Zero-Knowledge Proofs II

601.442/642 Modern Cryptography

26th March 2026

Recap: Proof Systems

Proof System

A proof system for a language $L \subset \{0,1\}^*$ is a pair of interactive algorithms (P, V), where V is a PPT algorithm, if it satisfies the following properties.

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Are both properties required?

Recap: The Power of Interaction and Randomness

- Proof system for languages beyond what is captured by non-interactive proofs.
  - Languages in NP have a non-interactive proof.
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Our goal is to construct zero-knowledge proofs.

Towards Zero-Knowledge

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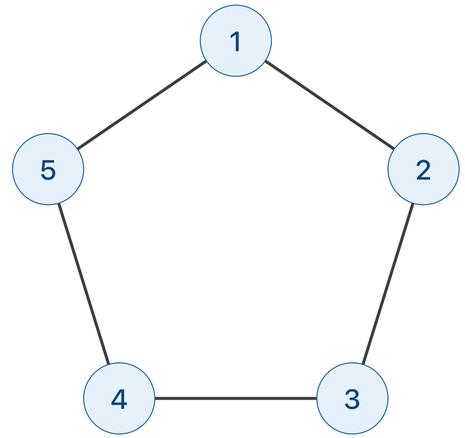
- When can we definitely say the verifier gains no knowledge?
  - **Key Idea:** If the verifier could have computed it on its own, it didn't learn it from the prover.
  - In other words, the verifier gains knowledge only if it receives the result of a computation that is infeasible for it.

Zero-Knowledge Proof for Graph Isomorphism

Zero-Knowledge Proof for Graph Isomorphism

- Graph $G = (V, E)$ consists of a vertex set V and edge set $E \subseteq V \times V$.



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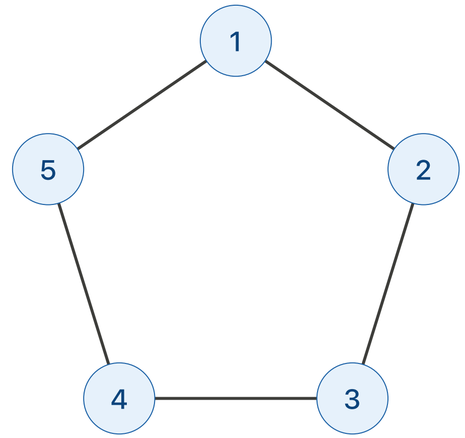

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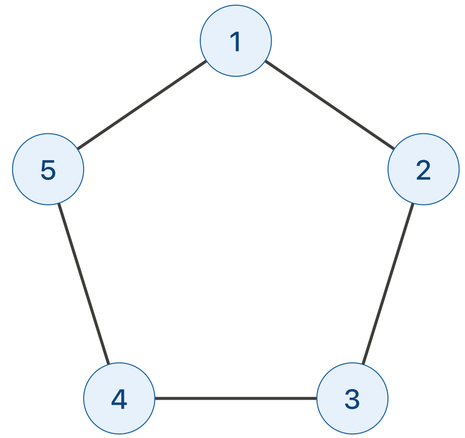

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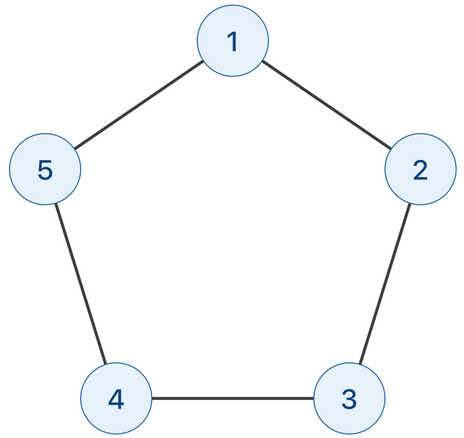

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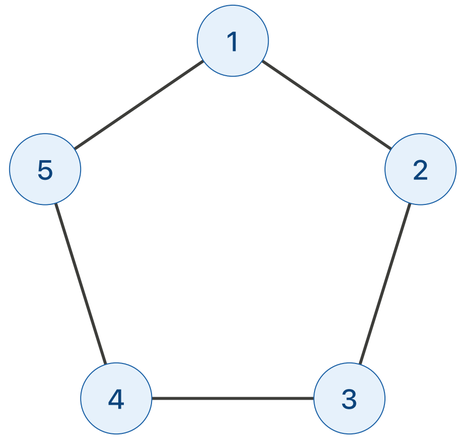

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- **Graph Isomorphism:** $G_0 = (V_0, E_0)$ and $G_1 = (V_1, E_1)$ are isomorphic if there exists a permutation $\phi \in \mathsf{Perm}_n$ such that
  - $V_1 = \{\phi(v) \mid v \in V_0\}$, and
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  - Alternatively, $G_1 = \phi(G_0)$.

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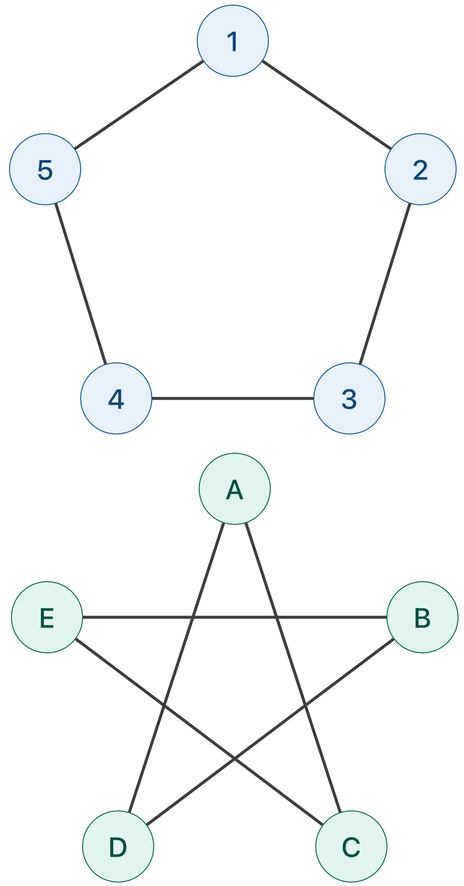

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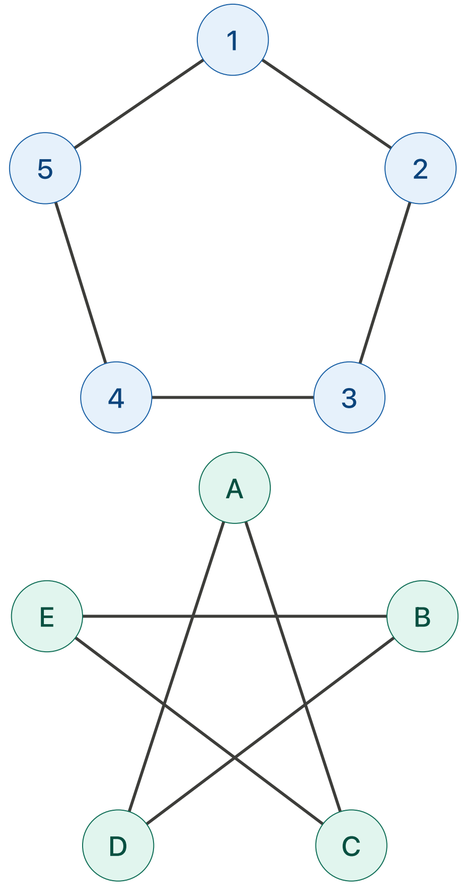
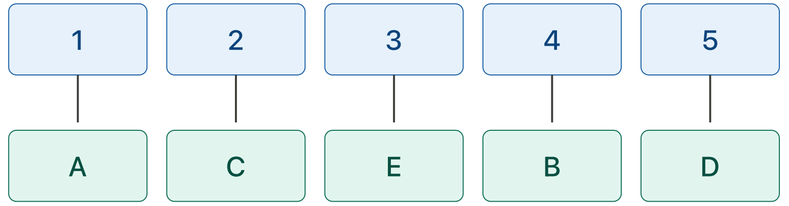


Isomorphism ϕ : G → H



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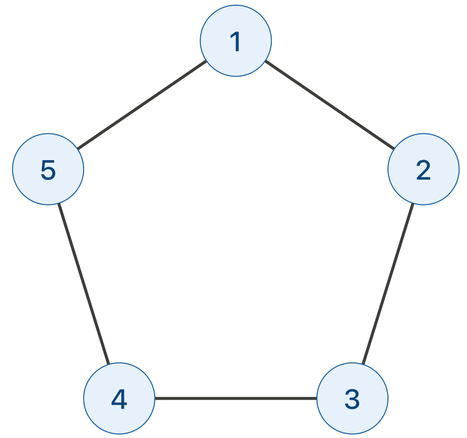

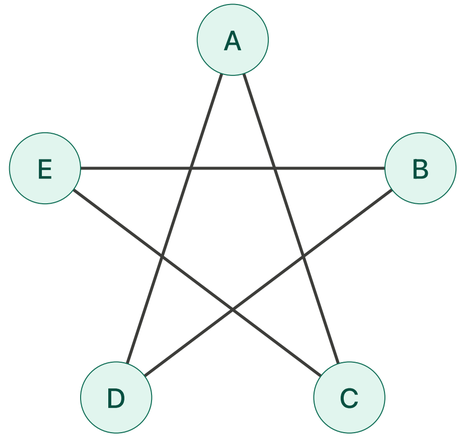

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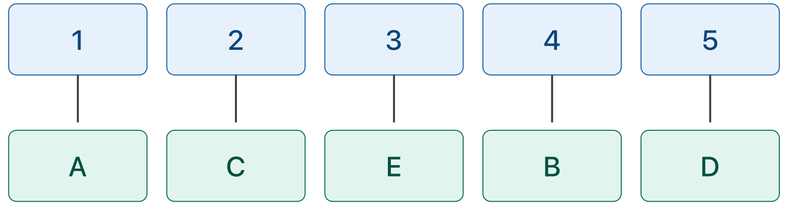

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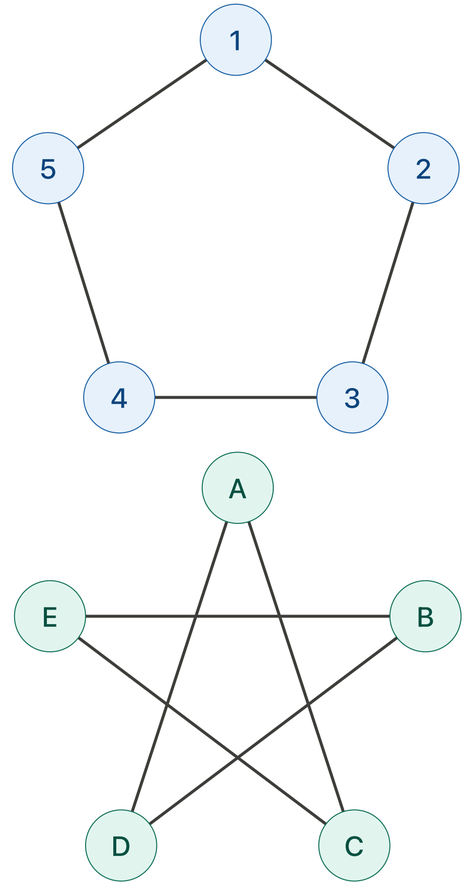

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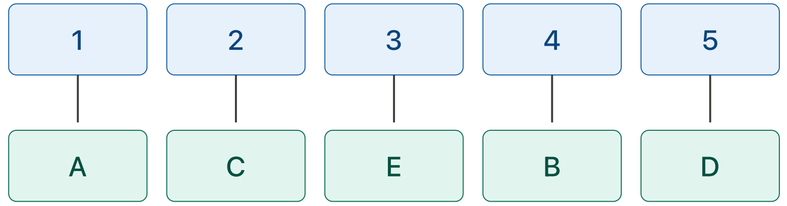

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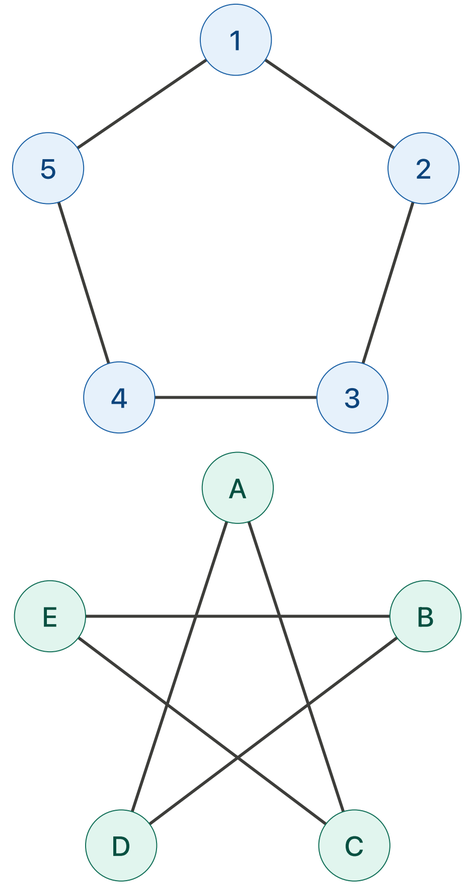

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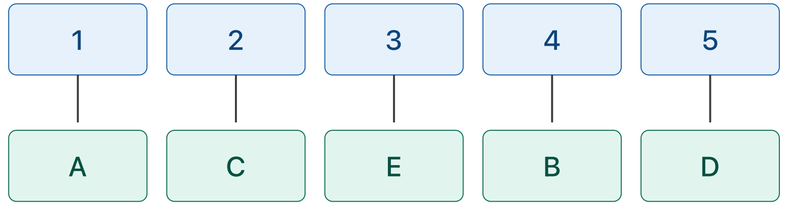

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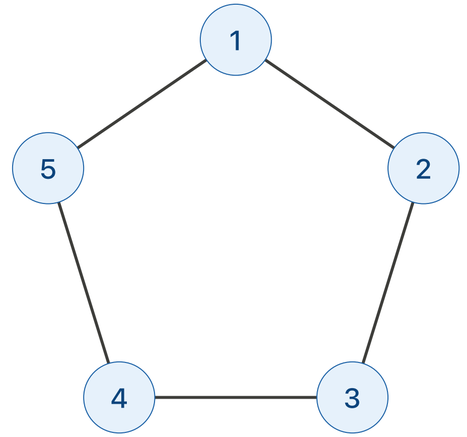

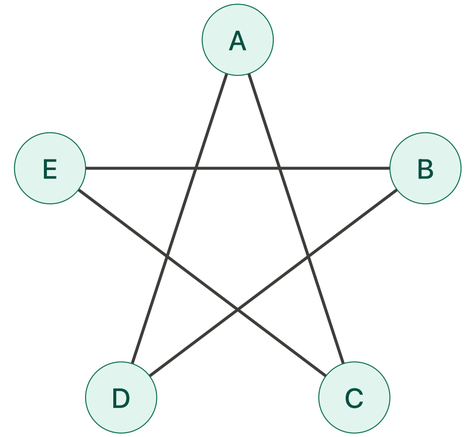

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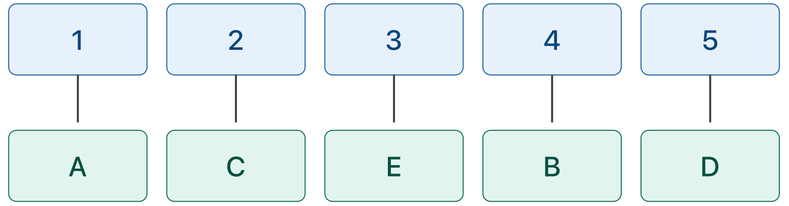

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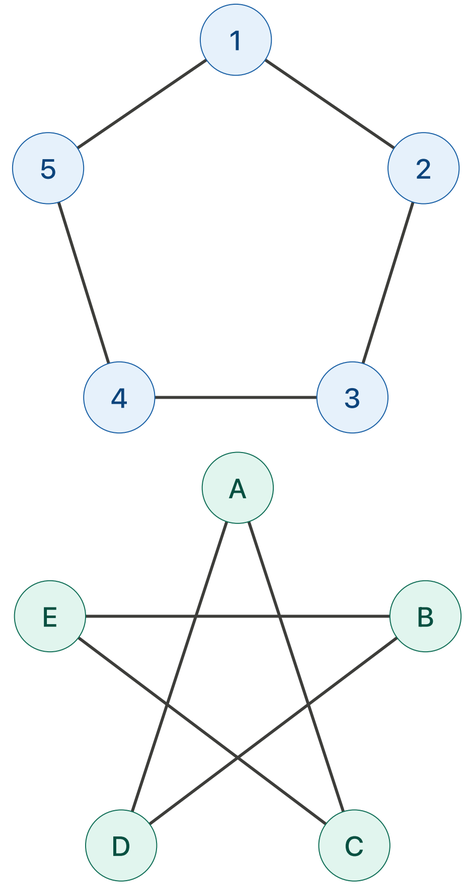

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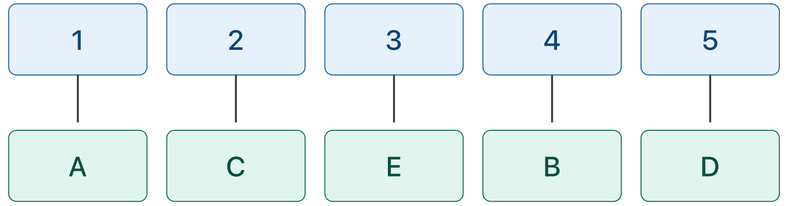

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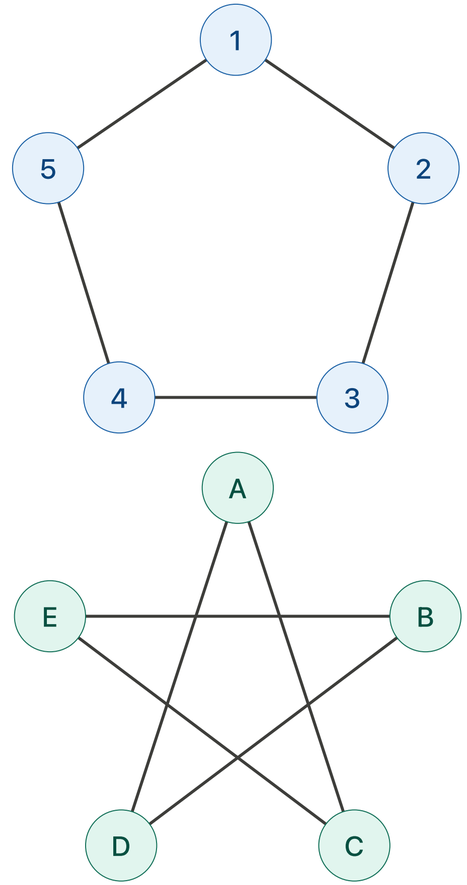
  - **Completeness:** Prover will use ϕ to convince the verifier.



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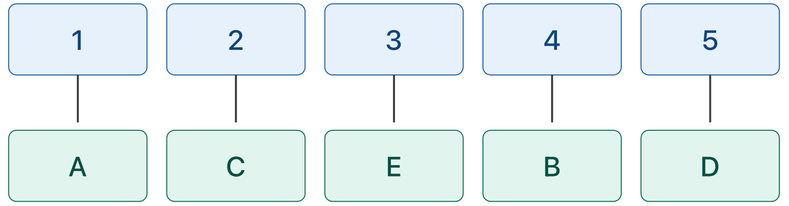

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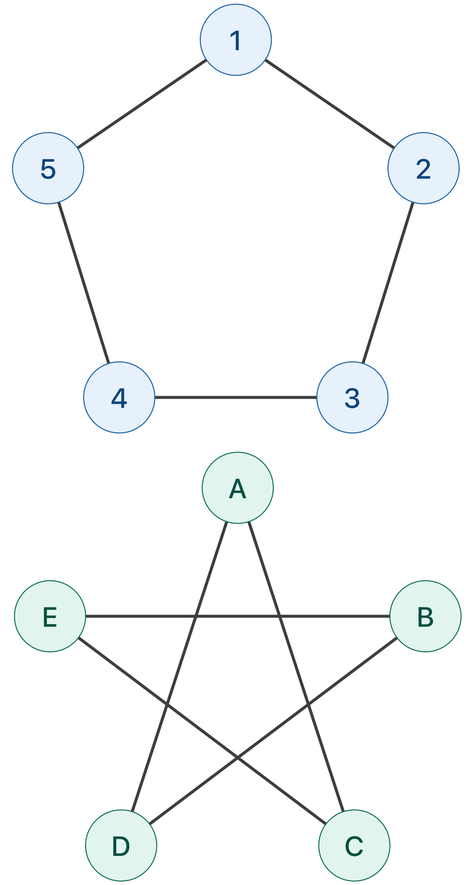

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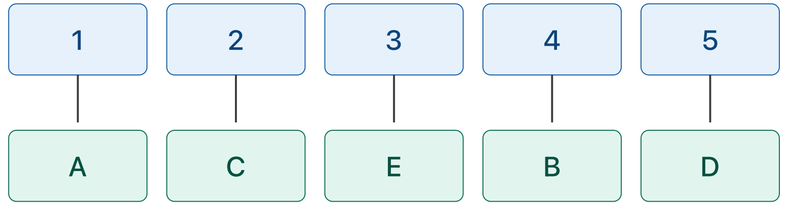

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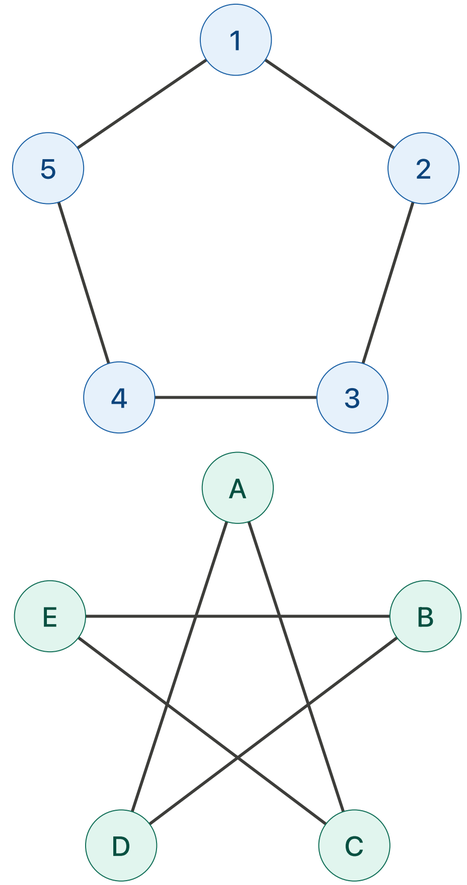
  - **Soundness:** If the graphs are not isomorphic, the verifier should reject with high probability.
  - **Zero-knowledge:** Verifier must not learn anything beyond the fact that the graphs are isomorphic.
    - Importantly, it shouldn't learn ϕ since it cannot compute it from G_0 and G_1.



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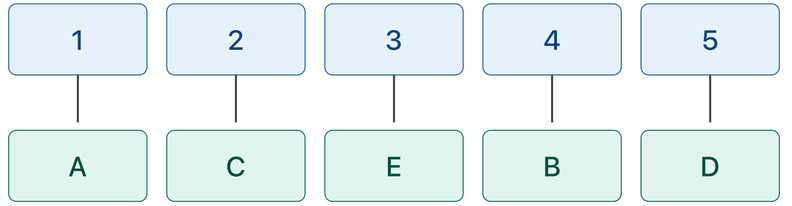

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Prover

Verifier

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Sample $b \xleftarrow{\$} \{0,1\}$

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Verifier

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$G' := \psi(G_1)$

Prove G' is isomorphic to G_b.

b

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Sample $\psi \xleftarrow{\$} \mathsf{Perm}_n$

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Sample $b \xleftarrow{\$} \{0,1\}$

Compute ϕ s.t.
$G_1 = \phi(G_0)$

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If $b = 0$, $\phi' :=$

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Accept if $G' = \phi'(G_b)$

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Zero-Knowledge Proof for Graph Isomorphism

- **Completeness**

Statement $x = (G_0, G_1)$

Prover

Verifier

Sample $\psi \xleftarrow{\$} \mathsf{Perm}_n$

$G' := \psi(G_1)$ →

Compute ϕ s.t. $G_1 = \phi(G_0)$

Sample $b \xleftarrow{\$} \{0,1\}$

← b

If $b = 0$, $\phi' := \psi \circ \phi$ →

If $b = 1$, $\phi' := \psi$

Accept if $G' = \phi'(G_b)$

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- **Completeness**
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$$\phi'(G_0) = \psi(\phi(G_0)) = \psi(G_1) = G'.$$

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- **Soundness**

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Zero-Knowledge Proof for Graph Isomorphism

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  - Repeat to amplify probability.

Zero-Knowledge Proof for Graph Isomorphism

- **Zero-Knowledge**

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Zero-Knowledge Proof for Graph Isomorphism

- **Zero-Knowledge**
  - Why does the verifier not gain any knowledge?

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  - Why does the verifier not gain any knowledge?
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    - The verifier could compute ψ from G'.
      - This is equivalent to solving graph isomorphism for (G', G_1).
      - But then it could solve graph isomorphism for (G_0, G_1) and compute ϕ by itself too.
      - Learning ϕ is not gaining knowledge in this case.

The Simulation Paradigm

Anything a party could compute from its own inputs is not knowledge it gains by interacting with an external entity.

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x

Prover

x

Verifier

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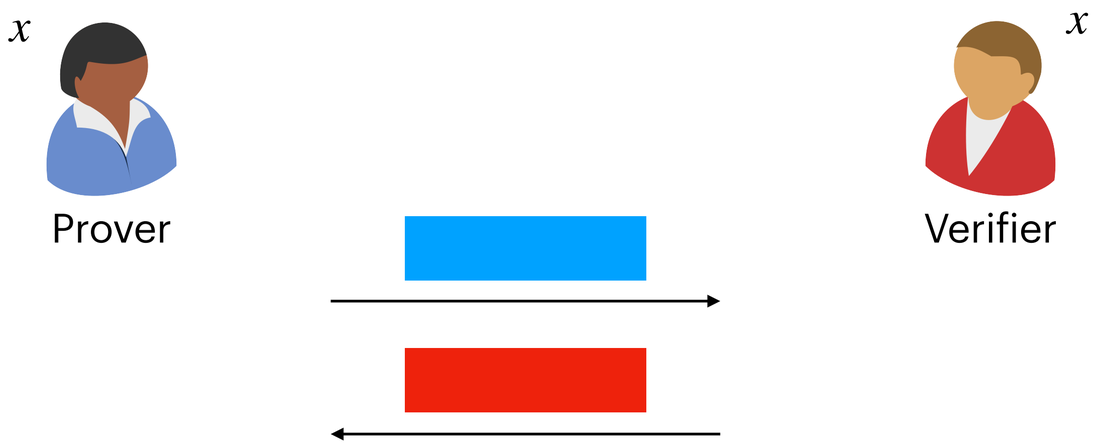

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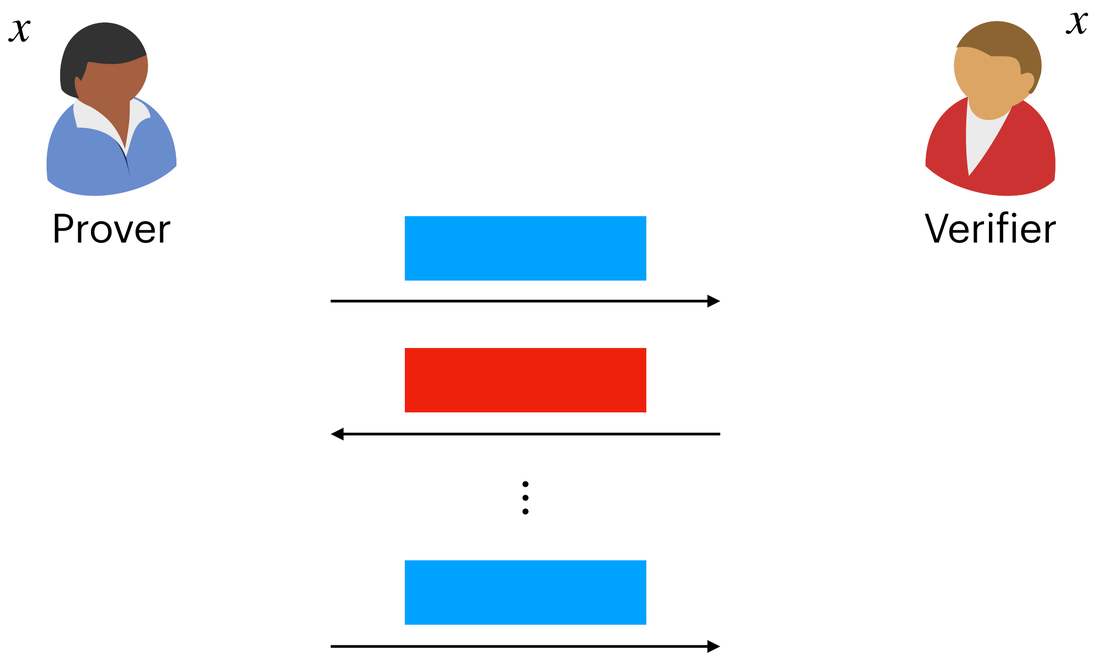

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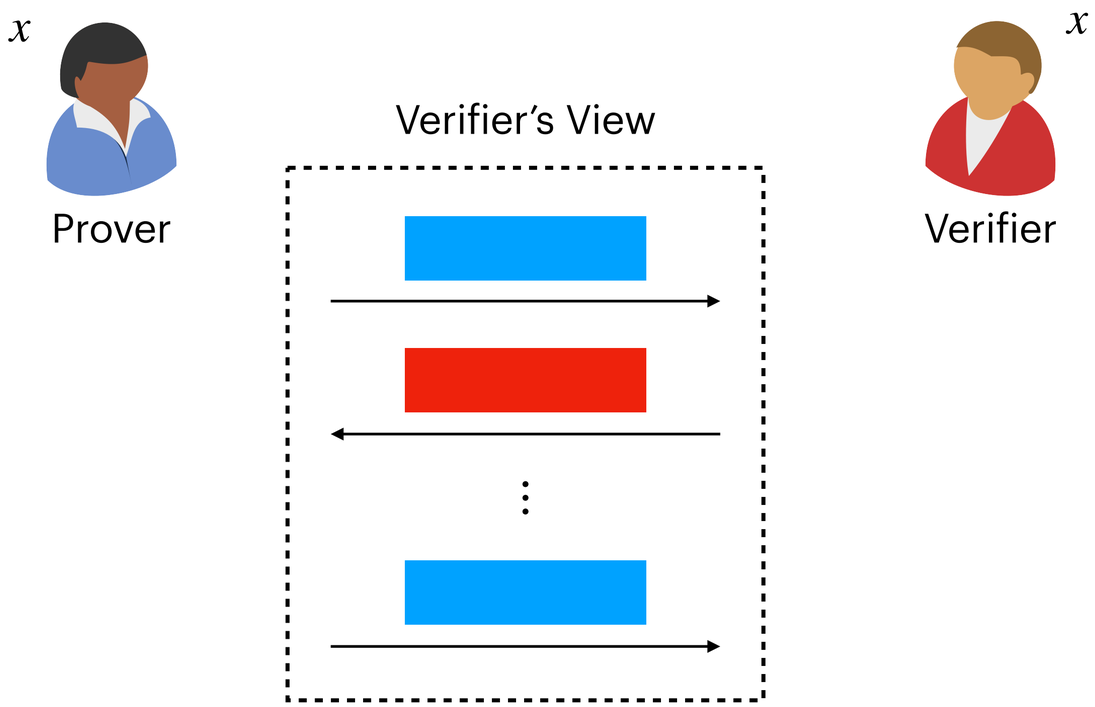

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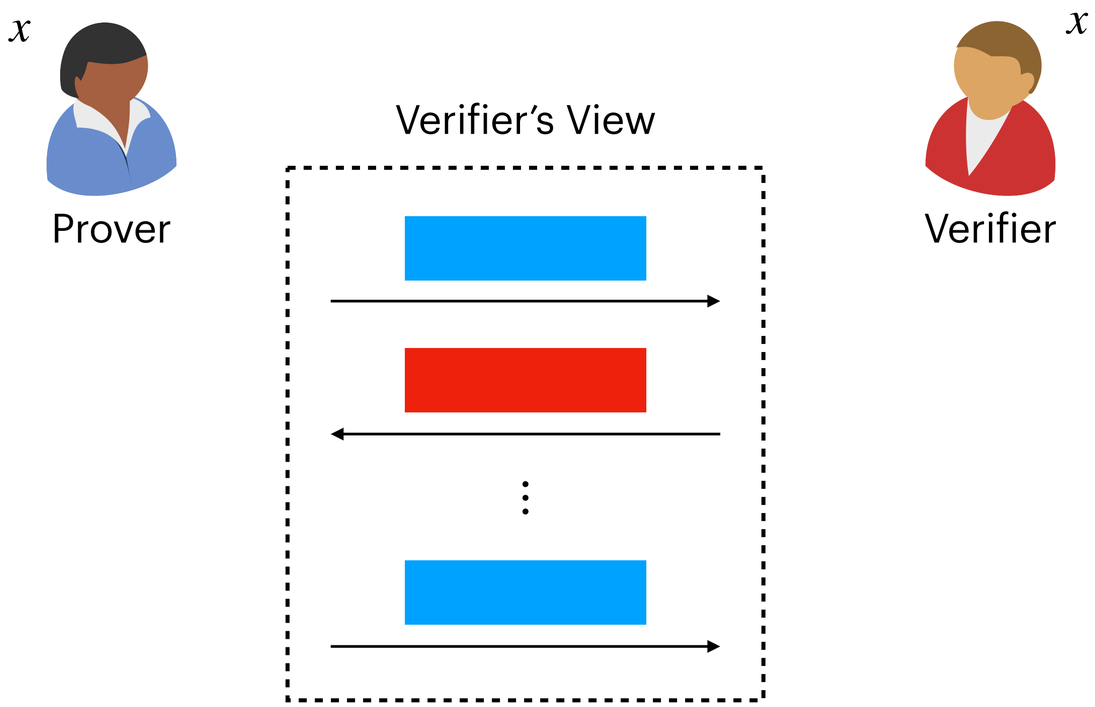

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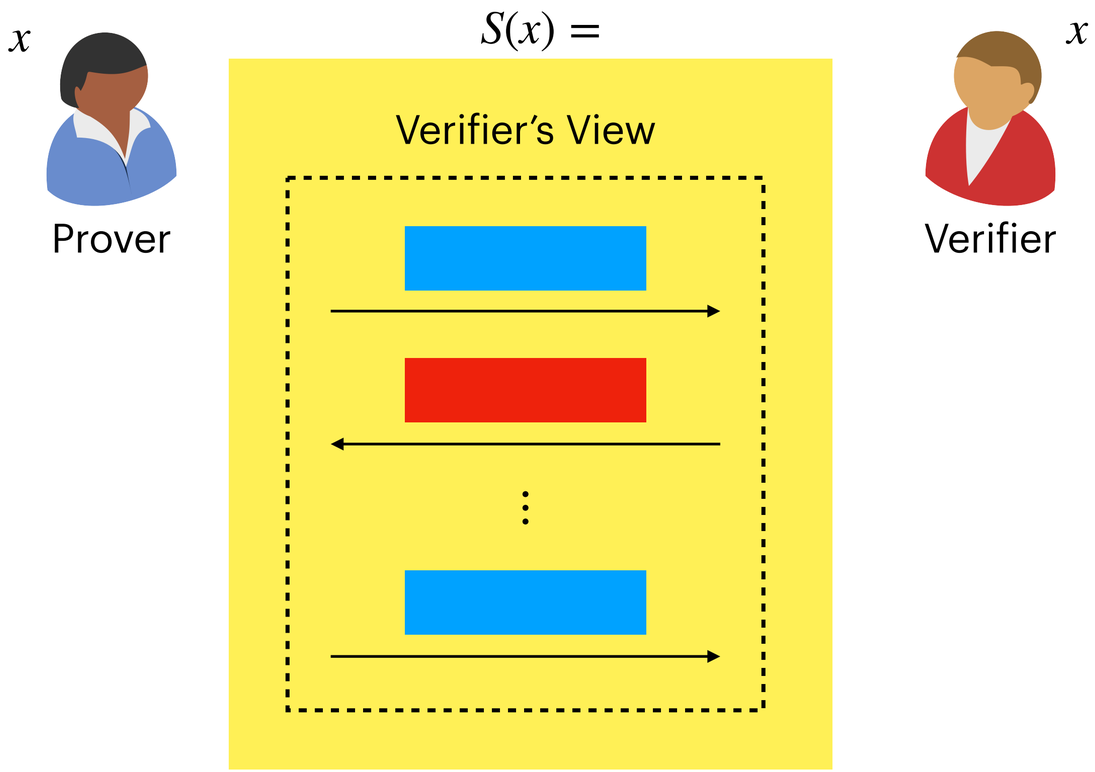

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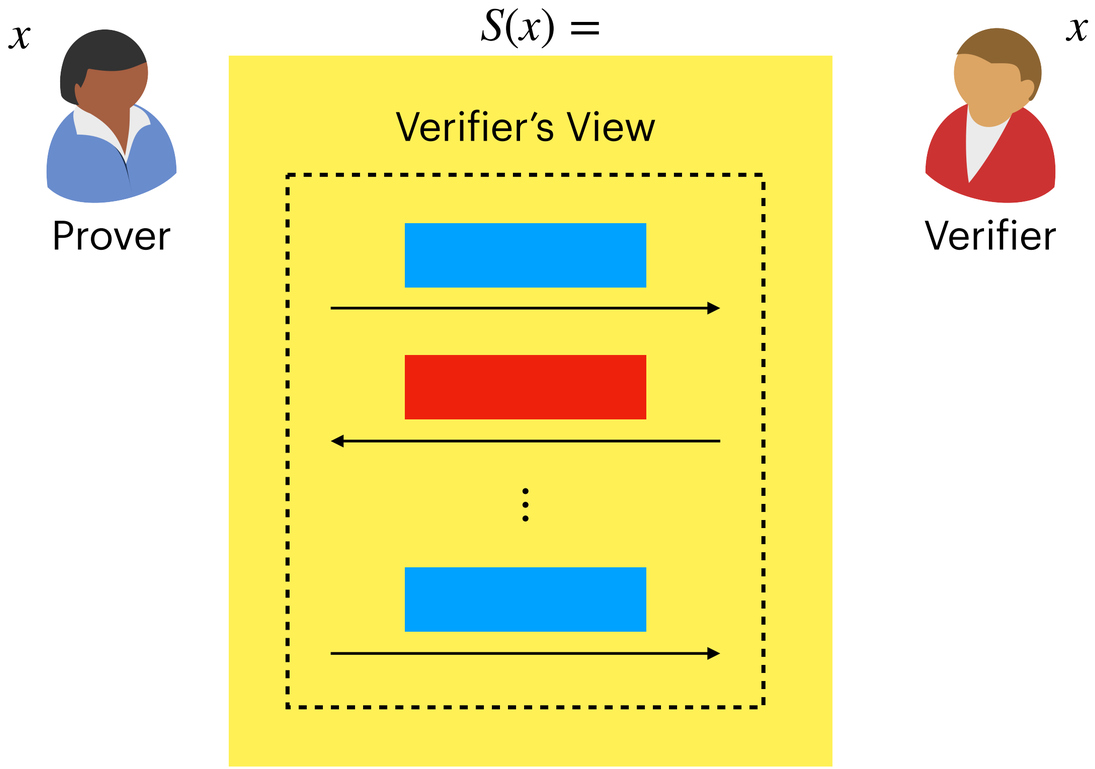

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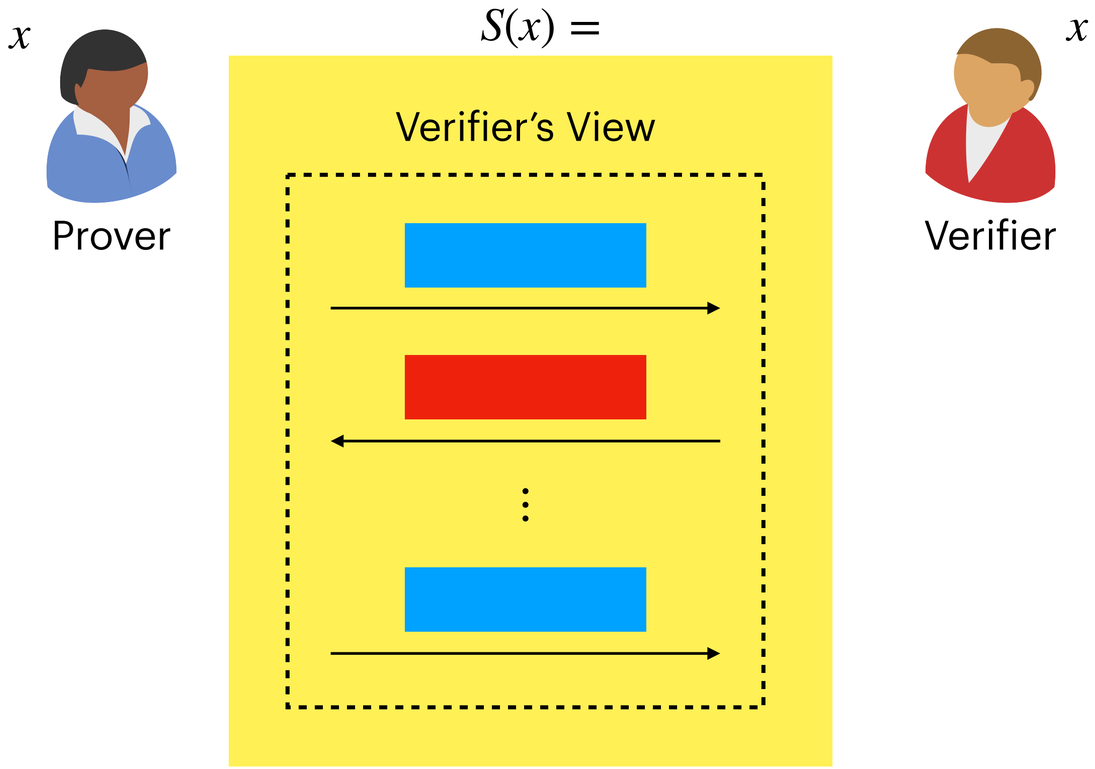

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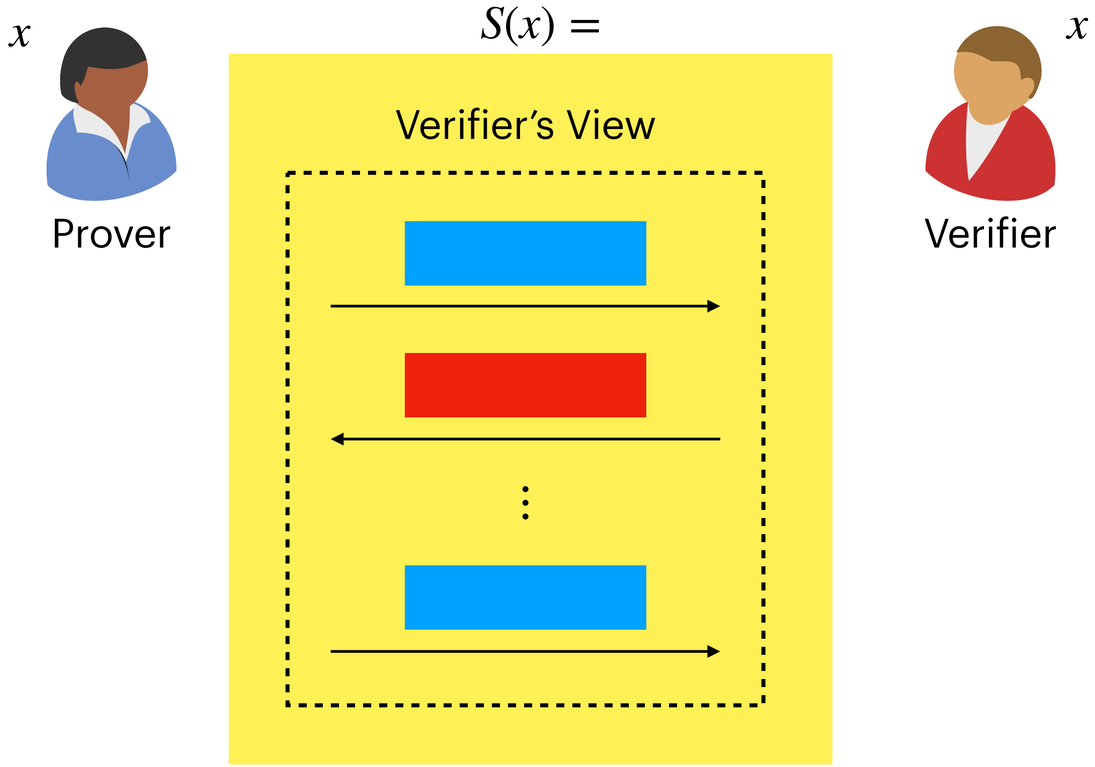
- The verifier is a PPT machine, the existence of such a simulator means it could have efficiently computed the view by itself.
  - Interacting with the prover provides no knowledge.

The Simulation Paradigm

- Paradox?
  - Protocol execution convinces the verifier of the validity of x.
  - The verifier could have generated the transcript by itself.



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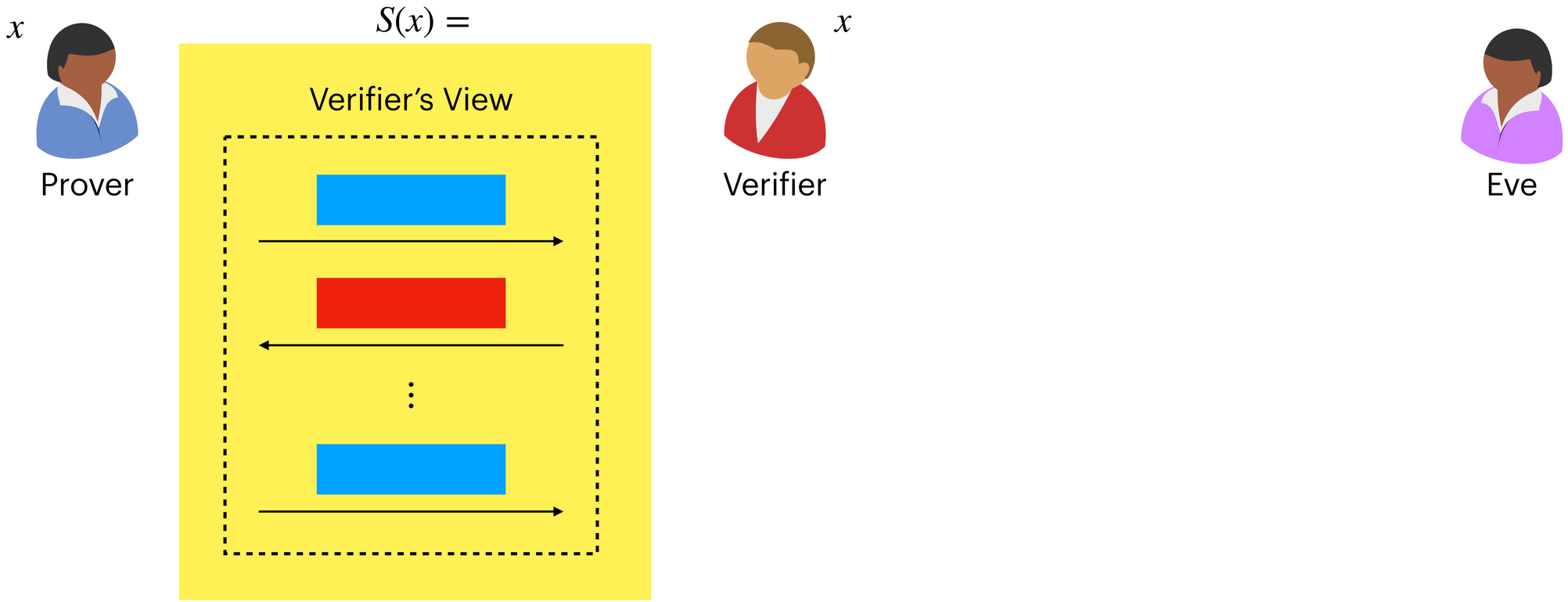

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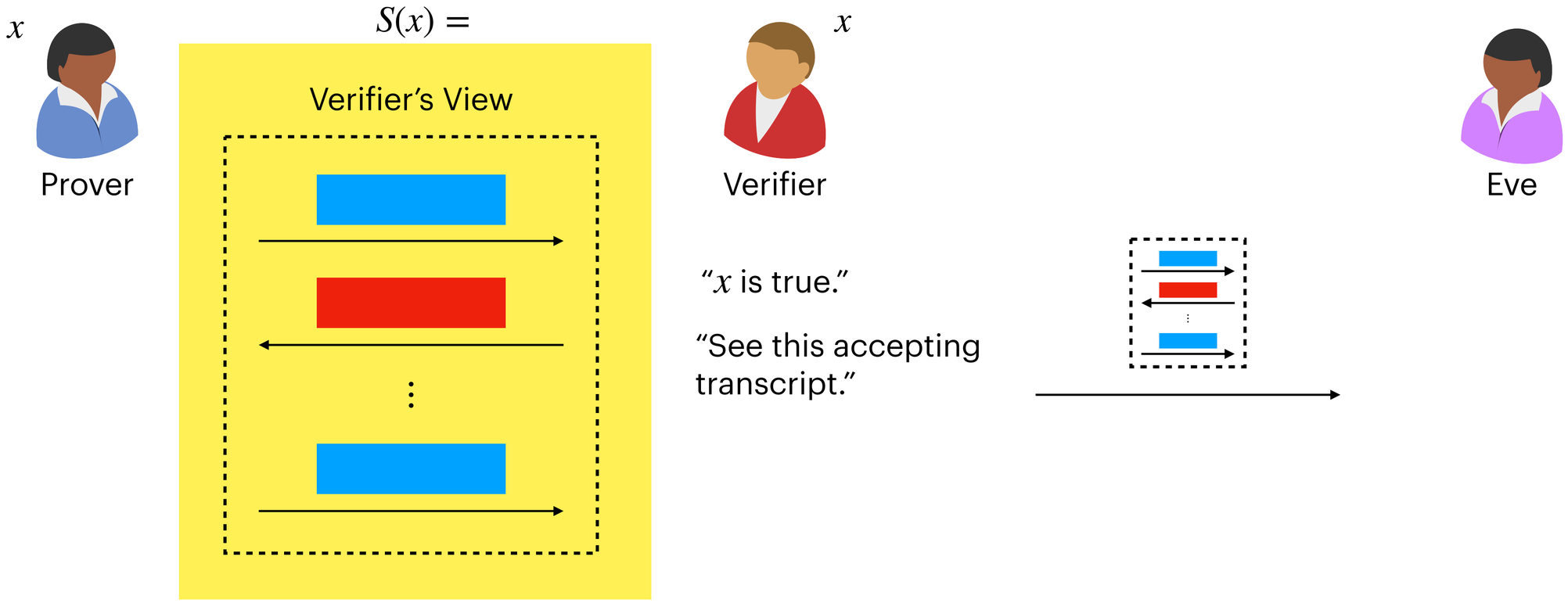

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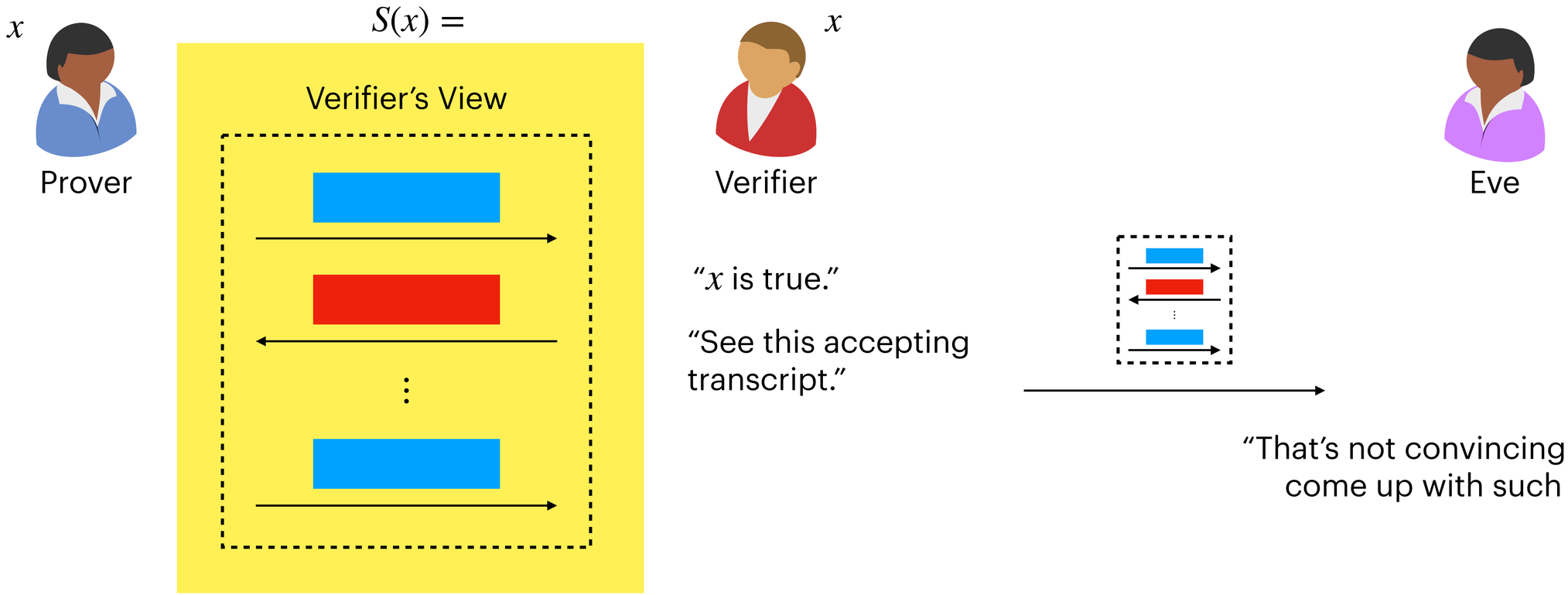

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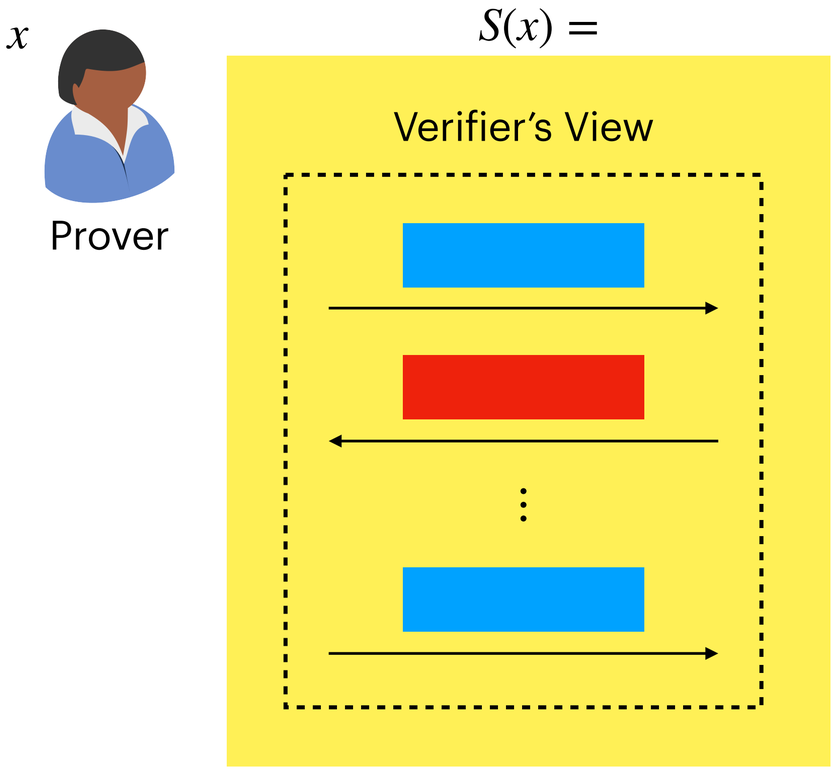
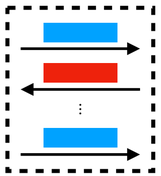


"x is true."

"See this accepting transcript."



"That's not convincing. Anyone can come up with such a transcript."

Verifier participates in a live conversation.

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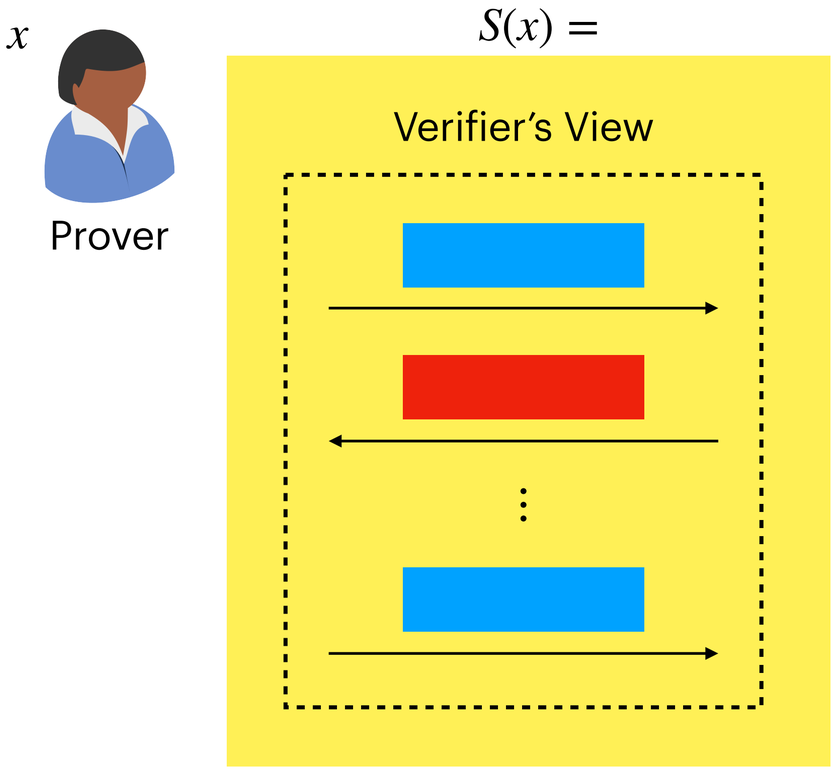

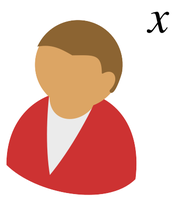

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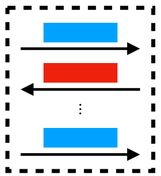

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- **Takeaway**
  - The prover must respond to the verifier's challenges on the fly. This is what convinces the verifier of the statement's validity.
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The Simulation Paradigm

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- **Analogy:** Think about a viral video.
  - If you witness the incident live, you are convinced it really happened.
  - If you only see the video, you remain skeptical; it could have been fabricated.

Zero-Knowledge Proof

Zero-Knowledge Proof System

A proof system (P, V) for a language $L \subset \{0,1\}^*$ is zero-knowledge if for every PPT algorithm $\widehat{V}$, there exists a PPT simulator S such that for every $x \in L$

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This was captured in our earlier definitions by considering a non-uniform adversary.

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- If the distributions are statistically close we call it statistical zero-knowledge.