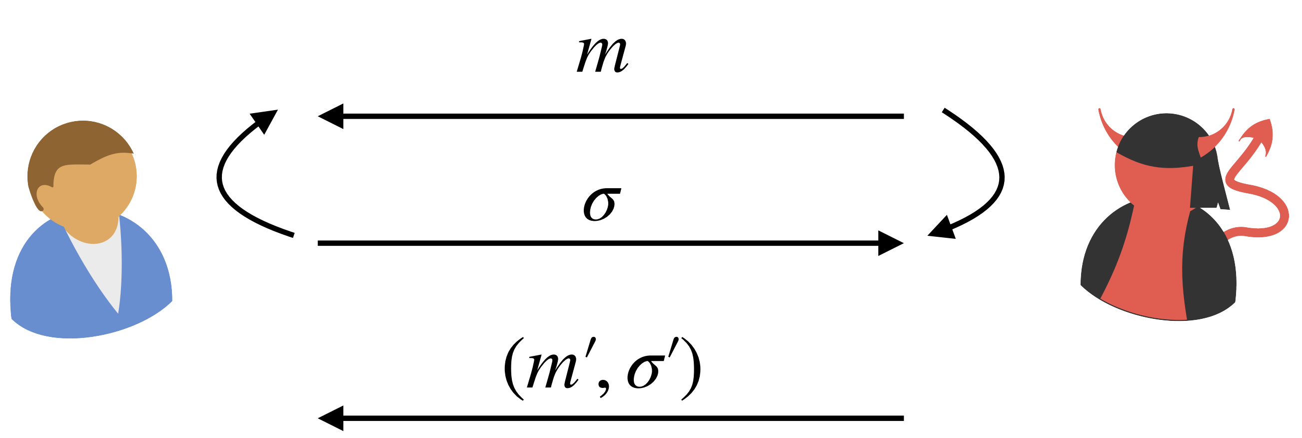

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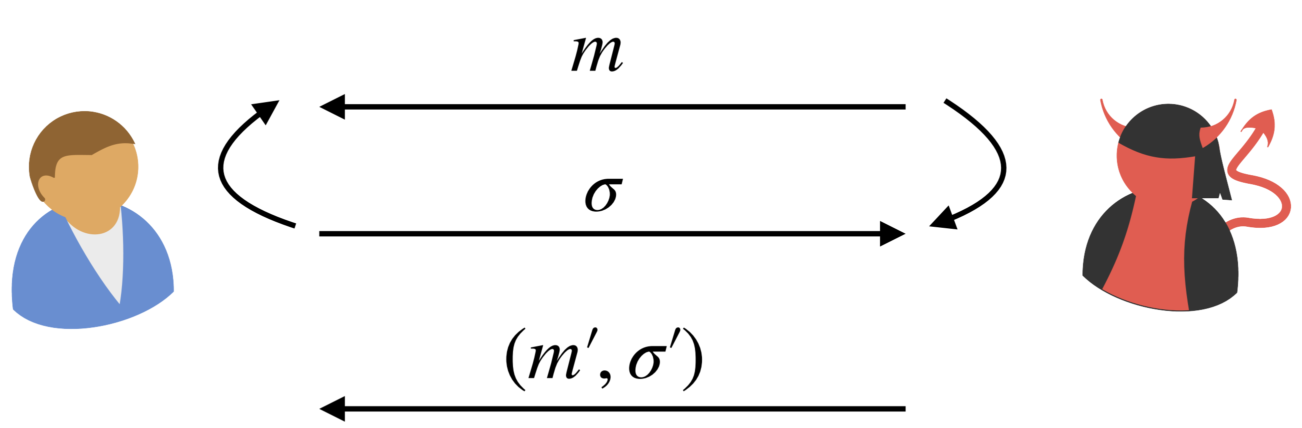

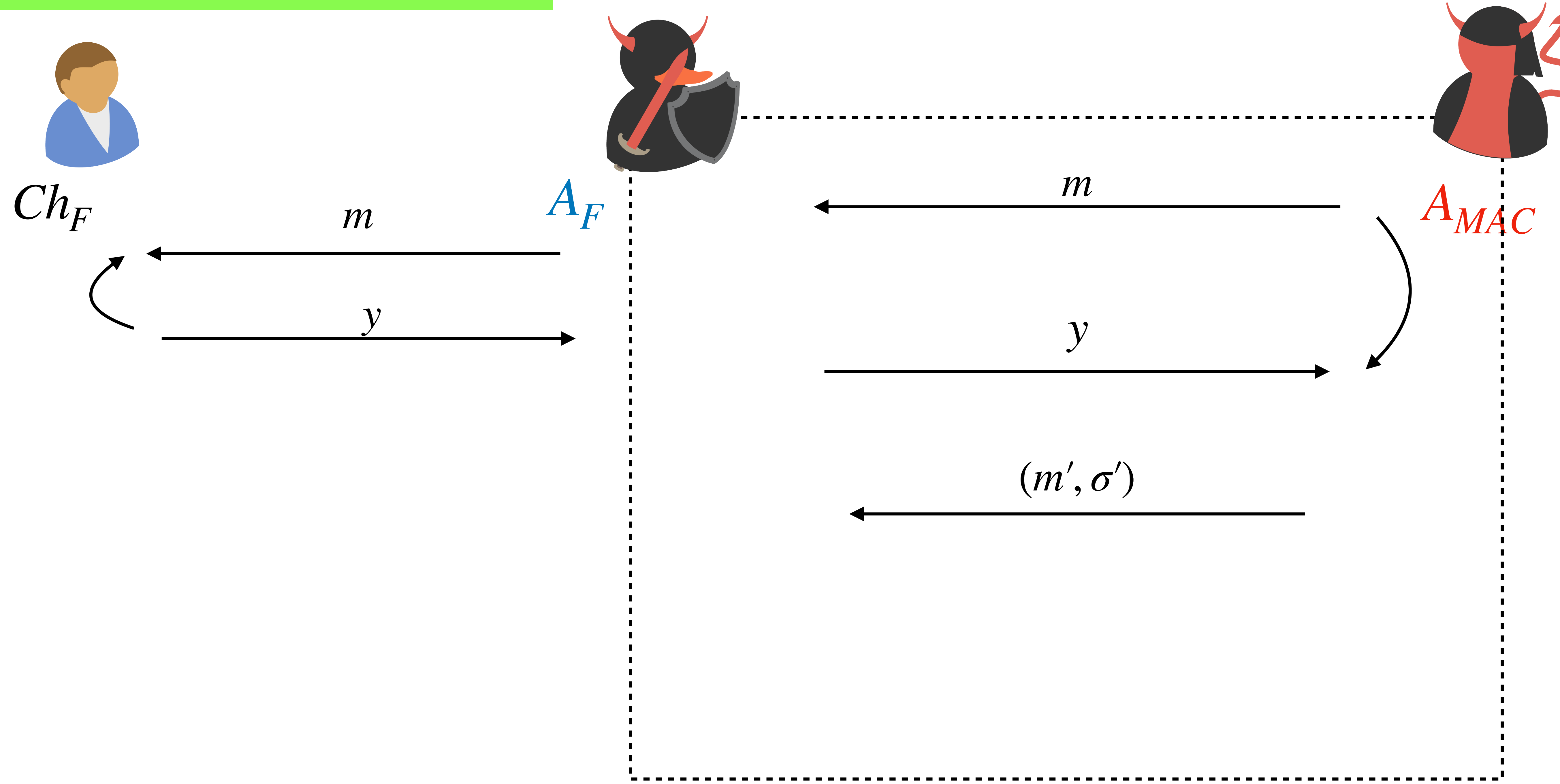

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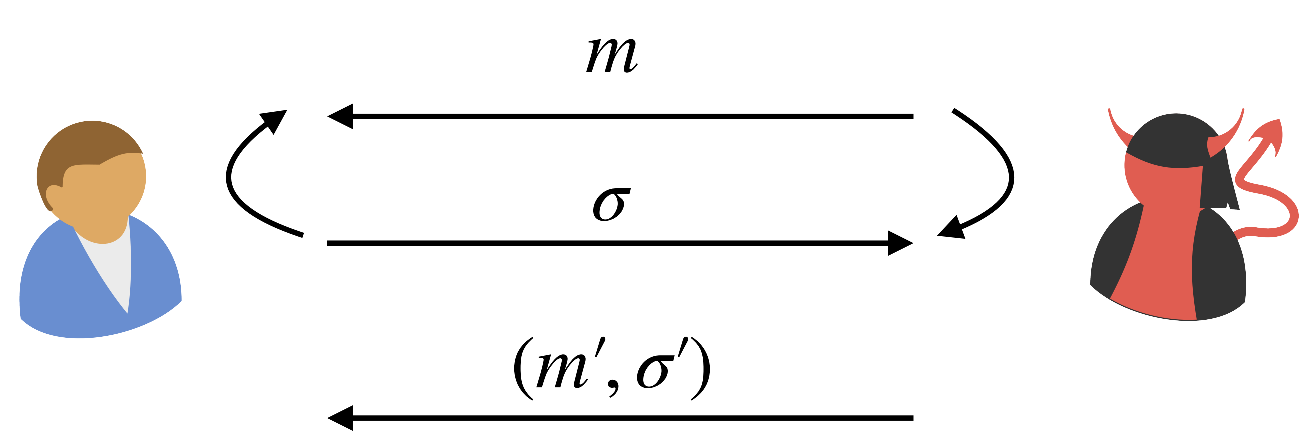

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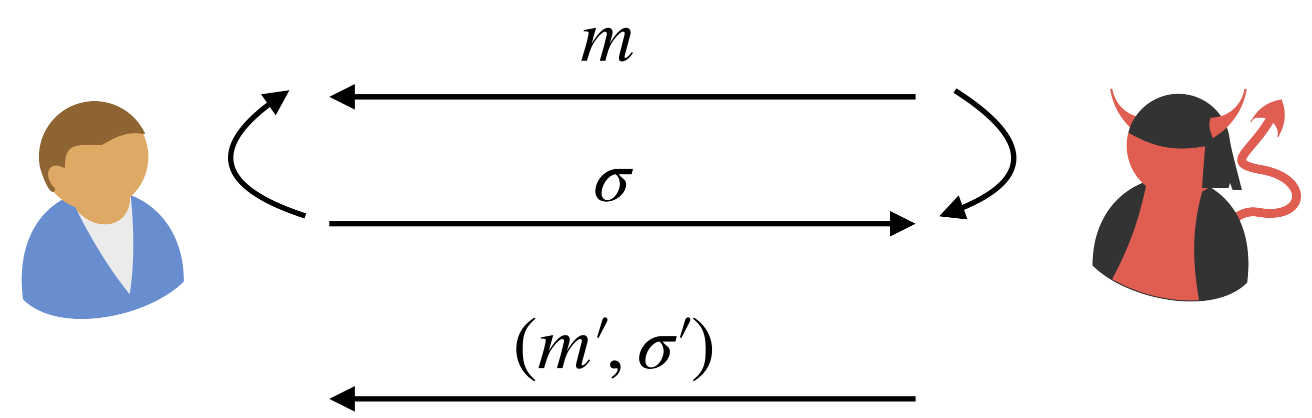

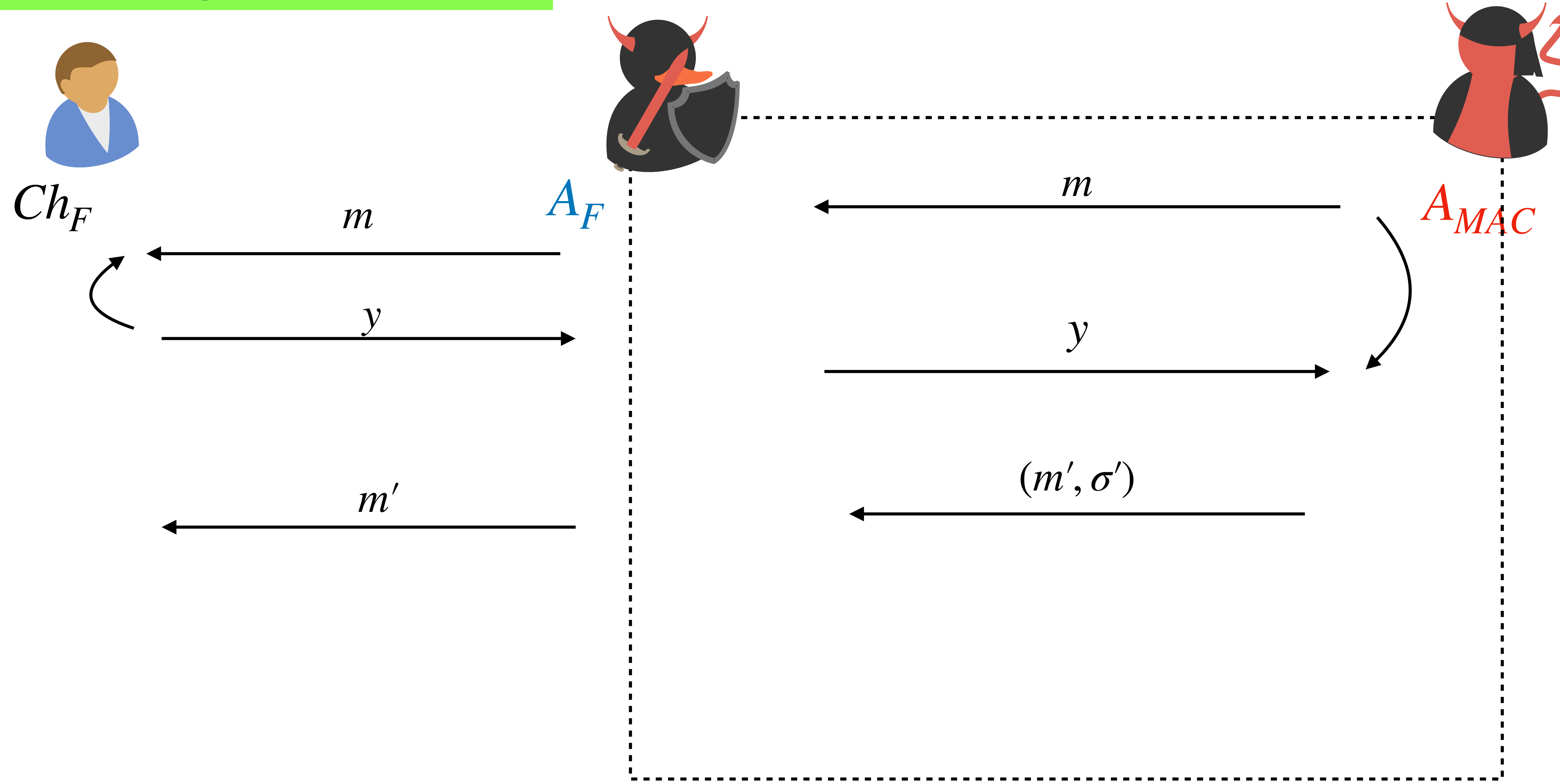

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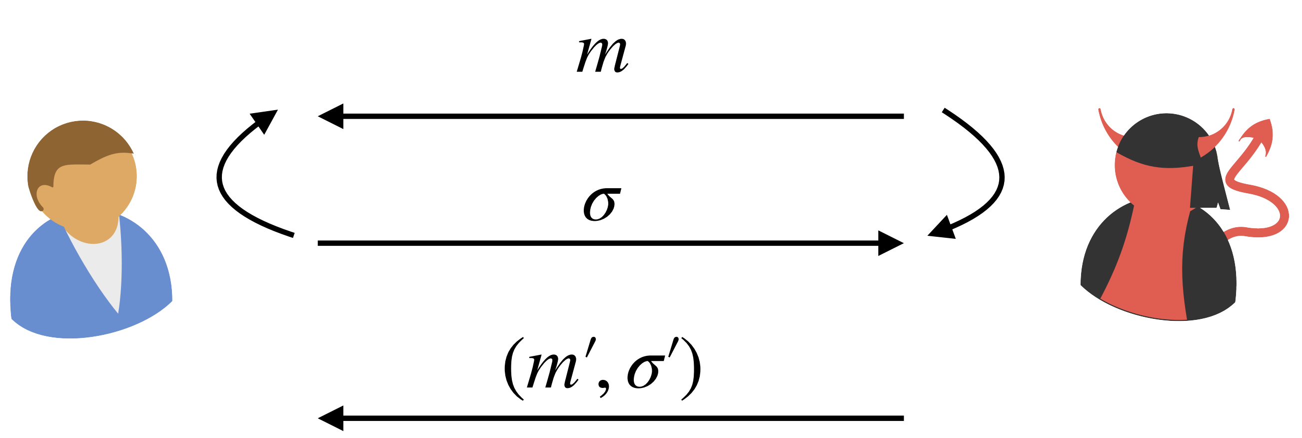

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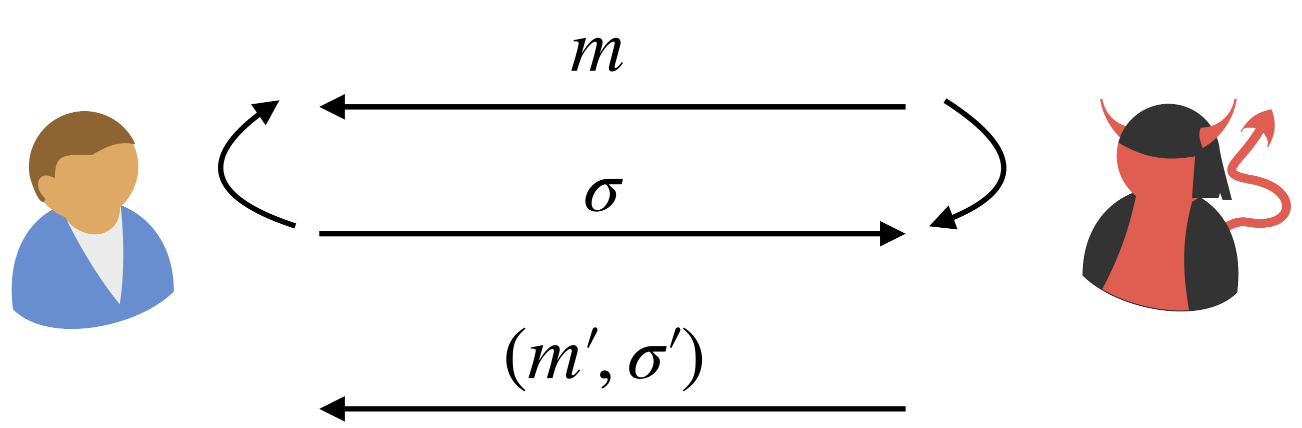
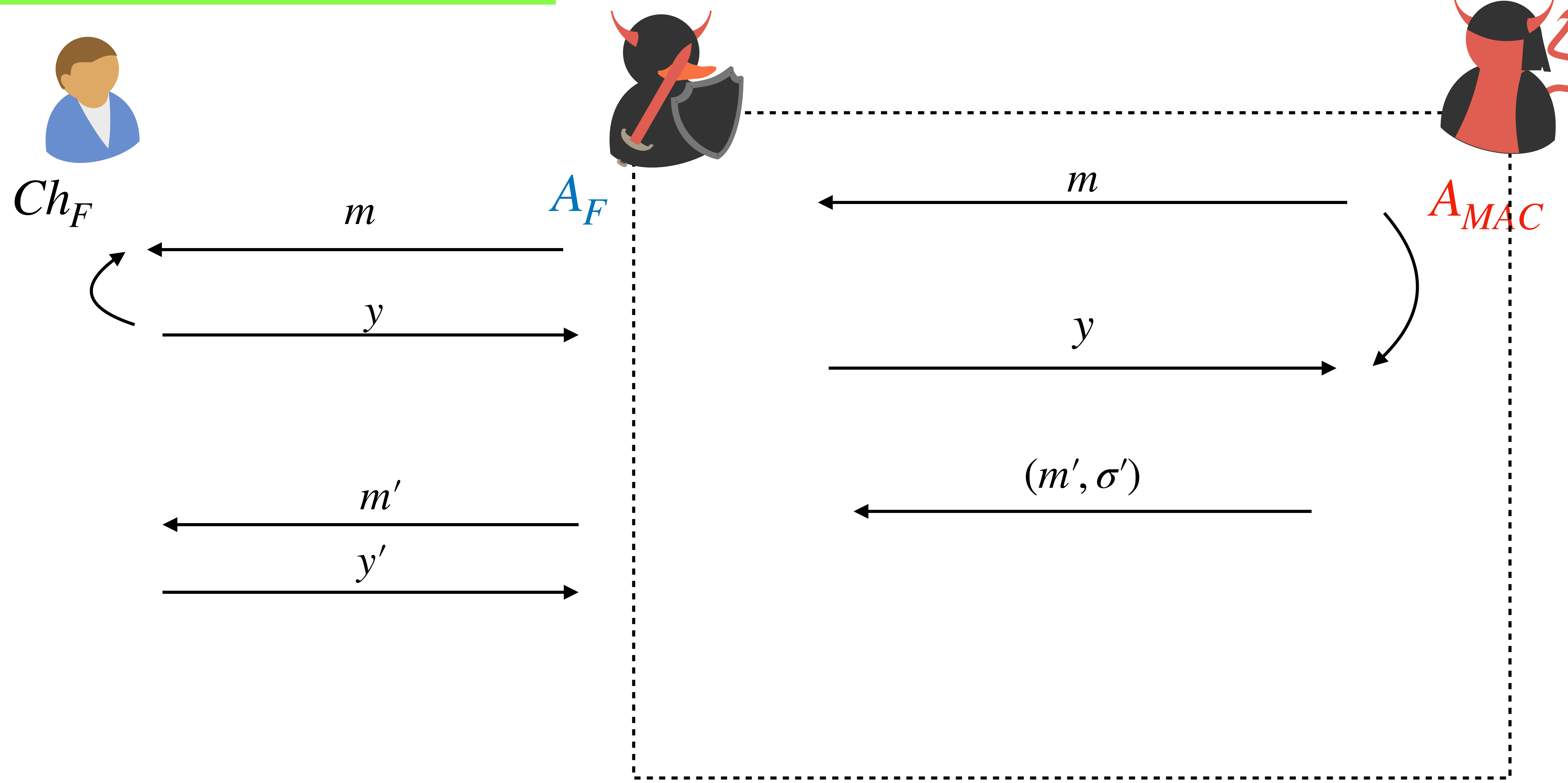


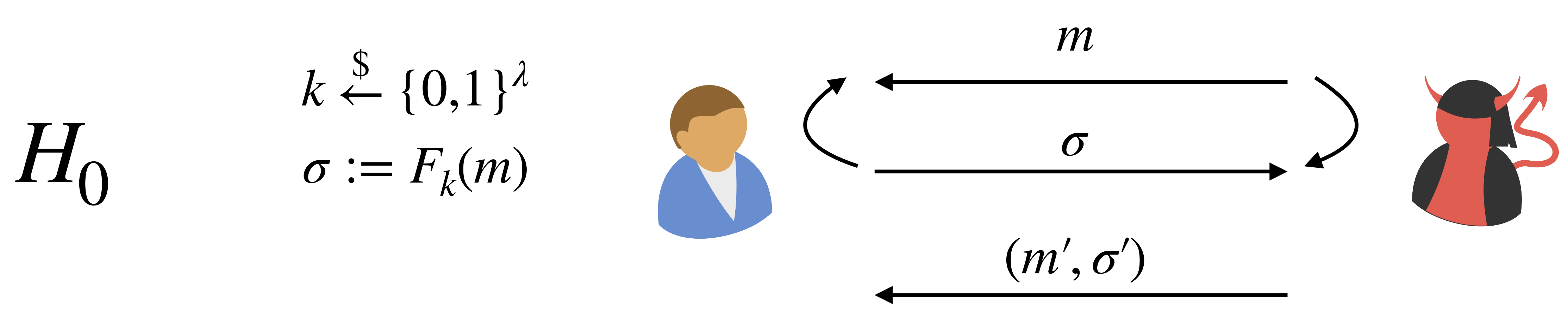


Proof of Security

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

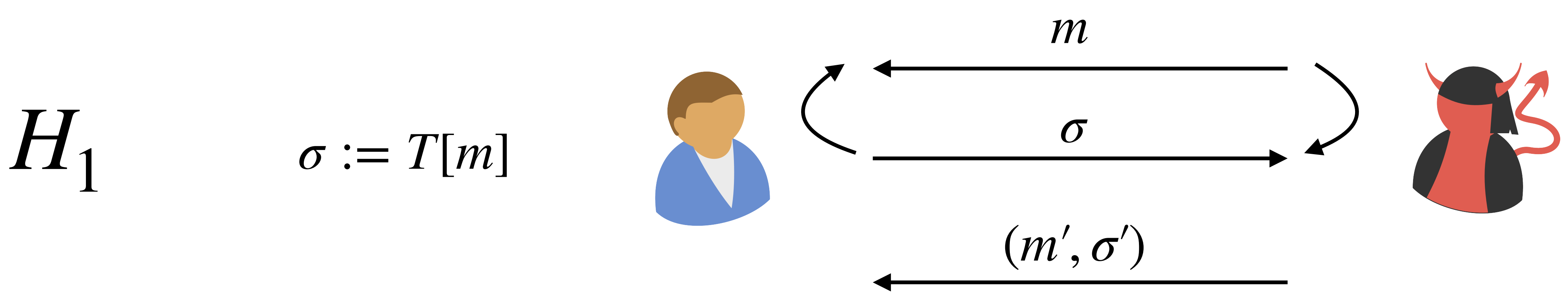
Tag(k, m) : $\sigma := F_k(m)$

Ver(k, m, σ) : $F_k(m) \stackrel{?}{=} \sigma$

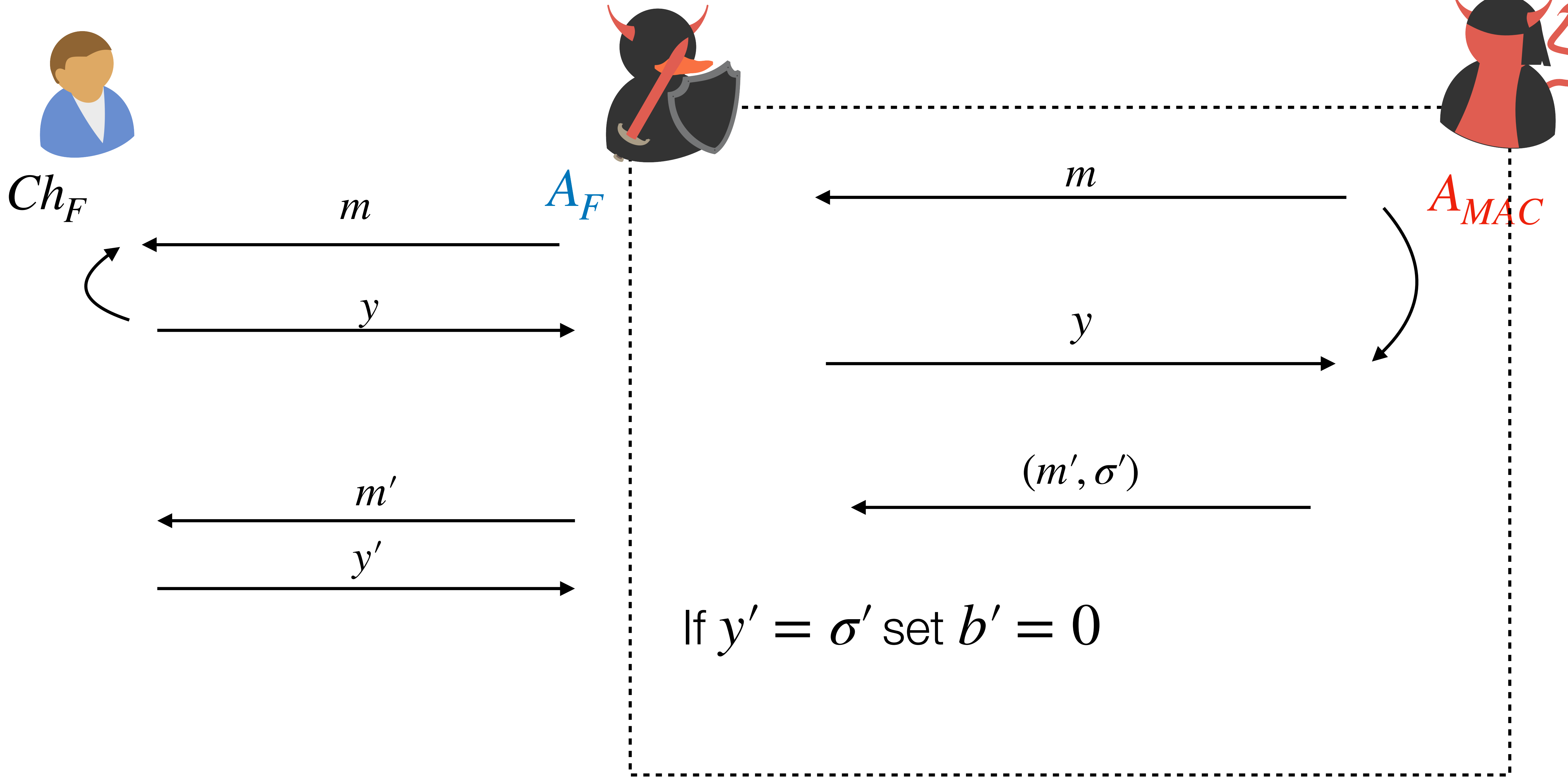


Wins if $F_k(m') = \sigma'$ and $\mathscr{A}$ never queried m'

Claim:
$$\Big| \Pr[\mathscr{A} \text{ wins in } H_0] - \Pr[\mathscr{A} \text{ wins in } H_1] \Big| \leq \mathsf{negl}(\lambda)$$



Wins if $T[m'] = \sigma'$ and $\mathscr{A}$ never queried m'



Proof of Security

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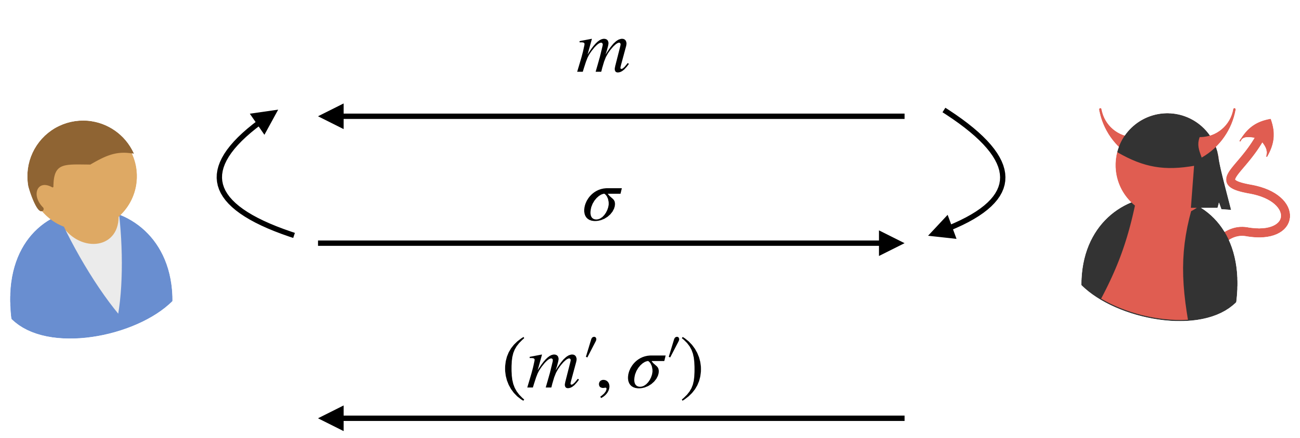
Wins if $F_k(m') = \sigma'$ and $\mathscr{A}$ never queried m'

H_0 $\quad k \xleftarrow{\$} \{0,1\}^\lambda$, $\sigma := F_k(m)$



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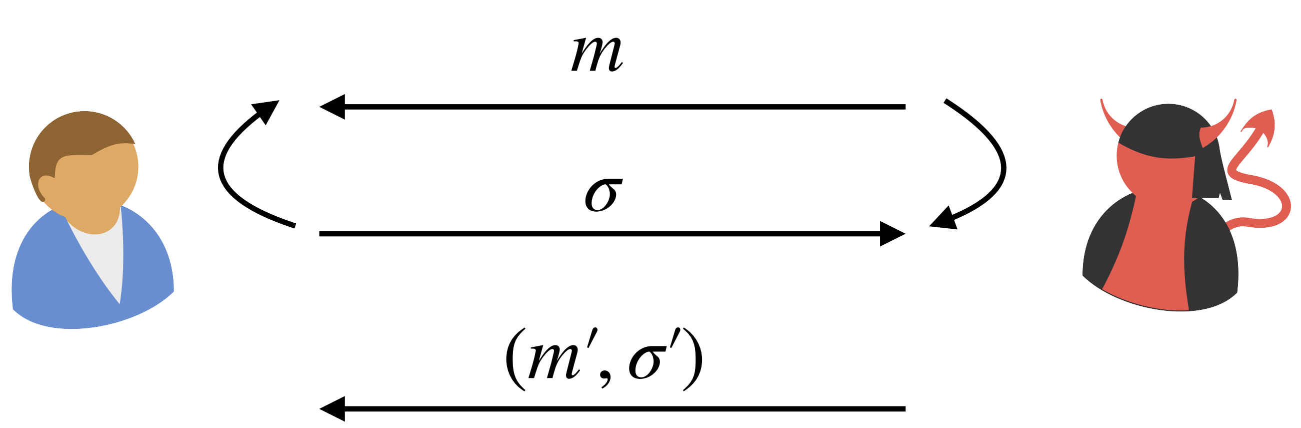
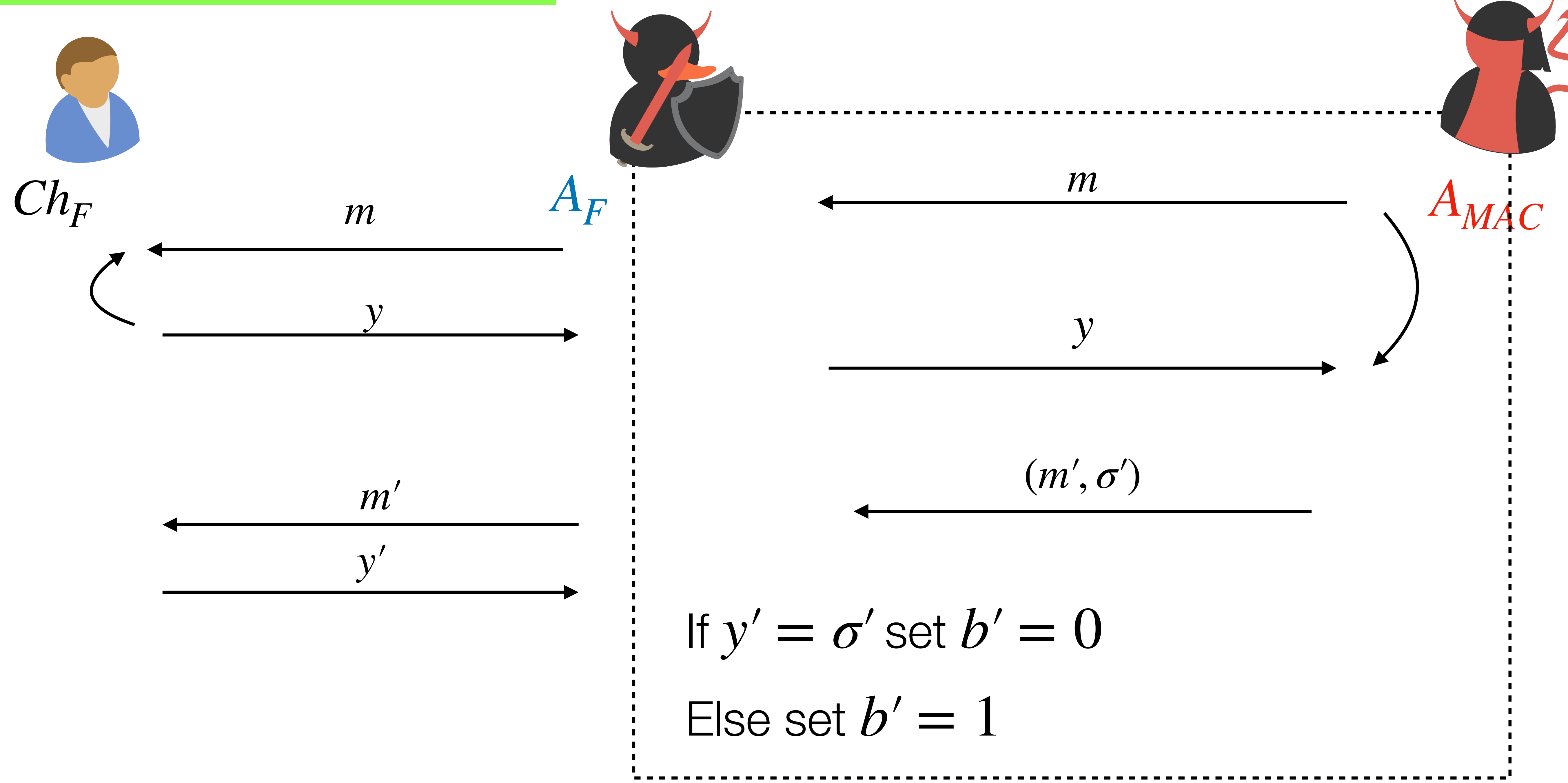
$$\left| \Pr[\mathscr{A} \text{ wins in } H_0] - \Pr[\mathscr{A} \text{ wins in } H_1] \right| \leq \text{negl}(\lambda)$$

Wins if $T[m'] = \sigma'$ and $\mathscr{A}$ never queried m'

H_1 $\quad \sigma := T[m]$





Proof of Security

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Wins if $F_k(m') = \sigma'$ and $\mathscr{A}$ never queried m'

H_0

$k \xleftarrow{\$} \{0,1\}^\lambda$
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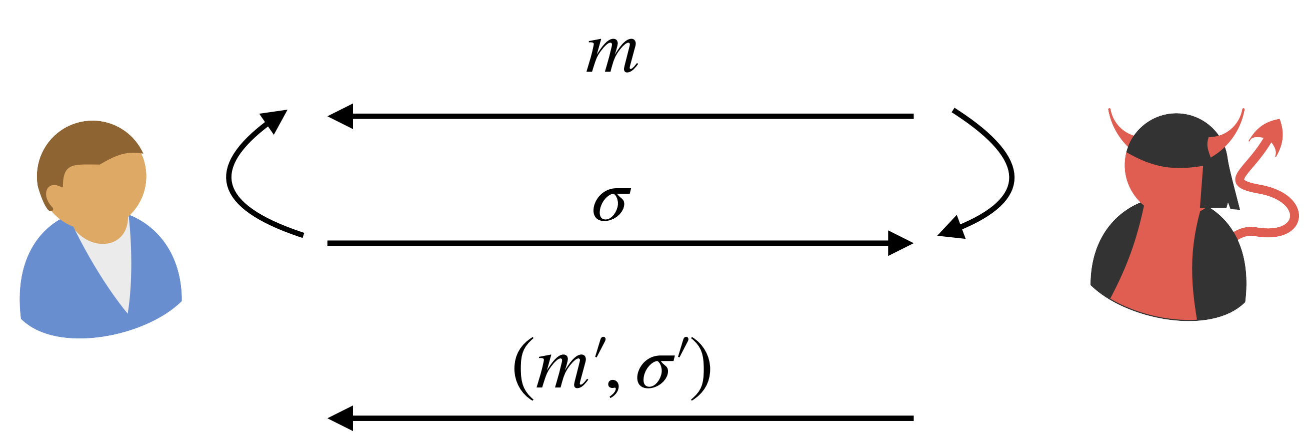
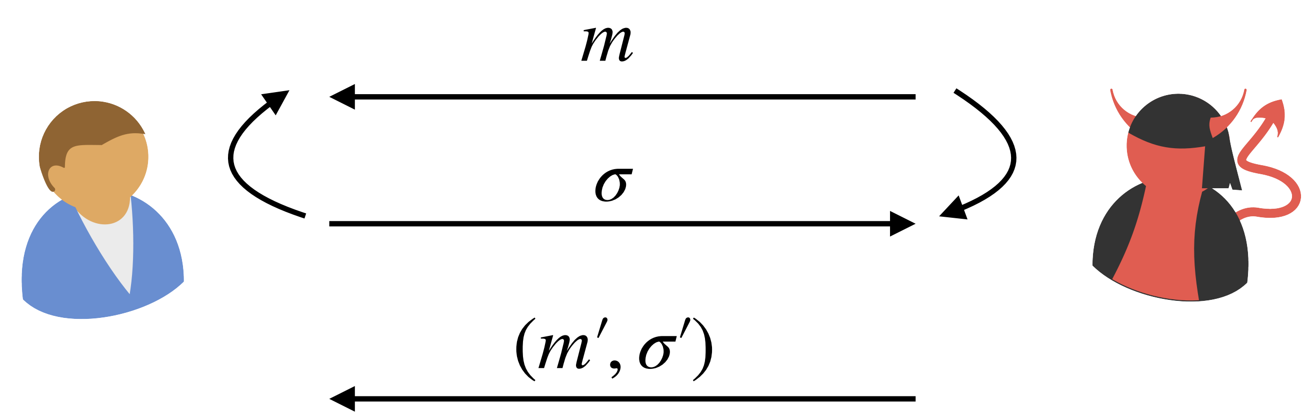


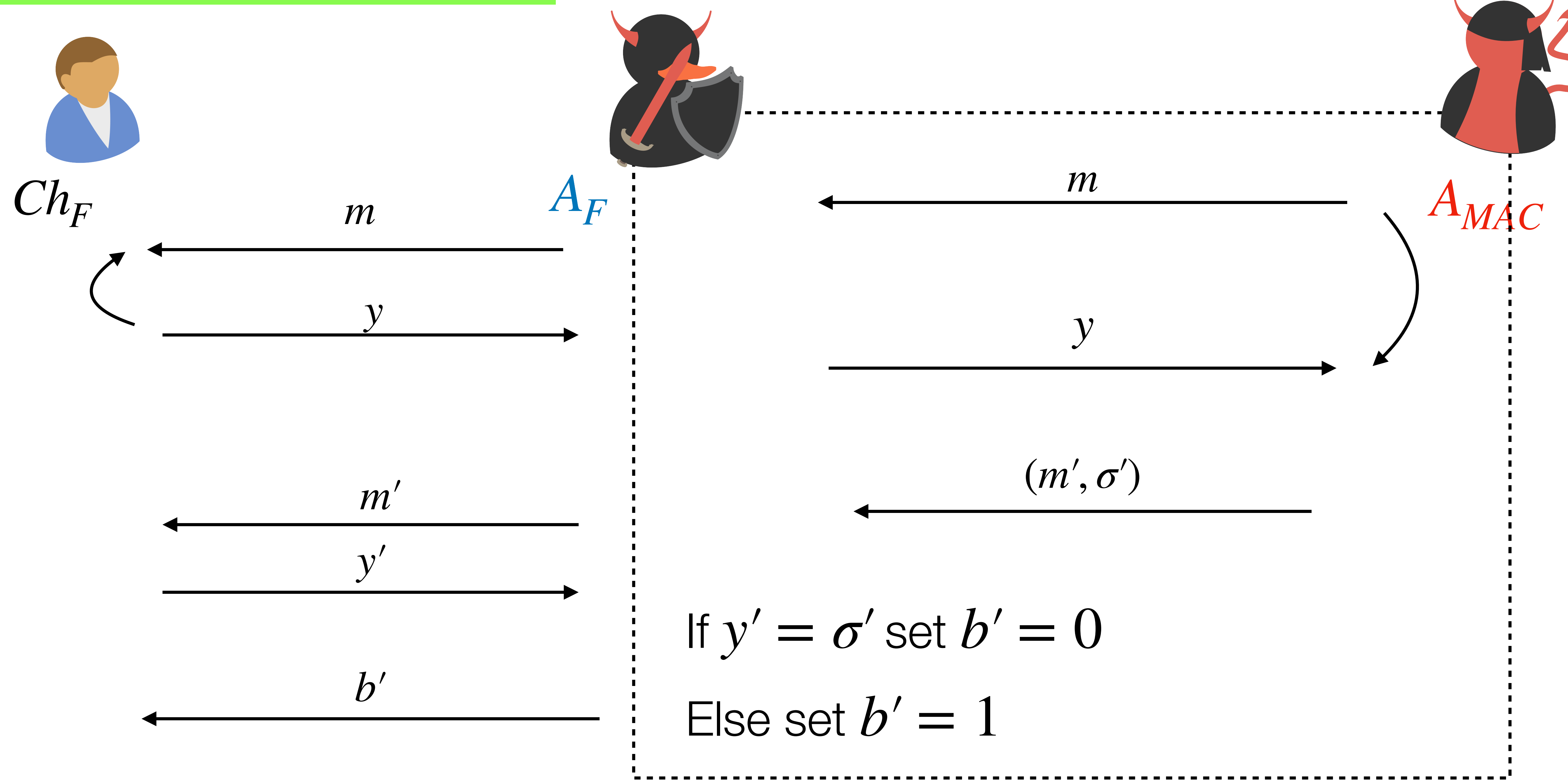
Claim:
$\left| \Pr[\mathscr{A} \text{ wins in } H_0] - \Pr[\mathscr{A} \text{ wins in } H_1] \right| \leq \text{negl}(\lambda)$

Wins if $T[m'] = \sigma'$ and $\mathscr{A}$ never queried m'

H_1

$\sigma := T[m]$





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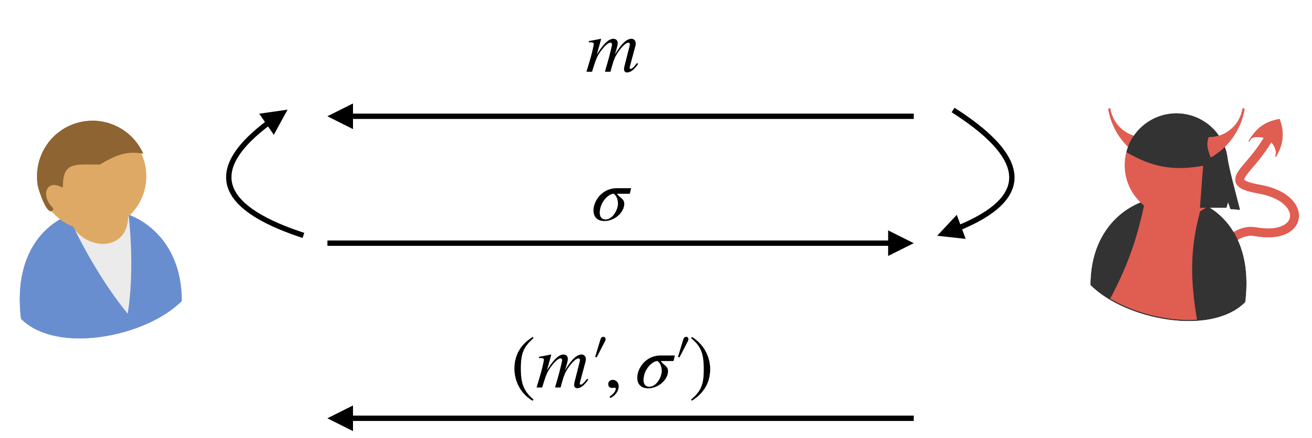

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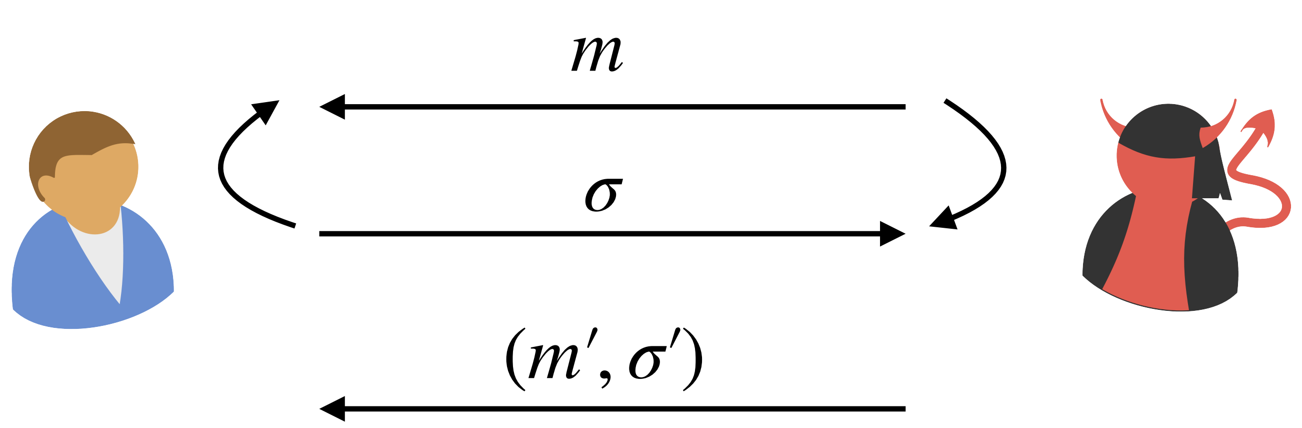
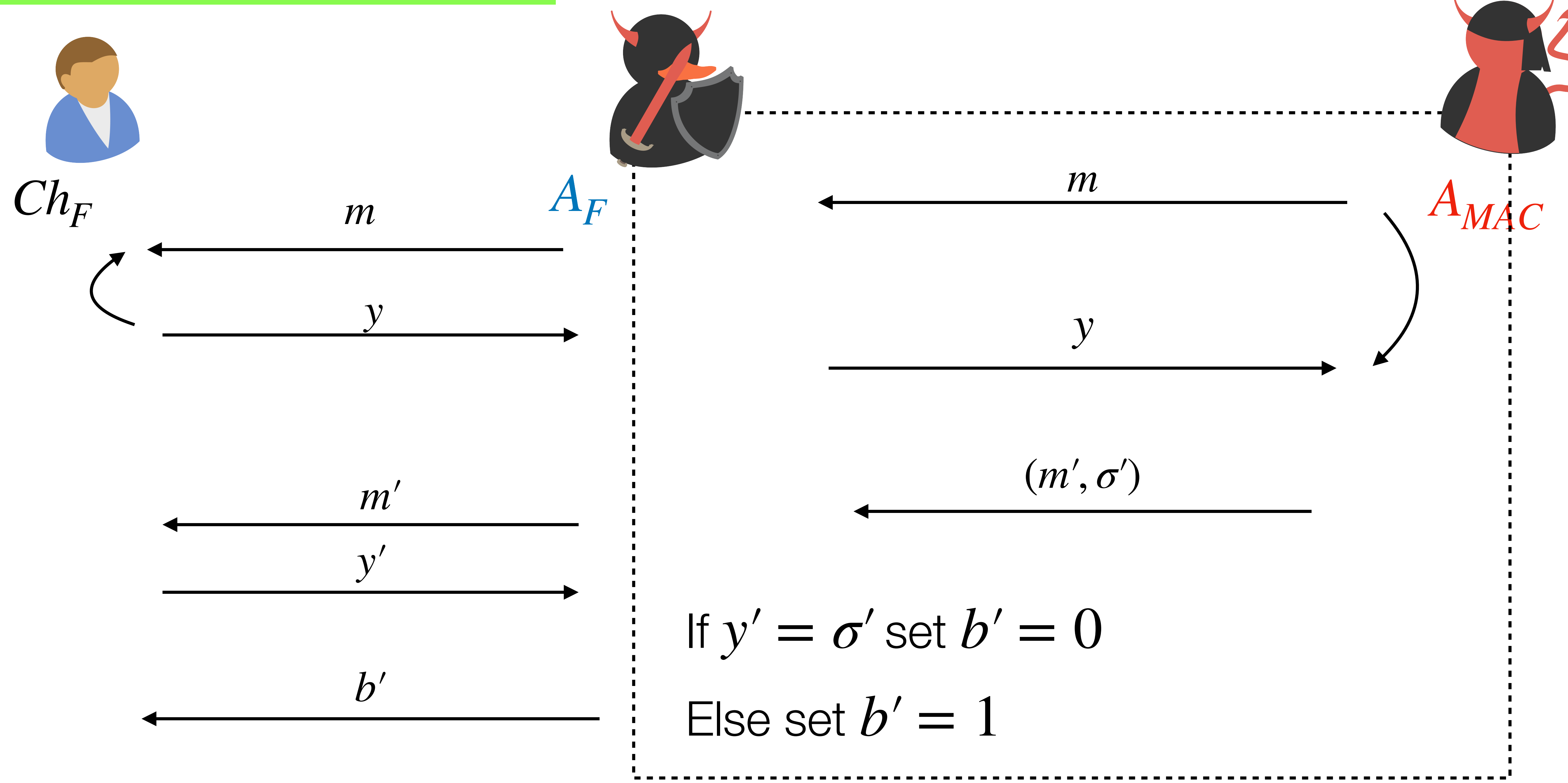


Wins if $T[m'] = \sigma'$ and $\mathscr{A}$ never queried m'



$\Pr[W_0] = \Pr[\mathscr{A}_F \text{ outputs } 0 \text{ in Game}_0]$

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

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H_0

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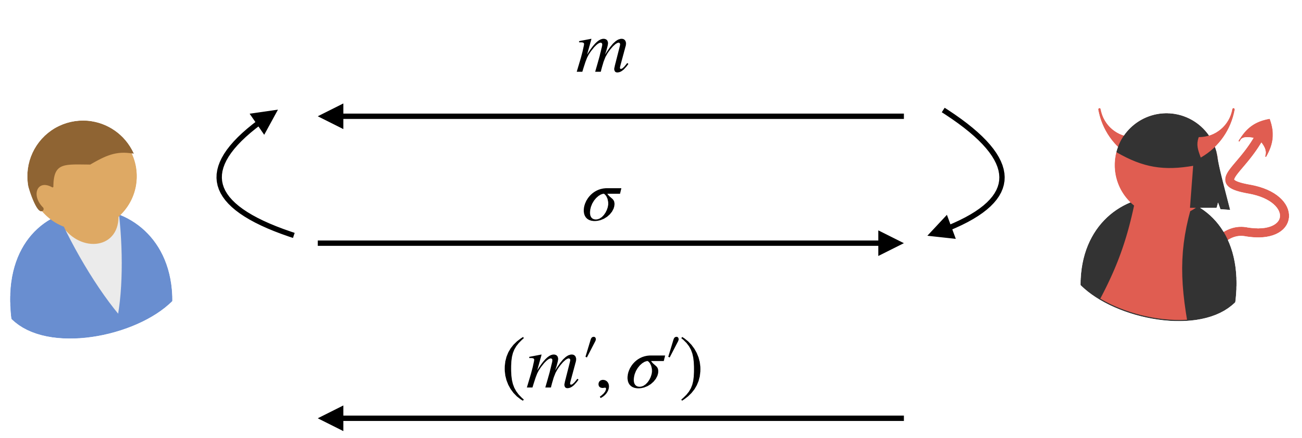

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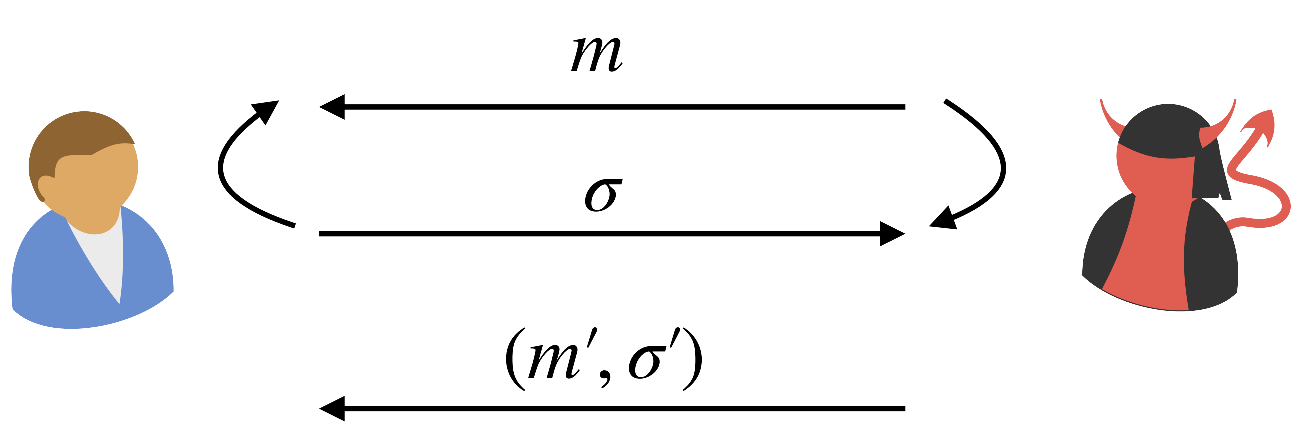
H_1

$\sigma := T[m]$



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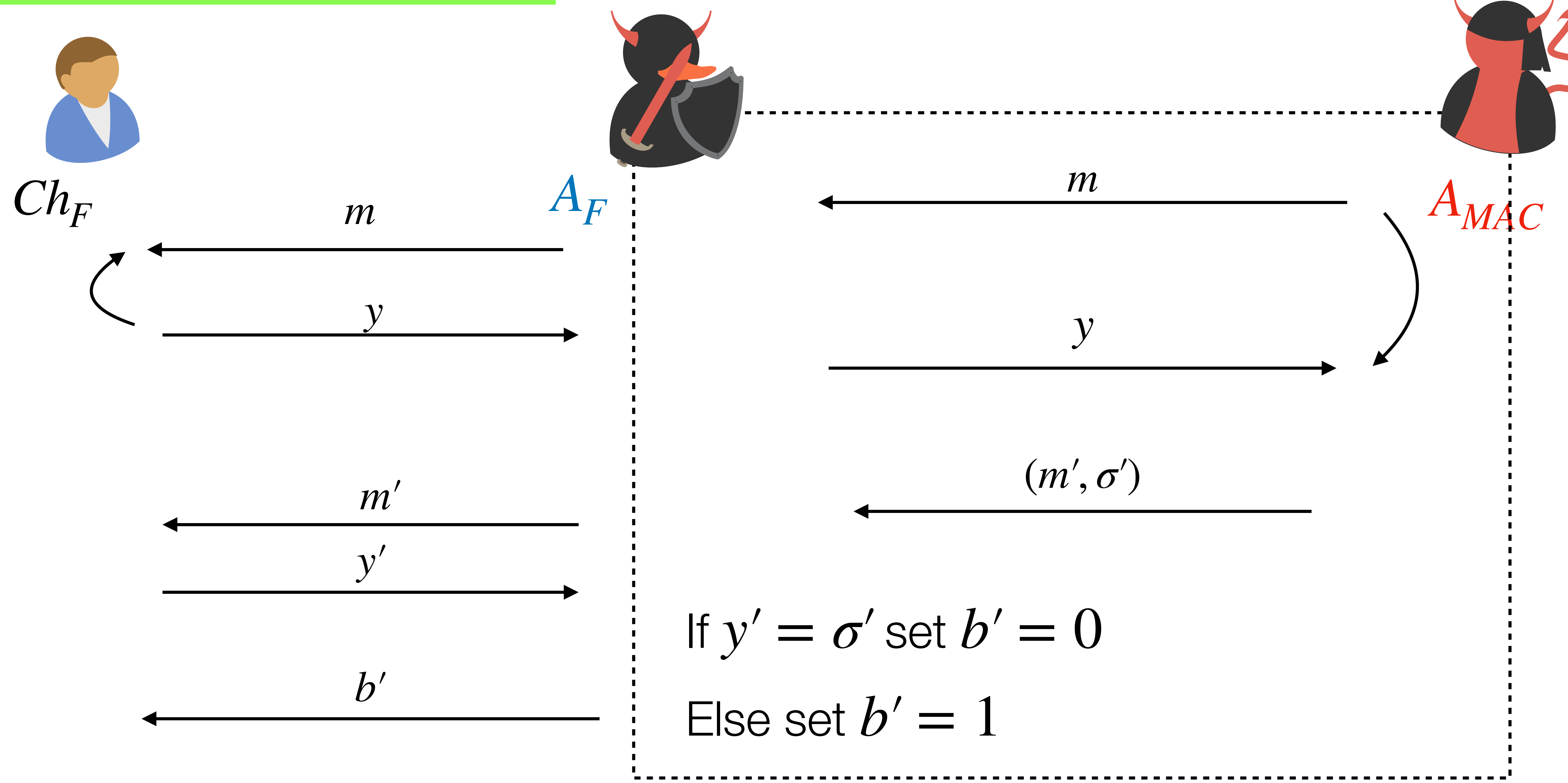

$\Pr[W_0] = \Pr[\mathscr{A}_F \text{ outputs } 0 \text{ in } \mathsf{Game}_0]$

$= \Pr[\mathscr{A}_{MAC} \text{ wins } H_0]$

Proof of Security

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

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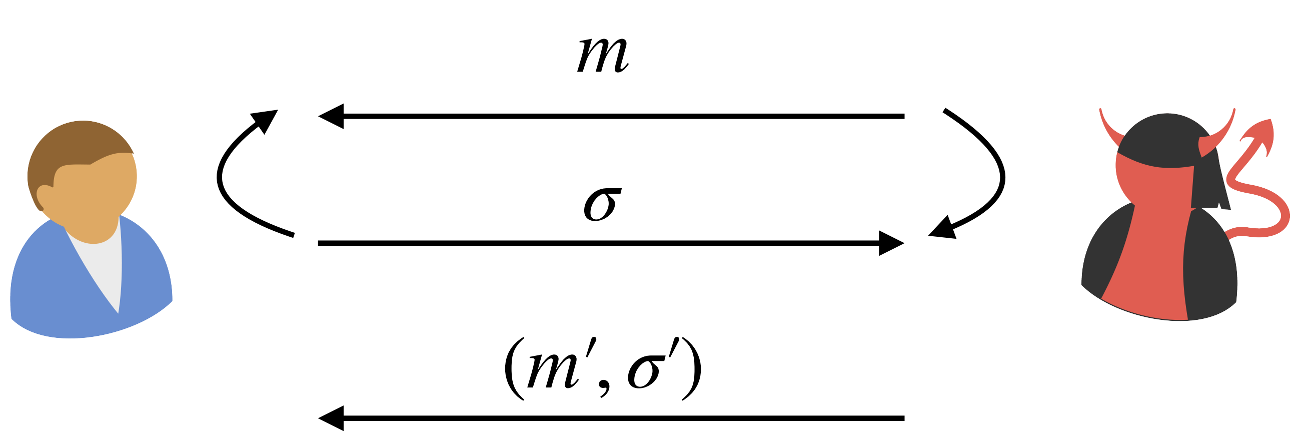

Wins if $F_k(m') = \sigma'$ and $\mathscr{A}$ never queried m'

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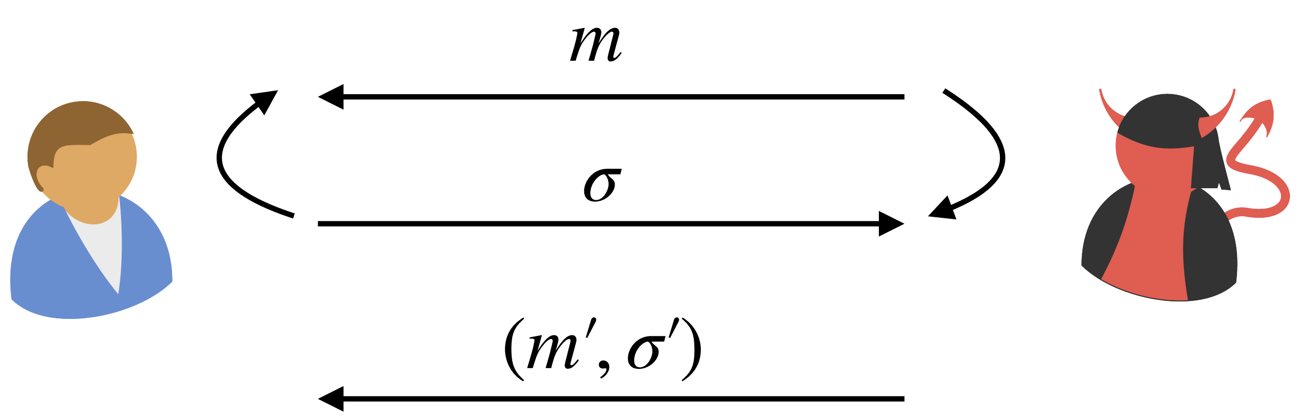
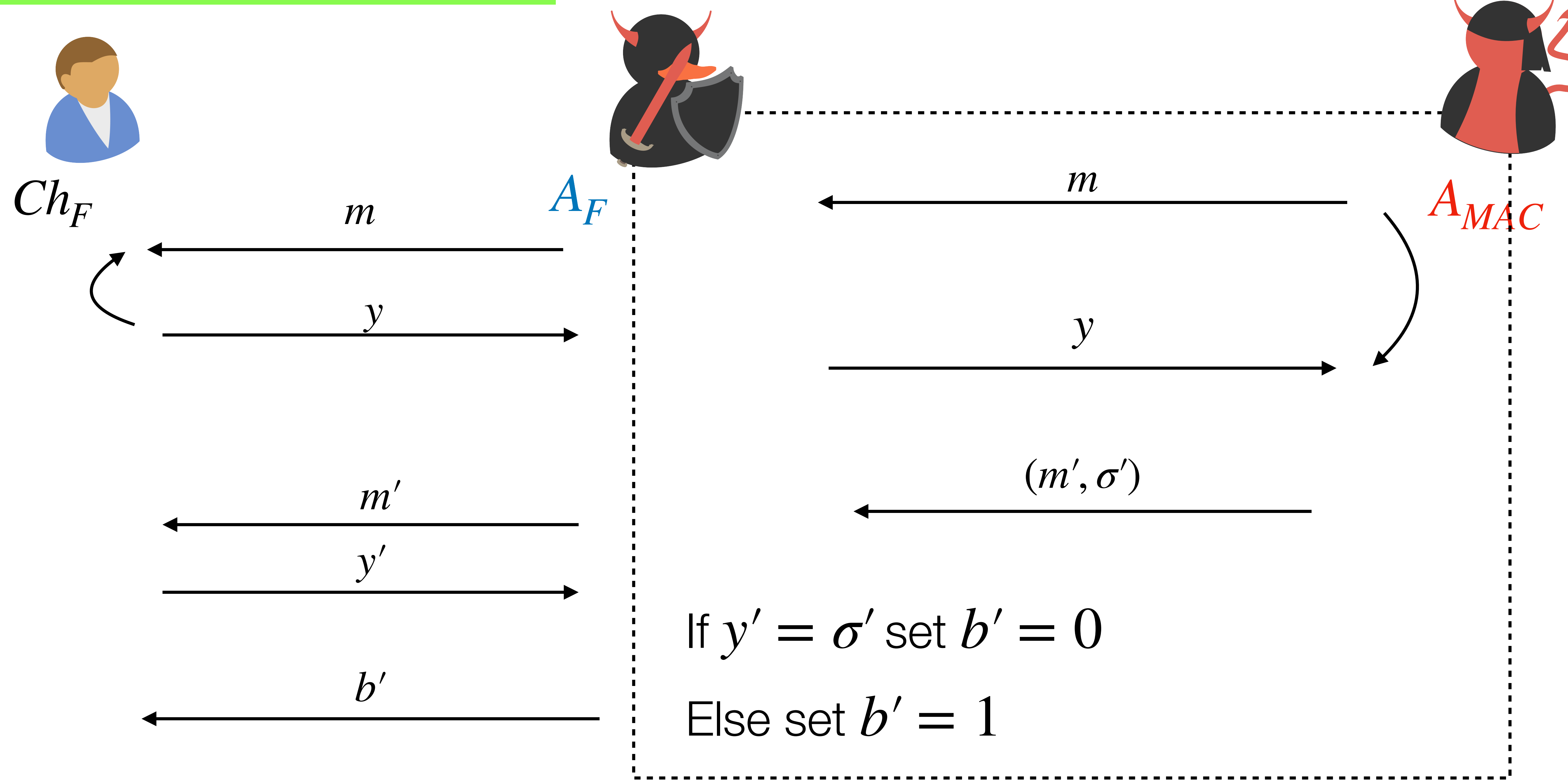
H_1

$\sigma := T[m]$



Wins if $T[m'] = \sigma'$ and $\mathscr{A}$ never queried m'



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H_0

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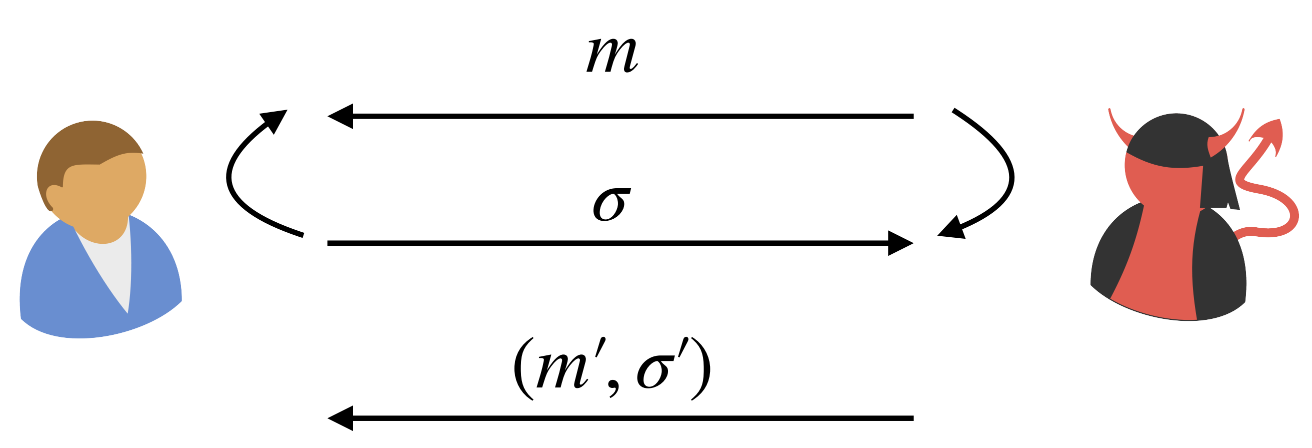

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Claim:
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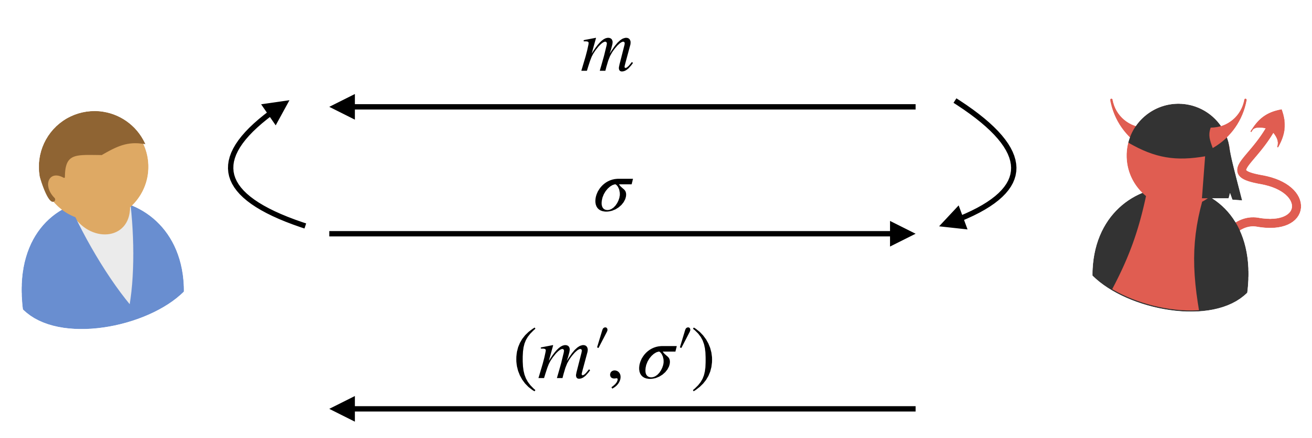
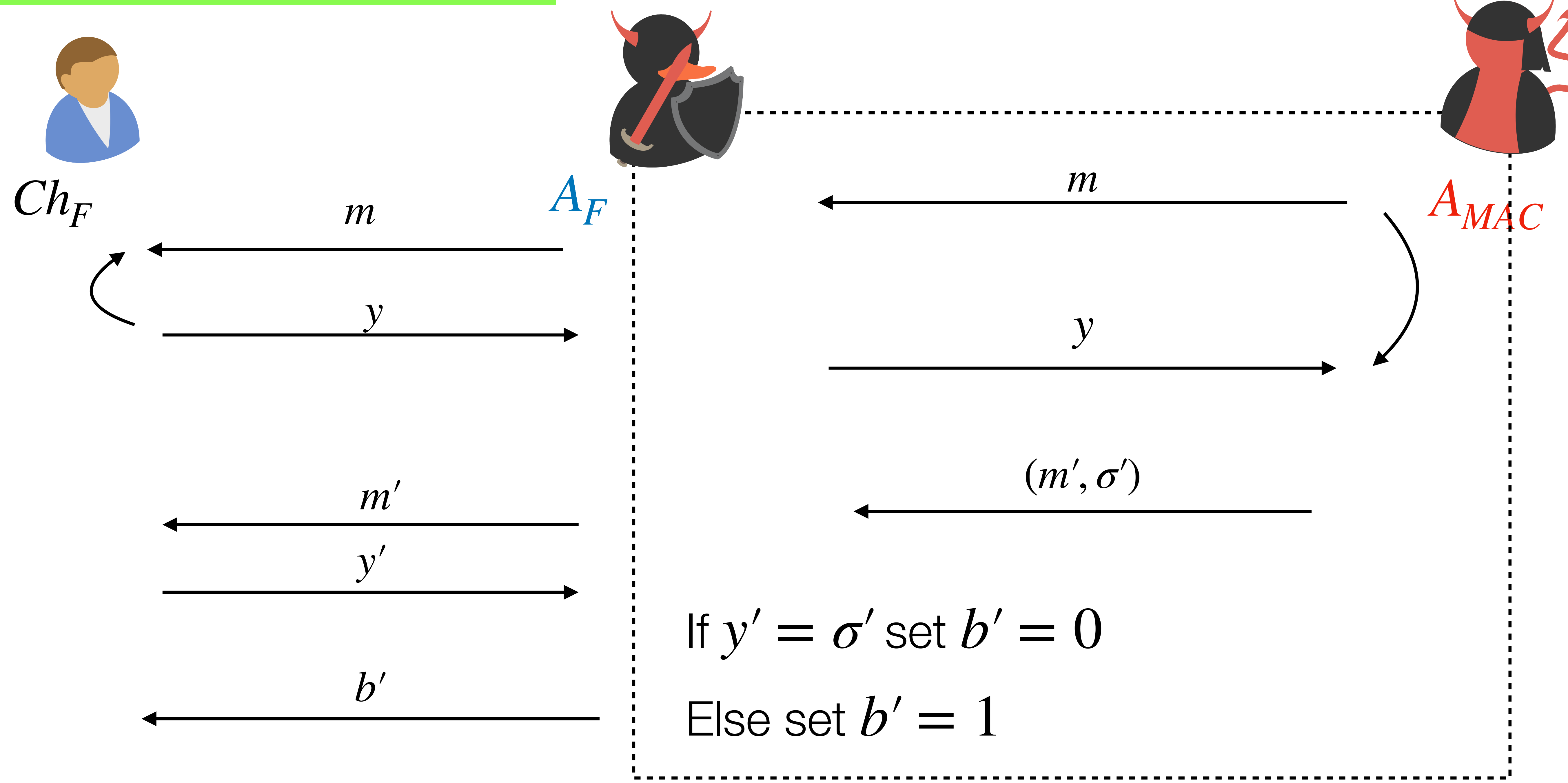
H_1

$\sigma := T[m]$



Wins if $T[m'] = \sigma'$ and $\mathscr{A}$ never queried m'



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$\Pr[W_1] = \Pr[\mathscr{A}_F \text{ outputs } 0 \text{ in } \mathsf{Game}_1]$

Proof of Security

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Wins if $F_k(m') = \sigma'$ and $\mathscr{A}$ never queried m'

H_0

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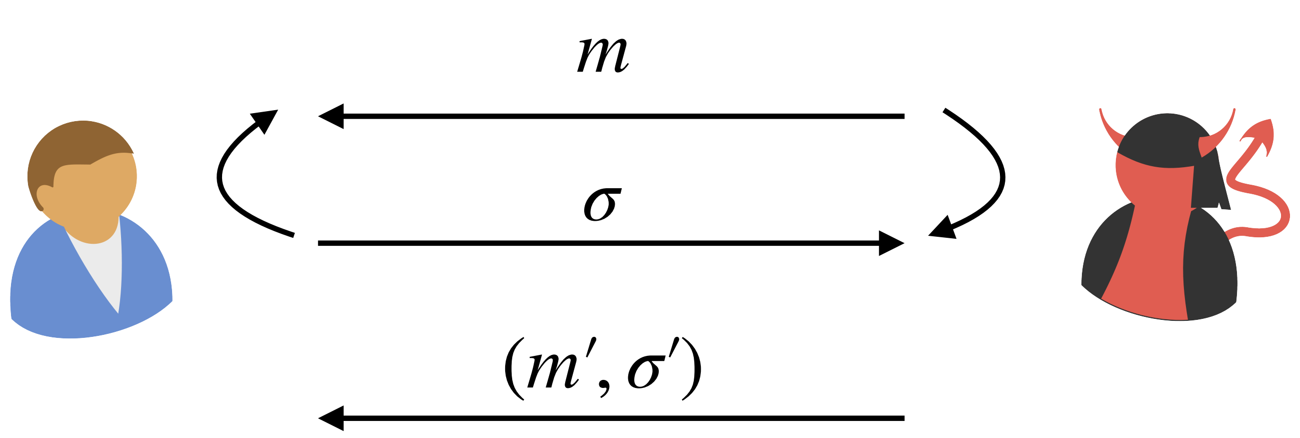
$\sigma := F_k(m)$



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H_1

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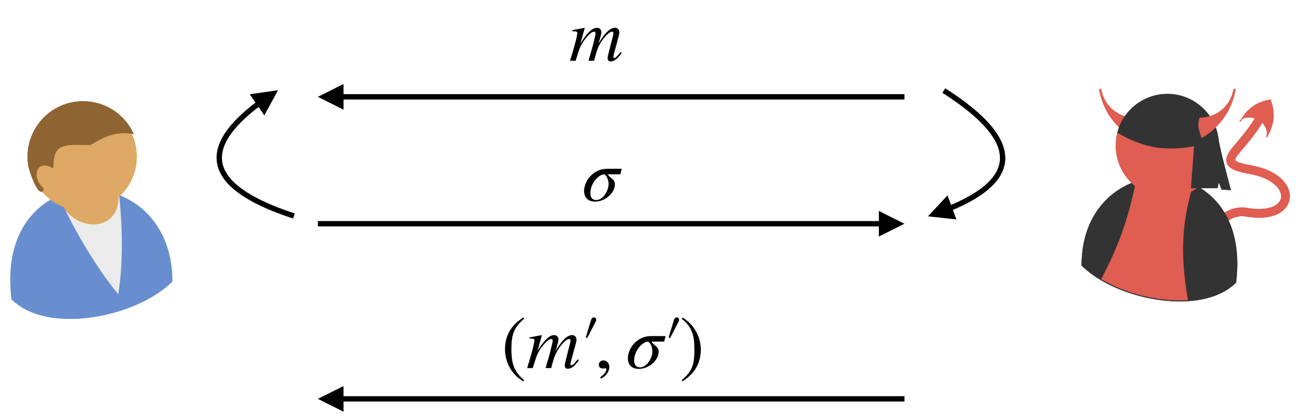

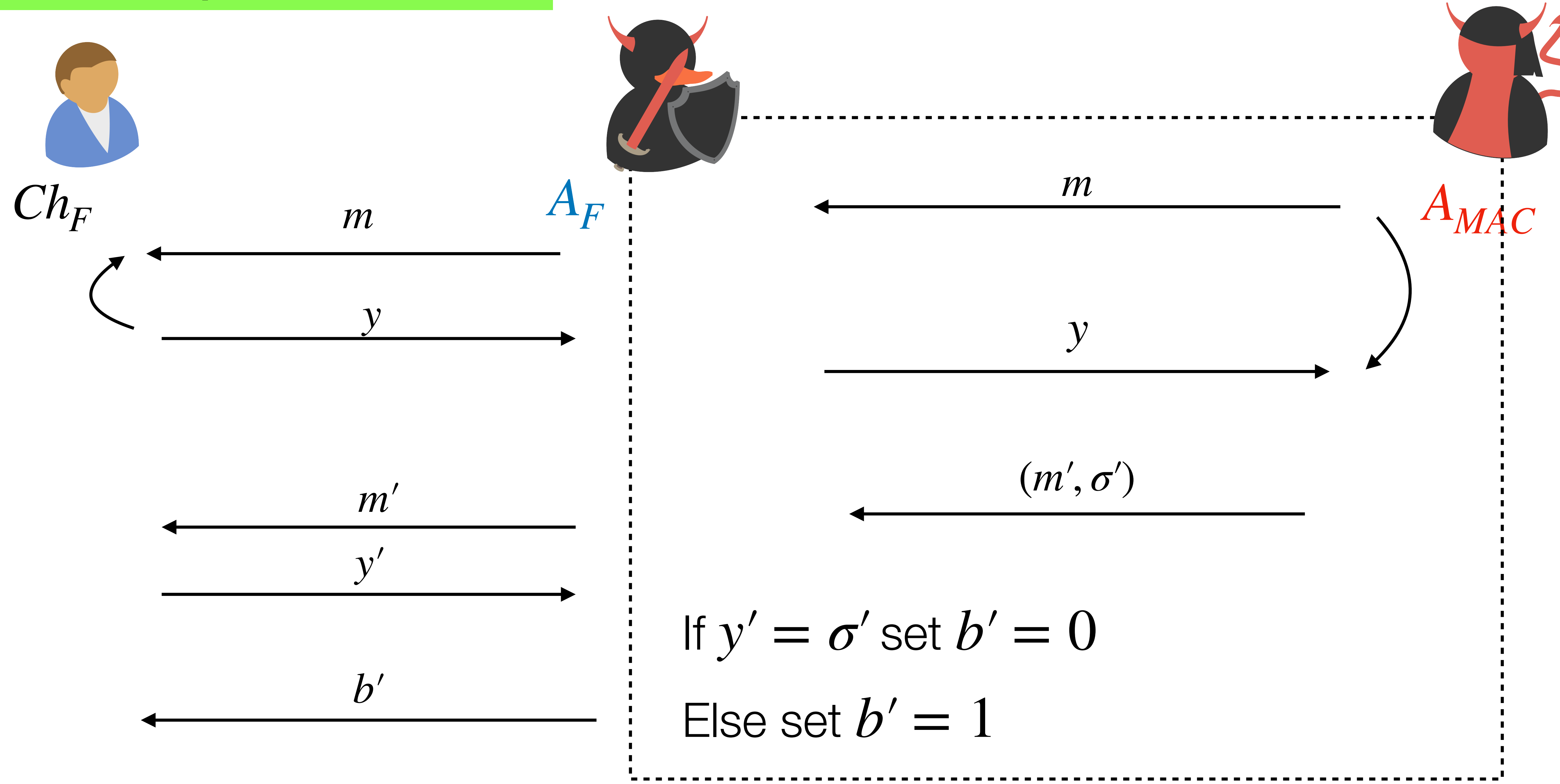

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Proof of Security

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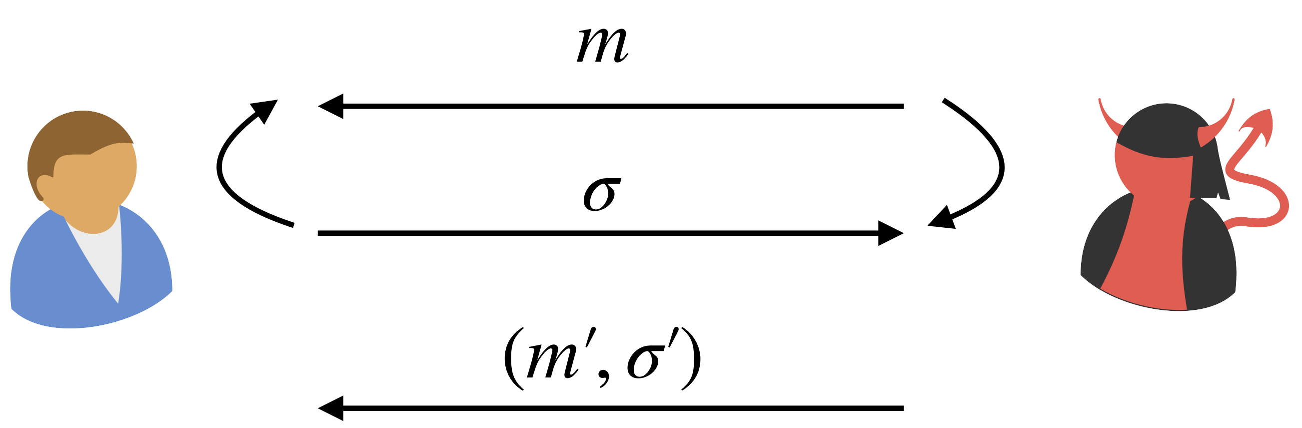

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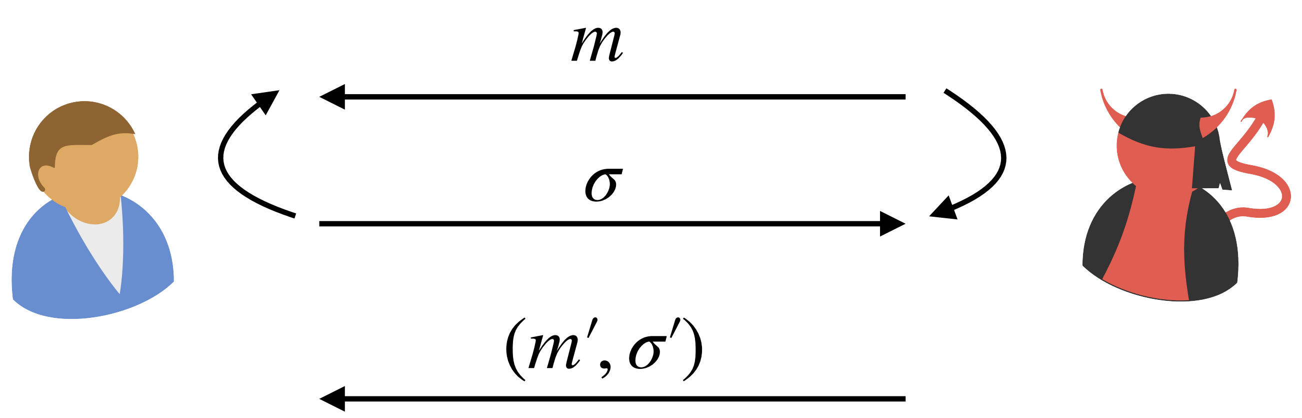

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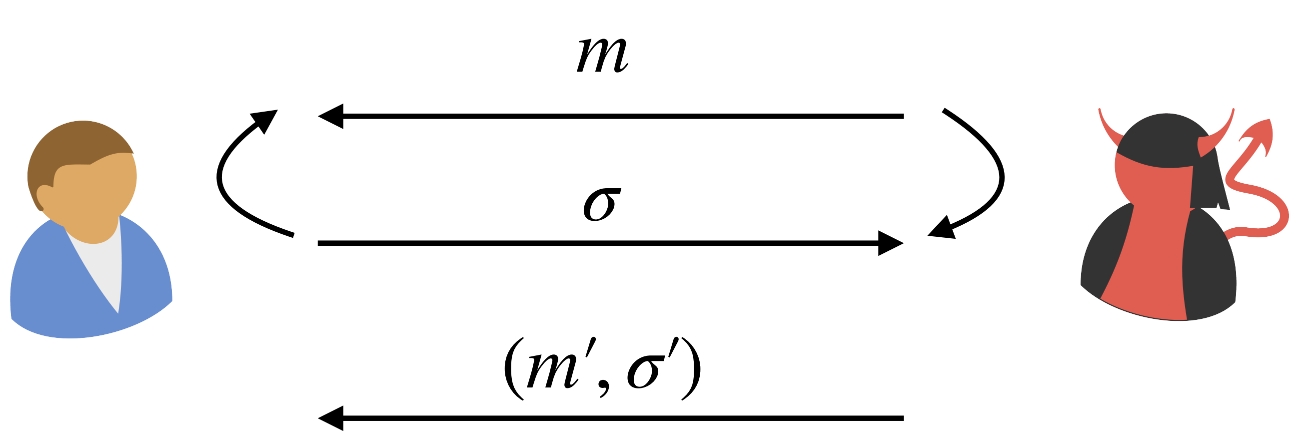

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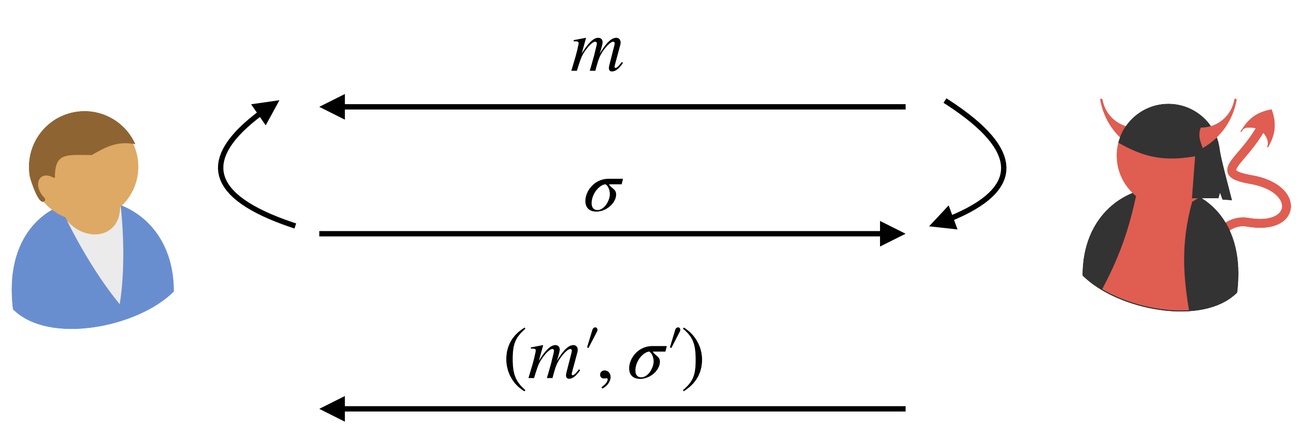

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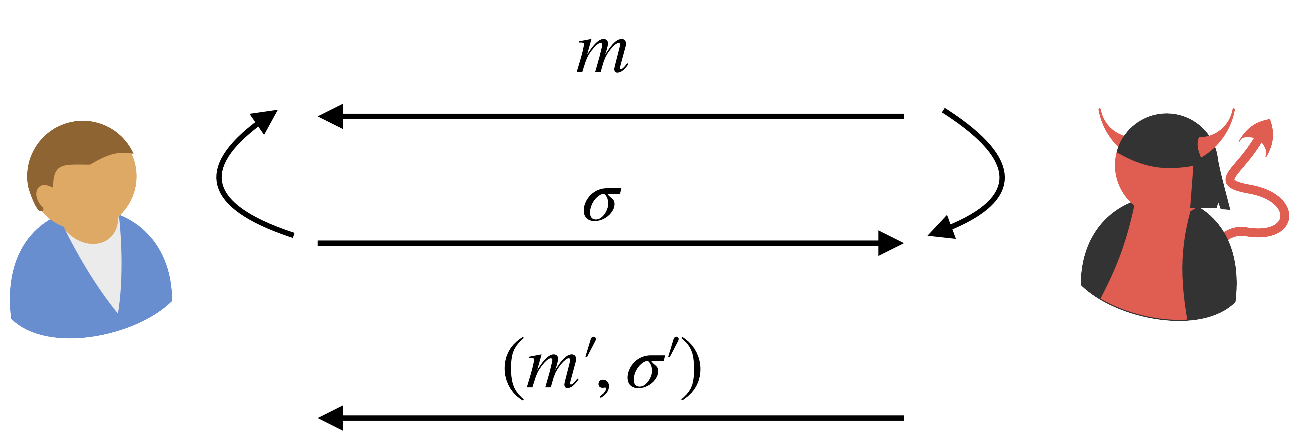

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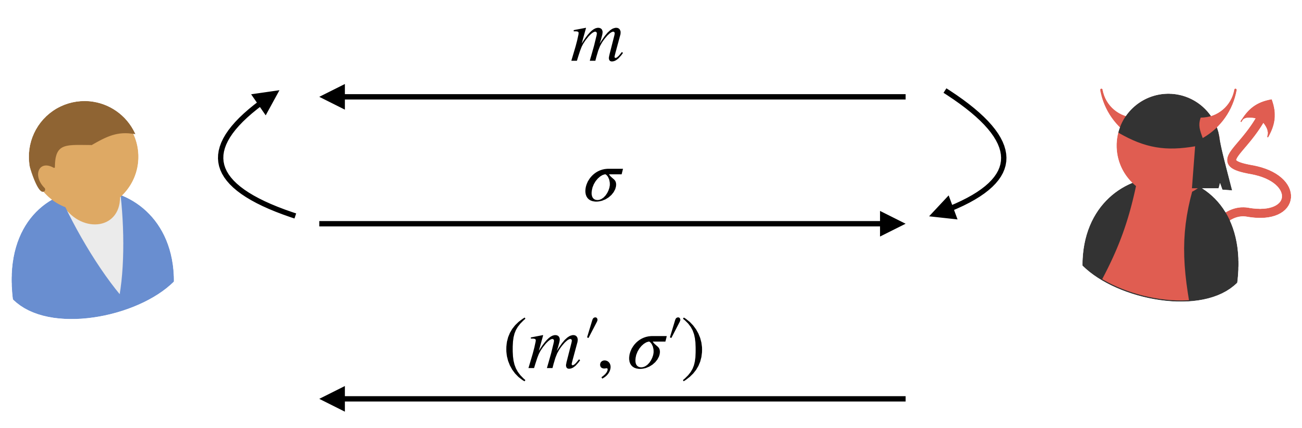

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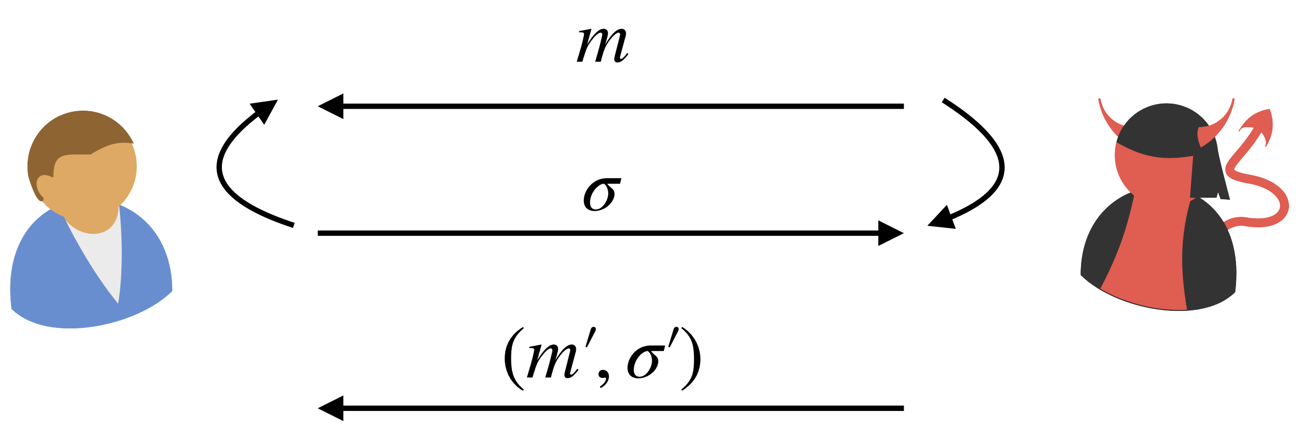

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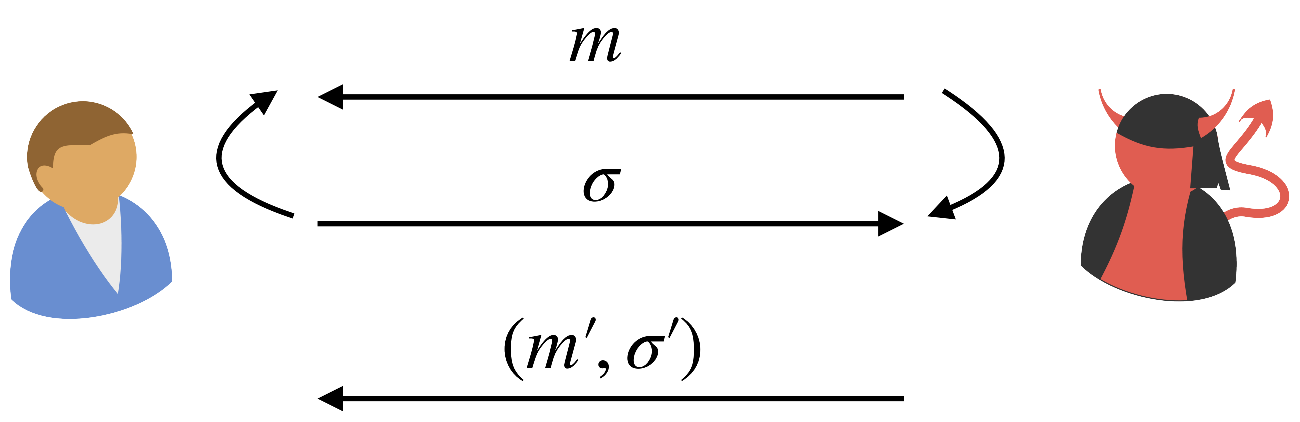
H_1

$\sigma := T[m]$



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Proof:

- Remember: T is just a table evaluating a random function!

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

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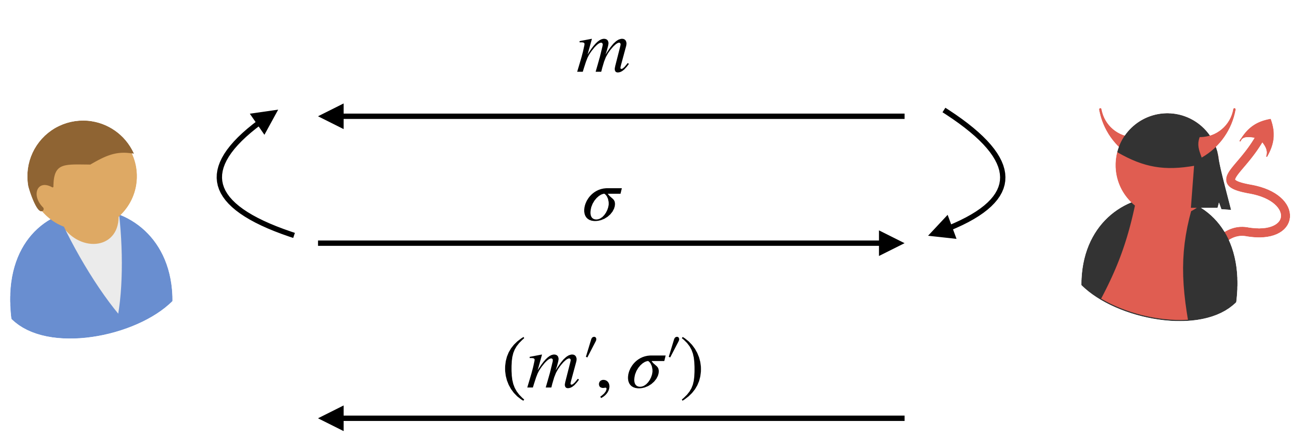

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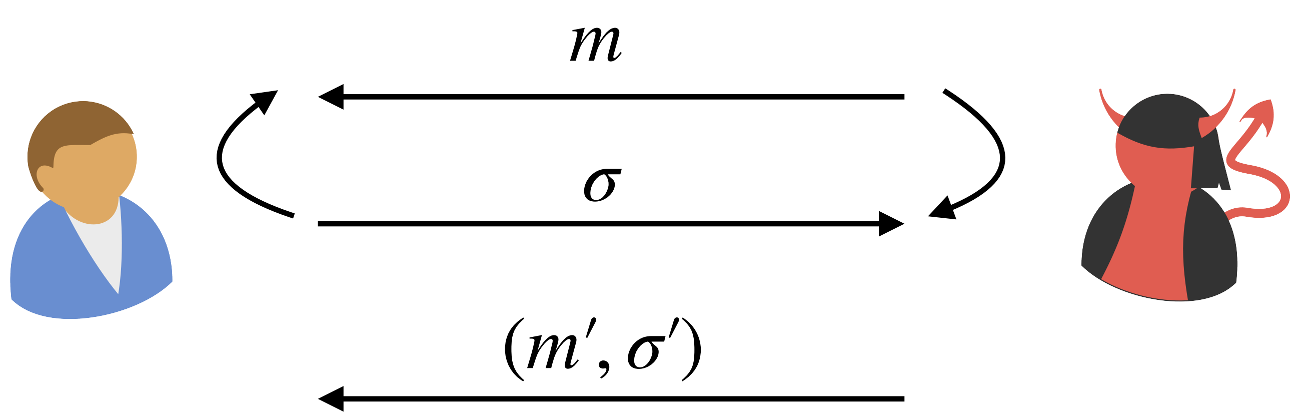

Wins if $T[m'] = \sigma'$ and $\mathscr{A}$ never queried m'

Claim: $\Pr[\mathscr{A} \text{ wins in } H_1] \leq \mathsf{negl}(\lambda)$

Proof:

- Remember: T is just a table evaluating a random function!
- So, $\Pr[T[m'] = \sigma'] = \dfrac{1}{2^\lambda} = \mathsf{negl}(\lambda)$

Proof of Security

KeyGen(1^λ) : $k \xleftarrow{\$} \{0,1\}^\lambda$

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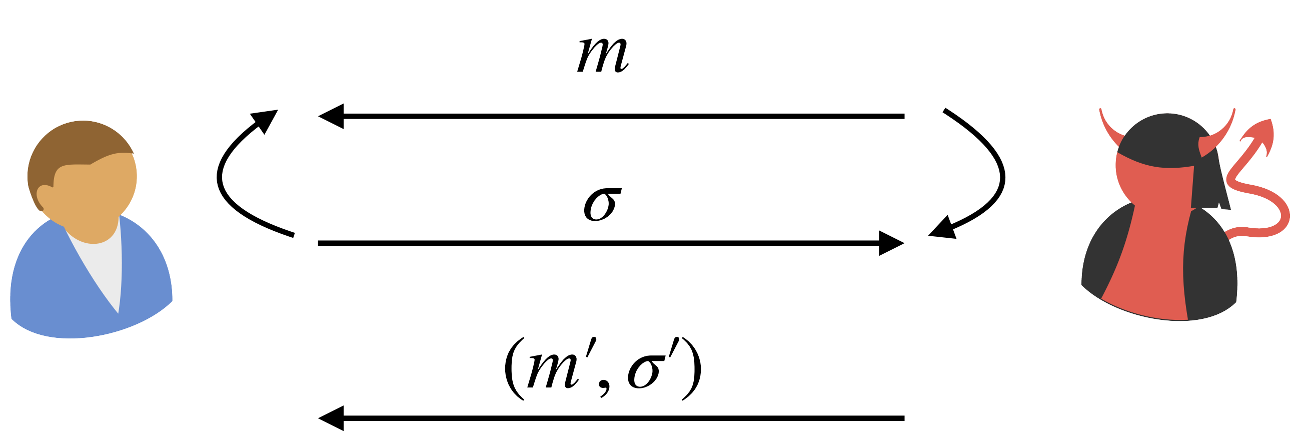

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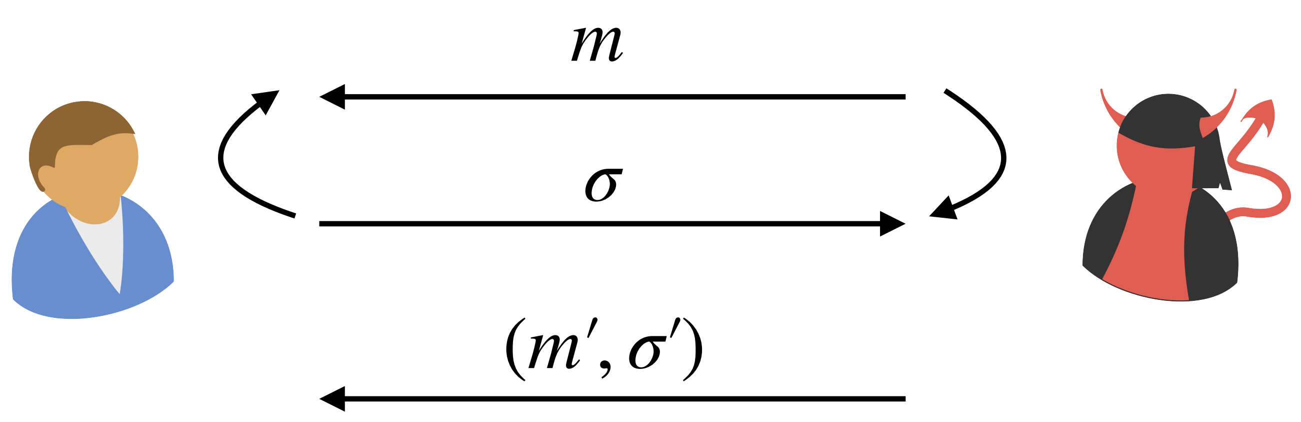

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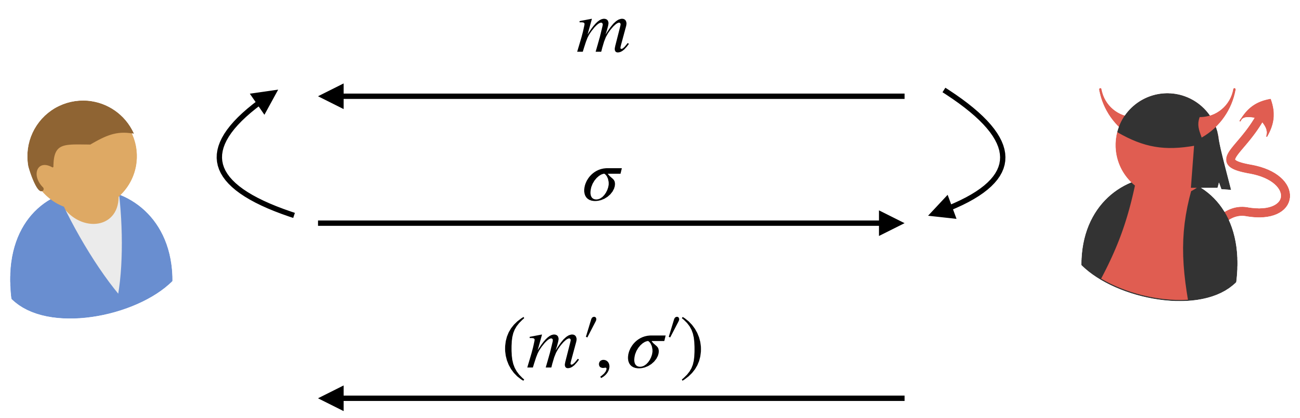

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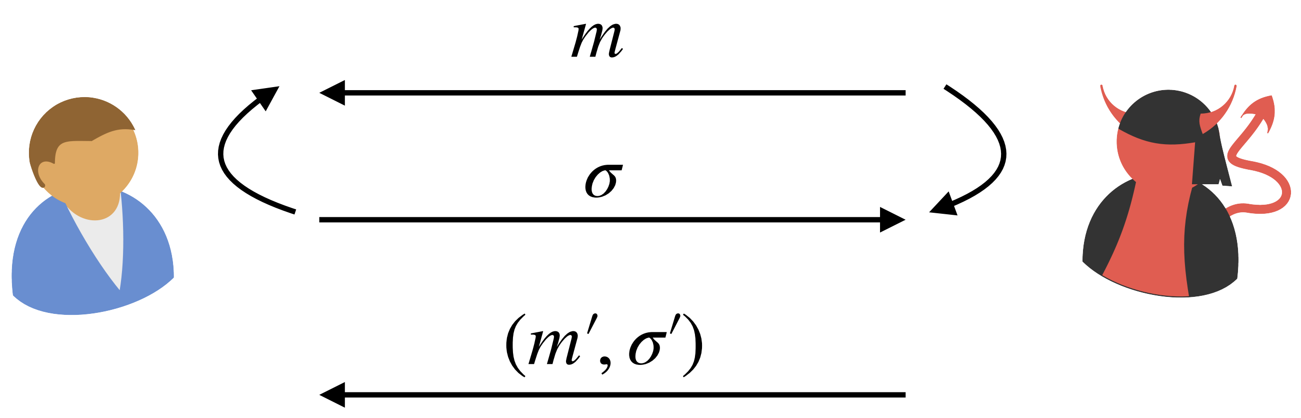

Wins if $T[m'] = \sigma'$ and $\mathscr{A}$ never queried m'

$$\Pr[\mathscr{A} \text{ wins in } H_1] \leq \mathsf{negl}(\lambda)$$

(Triangle inequality)

$$\Pr[\mathscr{A} \text{ wins in } H_0] \leq \mathsf{negl}(\lambda)$$

Hash Functions

$\mathsf{KeyGen}(1^\lambda) : k \xleftarrow{\$} \{0,1\}^\lambda$

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- Like with PRFs, we will consider *families* of hash functions (as otherwise non-uniform adversaries could just have collisions "built in").

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Collision-resistant Hash Function Family

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A deterministic family of functions $H = \{h_i : D_i \rightarrow R_i\}_{i \in I}$ is called a collision-resistant hash function family (CRHF) if it satisfies the following properties:

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$$\Pr\left[\begin{matrix} x \neq x' \wedge \\ h_i(x) = h_i(x') \end{matrix} : \begin{matrix} i \leftarrow \mathsf{Gen}(1^\lambda) \\ (x, x') \leftarrow \mathscr{A}(1^\lambda, i) \end{matrix}\right] \leq \mathsf{negl}(\lambda)$$

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$$\Pr\left[\mathscr{A} \text{ wins CRHFGame}\right] \leq \mathsf{negl}(\lambda)$$

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  - Unlikely to be constructed from OWF or OWP
  - Can be constructed from assumptions like factoring and discrete log (on their own!)

MAC Construction II

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Hash-and-MAC

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Let λ be the security parameter and, Let (KeyGen, Tag, Ver) be a MAC scheme with message space $\mathscr{M}$, and let $H_i = \{h_i : D \to \mathscr{M}\}$ be a CRHF family

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Proof of Security

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Proof of Security

H_0

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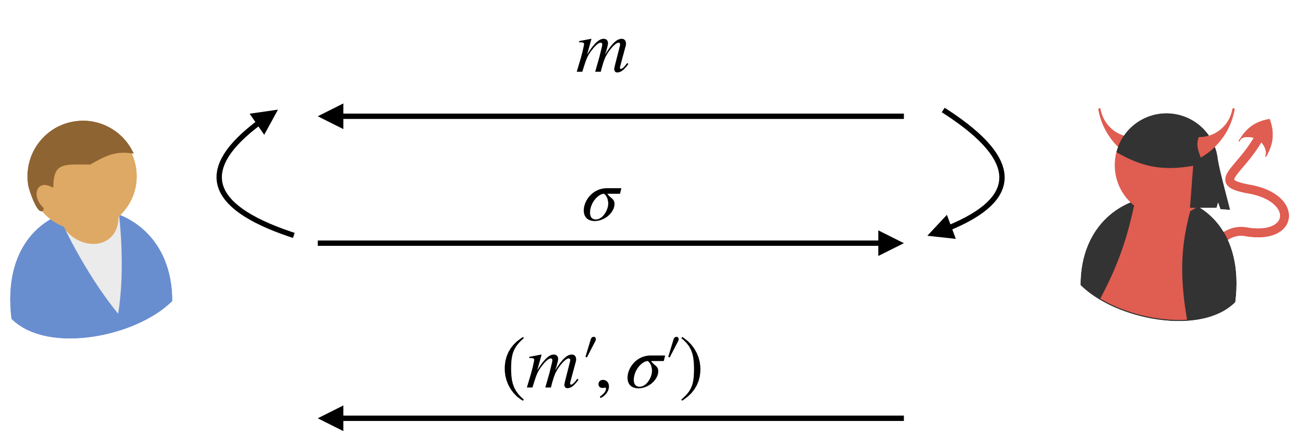

Wins if
$\textsf{Ver}(k, h_i(m'), \sigma') = 1$
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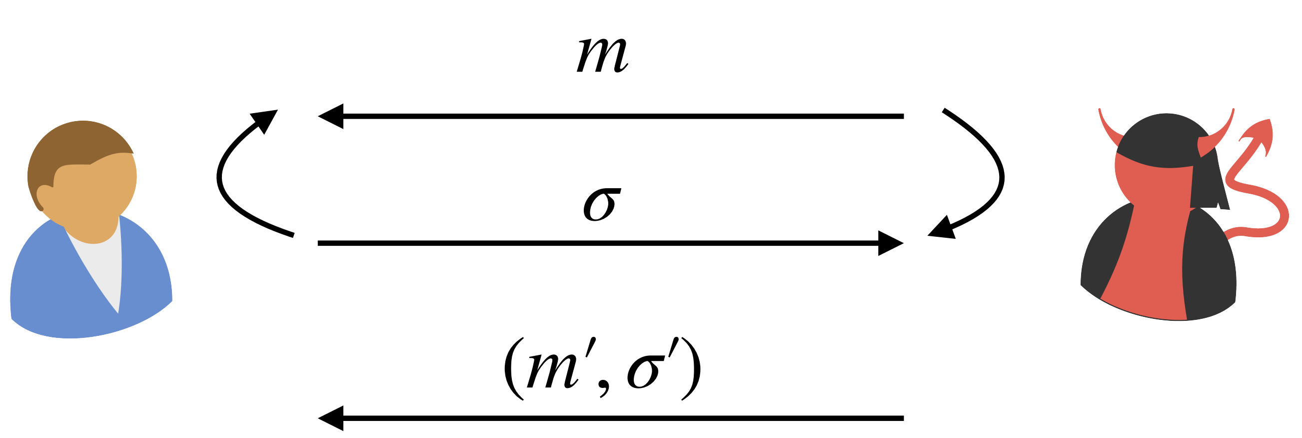

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Claim: If $\{h_i\}$ is a secure family of CHRFs **and** (KeyGen, Tag, Ver) is a secure MAC, then **Pr**$[\mathscr{A}$ wins in $H_0] \leq$ **negl**(λ)

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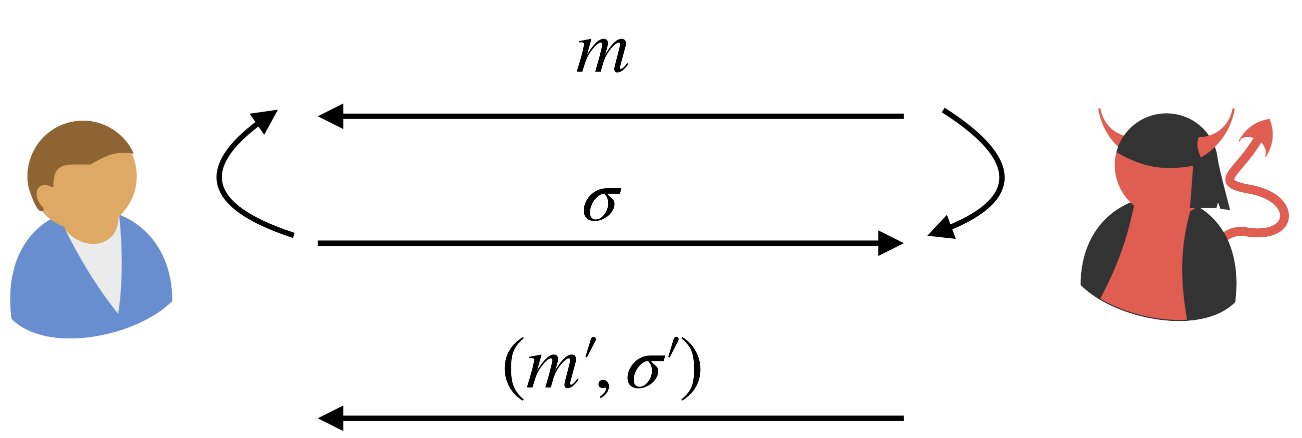

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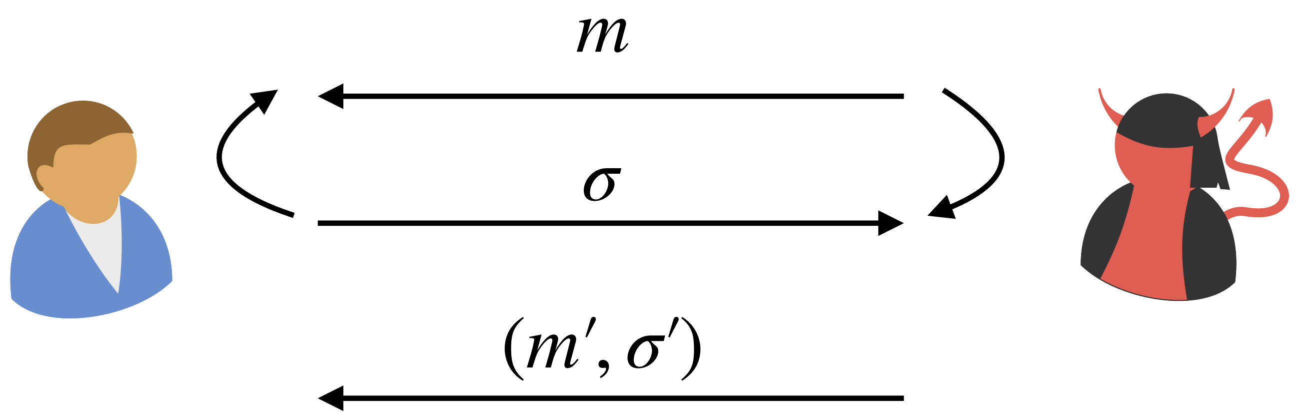

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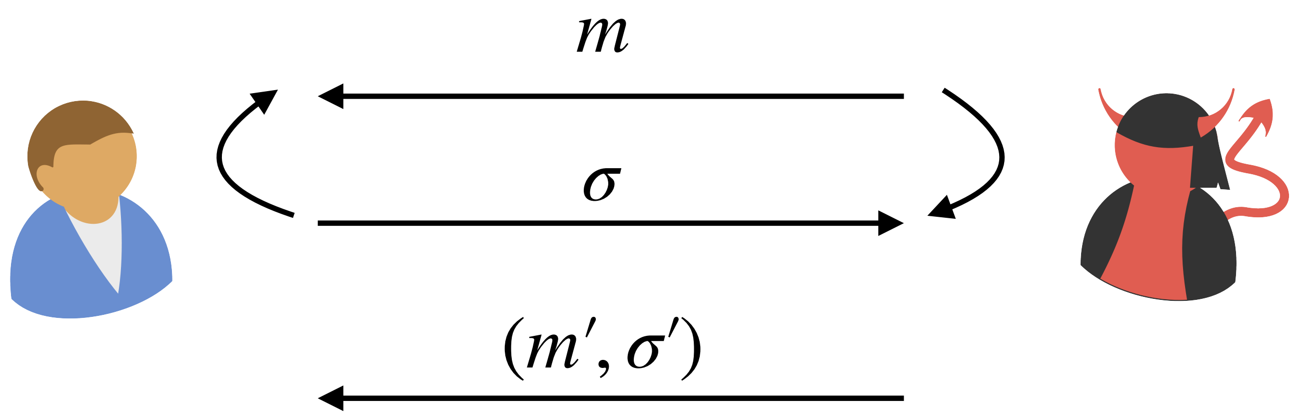

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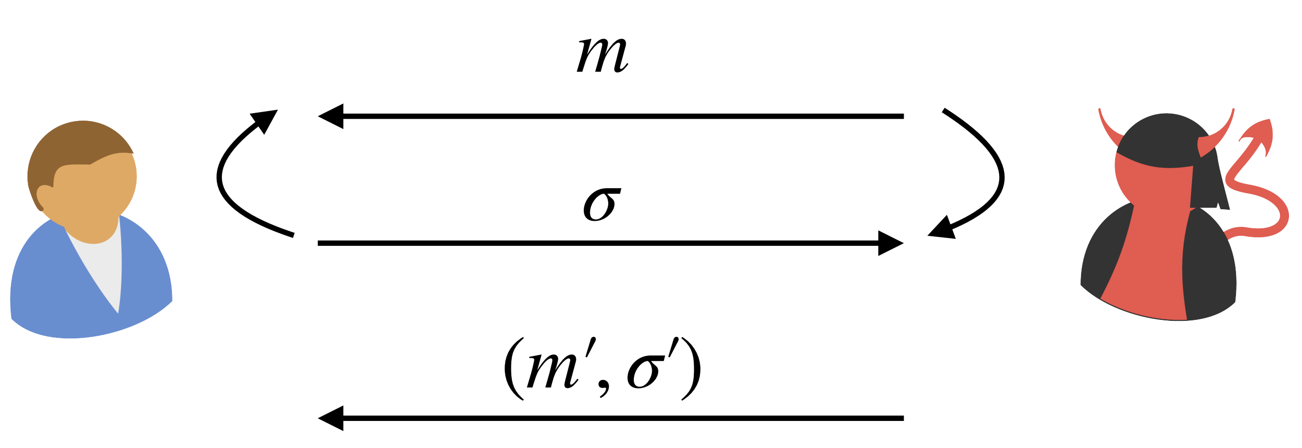

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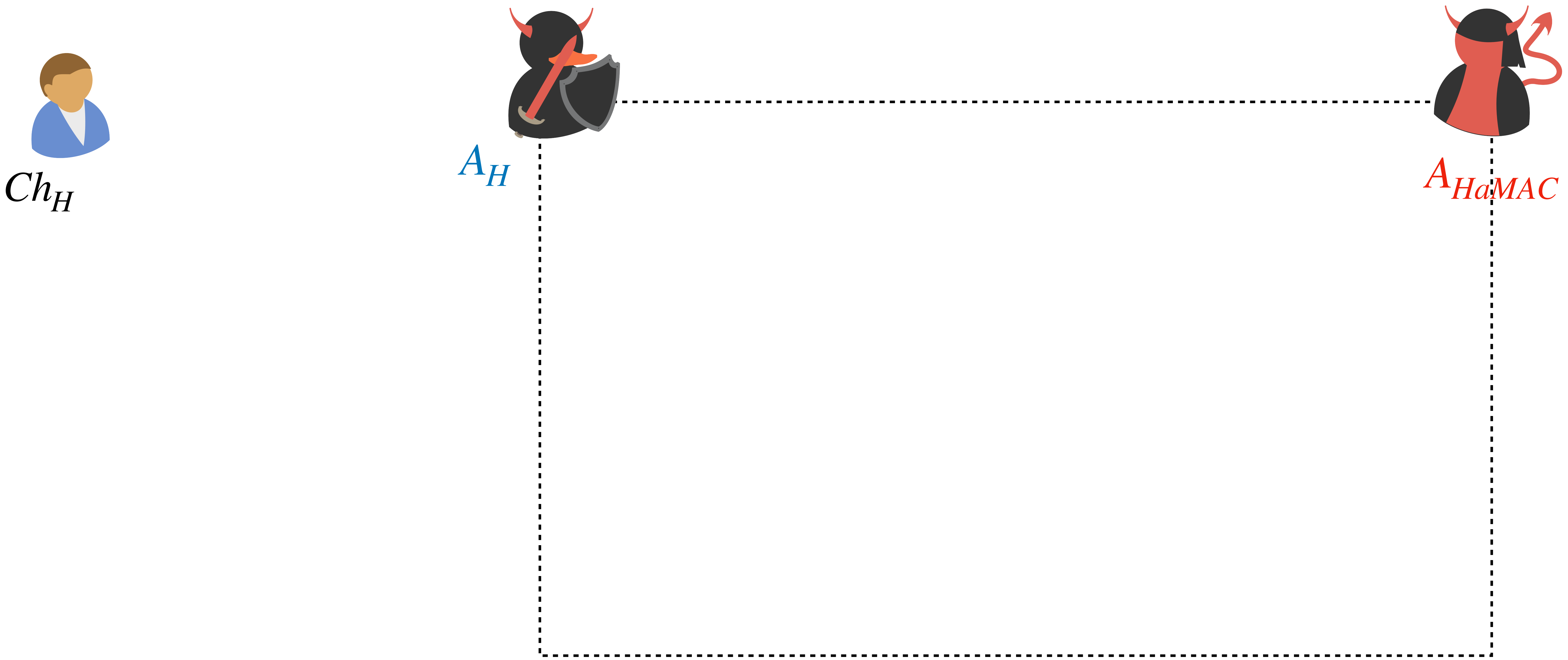

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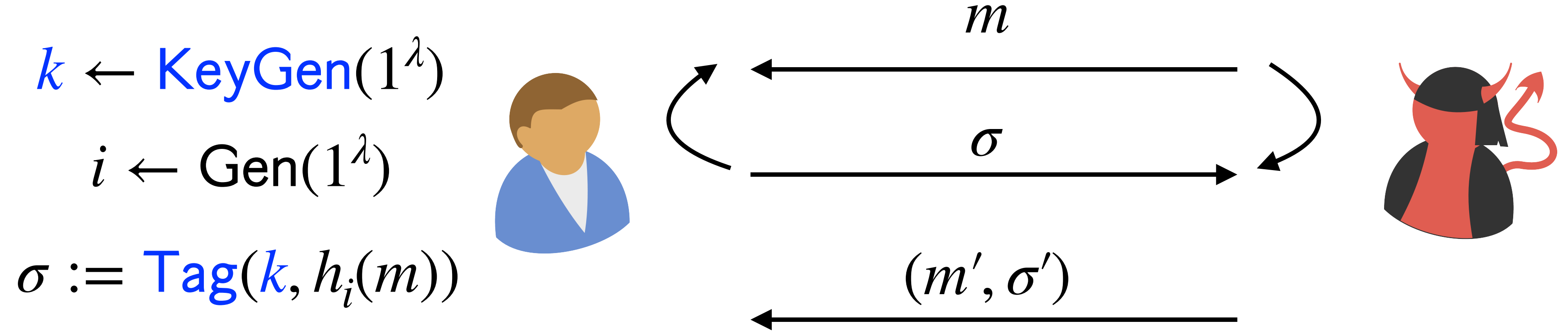

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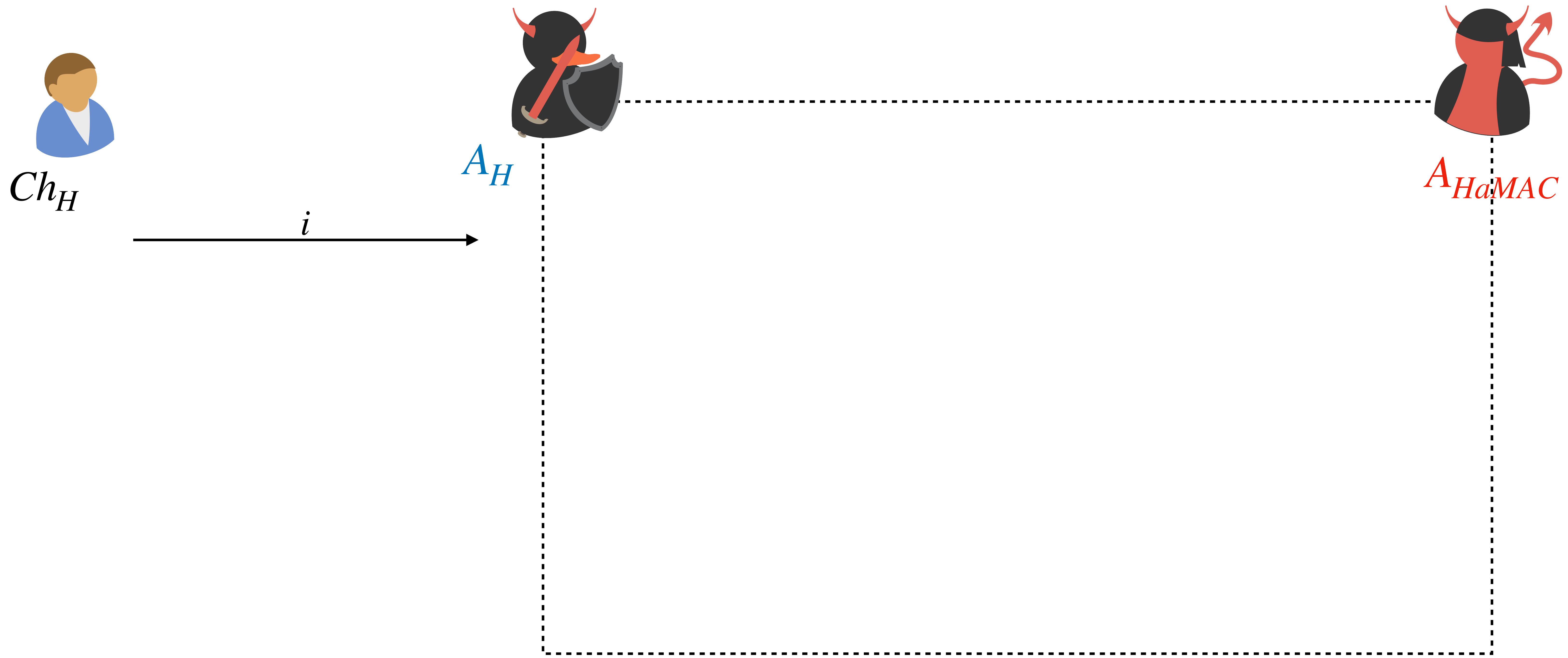

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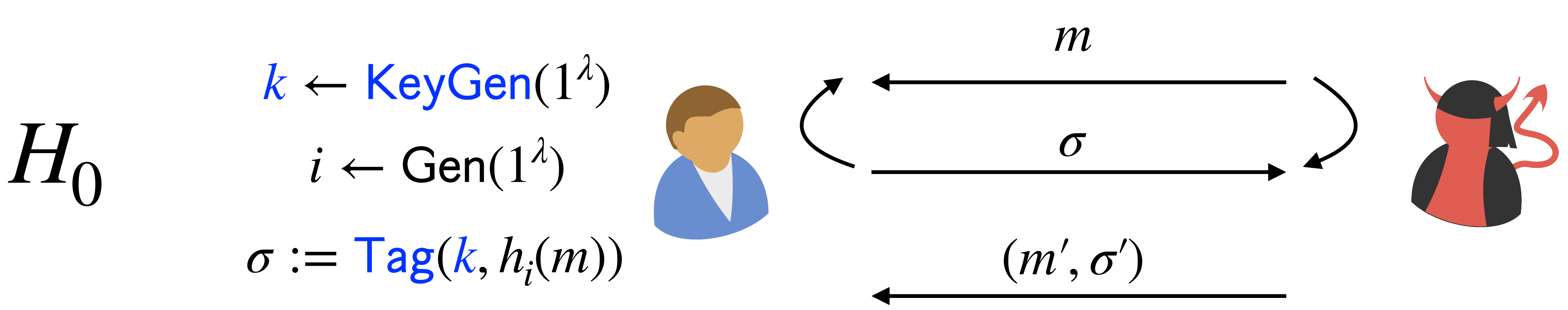

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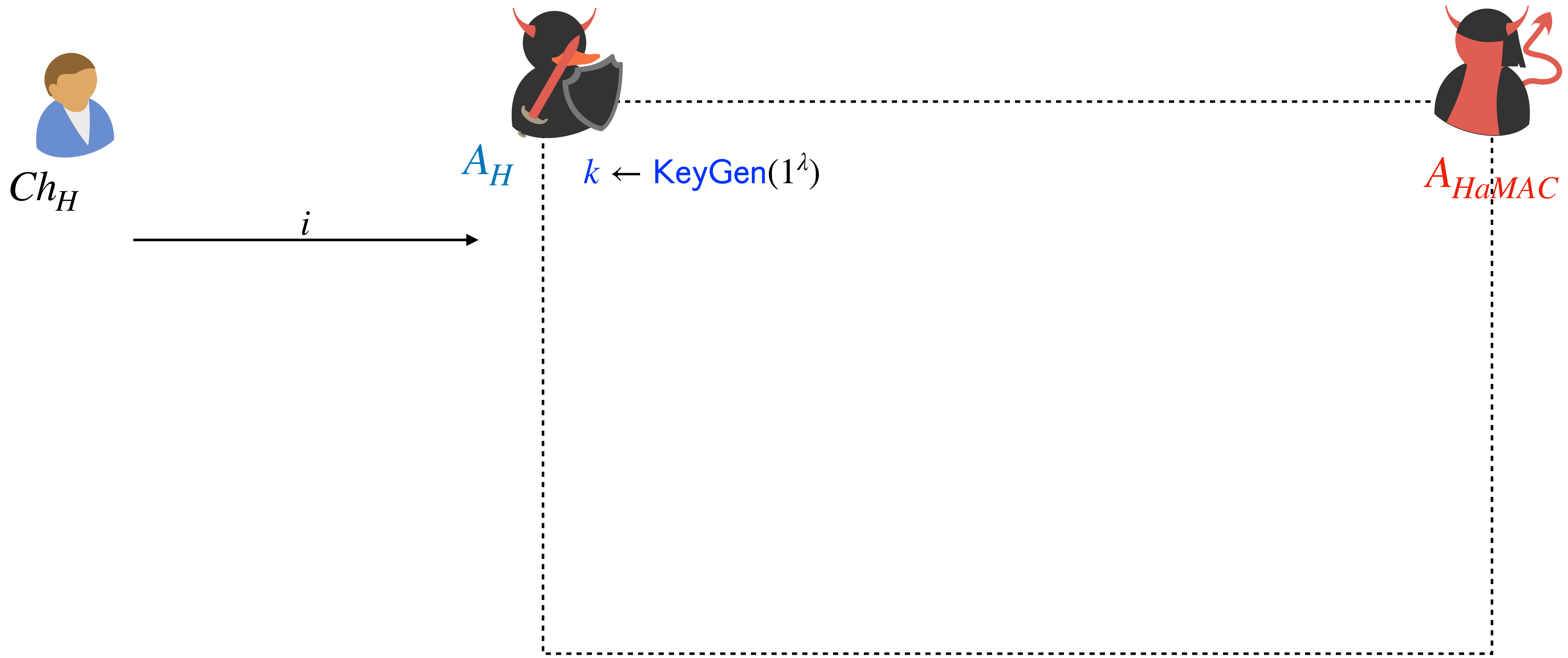

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m

σ

(m', σ')

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Ch_H

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A_H

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A_{HaMAC}

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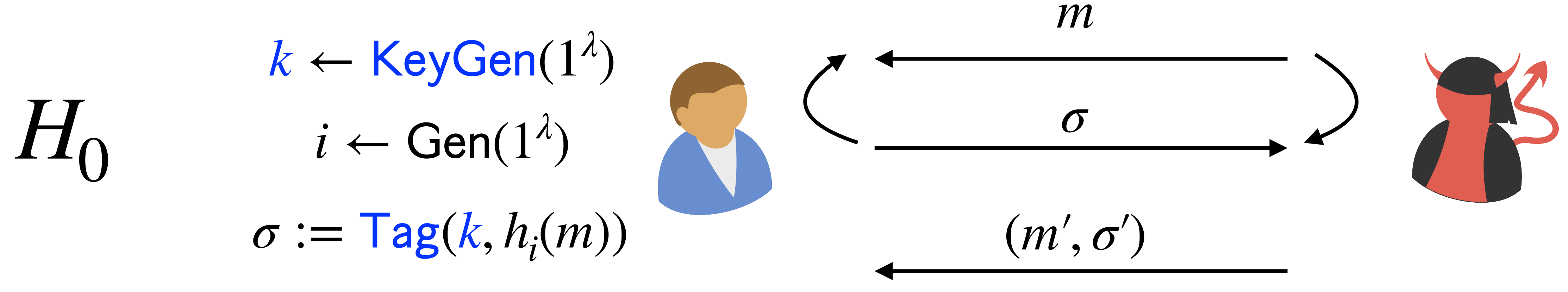

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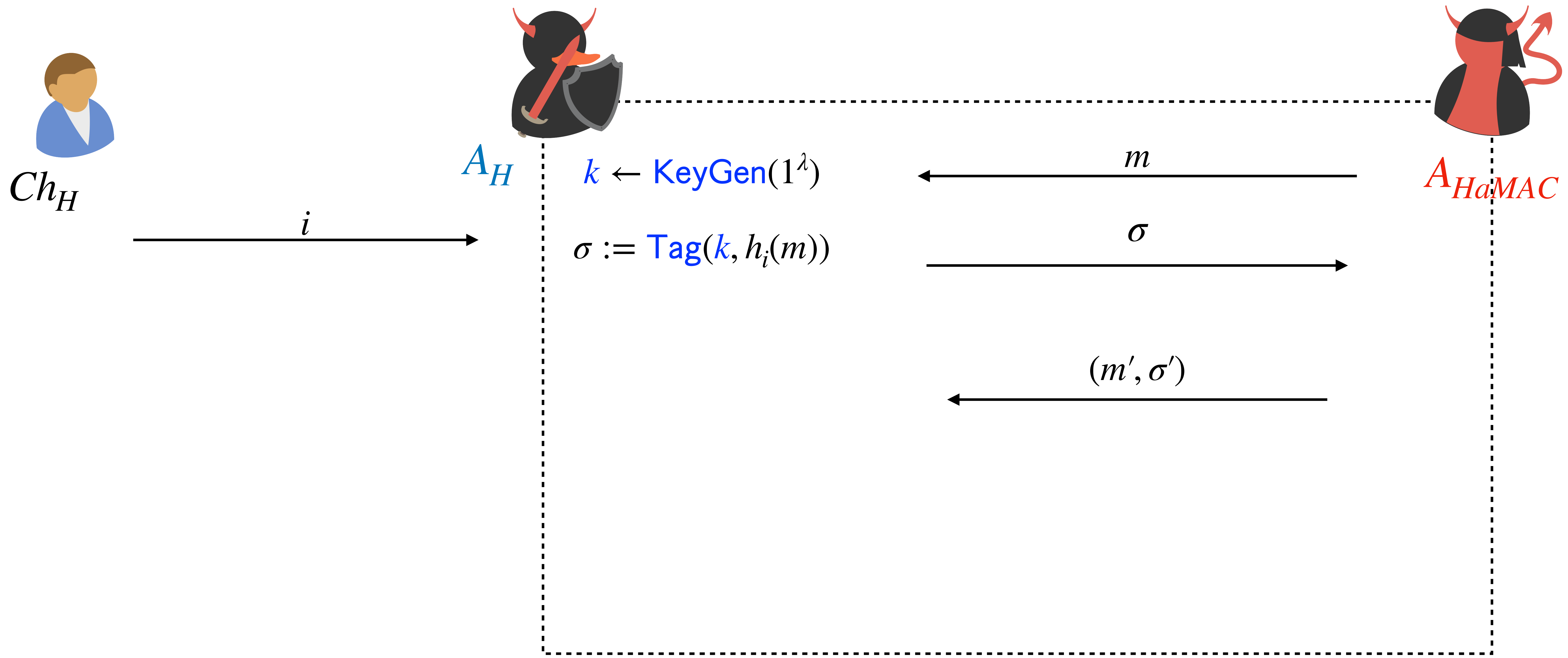

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Ch_H

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(m', σ')

If $\exists m$ such that $h_i(m') = h_i(m)$ return (m, m')

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : (k \leftarrow \mathsf{KeyGen}(1^\lambda), i)$

$\mathsf{Tag}(k, m) : \sigma := \mathsf{Tag}(k, h_i(m))$

$\mathsf{Ver}(k, m, \sigma) : \mathsf{Ver}(k, h_i(m), \sigma)$

H_0

$k \leftarrow \mathsf{KeyGen}(1^\lambda)$

$i \leftarrow \mathsf{Gen}(1^\lambda)$

$\sigma := \mathsf{Tag}(k, h_i(m))$

m

σ

(m', σ')

Wins if $\mathsf{Ver}(k, h_i(m'), \sigma') = 1$ and $\mathscr{A}$ never queried m'

Let **col** be the event that $h(m') = h(m)$ for some $m \neq m'$

Claim: $\Pr\left[\mathsf{col}\right] \leq \mathsf{negl}(\lambda)$

Ch_H

i

A_H

A_{HaMAC}

$k \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\sigma := \mathsf{Tag}(k, h_i(m))$

m

σ

(m', σ')

If $\exists m$ such that $h_i(m') = h_i(m)$ return (m, m')

Else return $(0,0)$

Proof of Security

$\mathsf{KeyGen}(1^\lambda) : (k \leftarrow \mathsf{KeyGen}(1^\lambda), i)$

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A_{HaMAC}

i

(m, m')

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m

σ

(m', σ')

If $\exists m$ such that $h_i(m') = h_i(m)$ return (m, m')

Else return $(0,0)$

Proof of Security

H_0

$k \leftarrow$ KeyGen(1^λ)

$i \leftarrow$ Gen(1^λ)

$\sigma :=$ Tag$(k, h_i(m))$

m ←

σ →

(m', σ') ←

Wins if
Ver$(k, h_i(m'), \sigma') = 1$
and $\mathscr{A}$ never
queried m'

KeyGen(1^λ) : $(k \leftarrow$ KeyGen$(1^\lambda), i)$

Tag(k, m) : $\sigma :=$ Tag$(k, h_i(m))$

Ver(k, m, σ) : Ver$(k, h_i(m), \sigma)$

Claim: Pr $\left[\text{col}\right] \leq$ negl(λ)

Let **col** be the event that $h(m') = h(m)$ for some $m \neq m'$

Ch_H

i →

(m, m') ←

A_H

$k \leftarrow$ KeyGen(1^λ)

$\sigma :=$ Tag$(k, h_i(m))$

If $\exists m$ such that $h_i(m') = h_i(m)$ return (m, m')

Else return $(0,0)$

A_{HaMAC}

m ←

σ →

(m', σ') ←

Pr[col] = Pr[$\mathscr{A}_H$ wins] $\leq$ negl(λ)

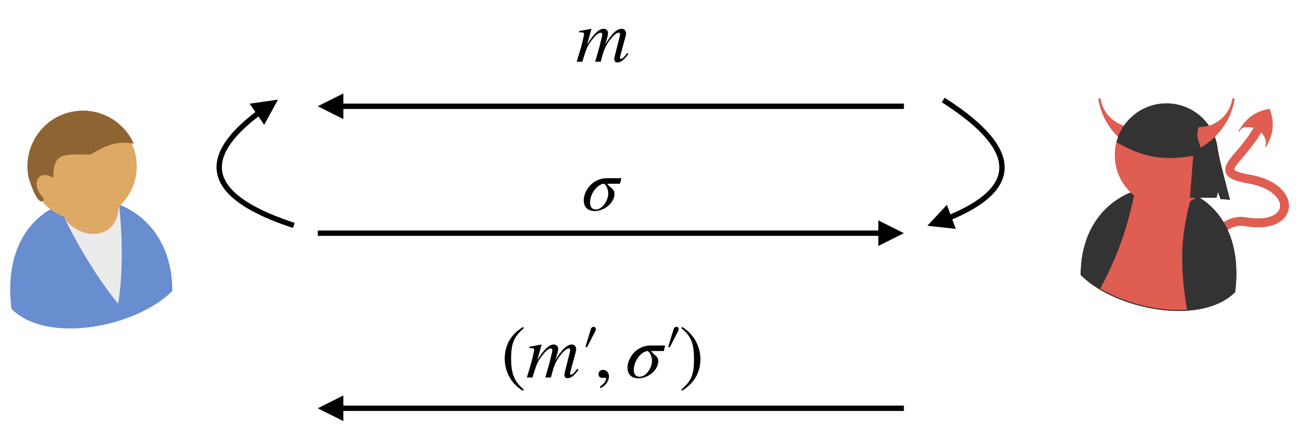
Proof of Security

H_0

$k \leftarrow \text{KeyGen}(1^\lambda)$

$i \leftarrow \text{Gen}(1^\lambda)$

$\sigma := \text{Tag}(k, h_i(m))$



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Let **col** be the event that $h(m') = h(m)$ for some $m \neq m'$

$$\Pr\left[\text{col}\right] \leq \text{negl}(\lambda)$$

Proof of Security

H_0

$k \leftarrow \text{KeyGen}(1^\lambda)$

$i \leftarrow \text{Gen}(1^\lambda)$

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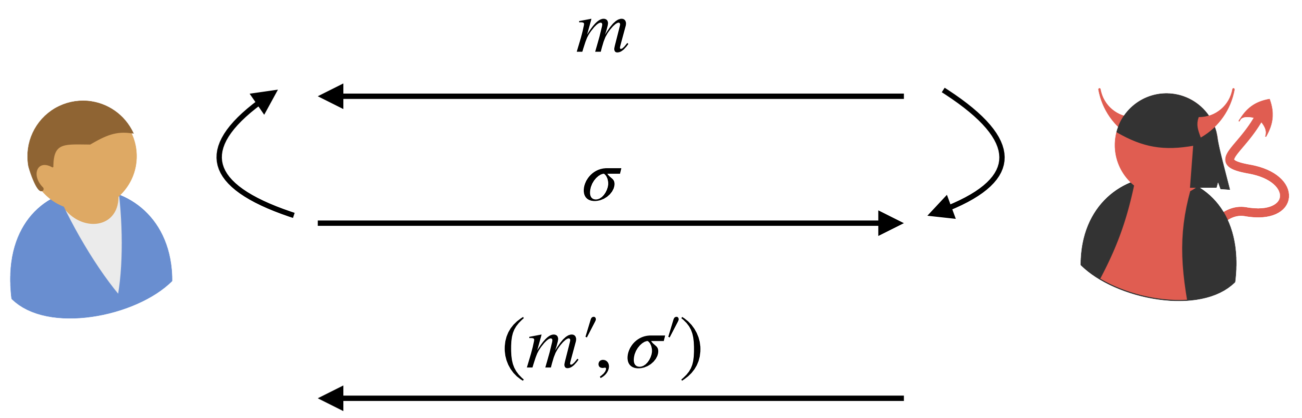

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$\Pr[\mathscr{A} \text{ wins in } H_0]$

Proof of Security

H_0

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$i \leftarrow \text{Gen}(1^\lambda)$

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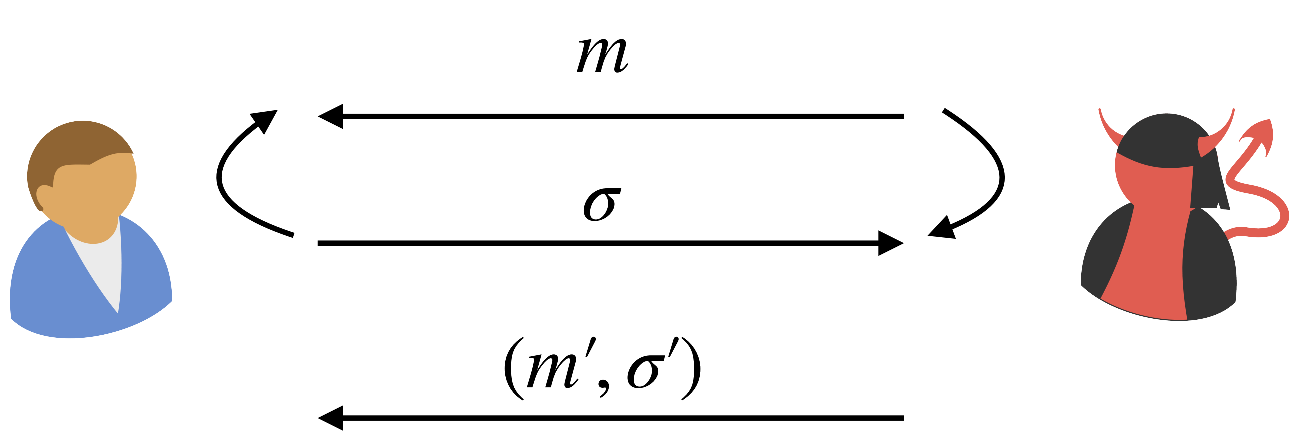

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Let **col** be the event that $h(m') = h(m)$ for some $m \neq m'$

$$\Pr\left[\text{col}\right] \leq \text{negl}(\lambda)$$

$$\Pr[\mathscr{A} \text{ wins in } H_0]$$

$$= \Pr[\mathscr{A} \text{ wins in } H_0 \mid \text{col}]\Pr[\text{col}] + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$$

Proof of Security

H_0

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$i \leftarrow$ Gen(1^λ)

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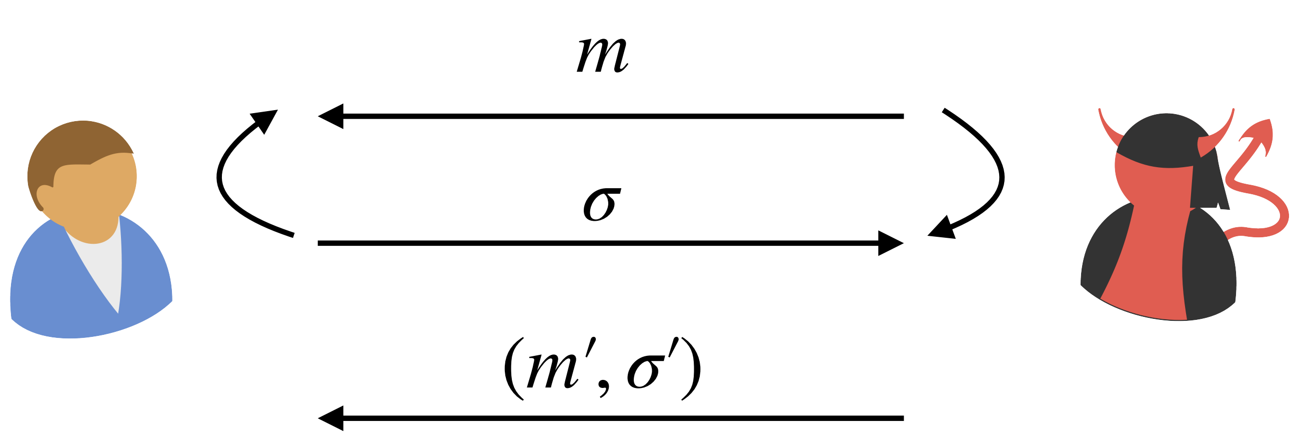

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Let **col** be the event that $h(m') = h(m)$ for some $m \neq m'$

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$$\Pr[\mathscr{A} \text{ wins in } H_0]$$

$$= \Pr[\mathscr{A} \text{ wins in } H_0 \mid \text{col}]\Pr[\text{col}] + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$$

$$\leq \text{negl}(\lambda) + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$$

Proof of Security

H_0

$k \leftarrow \text{KeyGen}(1^\lambda)$

$i \leftarrow \text{Gen}(1^\lambda)$

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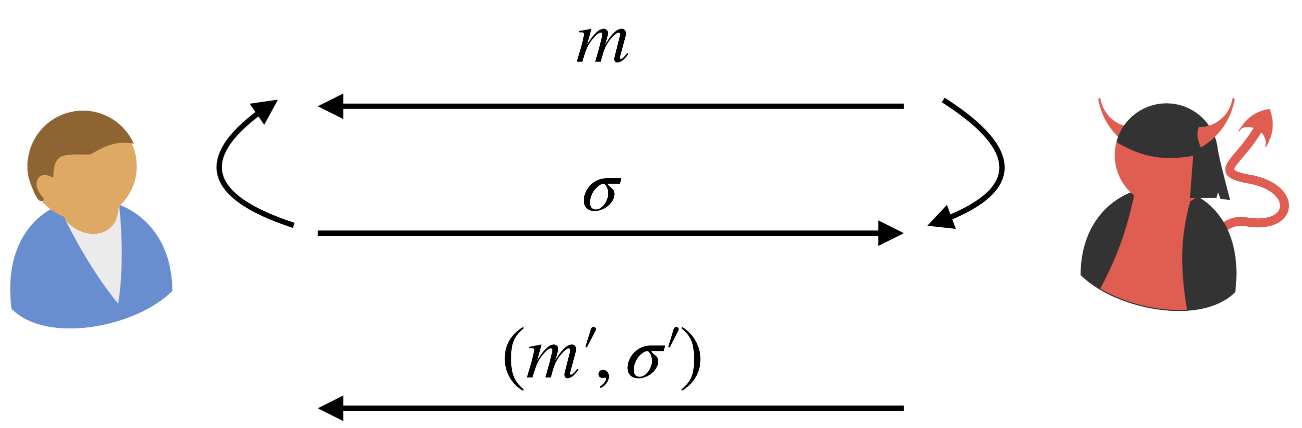

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$$\Pr[\mathscr{A} \text{ wins in } H_0]$$

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$$\leq \text{negl}(\lambda) + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$$

$$\leq \text{negl}(\lambda) + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]$$

Proof of Security

H_0

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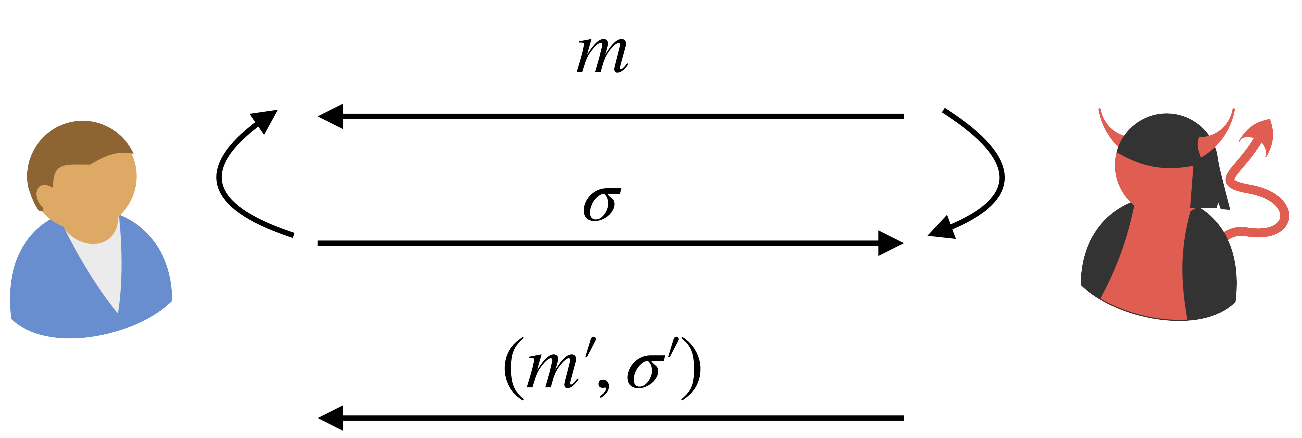

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$\text{Ver}(k, m, \sigma) : \text{Ver}(k, h_i(m), \sigma)$

Claim:
$\Pr\left[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}\right] \leq \text{negl}(\lambda)$

Let **col** be the event that $h(m') = h(m)$ for some $m \neq m'$

$$\Pr\left[\text{col}\right] \leq \text{negl}(\lambda)$$

$$\Pr[\mathscr{A} \text{ wins in } H_0]$$

$$= \Pr[\mathscr{A} \text{ wins in } H_0 \mid \text{col}]\Pr[\text{col}] + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$$

$$\leq \text{negl}(\lambda) + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$$

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Proof of Security

H_0

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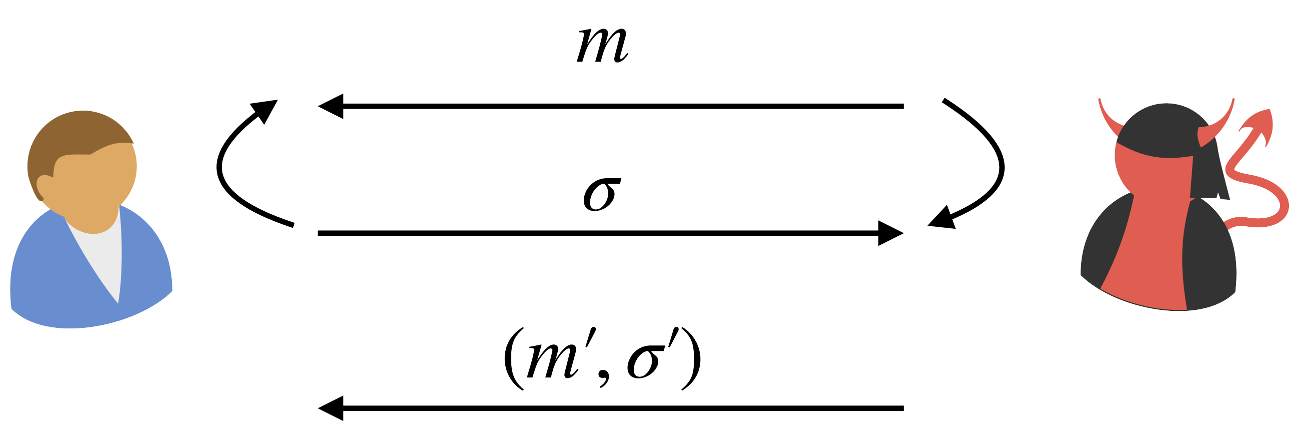

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Proof of Security

H_0

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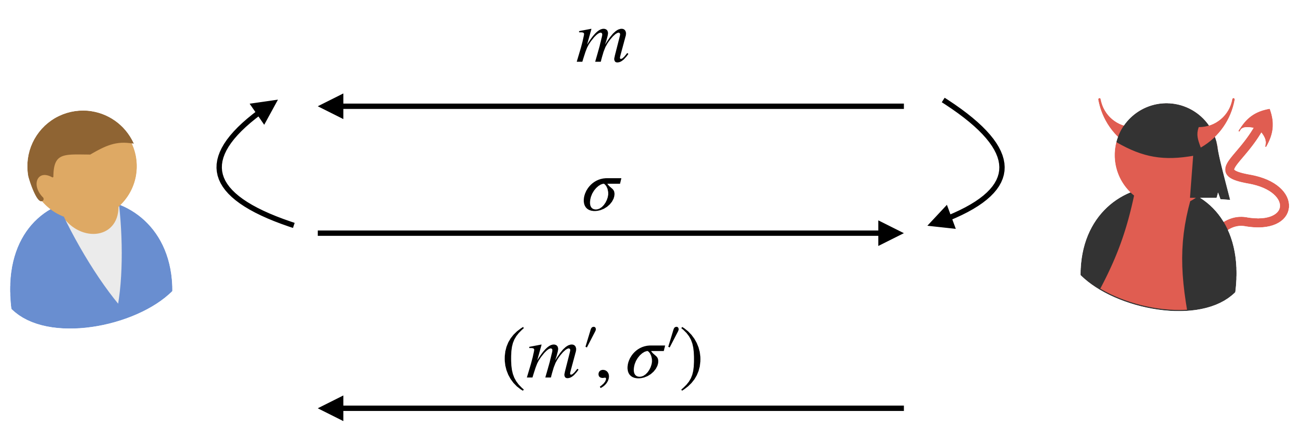

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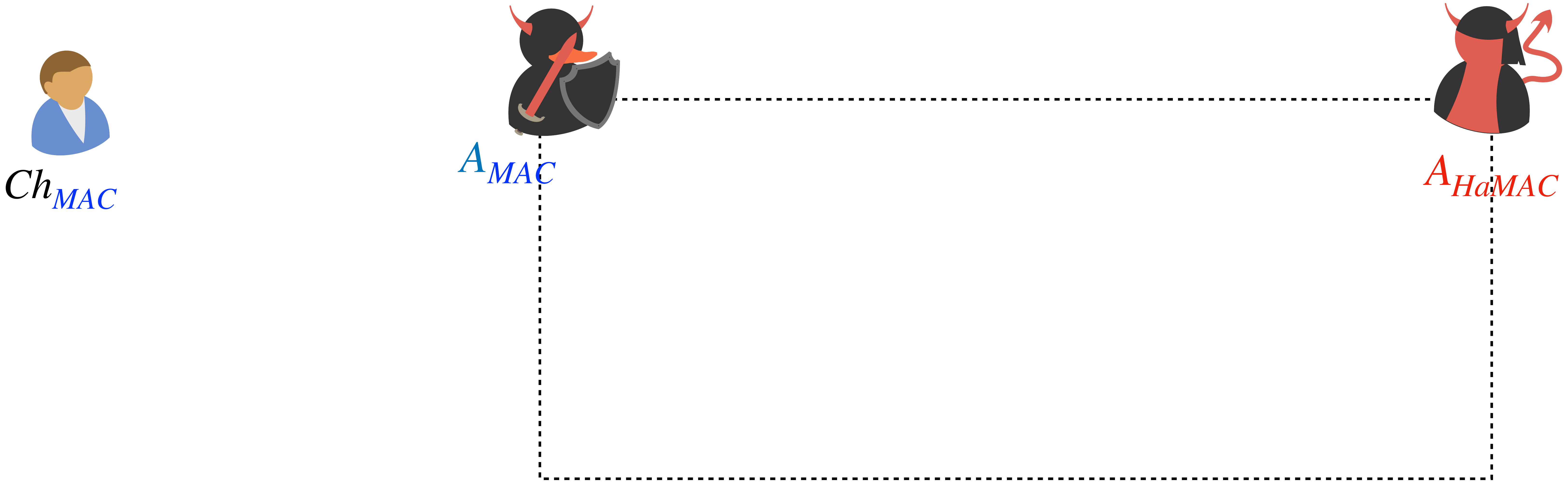

Proof of Security

H_0

$k \leftarrow$ KeyGen(1^λ)

$i \leftarrow$ Gen(1^λ)

$\sigma :=$ Tag$(k, h_i(m))$

m

σ

(m', σ')

Wins if
Ver$(k, h_i(m'), \sigma') = 1$
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KeyGen(1^λ) : $(k \leftarrow$ KeyGen$(1^\lambda), i)$

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Let **col** be the event that $h(m') = h(m)$ for some $m \neq m'$

Ch_{MAC}

A_{MAC}

$i \leftarrow$ Gen(1^λ)

A_{HaMAC}

Proof of Security

H_0

$k \leftarrow \mathsf{KeyGen}(1^\lambda)$

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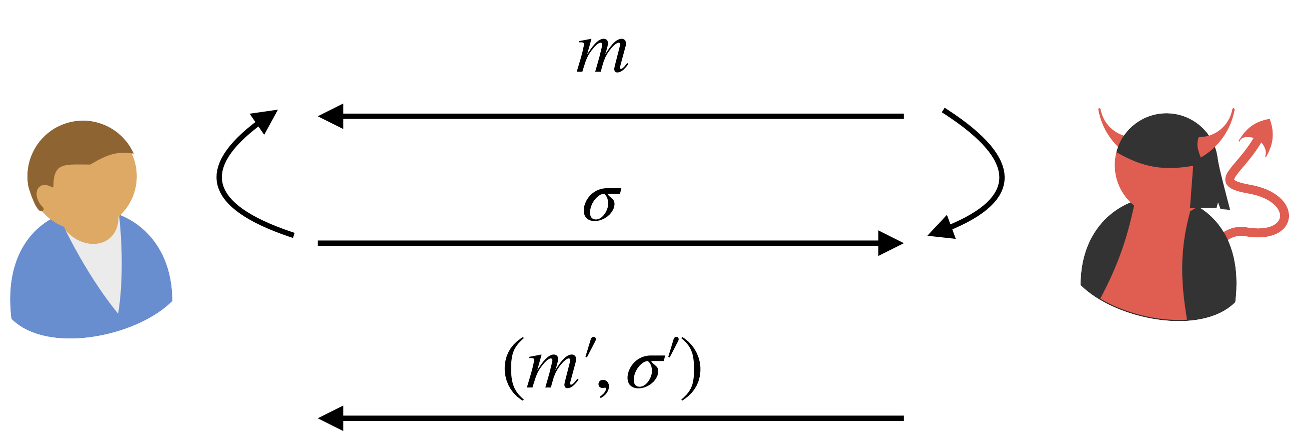

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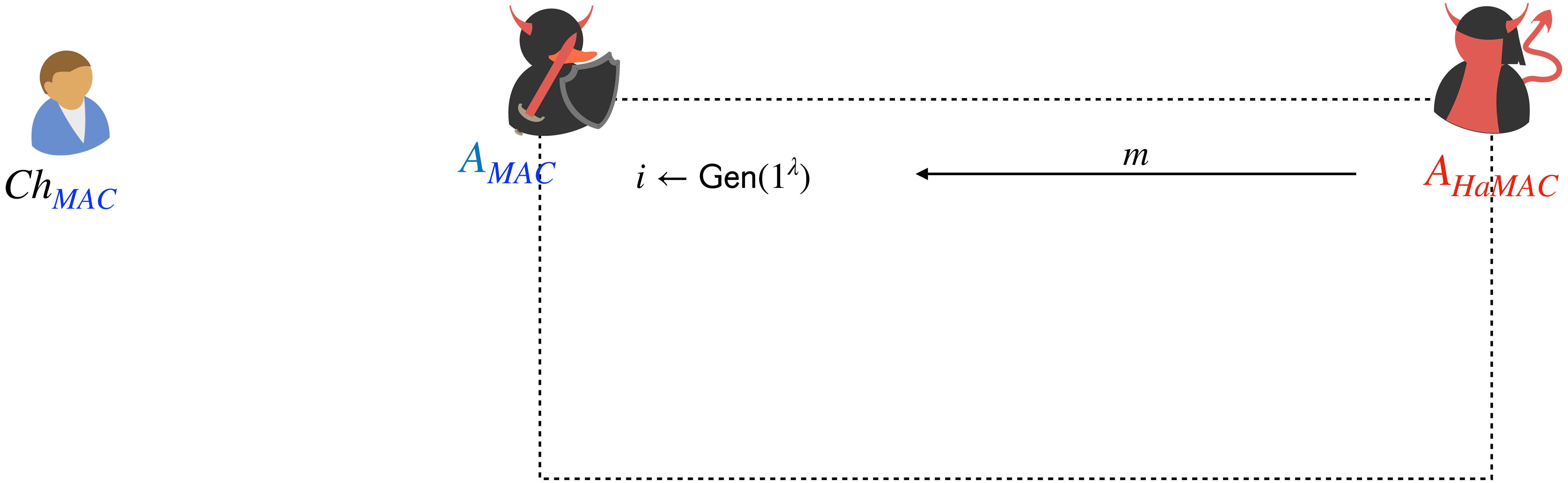

Proof of Security

H_0

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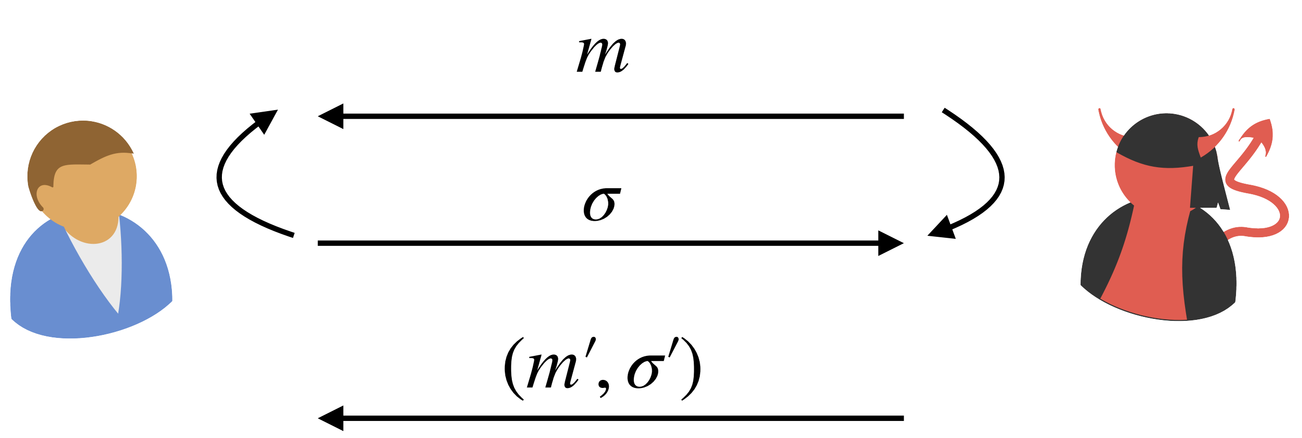

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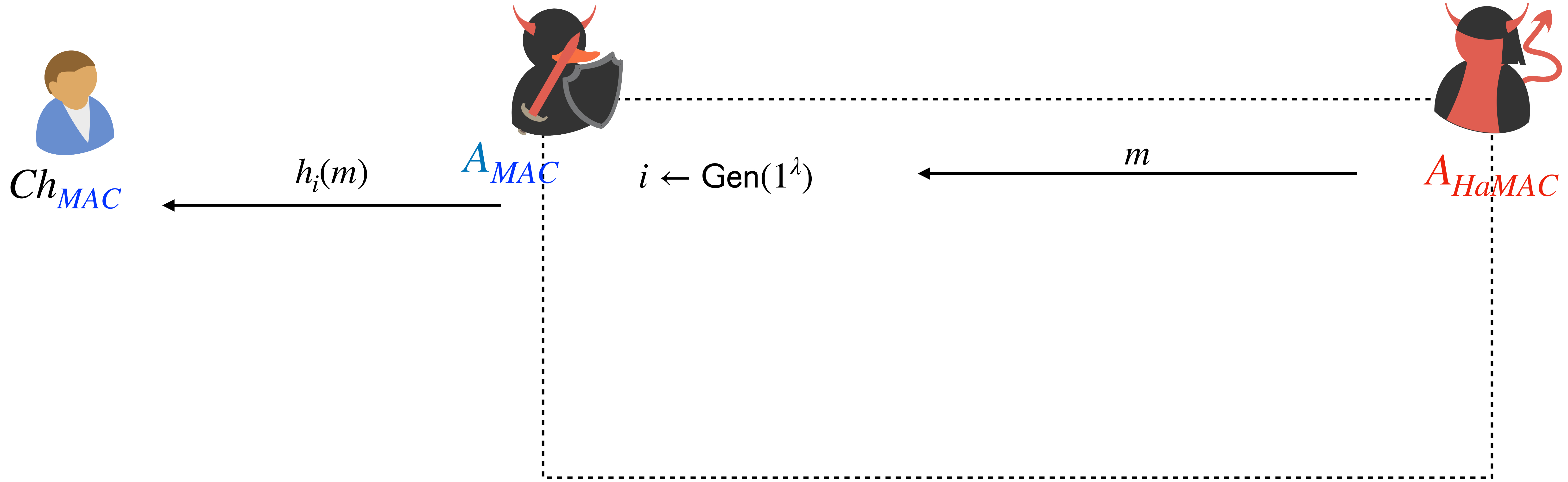

Proof of Security

H_0

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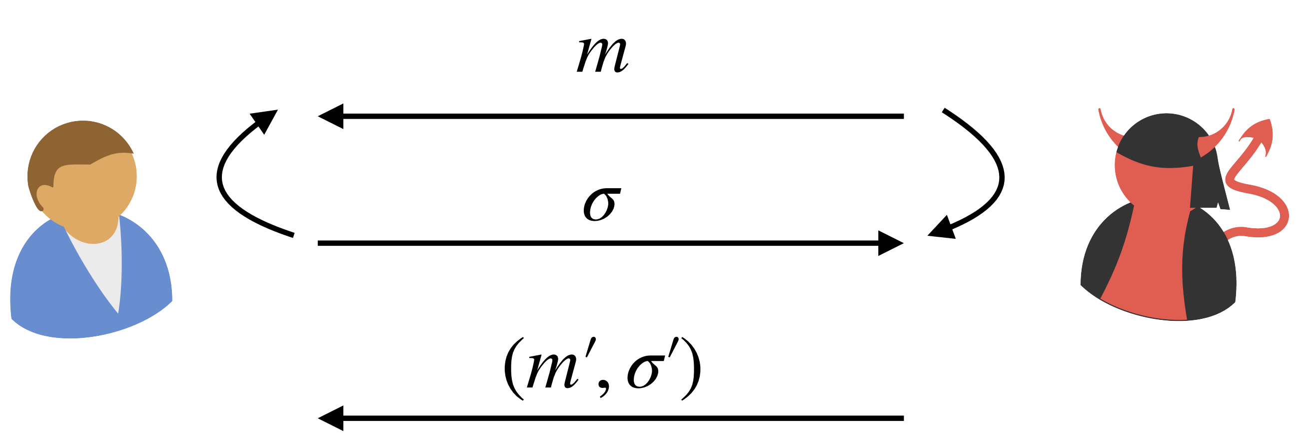

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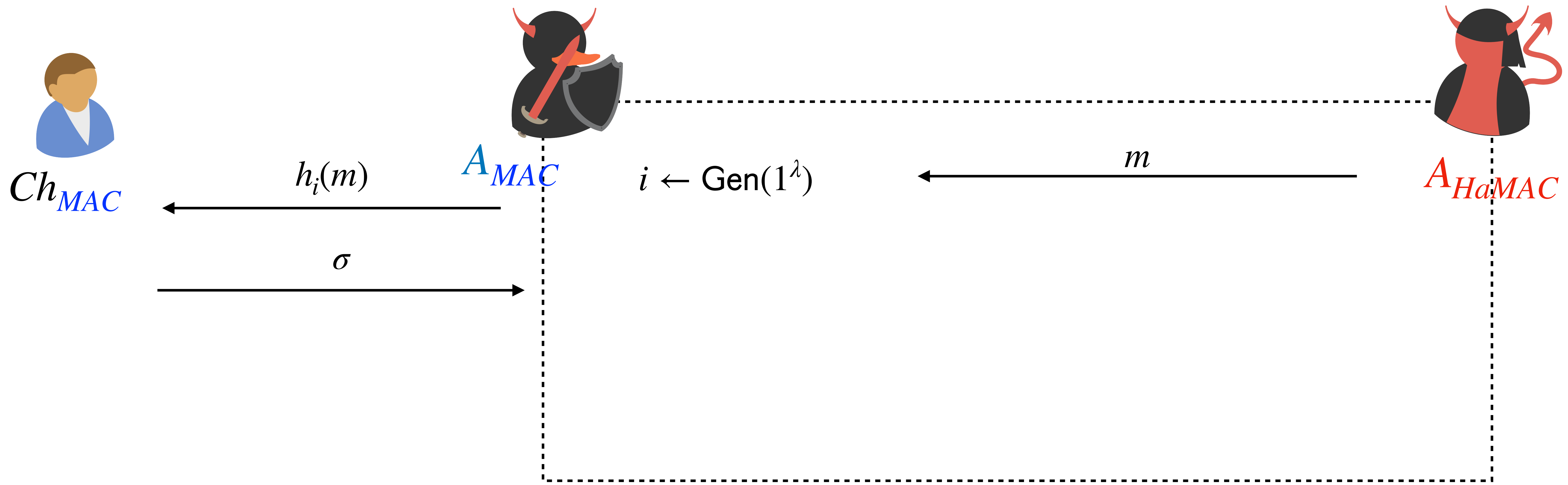
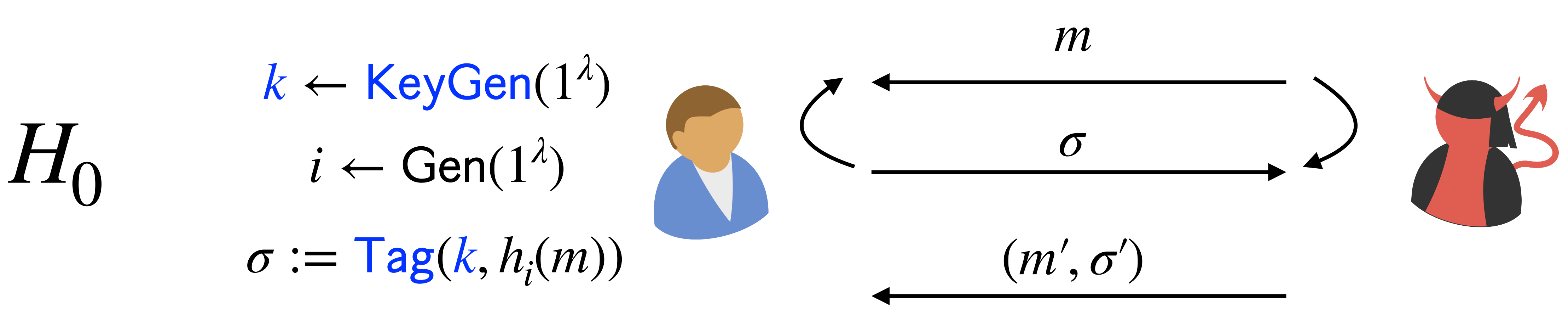


Proof of Security



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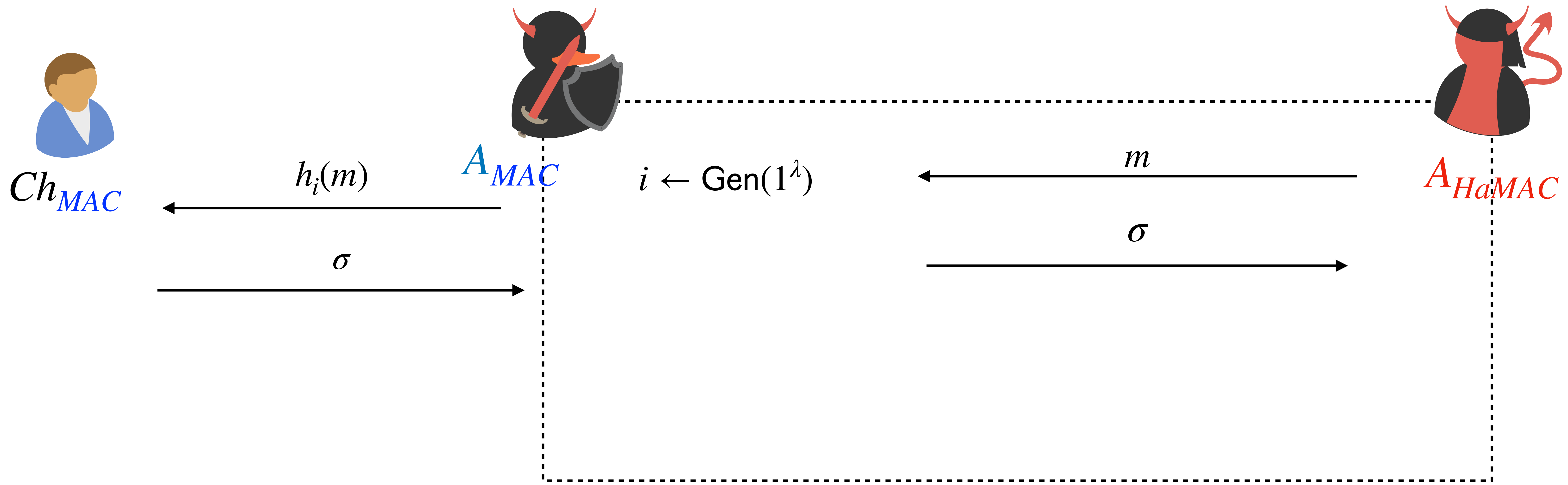
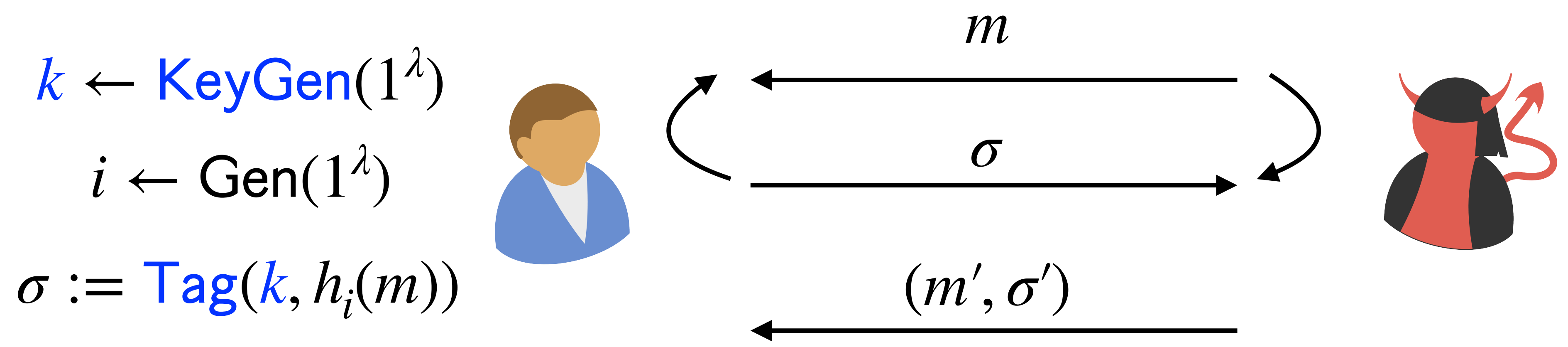


Proof of Security

H_0



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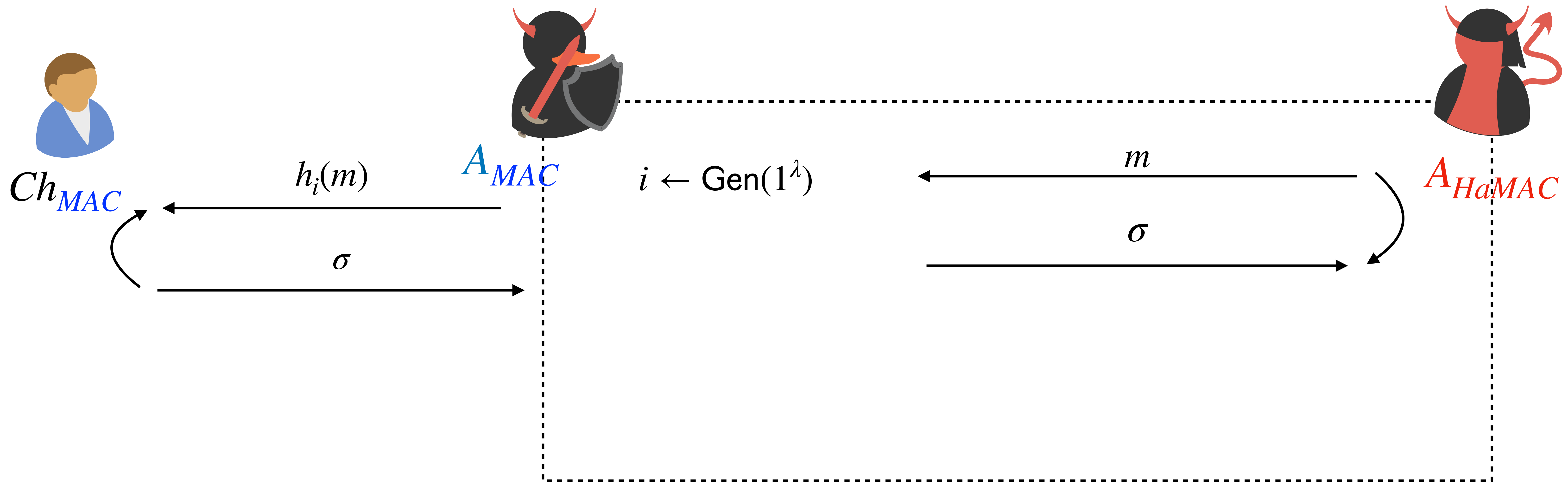

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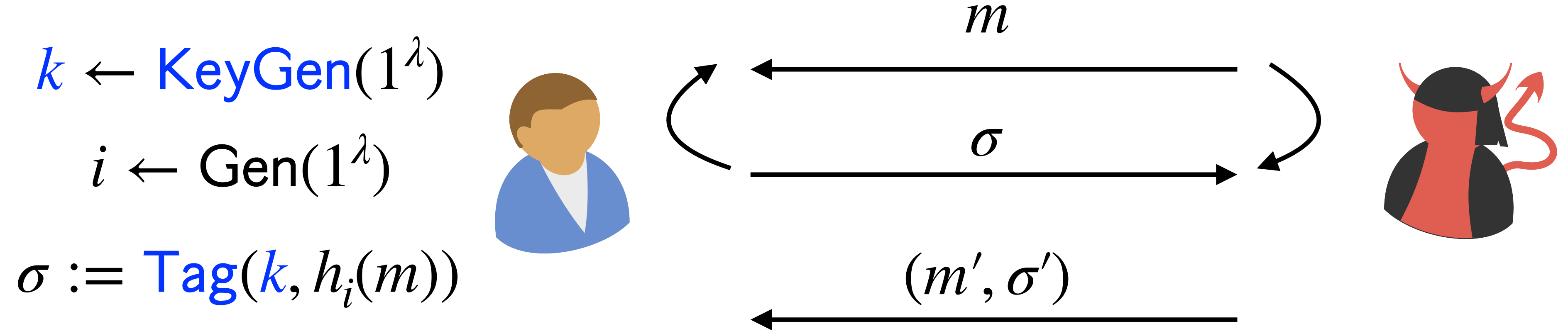
H_0



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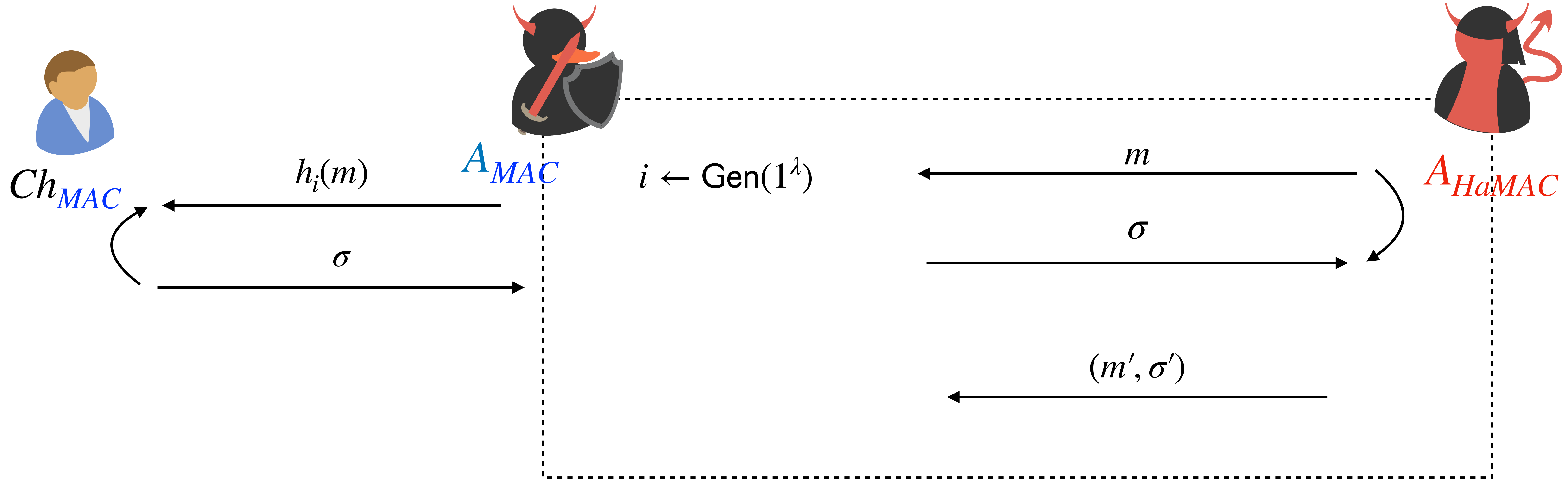


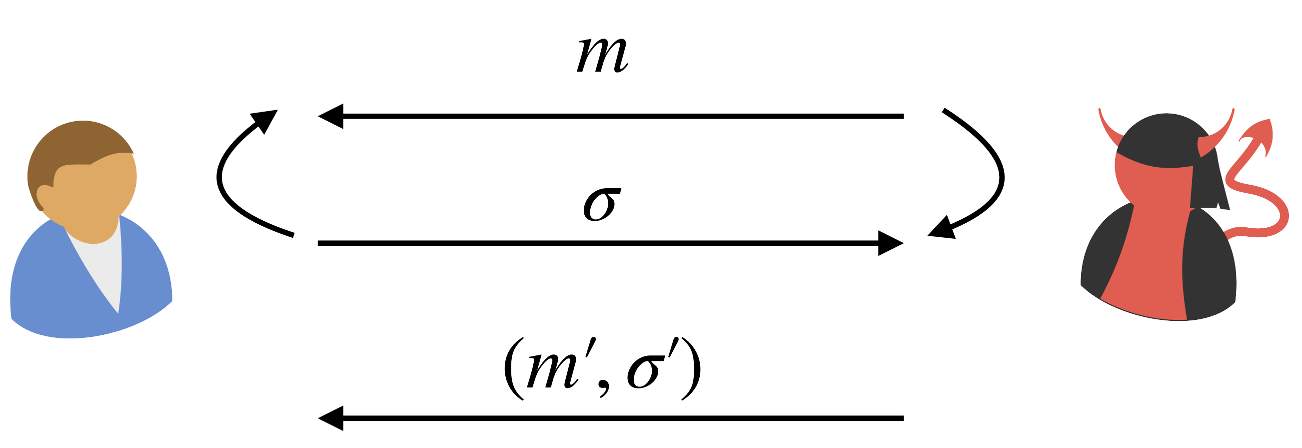
Proof of Security

H_0

$k \leftarrow$ KeyGen(1^λ)

$i \leftarrow$ Gen(1^λ)

$\sigma :=$ Tag$(k, h_i(m))$



Wins if
Ver$(k, h_i(m'), \sigma') = 1$
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KeyGen(1^λ) : $(k \leftarrow$ KeyGen$(1^\lambda), i)$

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Claim:
Pr $\left[\mathscr{A} \text{ wins in } H_0 \mid \neg\textbf{col}\right] \leq$ negl(λ)

Let **col** be the event that
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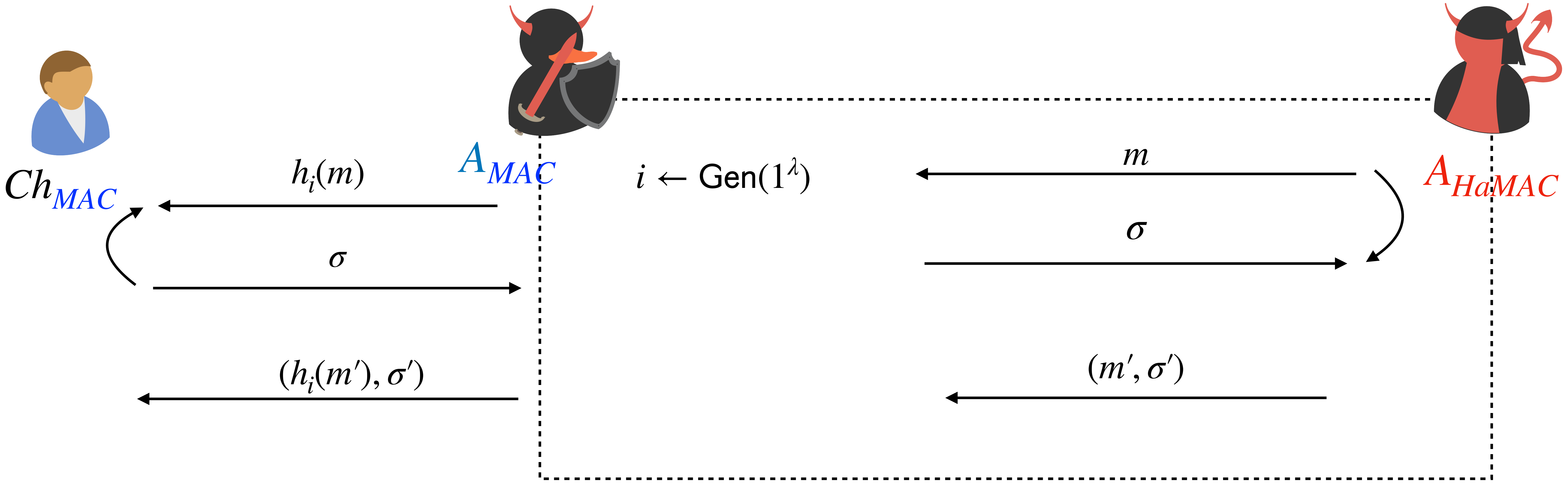

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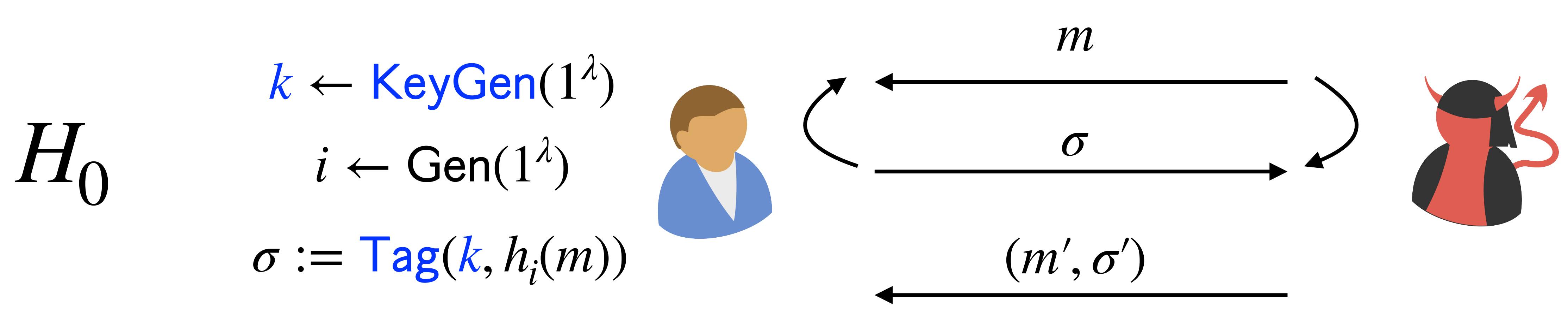
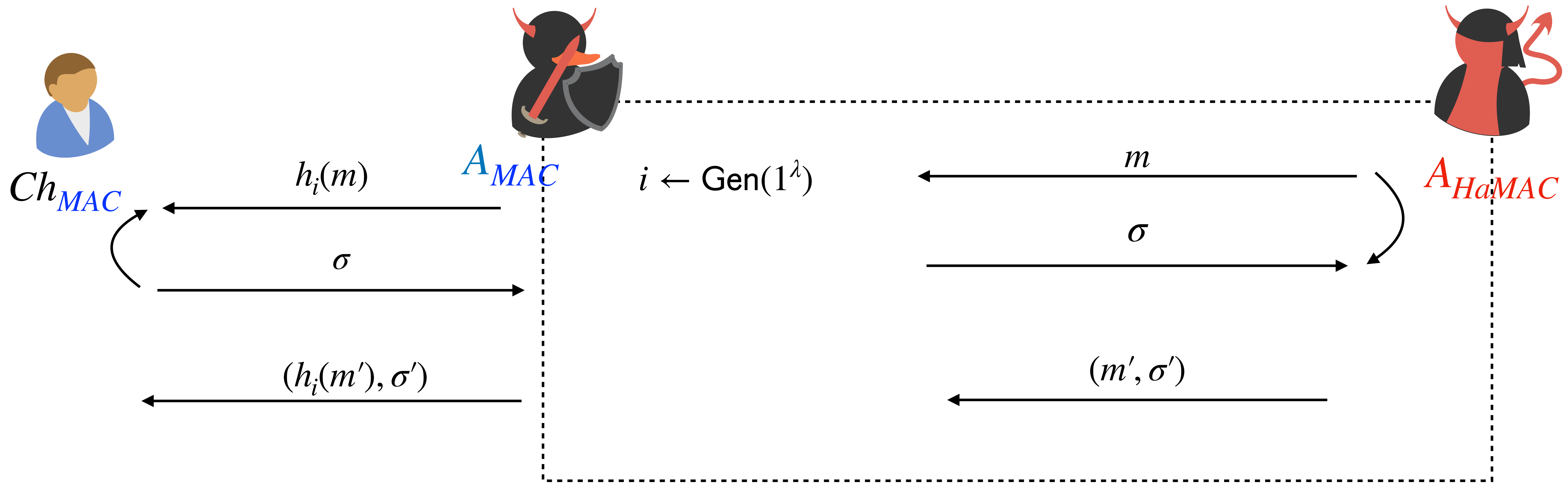
$\mathsf{Ver}(k, m, \sigma) : \mathsf{Ver}(k, h_i(m), \sigma)$



Wins if $\mathsf{Ver}(k, h_i(m'), \sigma') = 1$ and $\mathscr{A}$ never queried m'

Let **col** be the event that $h(m') = h(m)$ for some $m \neq m'$

Claim:
$\Pr\left[\mathscr{A} \text{ wins in } H_0 \mid \neg\mathsf{col}\right] \leq \mathsf{negl}(\lambda)$



$\Pr[\mathscr{A} \text{ wins} \mid \neg\mathsf{col}] = \Pr[\mathscr{A}_{MAC} \text{ wins}] \leq \mathsf{negl}(\lambda)$

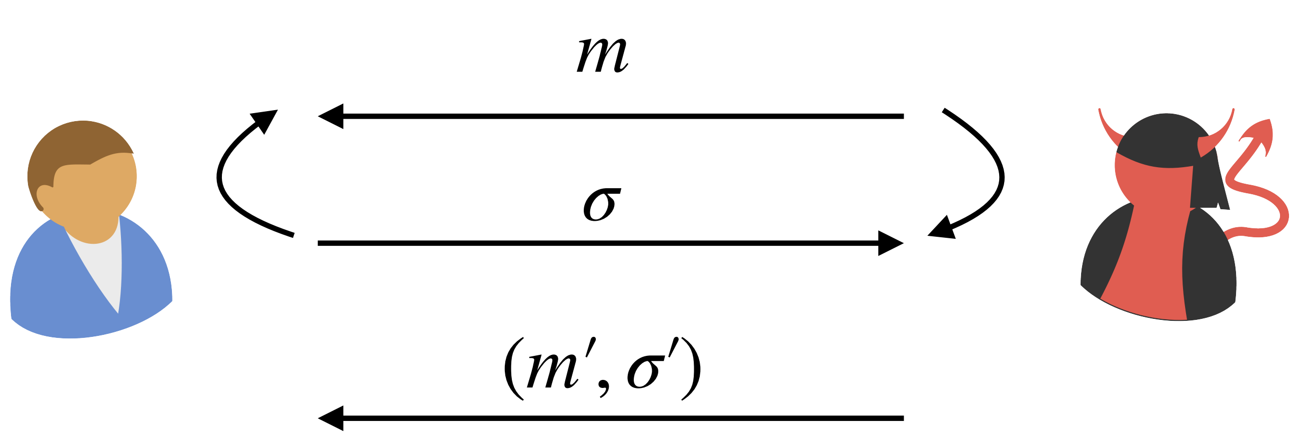
Proof of Security

H_0

$k \leftarrow \text{KeyGen}(1^\lambda)$

$i \leftarrow \text{Gen}(1^\lambda)$

$\sigma := \text{Tag}(k, h_i(m))$



Wins if
$\text{Ver}(k, h_i(m'), \sigma') = 1$
and $\mathscr{A}$ never
queried m'

$\text{KeyGen}(1^\lambda) : (k \leftarrow \text{KeyGen}(1^\lambda), i)$

$\text{Tag}(k, m) : \sigma := \text{Tag}(k, h_i(m))$

$\text{Ver}(k, m, \sigma) : \text{Ver}(k, h_i(m), \sigma)$

Let **col** be the event that
$h(m') = h(m)$ for some $m \neq m'$

$$\Pr\left[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}\right] \leq \text{negl}(\lambda)$$

$$\Pr\left[\text{col}\right] \leq \text{negl}(\lambda)$$

$$\Pr[\mathscr{A} \text{ wins in } H_0]$$

$$= \Pr[\mathscr{A} \text{ wins in } H_0 \mid \text{col}]\Pr[\text{col}] + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$$

$$\leq \text{negl}(\lambda) + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$$

$$\leq \text{negl}(\lambda) + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]$$

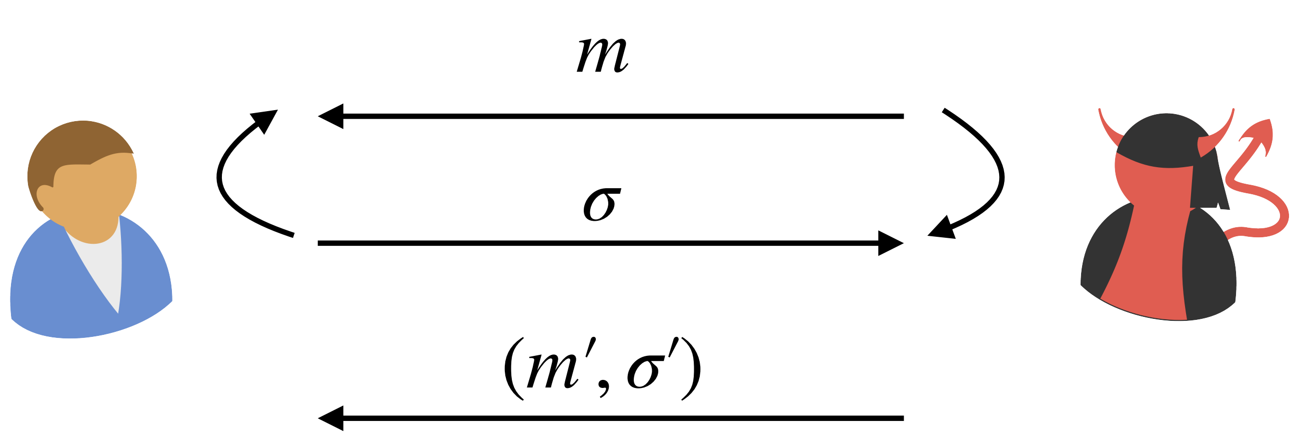
Proof of Security

H_0

$k \leftarrow$ KeyGen(1^λ)

$i \leftarrow$ Gen(1^λ)

$\sigma :=$ Tag$(k, h_i(m))$



Wins if
Ver$(k, h_i(m'), \sigma') = 1$
and $\mathscr{A}$ never
queried m'

KeyGen(1^λ) : $(k \leftarrow$ KeyGen$(1^\lambda), i)$

Tag(k, m) : $\sigma :=$ Tag$(k, h_i(m))$

Ver(k, m, σ) : Ver$(k, h_i(m), \sigma)$

Let **col** be the event that $h(m') = h(m)$ for some $m \neq m'$

$$\Pr\left[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}\right] \leq \text{negl}(\lambda)$$

$$\Pr\left[\text{col}\right] \leq \text{negl}(\lambda)$$

$\Pr[\mathscr{A}$ wins in $H_0]$

$= \Pr[\mathscr{A} \text{ wins in } H_0 \mid \text{col}]\Pr[\text{col}] + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$

$\leq \text{negl}(\lambda) + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]\Pr[\neg\text{col}]$

$\leq \text{negl}(\lambda) + \Pr[\mathscr{A} \text{ wins in } H_0 \mid \neg\text{col}]$

$\leq \text{negl}(\lambda) + \text{negl}(\lambda)$