CCA Security I

601.442/642 Modern Cryptography

9th April 2026

Logistics

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- Extra-credit HW out next week.

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- Discuss how it may not be sufficient in practice
- Introduce stronger IND-CCA security
- Describe an IND-CCA secure symmetric encryption scheme

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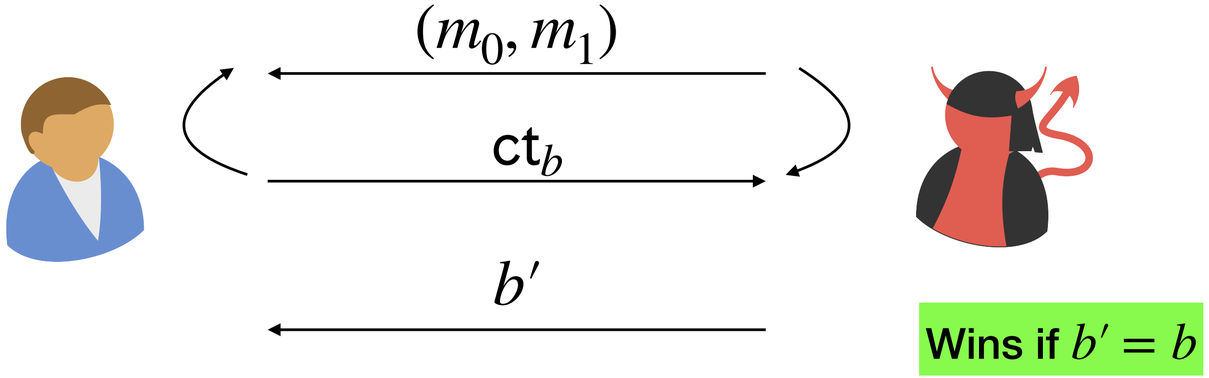

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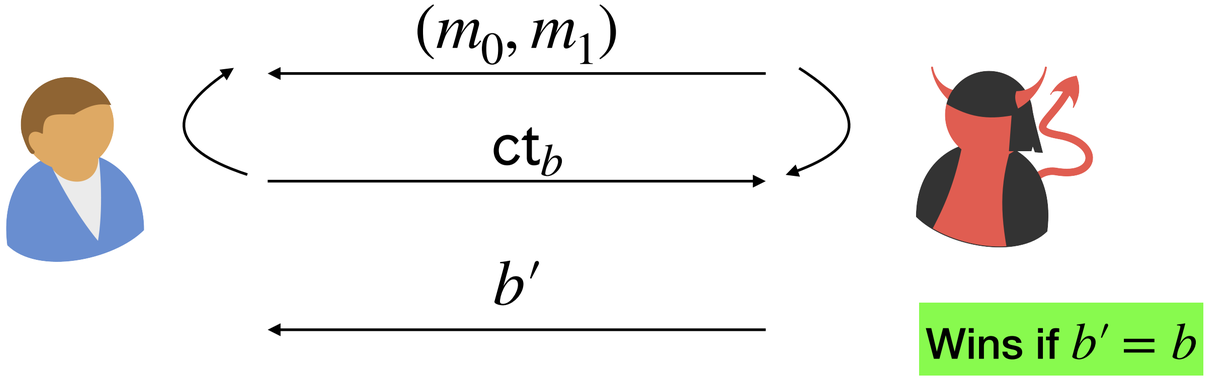

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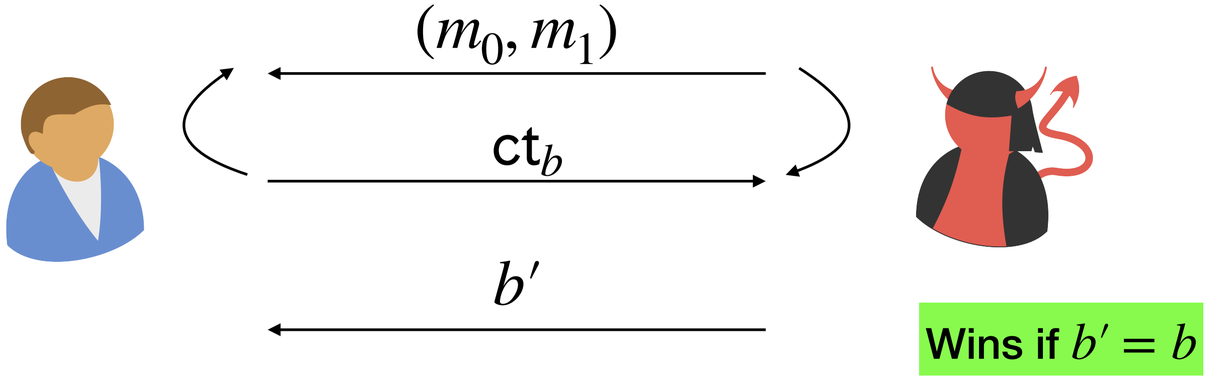

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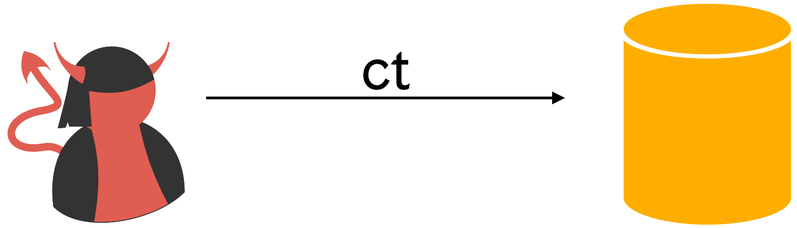

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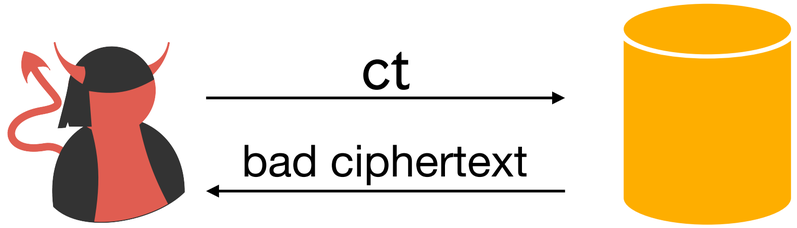

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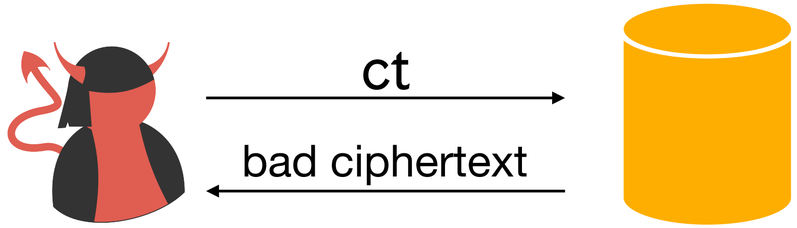

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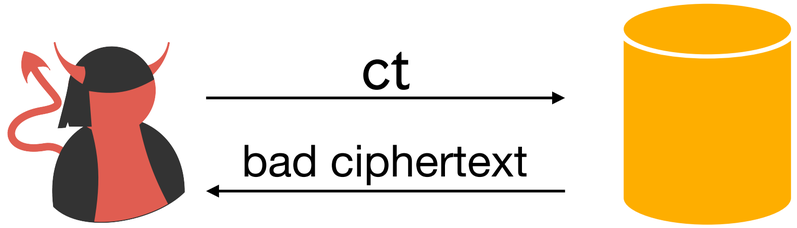

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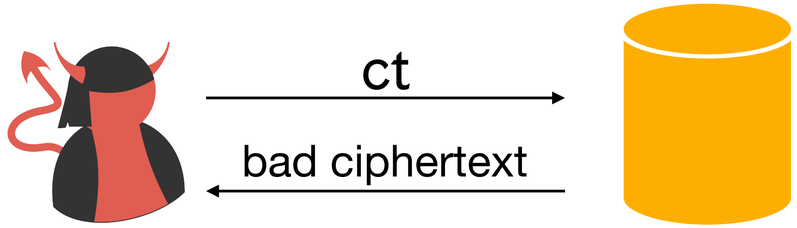

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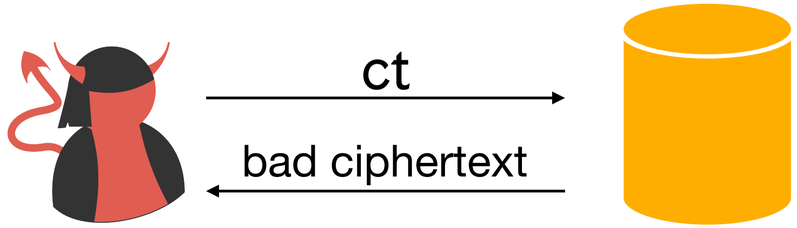

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  - This has been used to completely break schemes in real life!
  - The problem was these schemes were proved to be secure *only* if the adversary can't learn anything about the ciphertexts.

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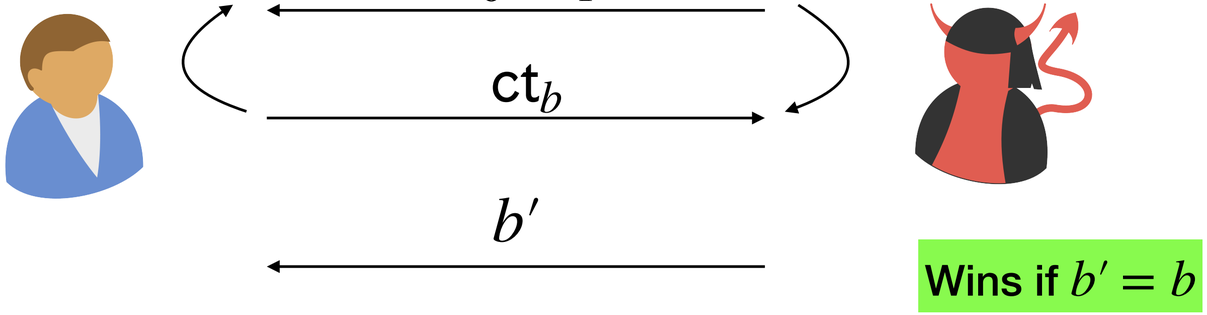

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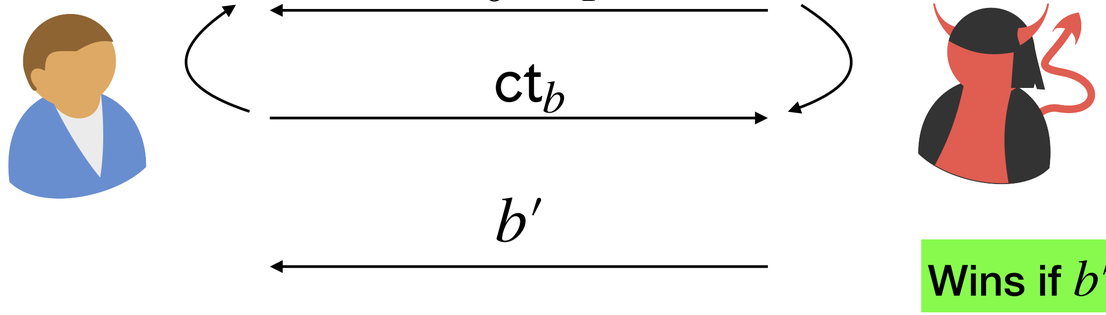

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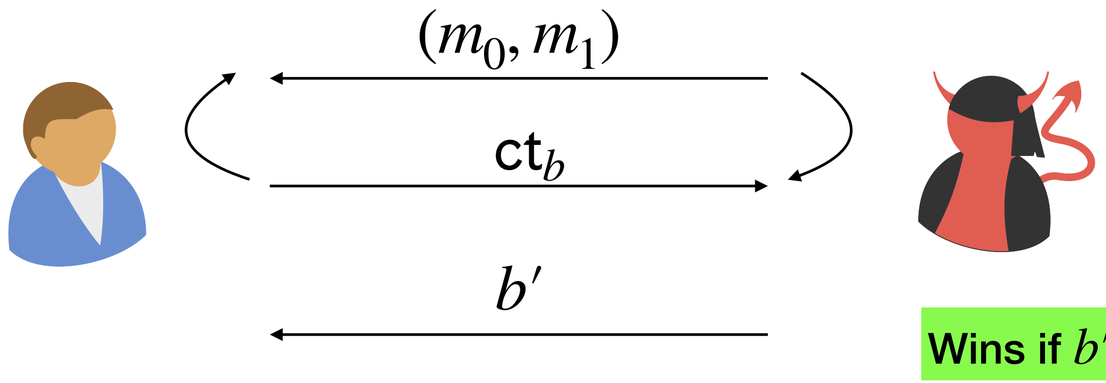

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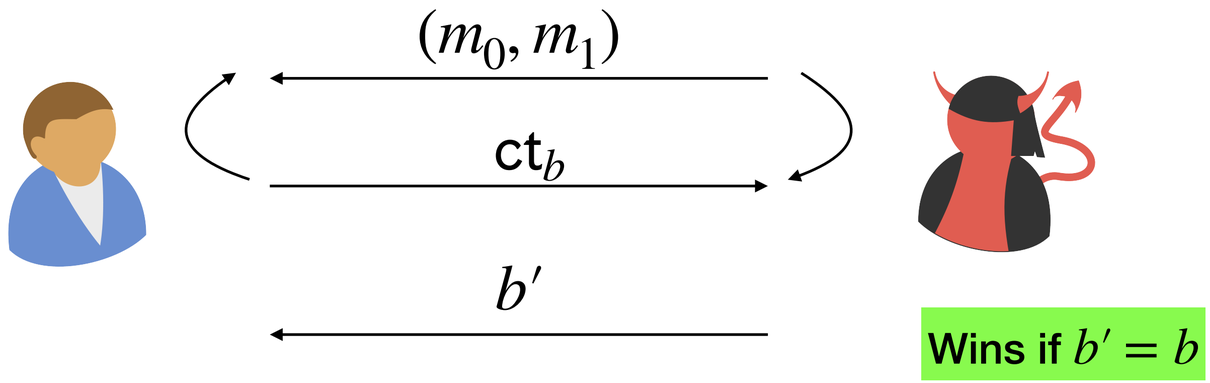

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- The *stronger* the adversary is, i.e. the more it can learn, the more secure our scheme is when we prove that it satisfies the definition.
- What is the most information an adversary can learn about a ciphertext?
- The plaintext it encrypts!

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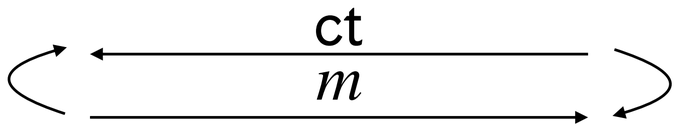

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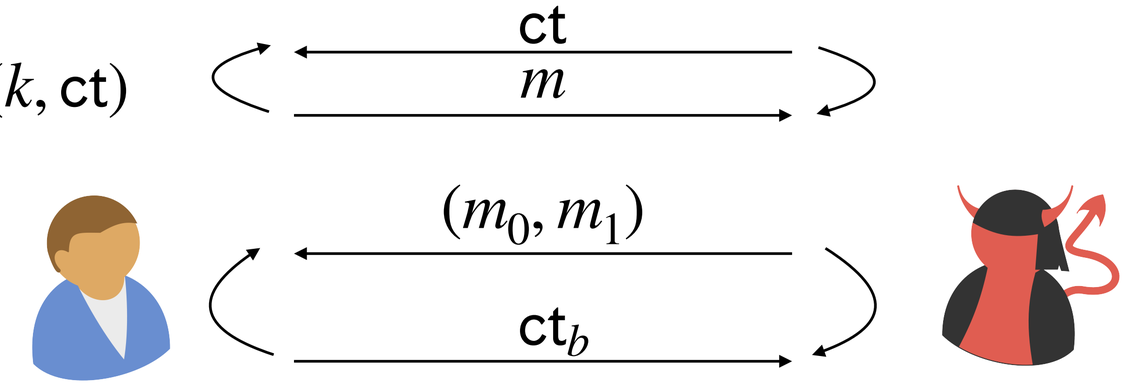

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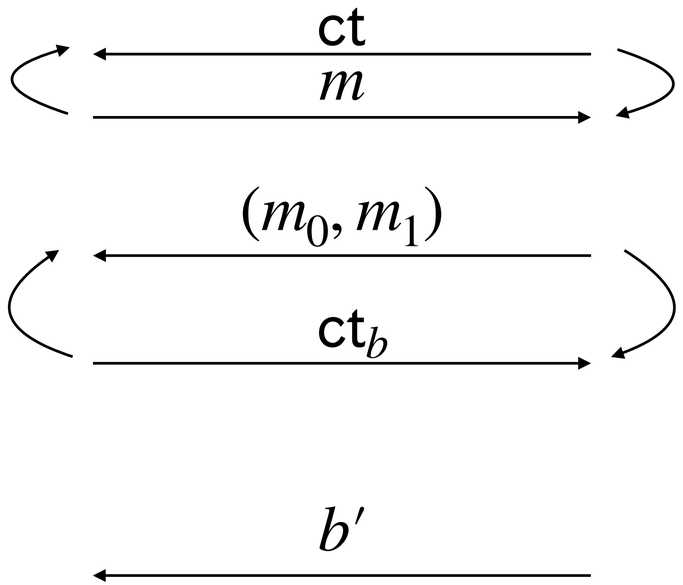

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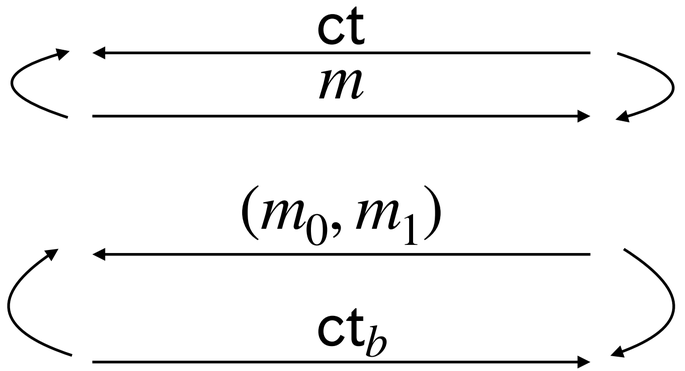

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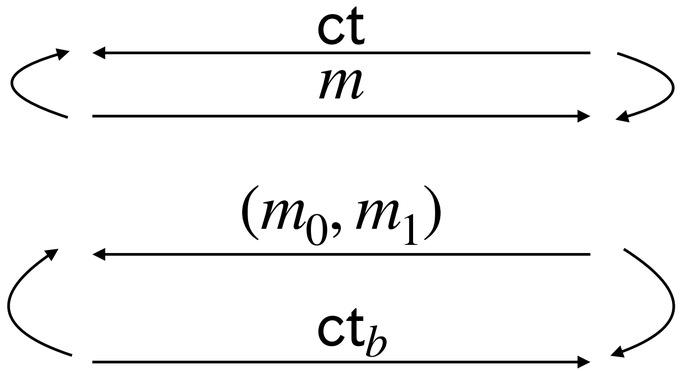

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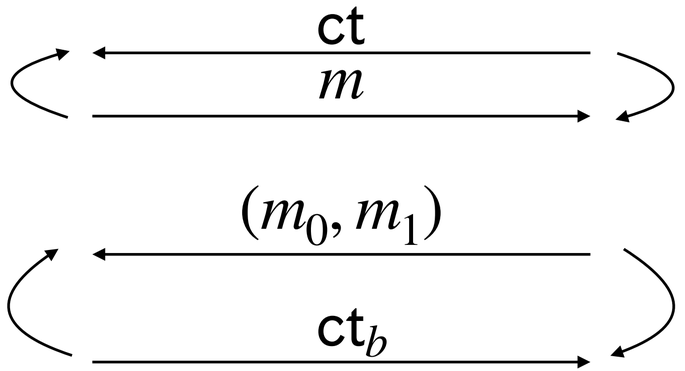

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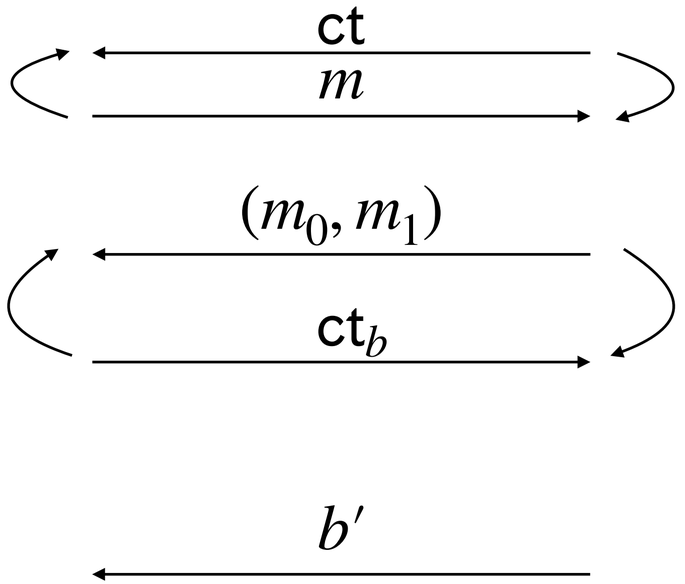

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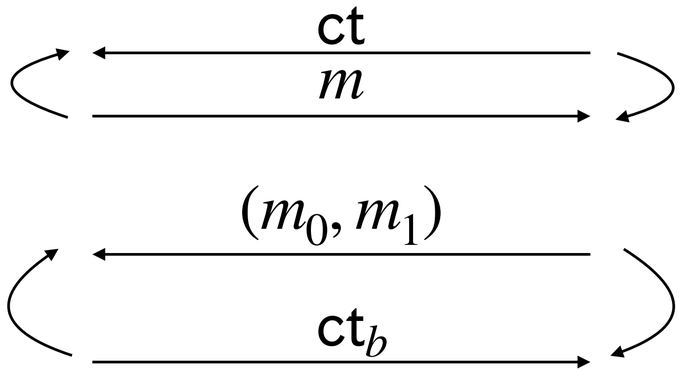

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- But also need to make sure they don't just decrypt $\mathbf{ct}_b$!

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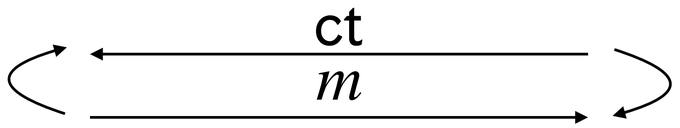

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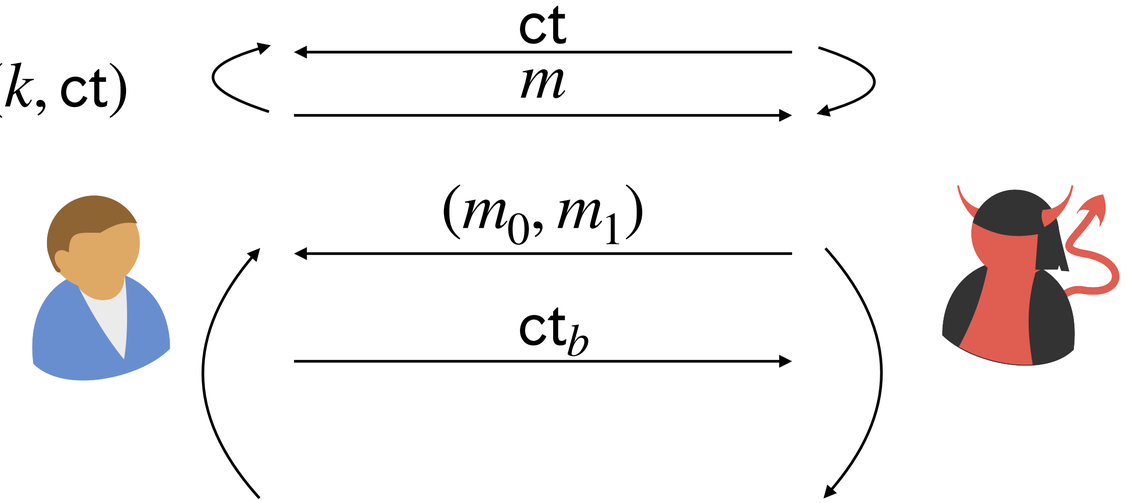

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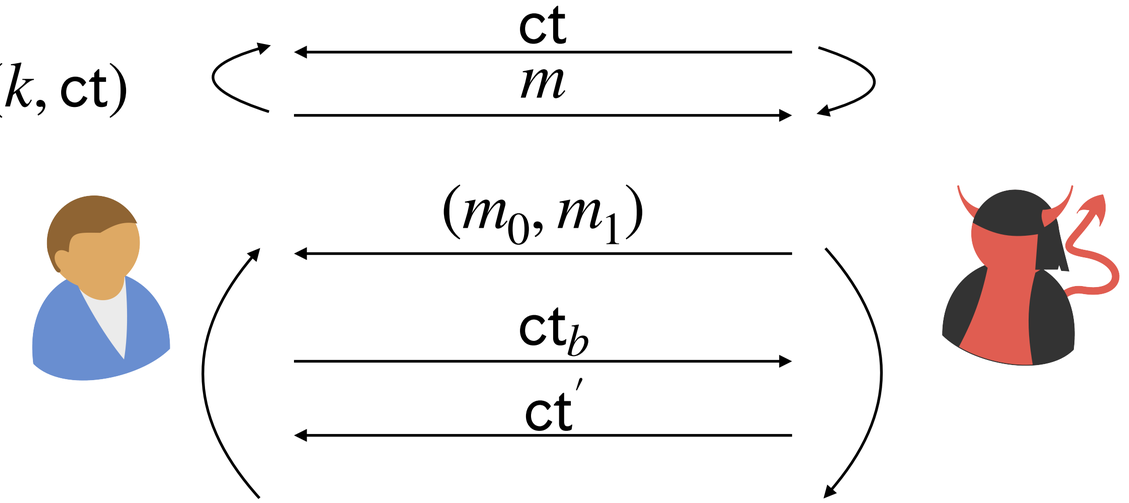

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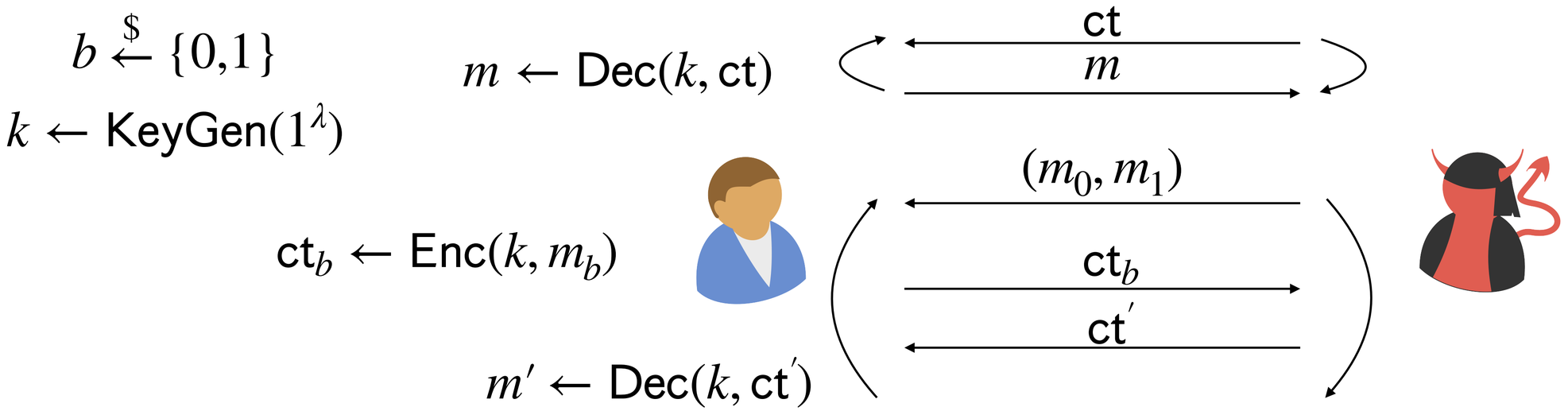

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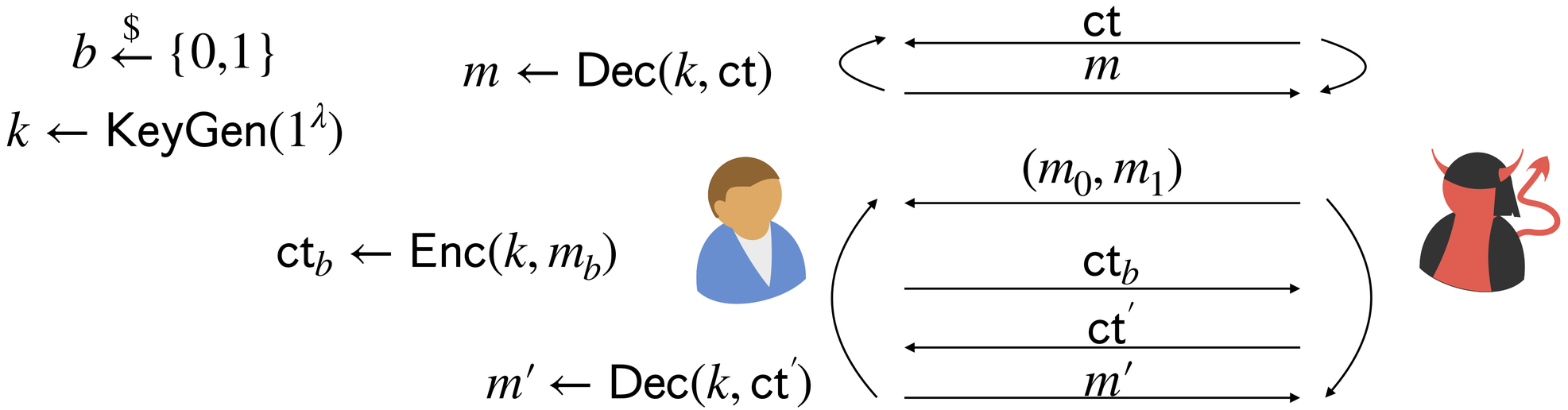

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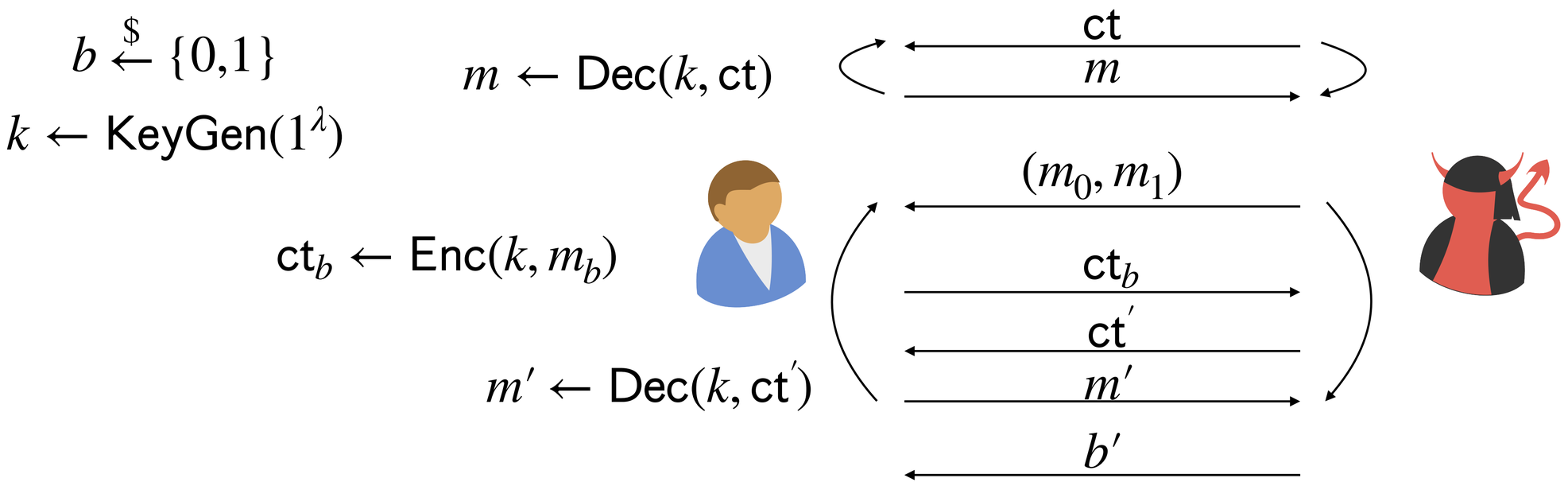

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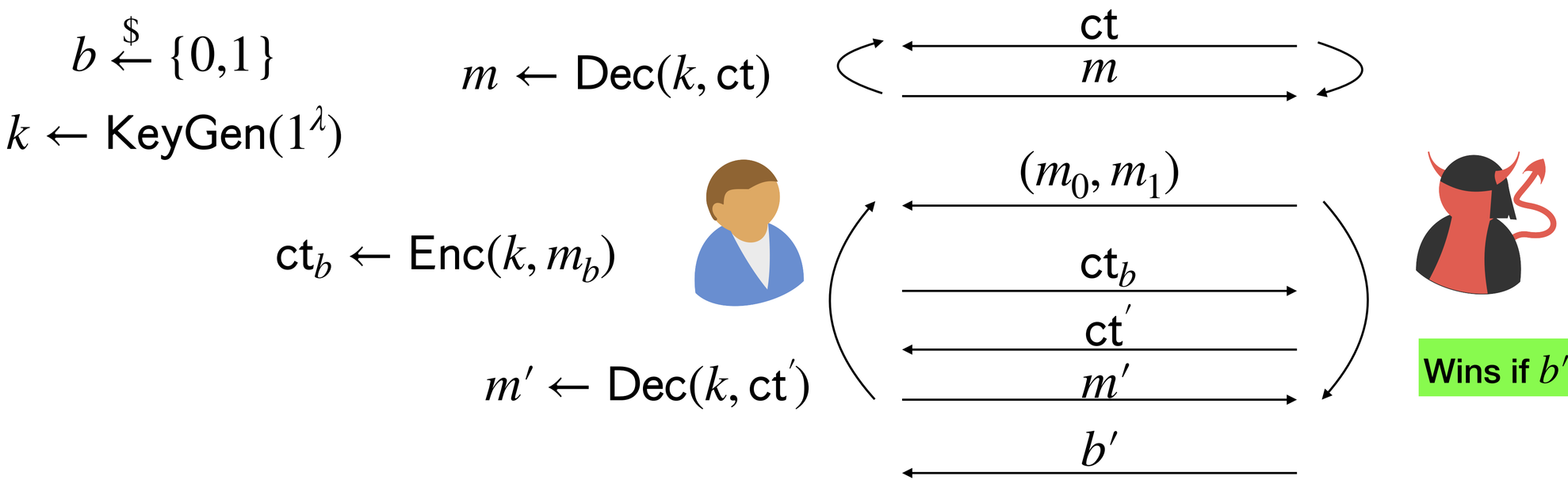

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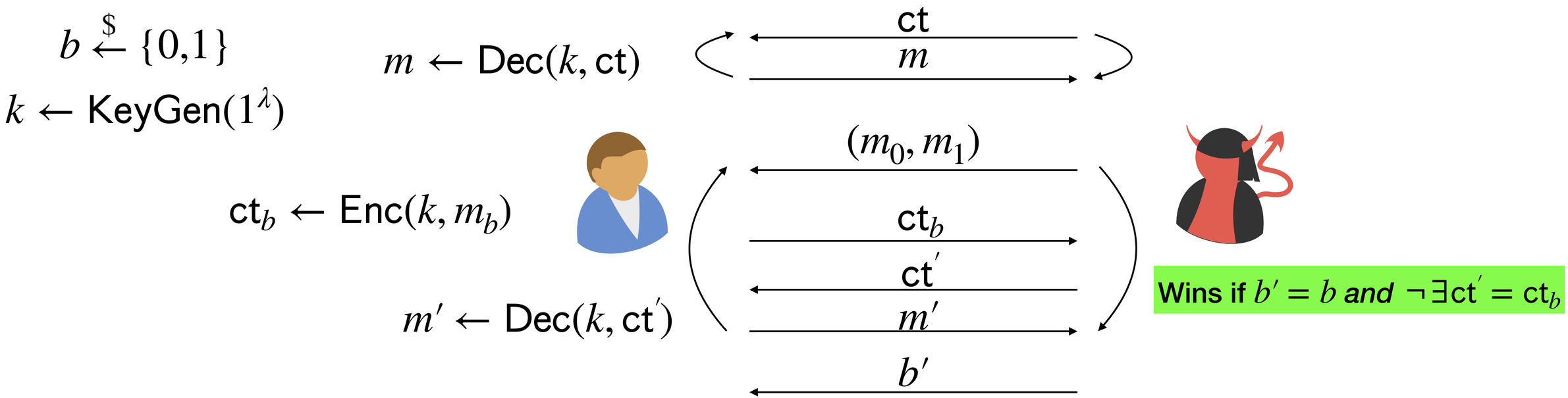

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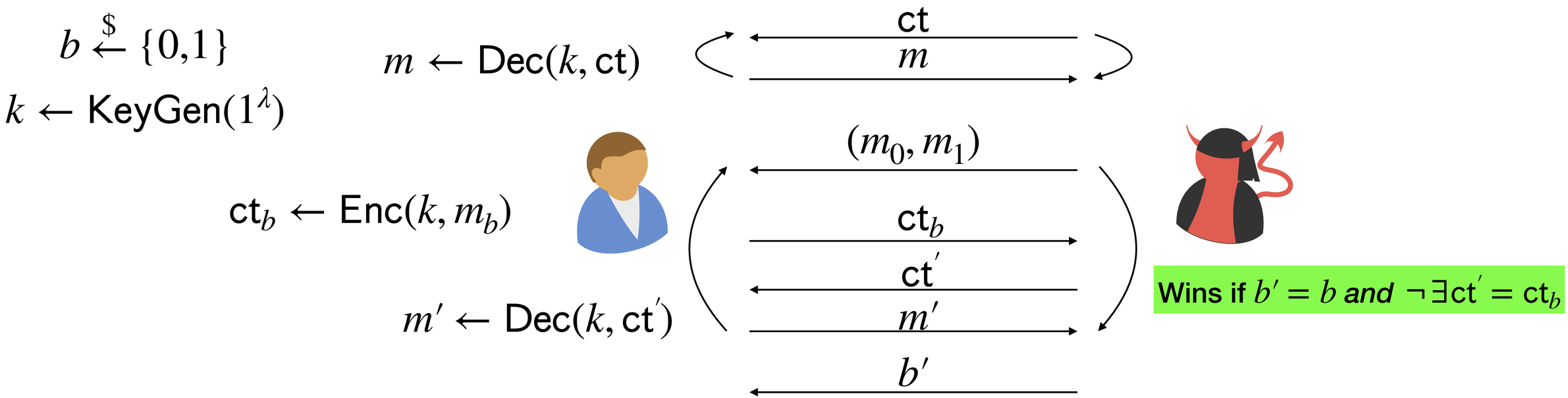

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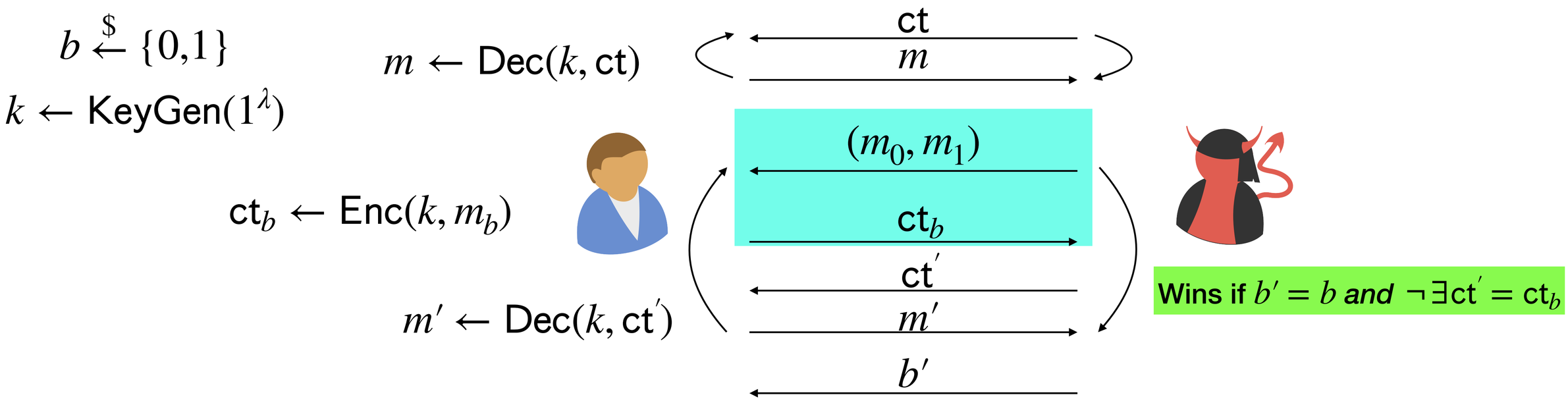

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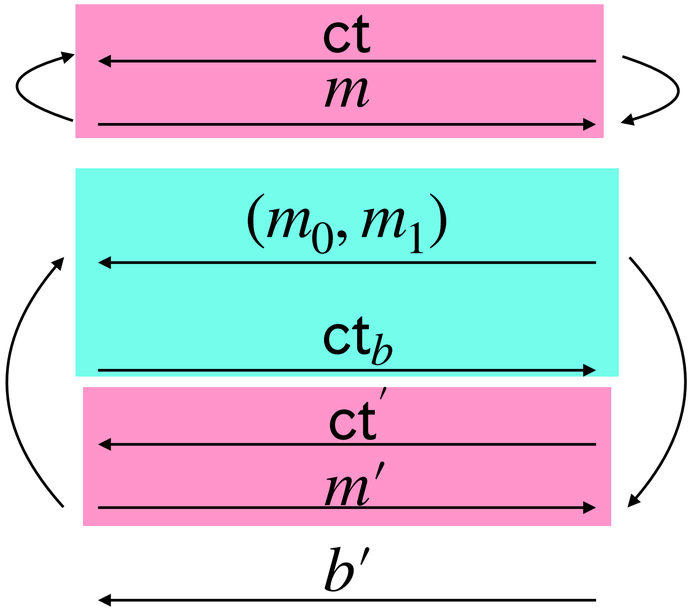

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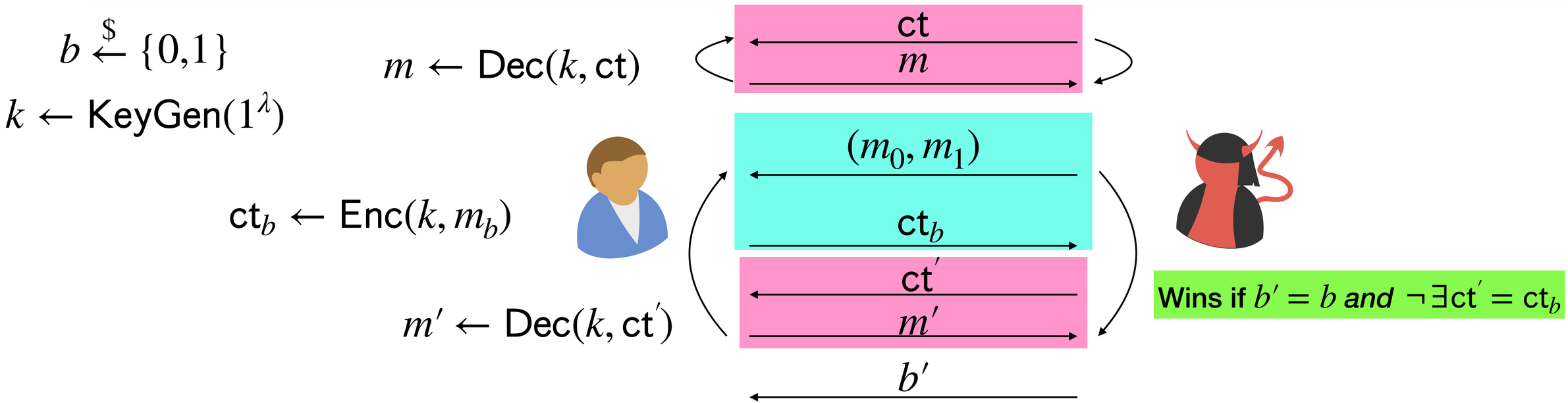

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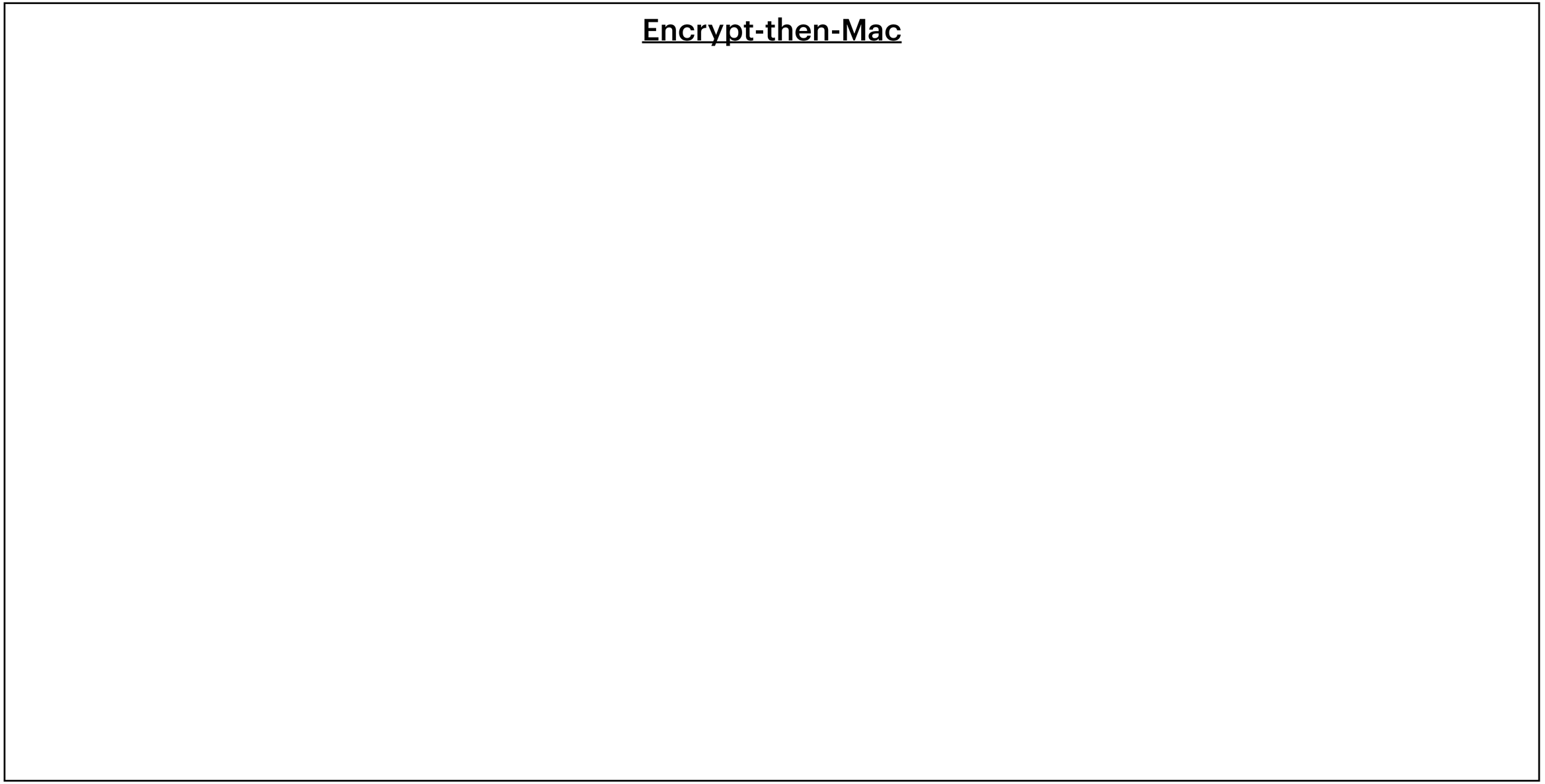
- How do we fix this? Prevent $\mathcal{A}$ from be able to make new *valid* ciphertexts.
- If **Dec** just rejects ciphertexts that weren't output by **Enc**, the decryption oracle is useless!

Encrypt-then-Mac



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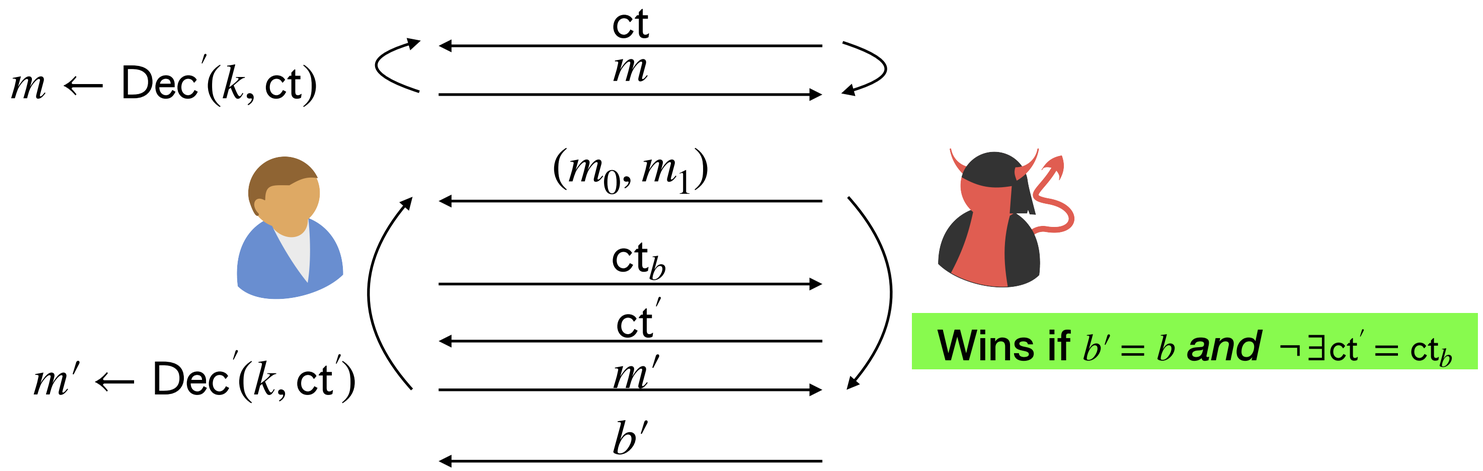

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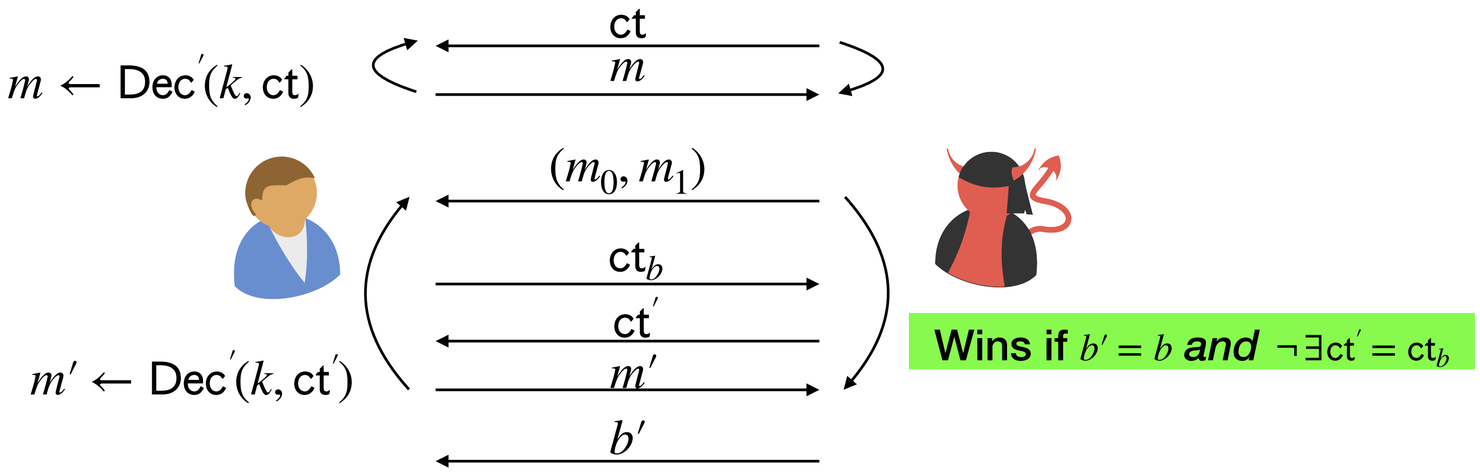

Proof of Security

- $\mathsf{Dec}'((k_{MAC}, k_{Enc}), (\mathsf{ct}, \sigma))$:
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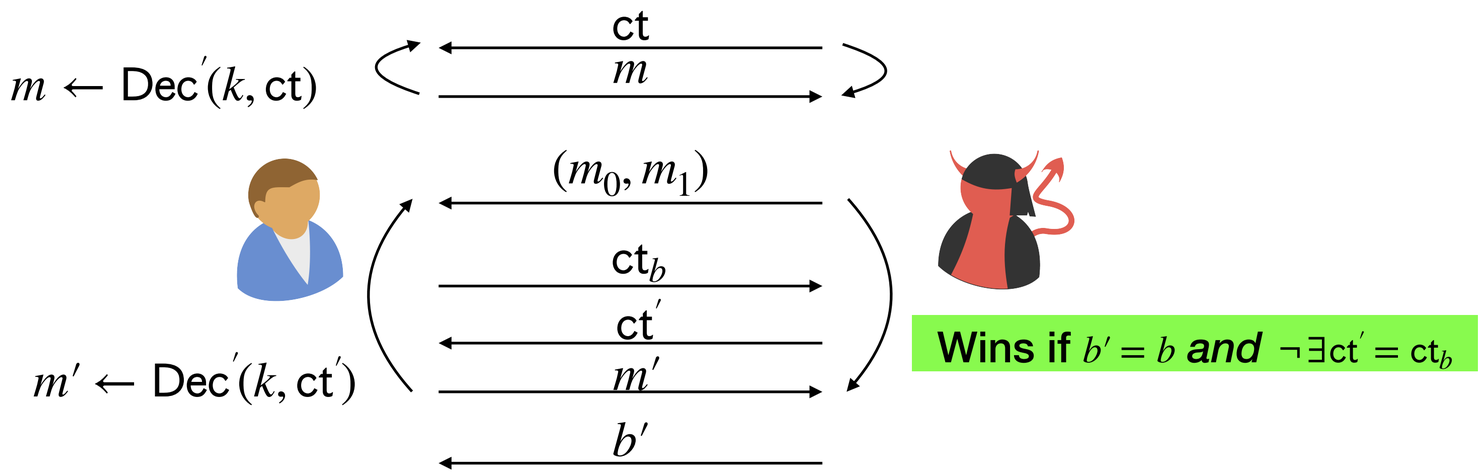
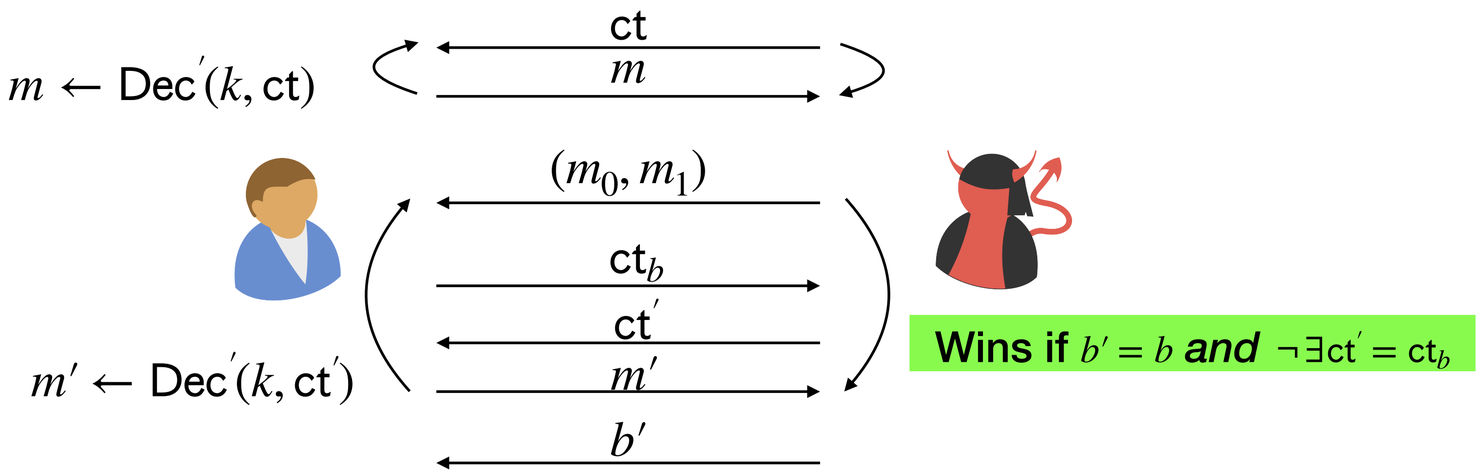


Game 1



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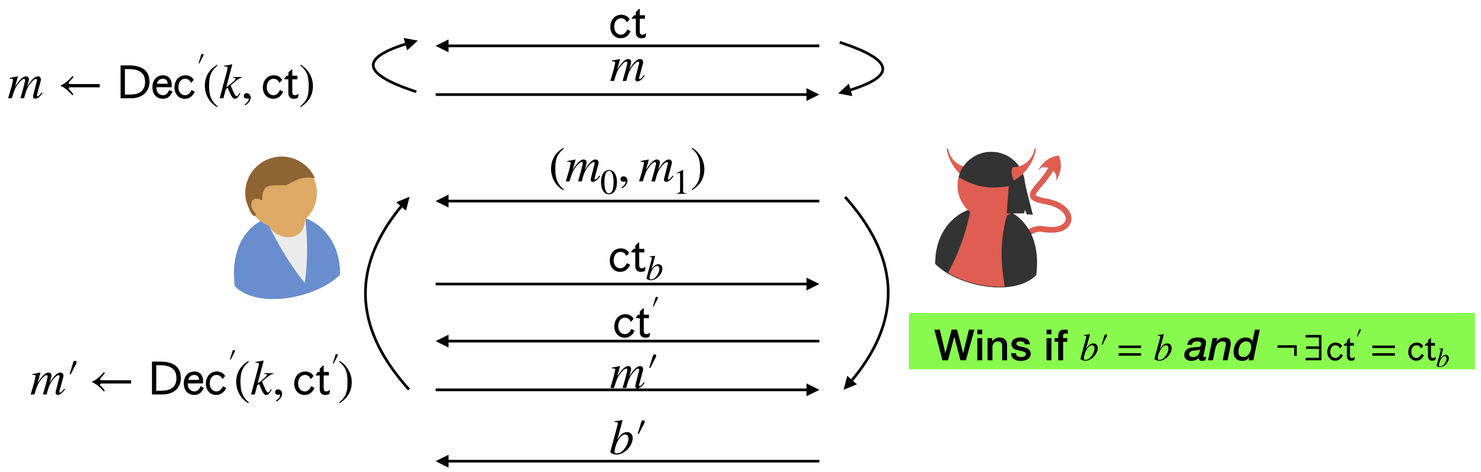

Wins if $b' = b$ *and* $\neg \exists \mathsf{ct}' = \mathsf{ct}_b$

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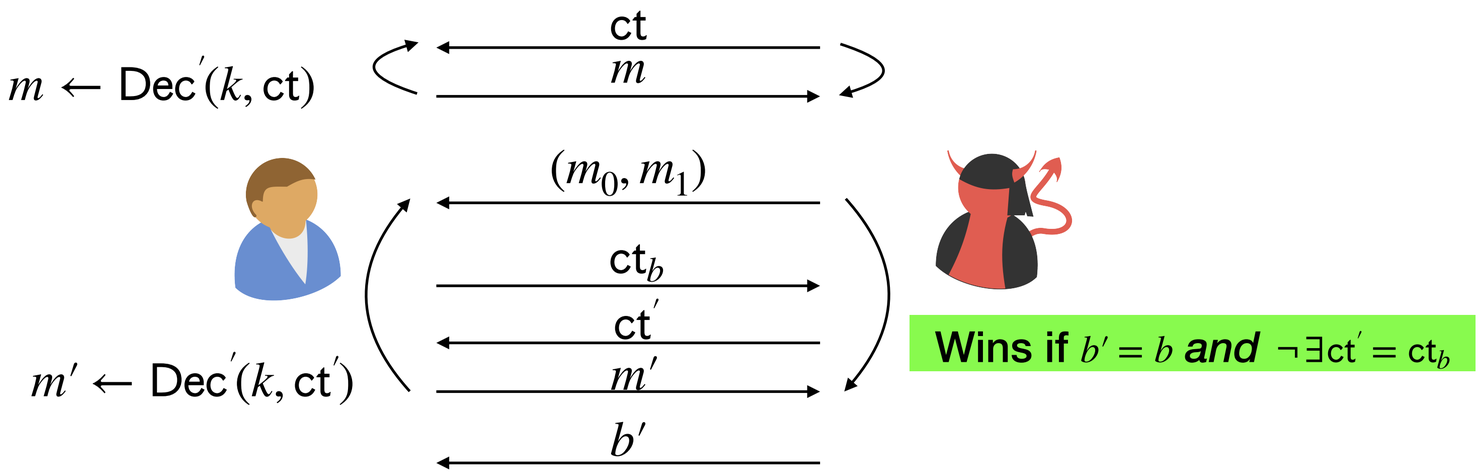

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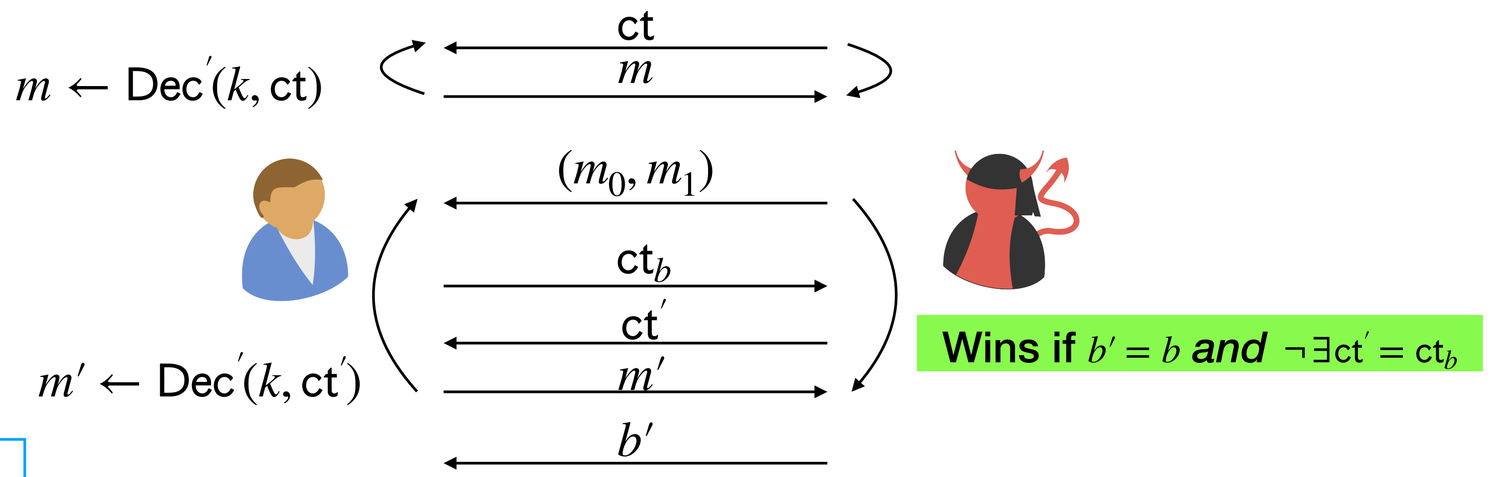

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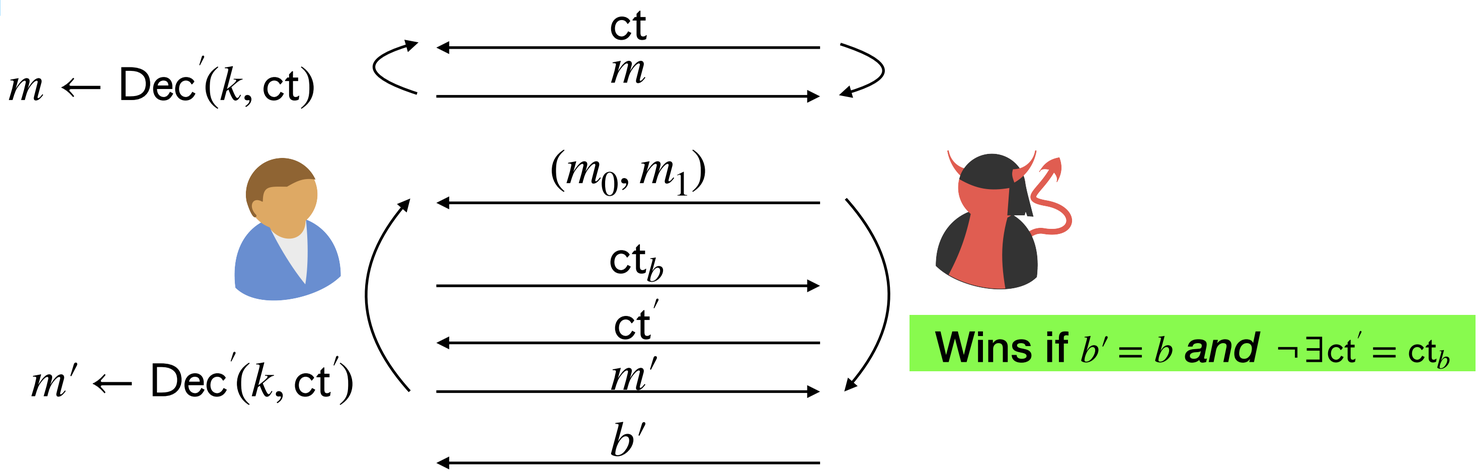

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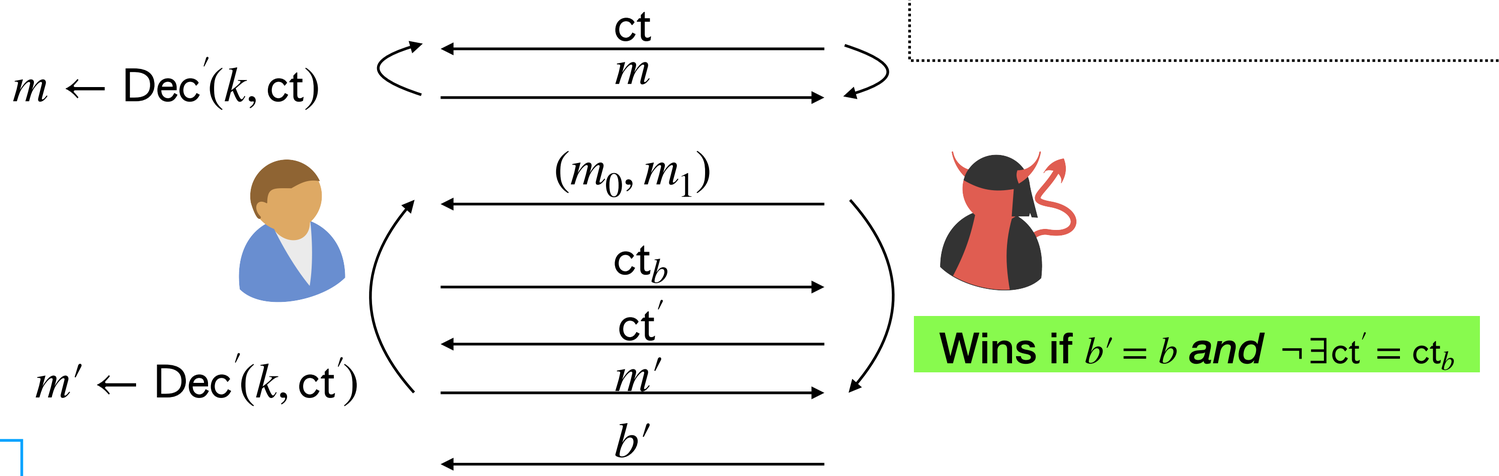
These games are only different to a winning adversary if it can create new signatures.

Game 0



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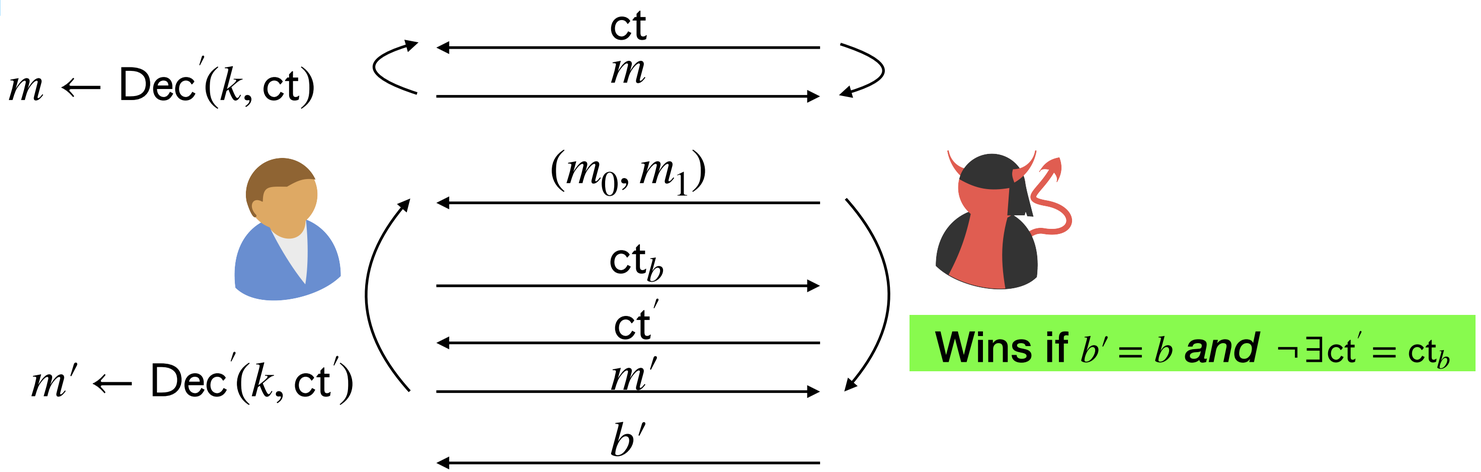

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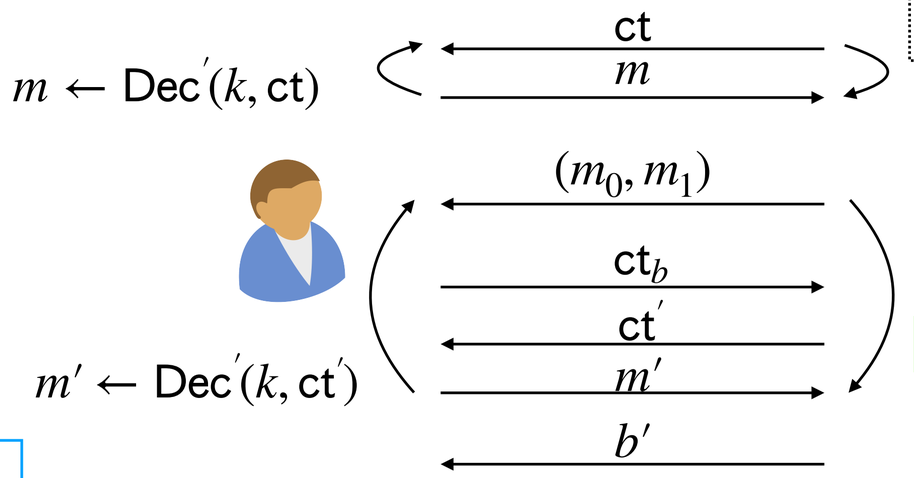

But, by strong UF-CMA security it can't!

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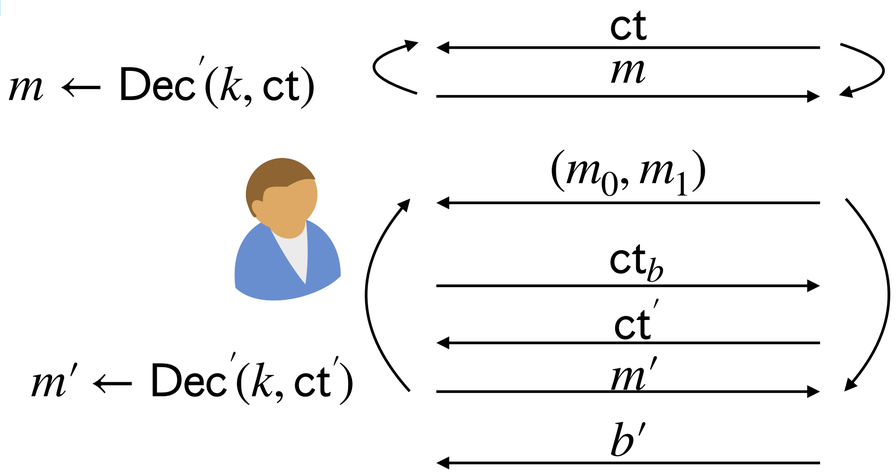
Game 1



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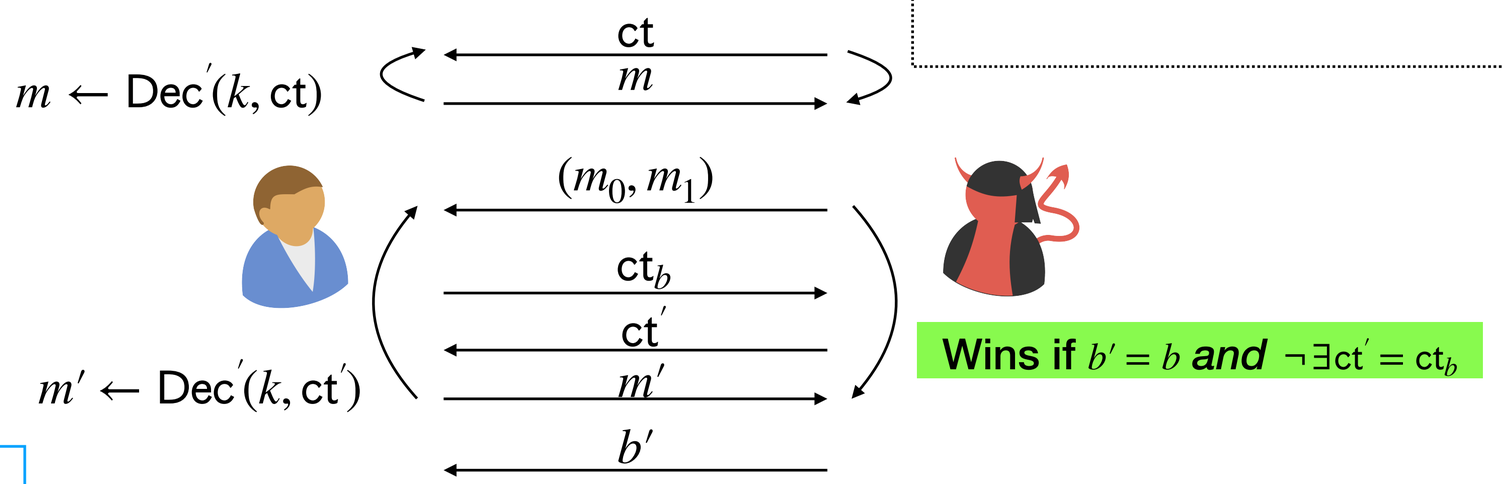

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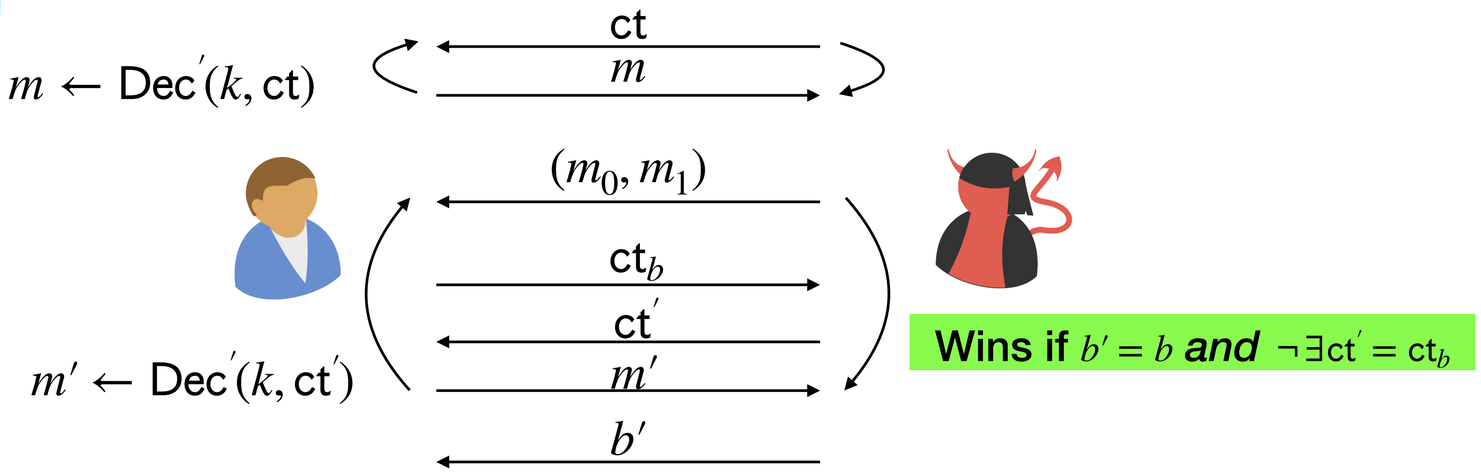

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So
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By the strong UF-CMA security of MAC

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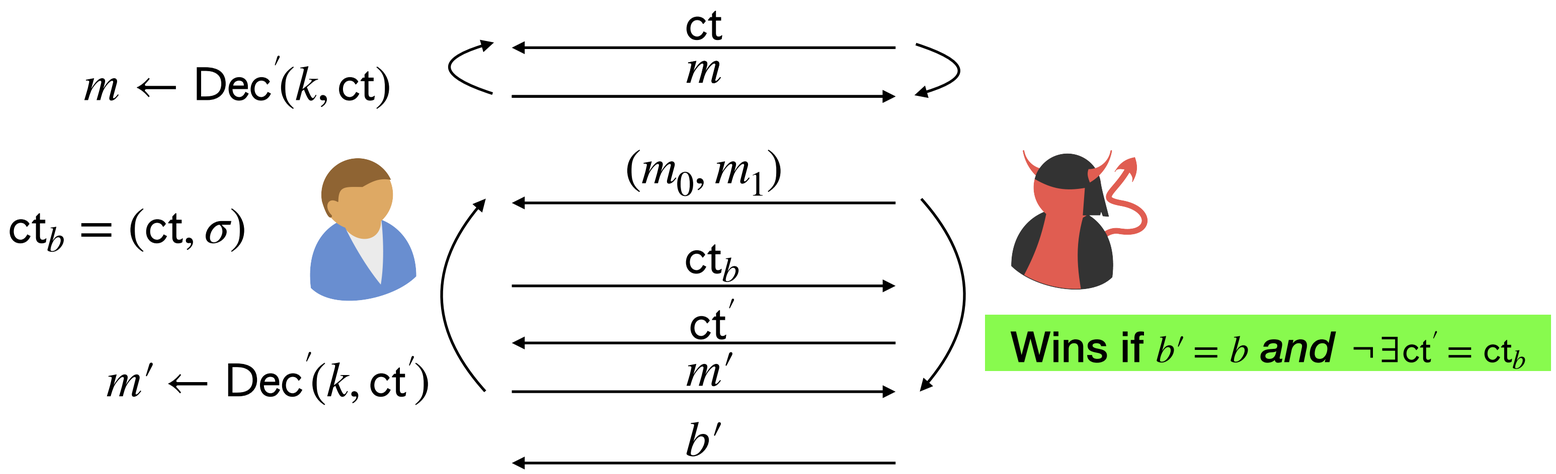

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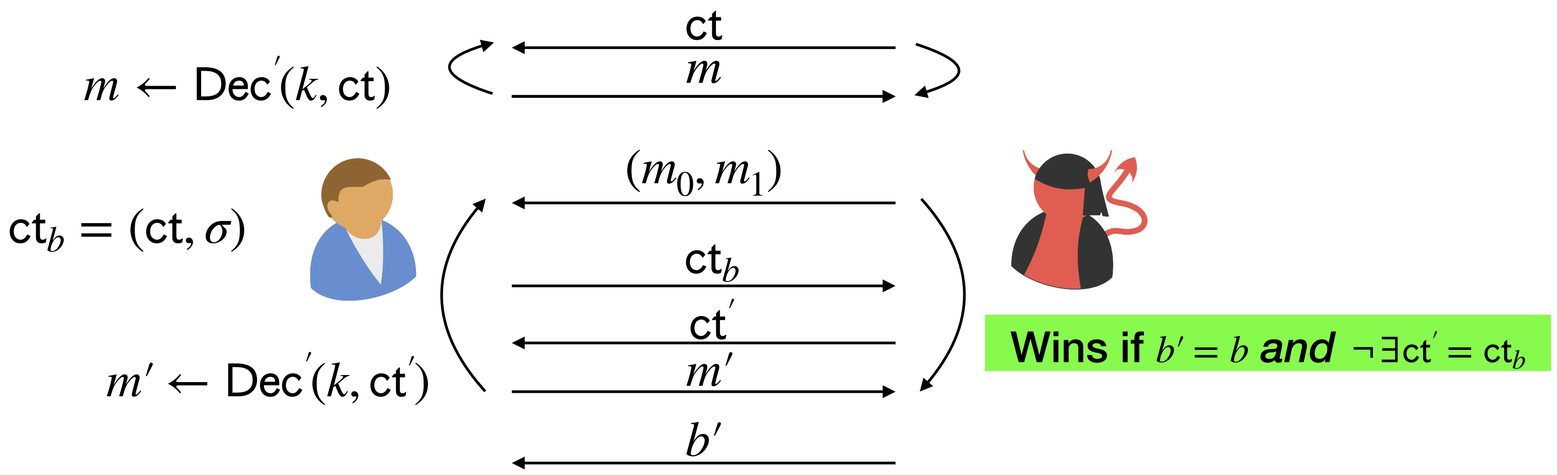

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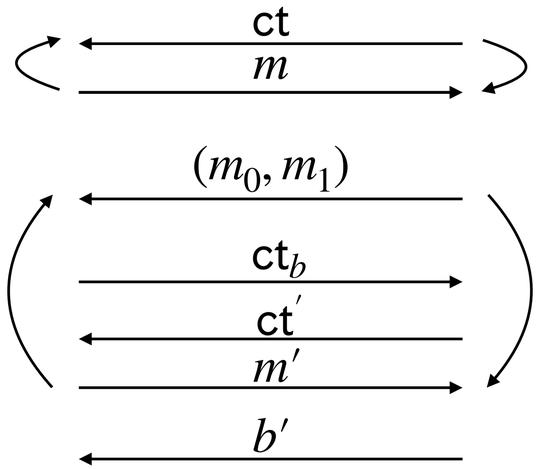
Ch_{CPA}

Game 1

$m \leftarrow \mathsf{Dec}'(k, \mathsf{ct})$

$\mathsf{ct}_b = (\mathsf{ct}, \sigma)$

$m' \leftarrow \mathsf{Dec}'(k, \mathsf{ct}')$



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B_{CPA}

A_{Game_1}

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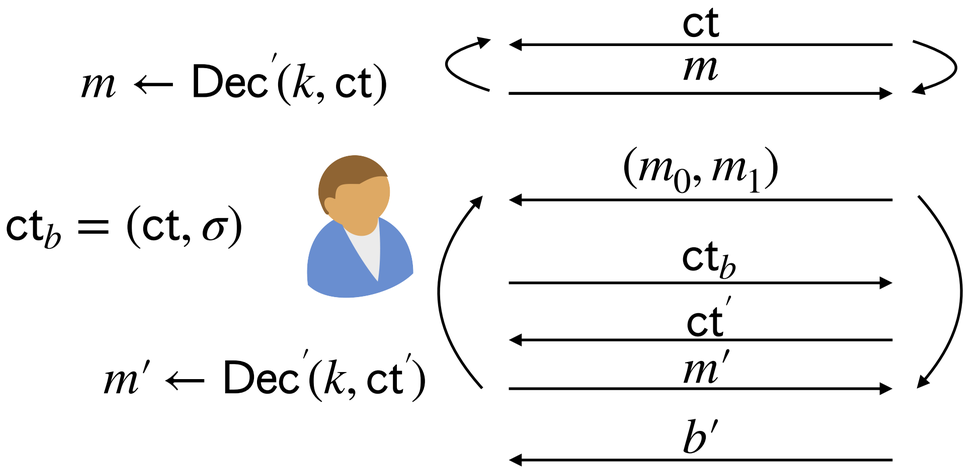

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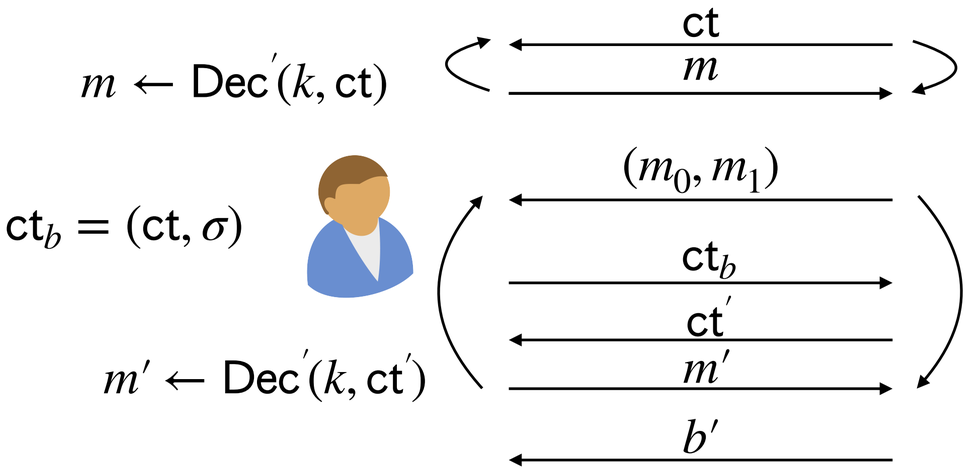

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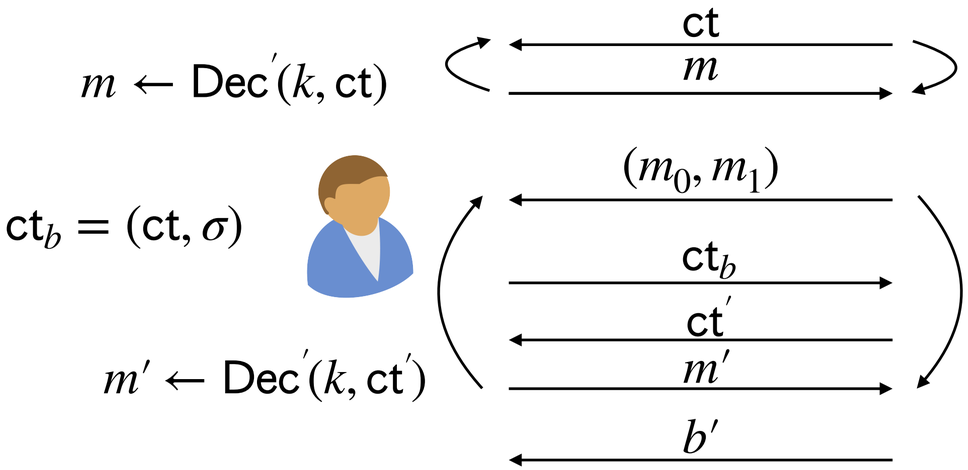

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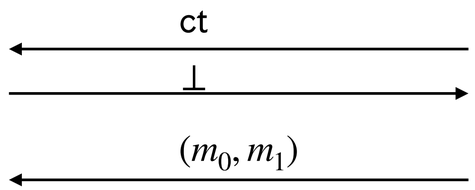

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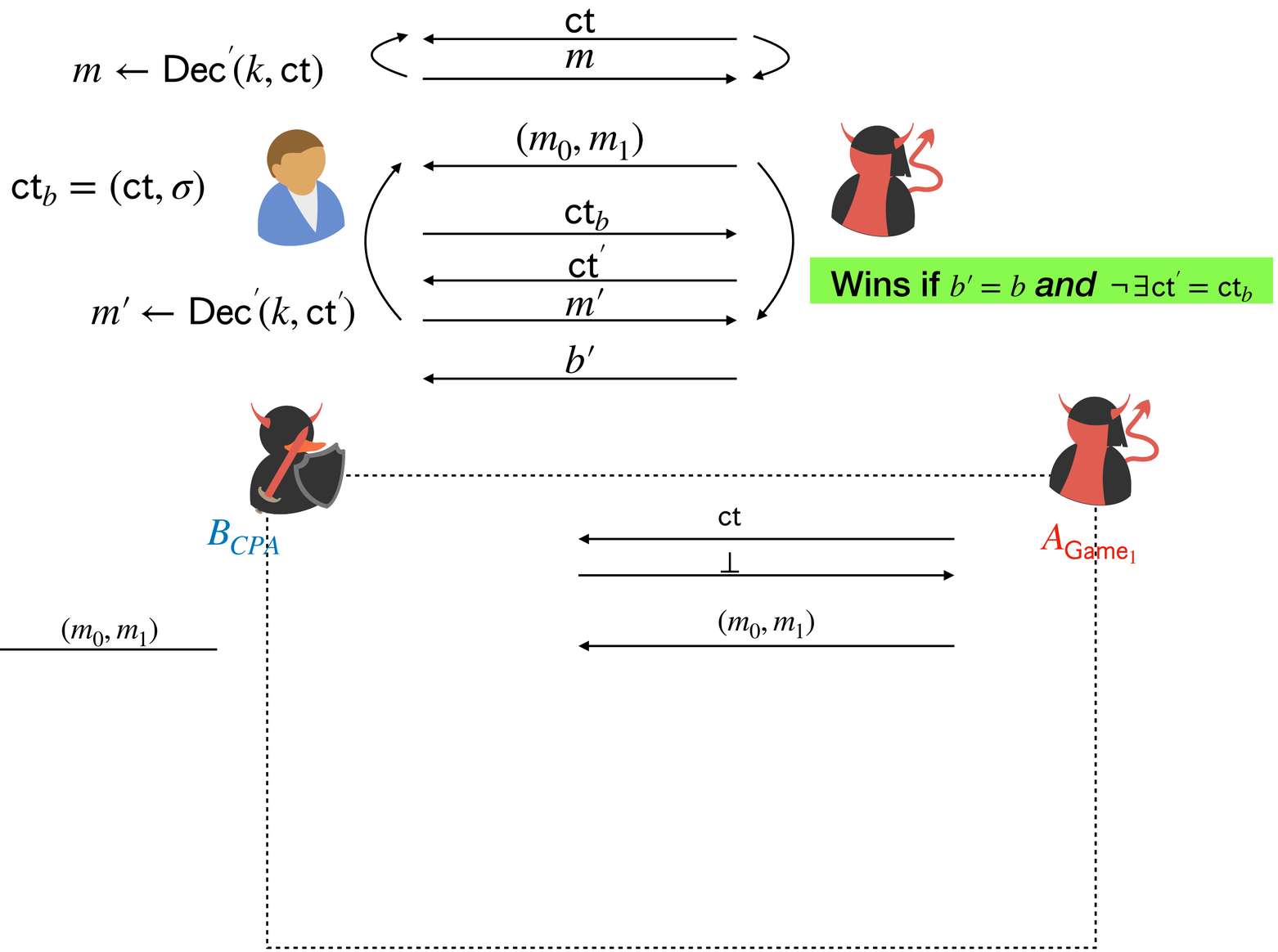

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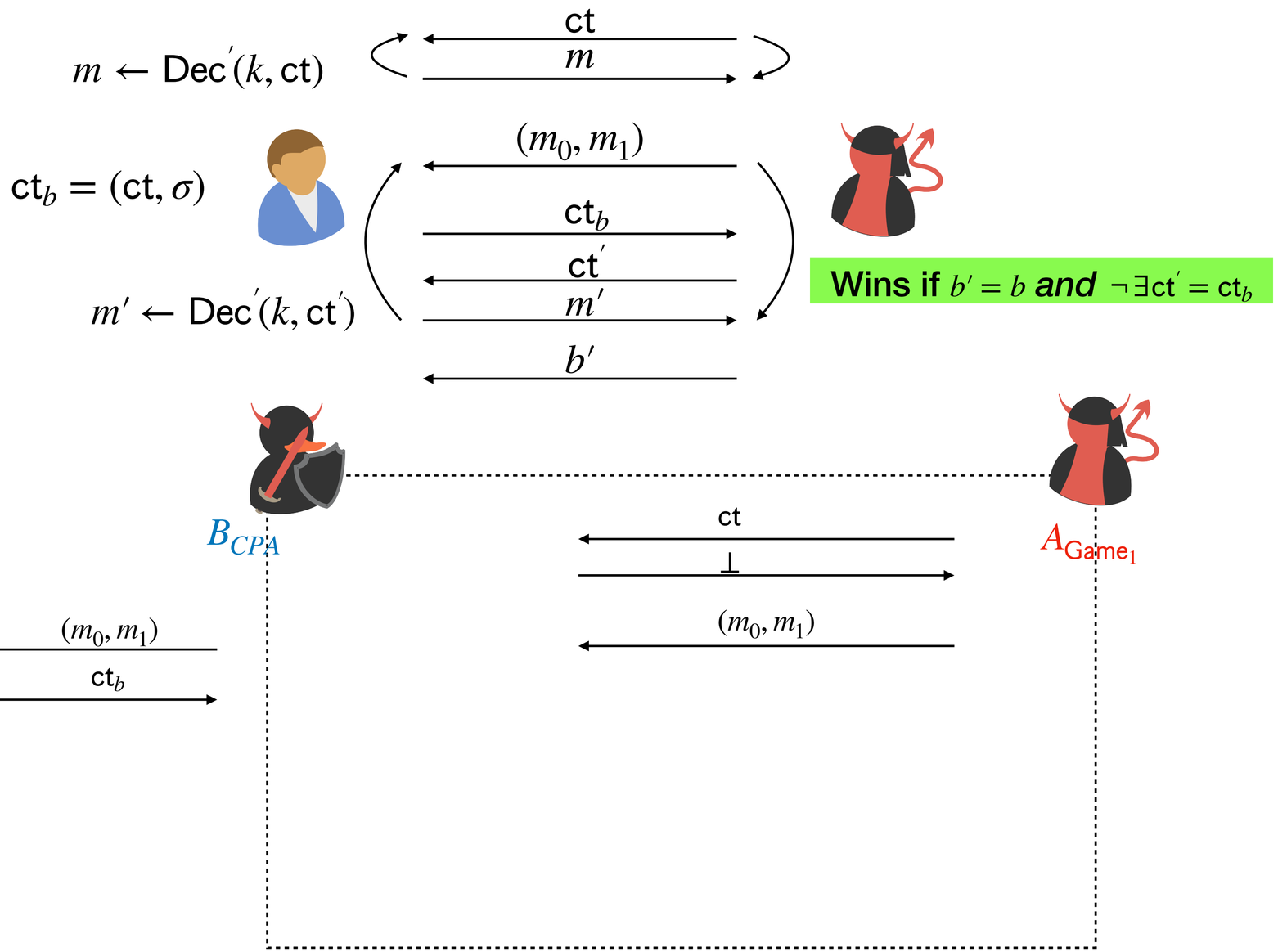

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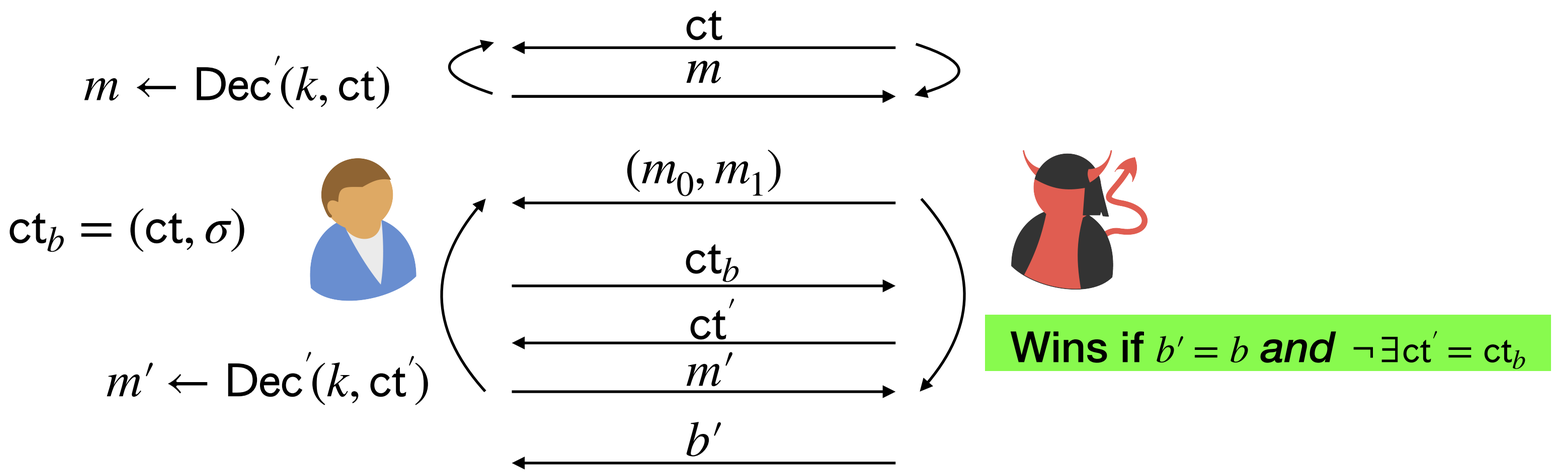

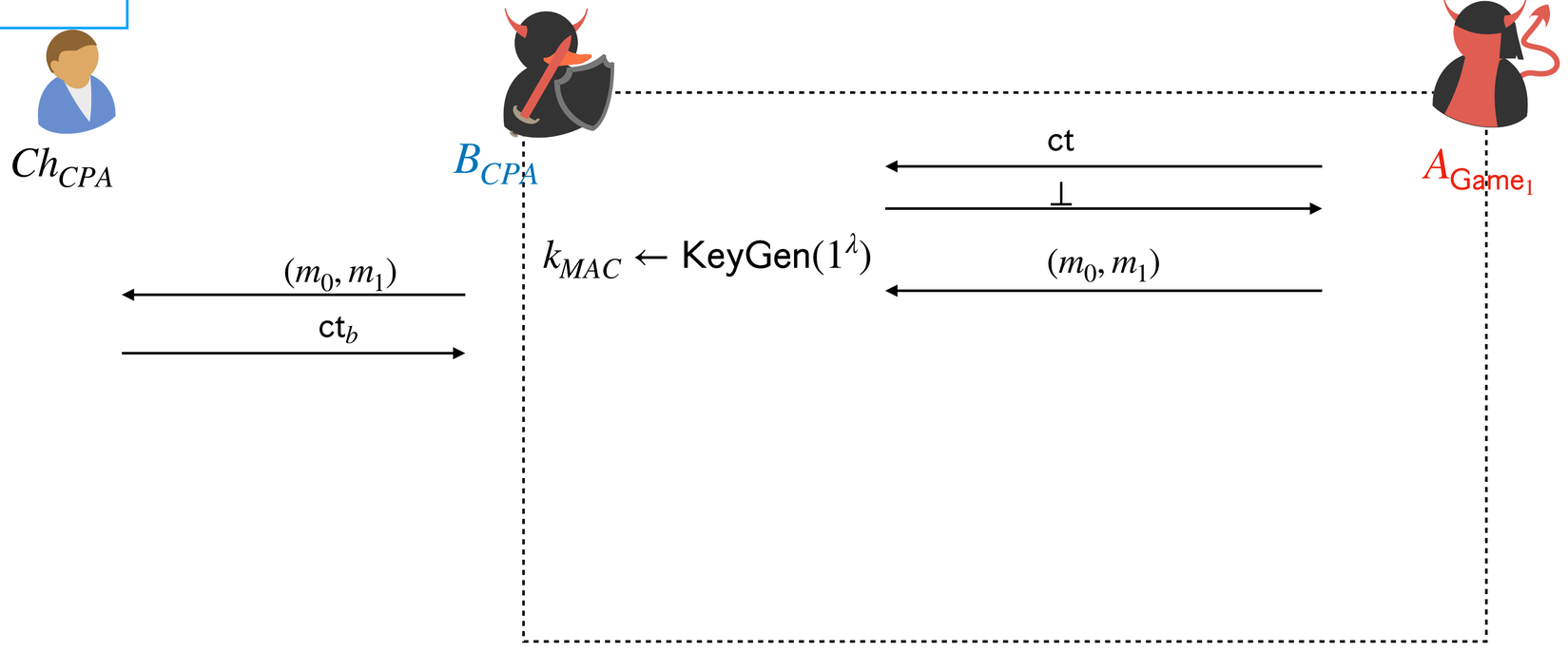

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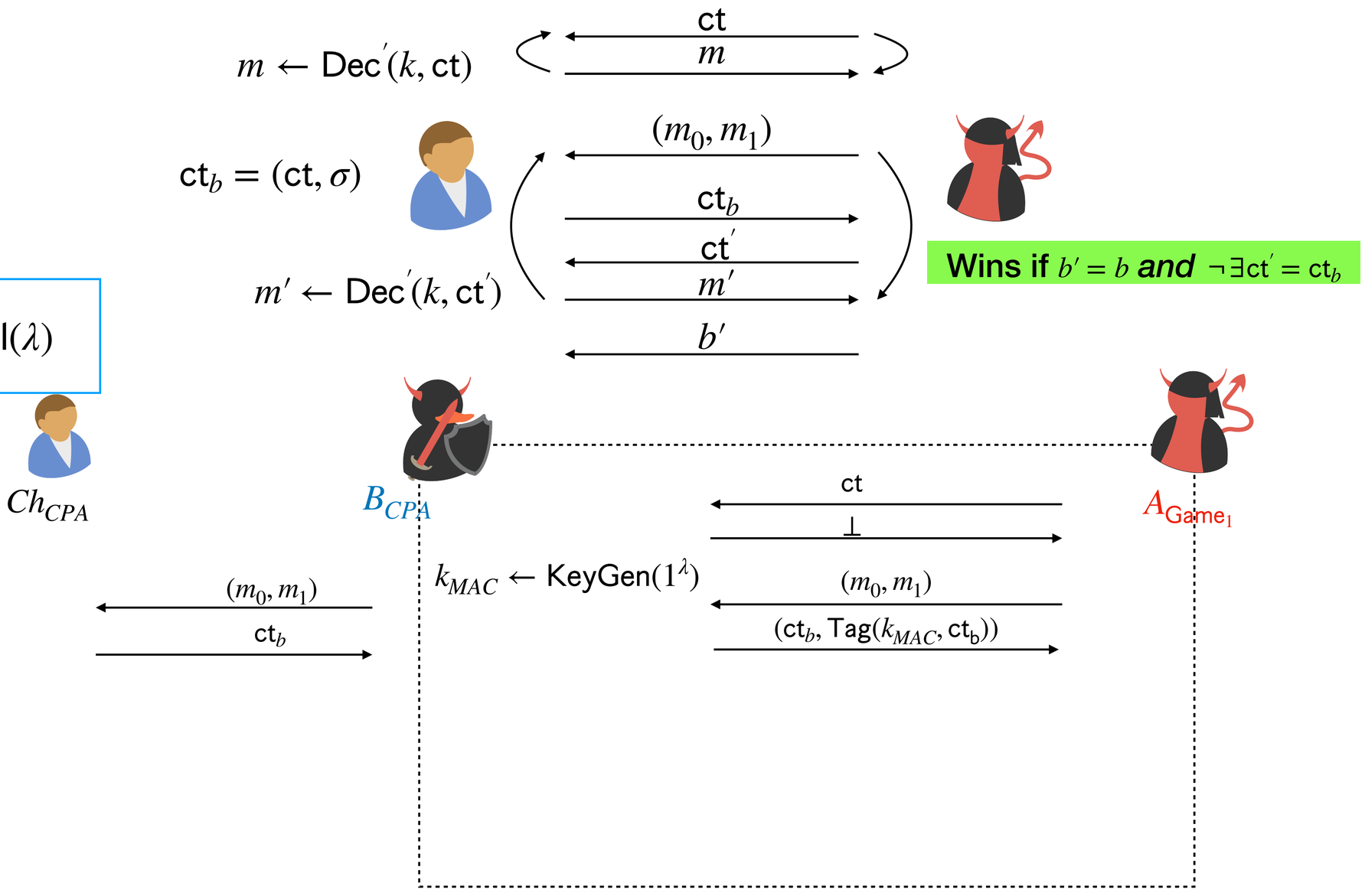

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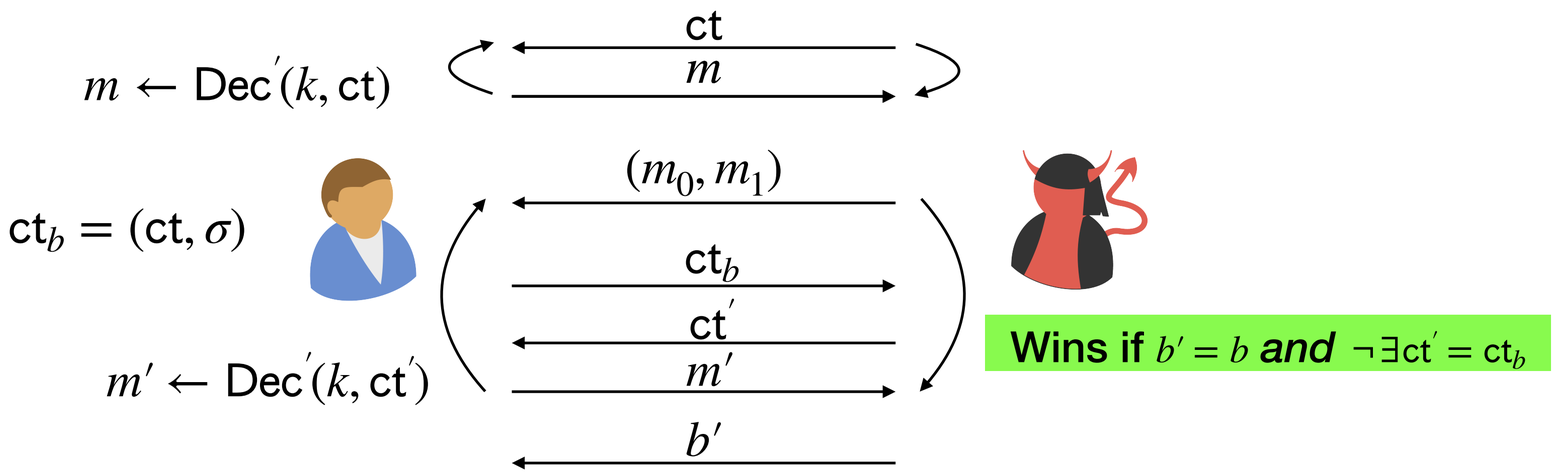

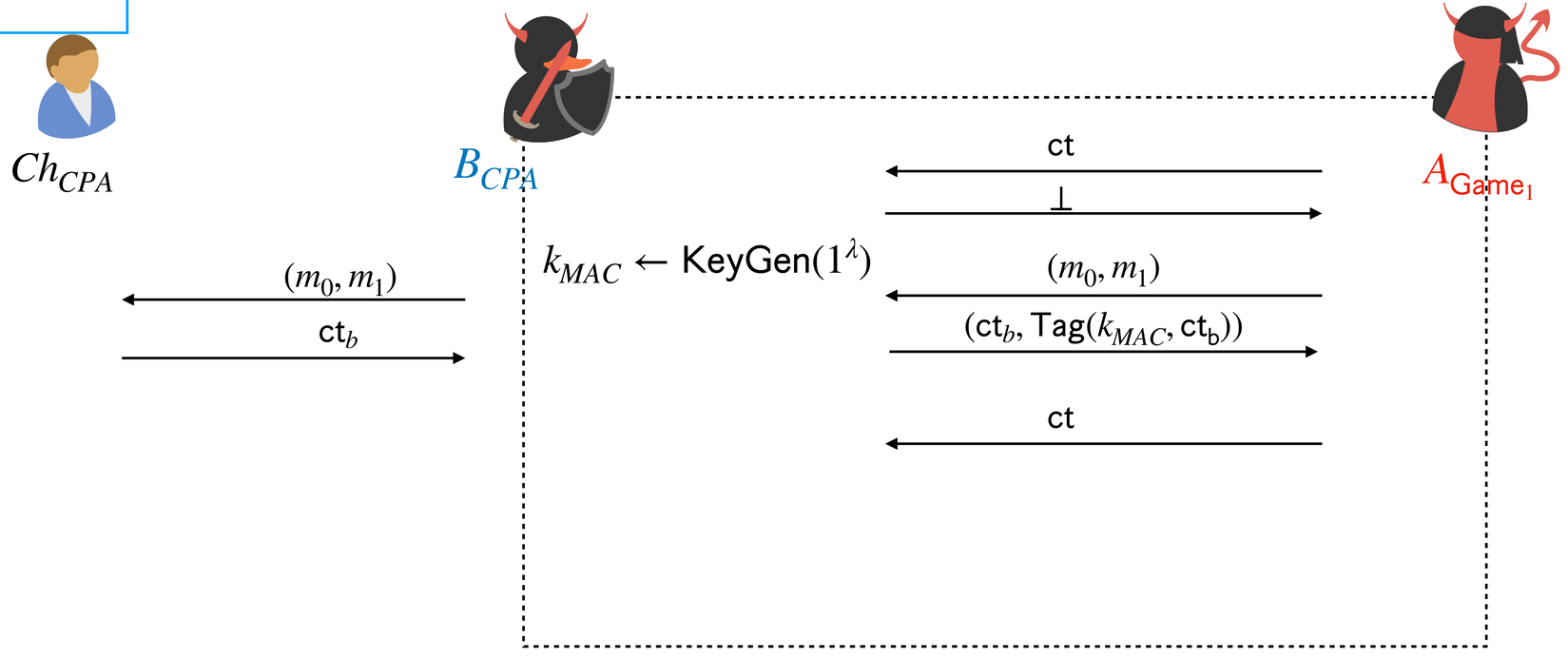

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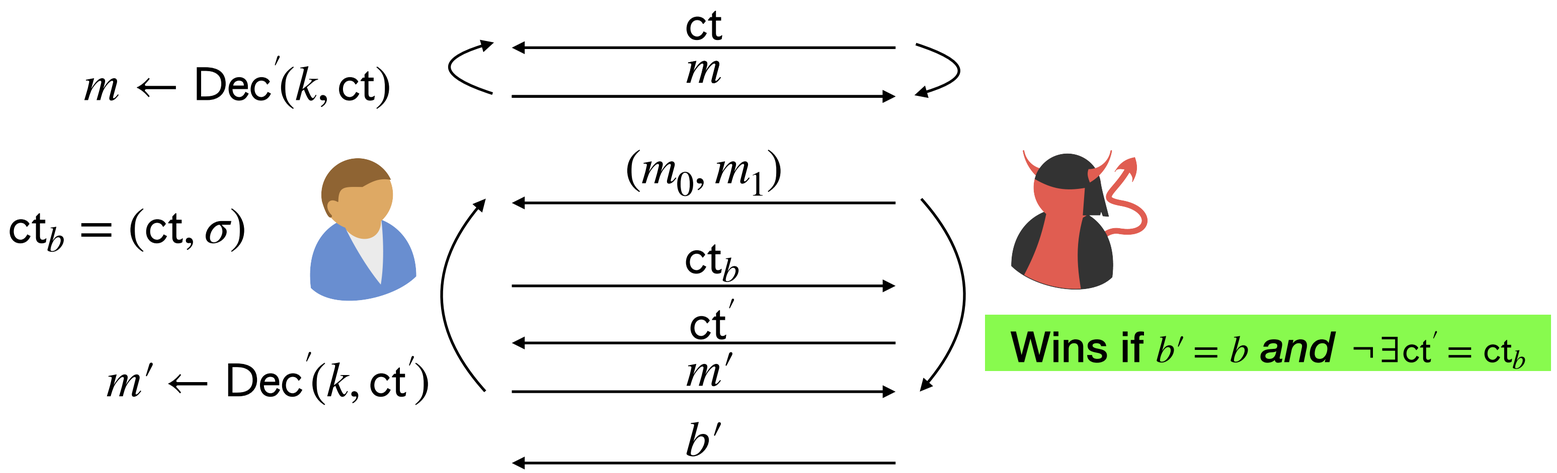

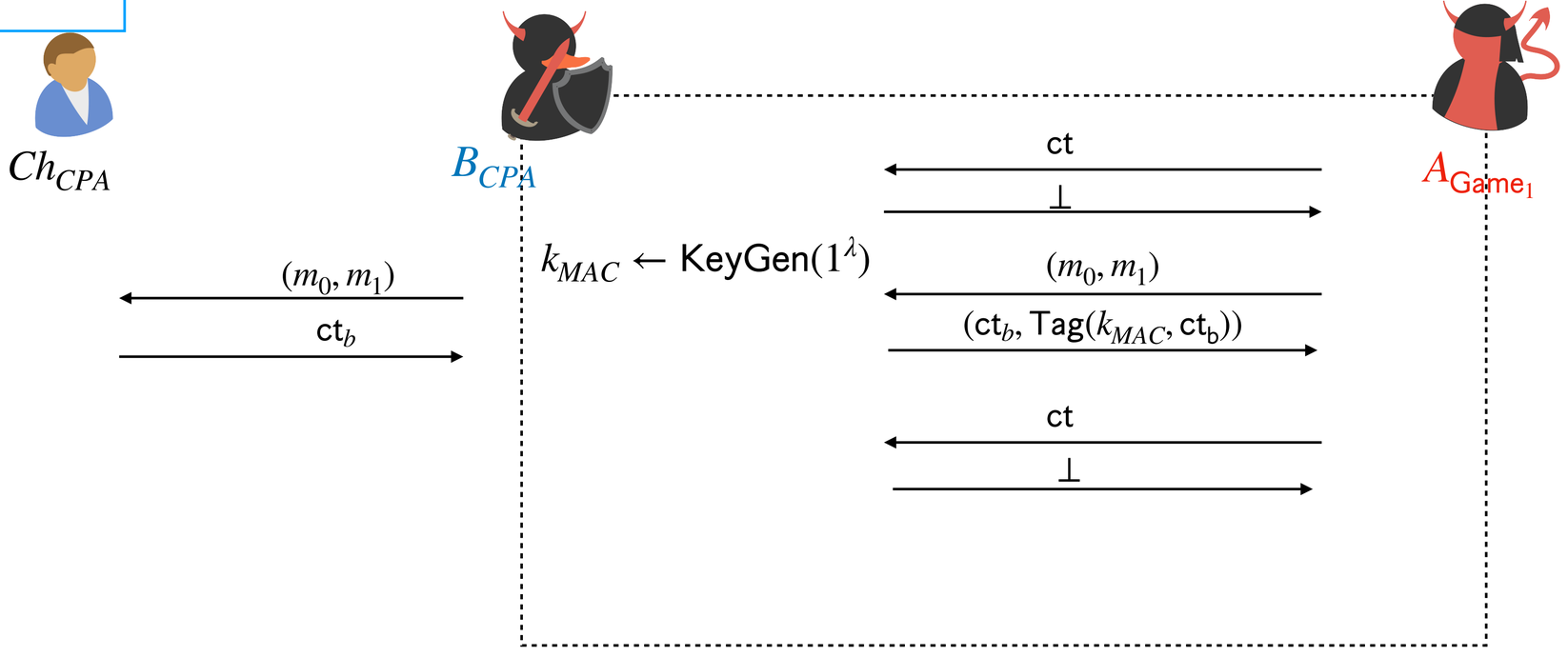

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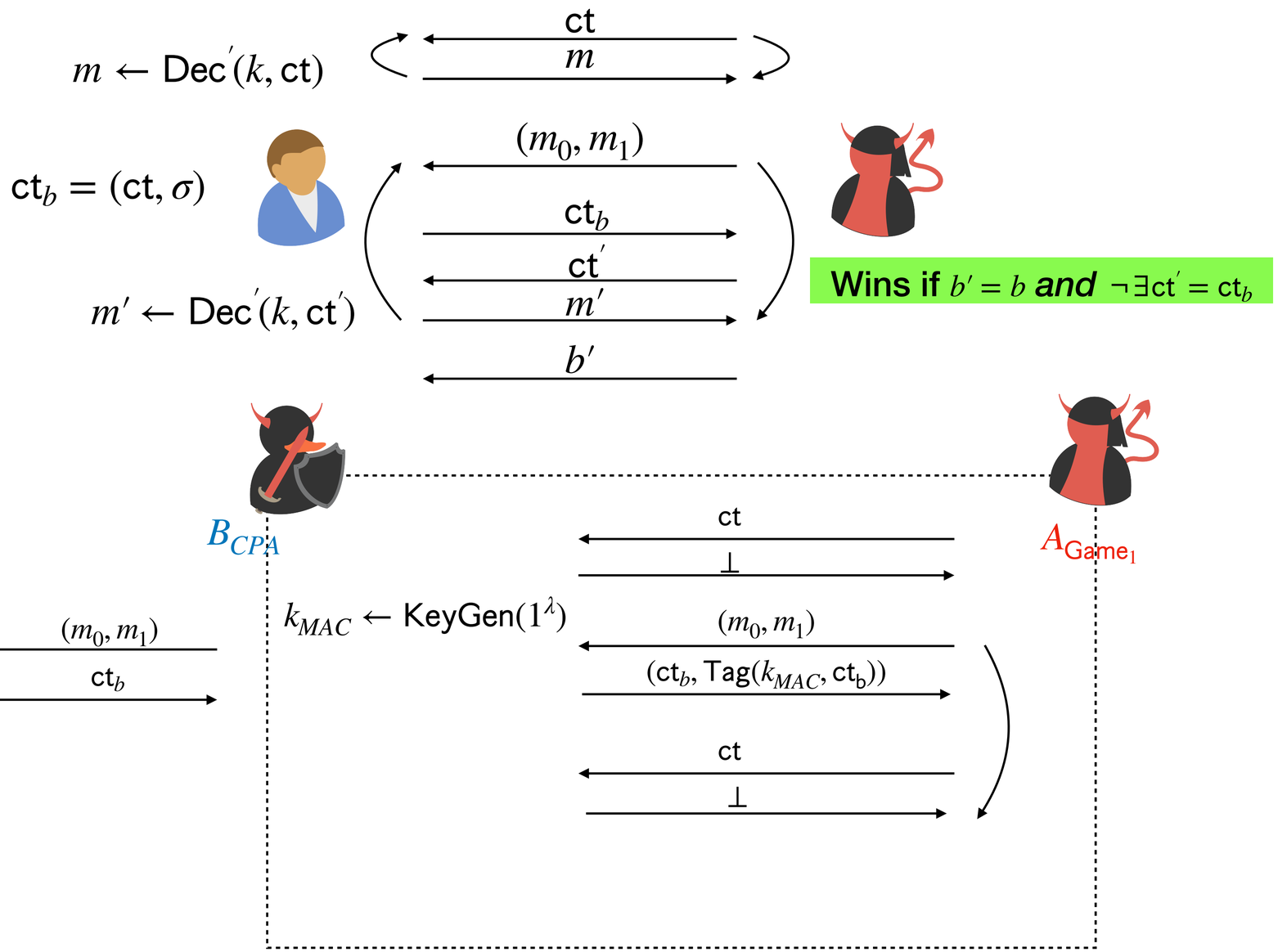

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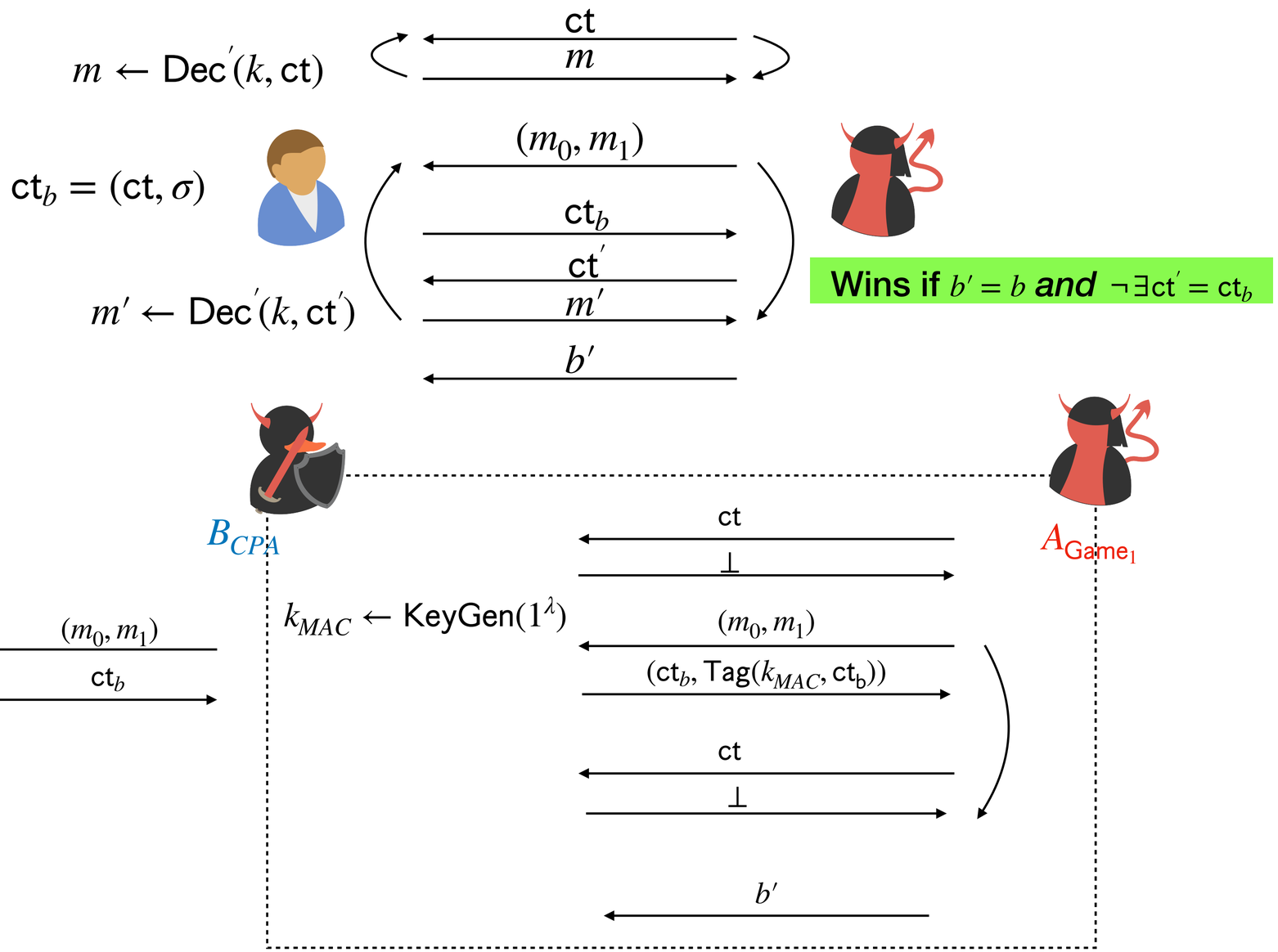

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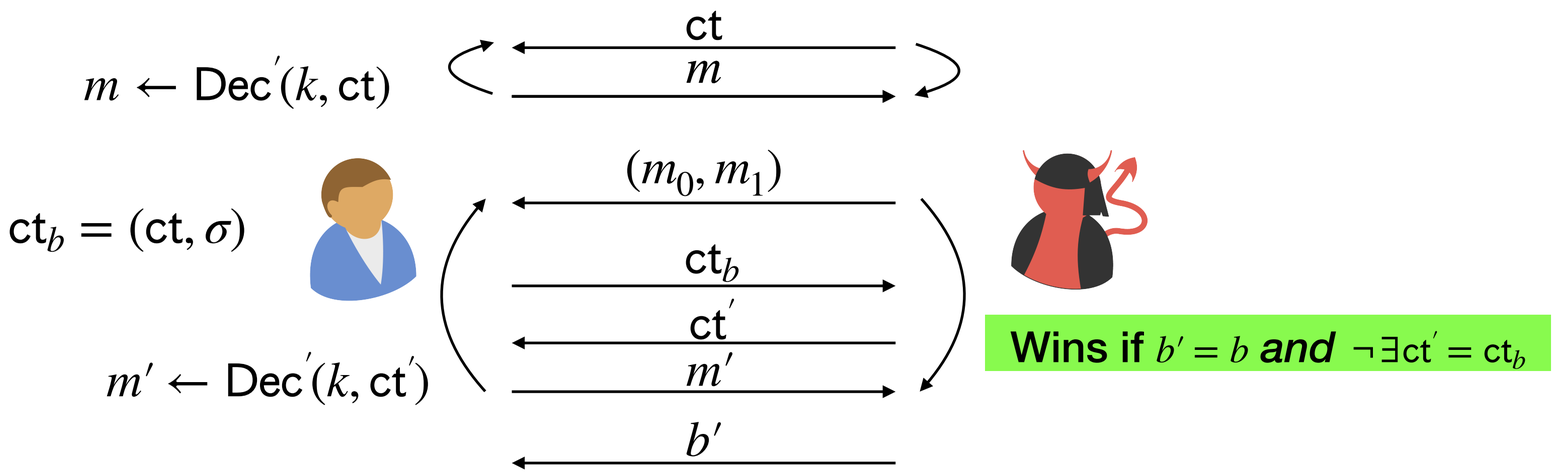

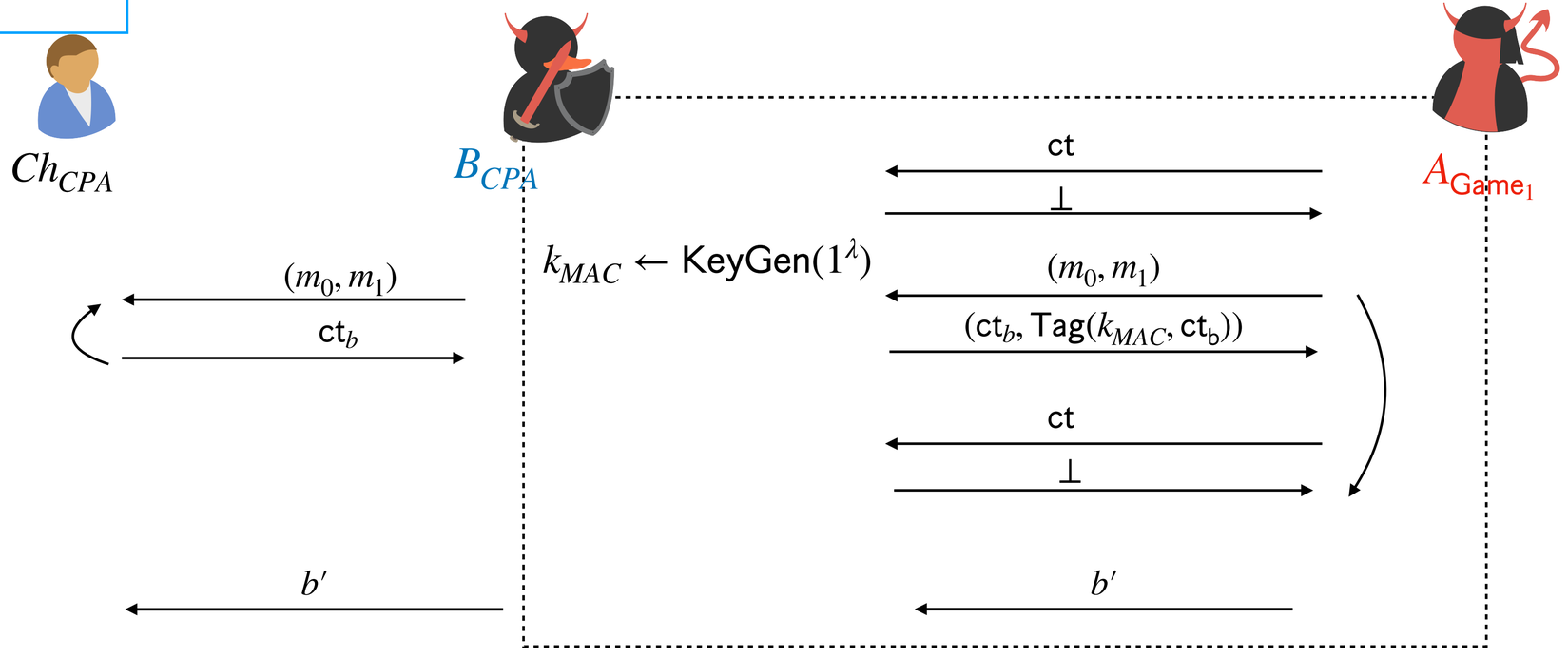

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Input mapping: $\mathscr{A}$ sees $\perp$ from decryption queries and ciphertexts with MACs, just like it expects!

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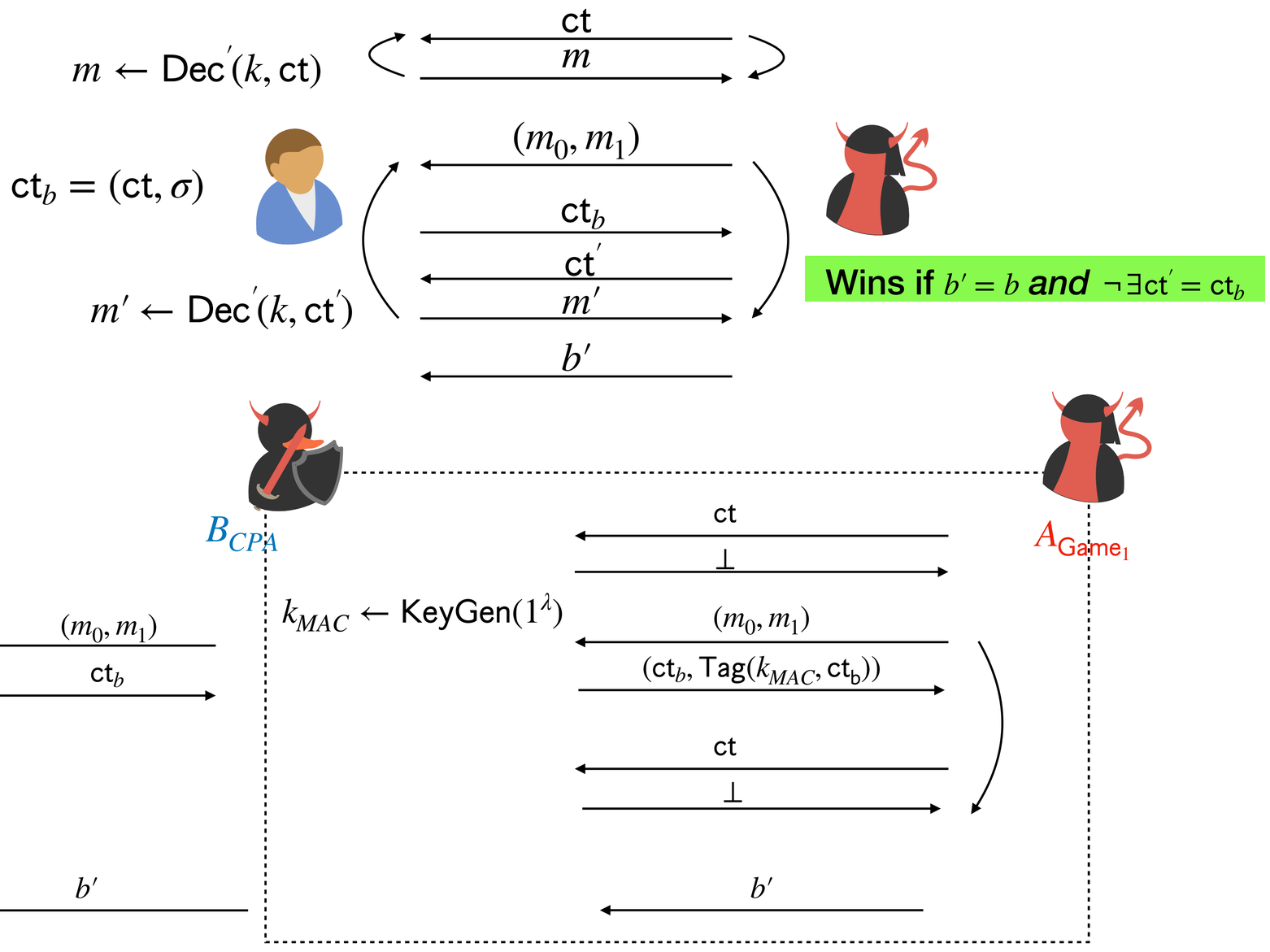

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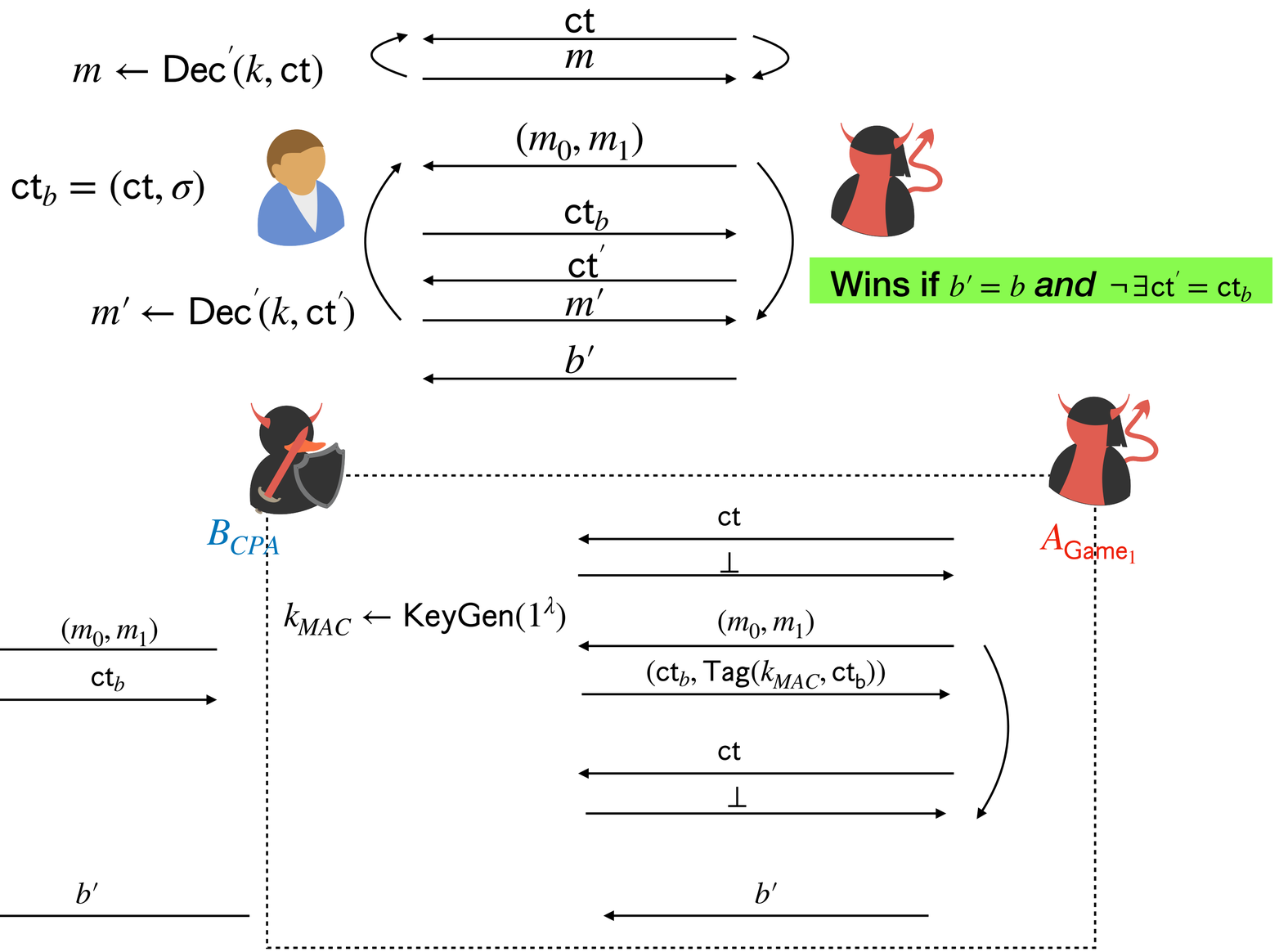
Therefore:
$\mathsf{Pr}[\mathscr{A}\ \text{wins}\ \mathsf{Game}_1] = \mathsf{Pr}[\mathscr{B}\ \text{wins}\ CPA] = \mathsf{negl}(\lambda)$

Game 1



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- We will see a scheme that uses NIZKs to prove that ciphertexts are well-formed.

Recap: NIZKs

NIZKs

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Let $L \subset \{0,1\}^*$ be a language.

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Recap: NIZKs

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Let $L \subset \{0,1\}^*$ be a language.

- $\mathsf{CRSGen}(1^\lambda) \to \mathsf{crs}$
- $P(x, \mathsf{crs}) \to \pi$
- $V(\pi, \mathsf{crs}) \to \{0,1\}$

Completeness: for all $x \in L$ and for all P

$$\Pr\left[V(\pi) = 1 : \begin{matrix} \mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda) \\ \pi \leftarrow P(x, \mathsf{crs}) \end{matrix}\right] = 1$$

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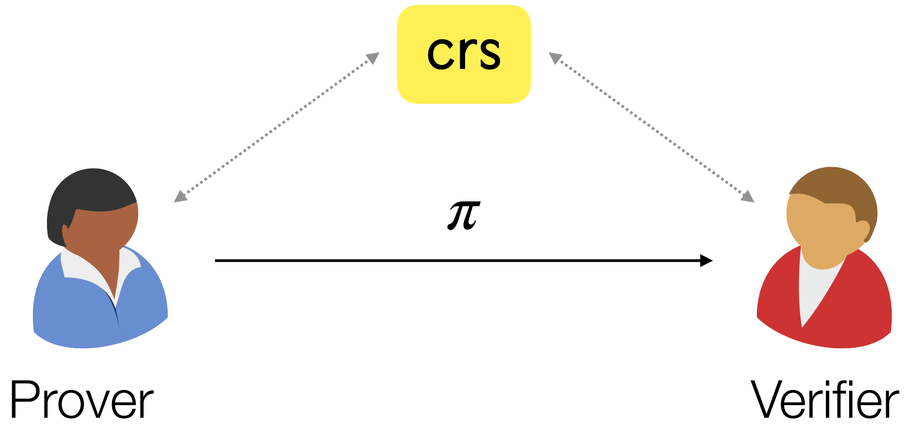

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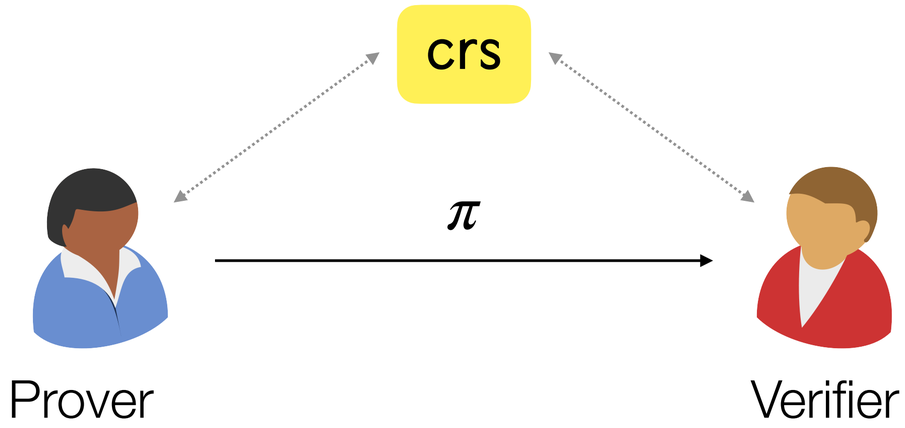

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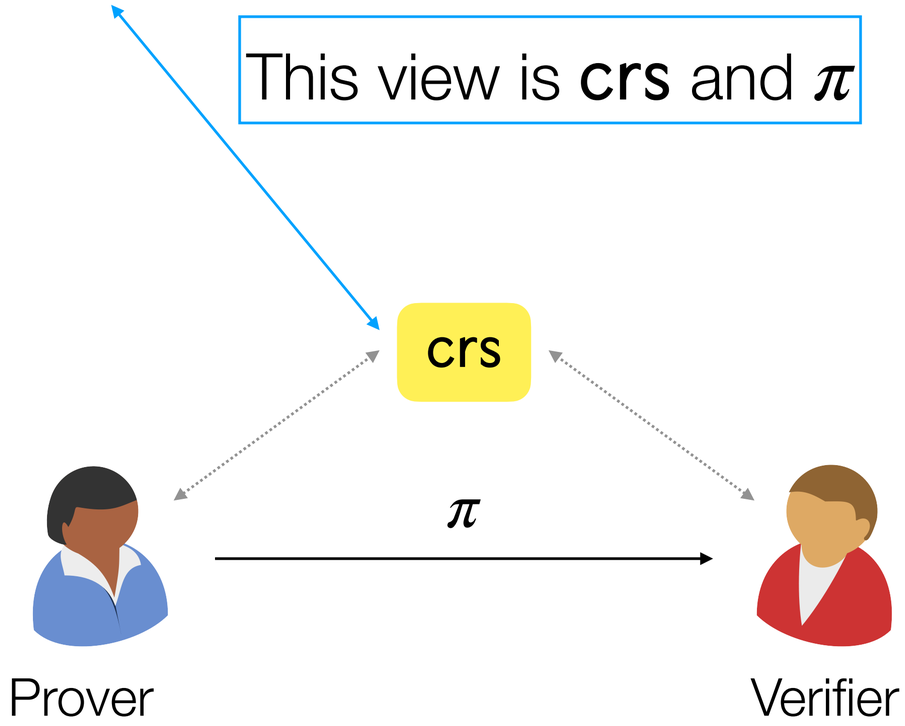

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This view is **crs** and π

We will split S into one that generates **crs**, and one that generates π