CCA Security II

601.442/642 Modern Cryptography

14th April 2026

Logistics

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- HW 8 due Thursday

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- Please do your course evaluations! This is our first time teaching the course so feedback is extremely valuable

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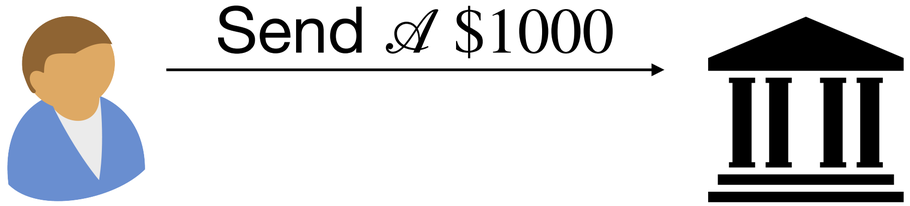

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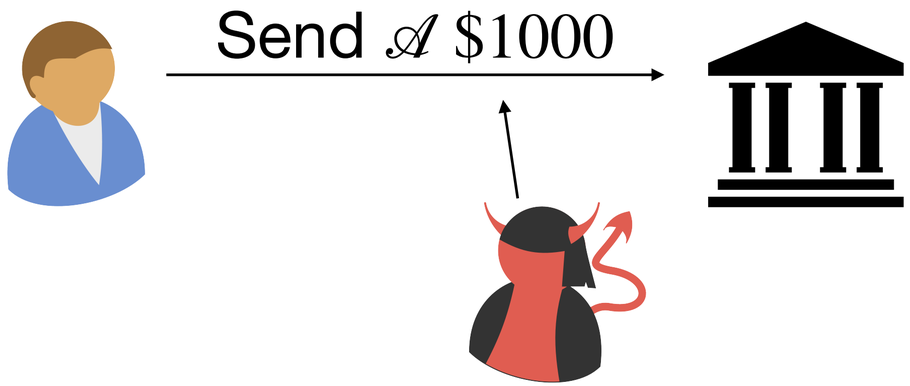

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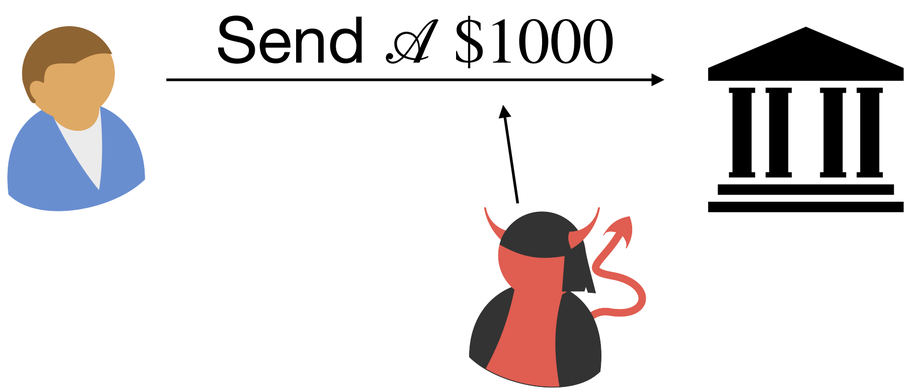

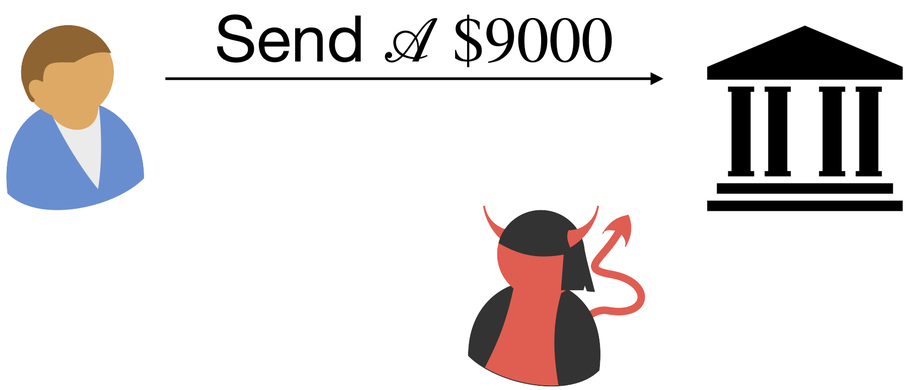

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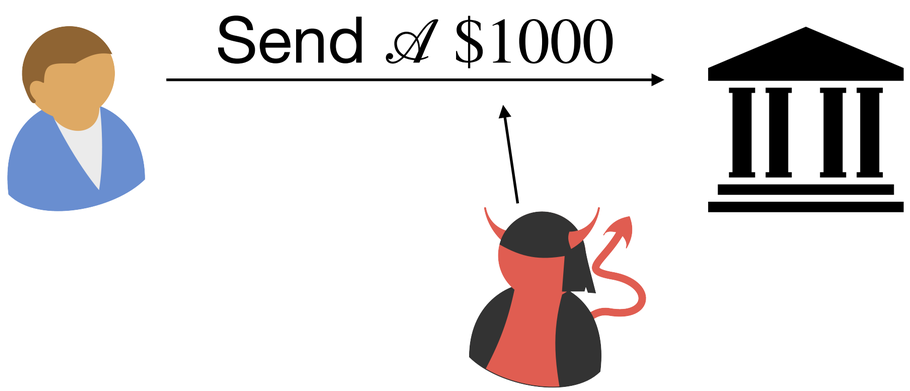
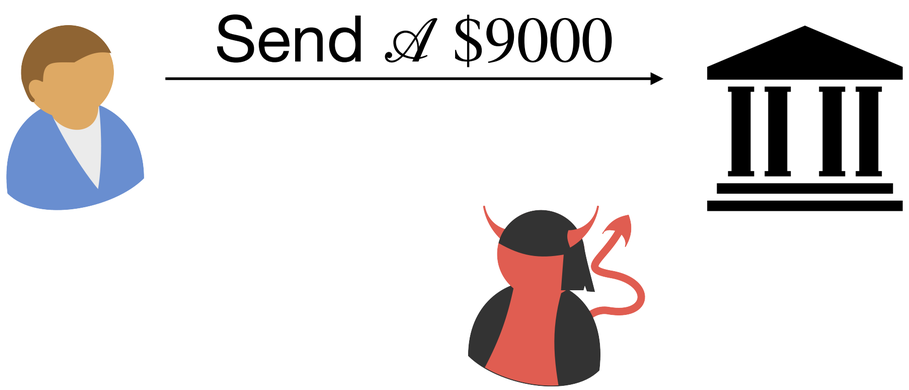
Adversary can't *read* the ciphertext, but can edit it!





Recap: NIZKs

NIZKs

Let $L \subset \{0,1\}^*$ be a language.

- $\mathsf{CRSGen}(1^\lambda) \to \mathsf{crs}$
- $P(x, \mathsf{crs}) \to \pi$
- $V(\pi, \mathsf{crs}) \to \{0,1\}$

Completeness: for all $x \in L$ and for all P

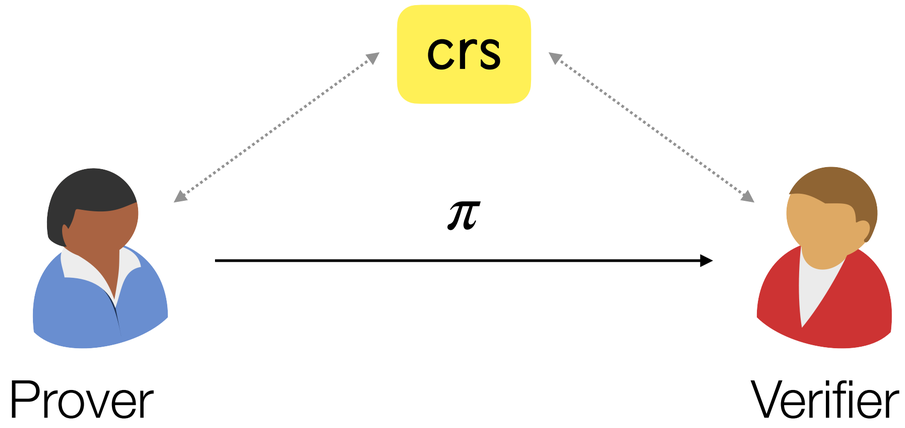
$$\Pr\left[V(\pi) = 1 : \begin{matrix}\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda) \\ \pi \leftarrow P(x, \mathsf{crs})\end{matrix}\right] = 1$$

Soundness: for all $x \notin L$ and for all $\hat{P}$

$$\Pr\left[V(\pi, \mathsf{crs}) : \begin{matrix}\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda) \\ \pi \leftarrow \hat{P}(x, \mathsf{crs})\end{matrix}\right] \leq \mathsf{negl}(|x|)$$

Zero Knowledge: for all PPT $\hat{V}$, there exists a PPT simulator S such that for all $x \in L$ and $z \in \{0,1\}^*$

$$\left\{\mathsf{View}_{\hat{V}}\left[P(x, \mathsf{crs}) \leftrightarrow \hat{V}(x, \mathsf{crs}, z)\right]\right\} \overset{c}{\approx} \{S(x, z)\}$$



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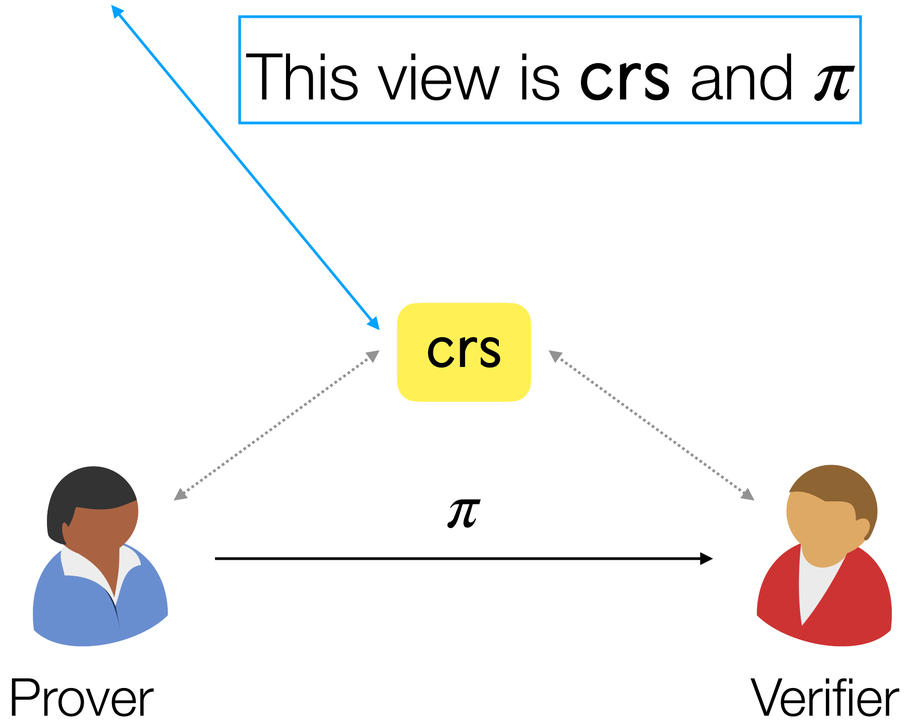

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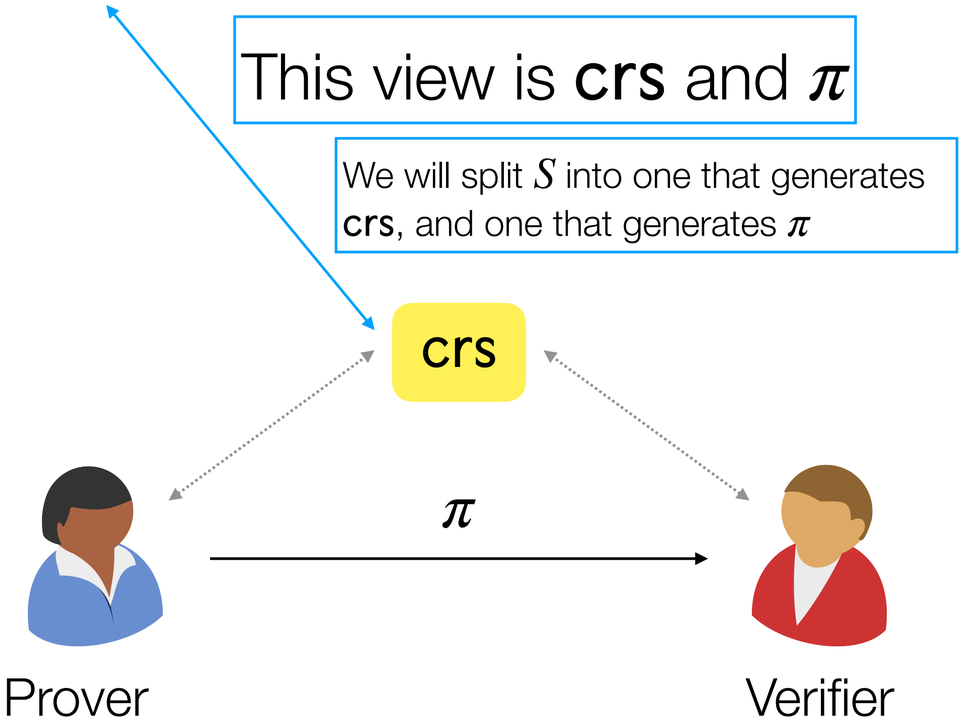

Re NIZK

Says that simulators can always make proofs for *true* statements.

Doesn't say they *can't* make proofs for false statements, just that we can't guarantee it.

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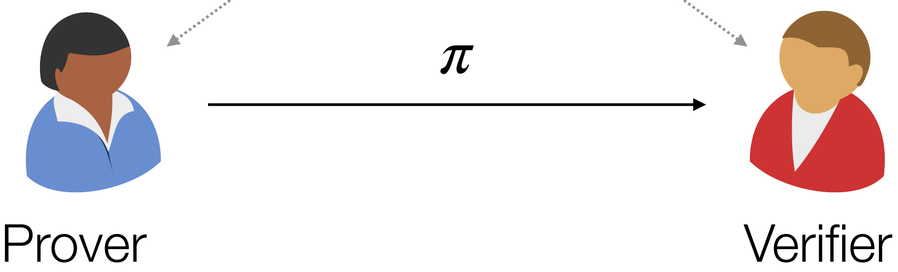

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- Sometimes they might be able to lie about something that should be *secret*
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- In this case we will use *zero-knowledge* proofs. Then the secret isn't leaked!

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Specifying the randomness used.

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A public key encryption scheme (KeyGen, Enc, Dec) satisfies *indistinguishability under chosen-ciphertext attack (1)* (IND-CCA-1) if for all NUPPT $\mathscr{A}$, there exists a negligible function $\nu(\cdot)$ such that $\forall \lambda \in \mathbb{N}$

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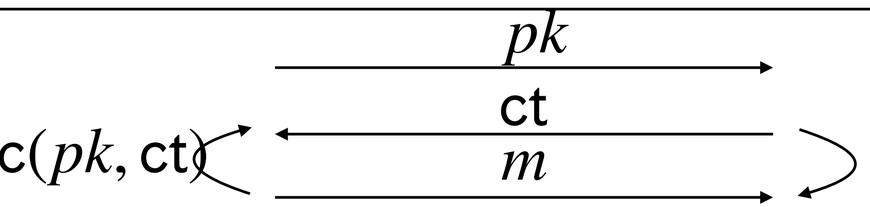

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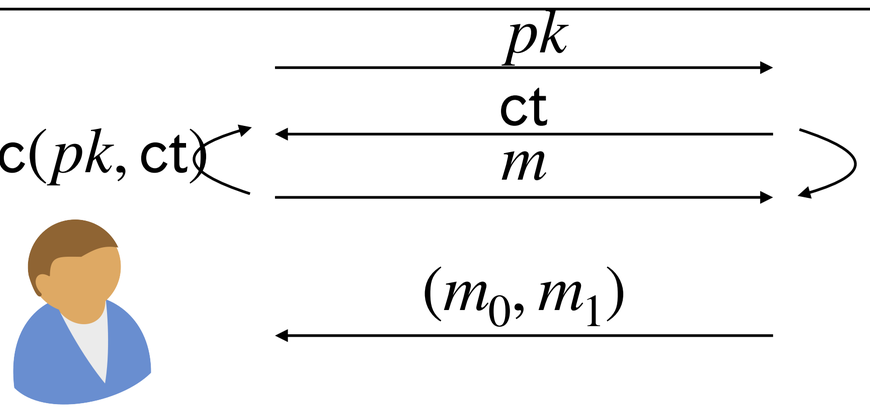

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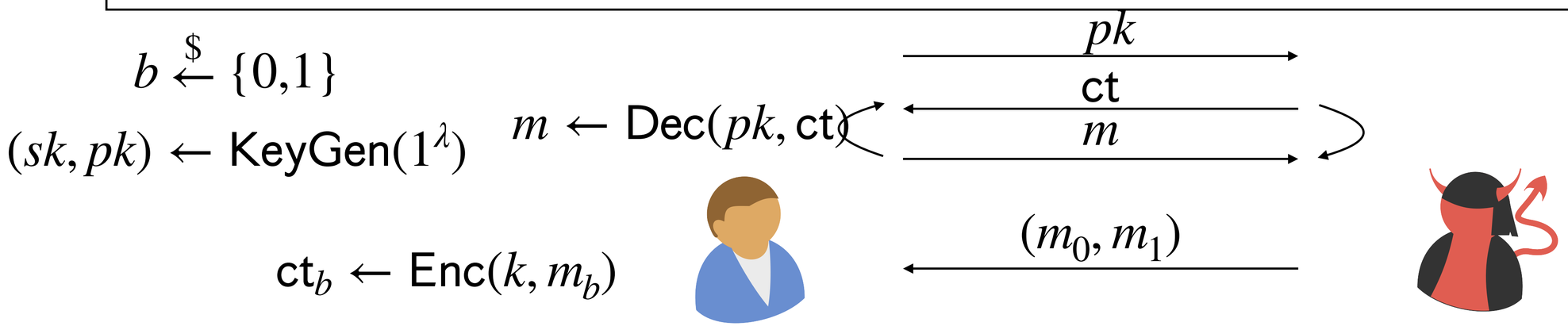

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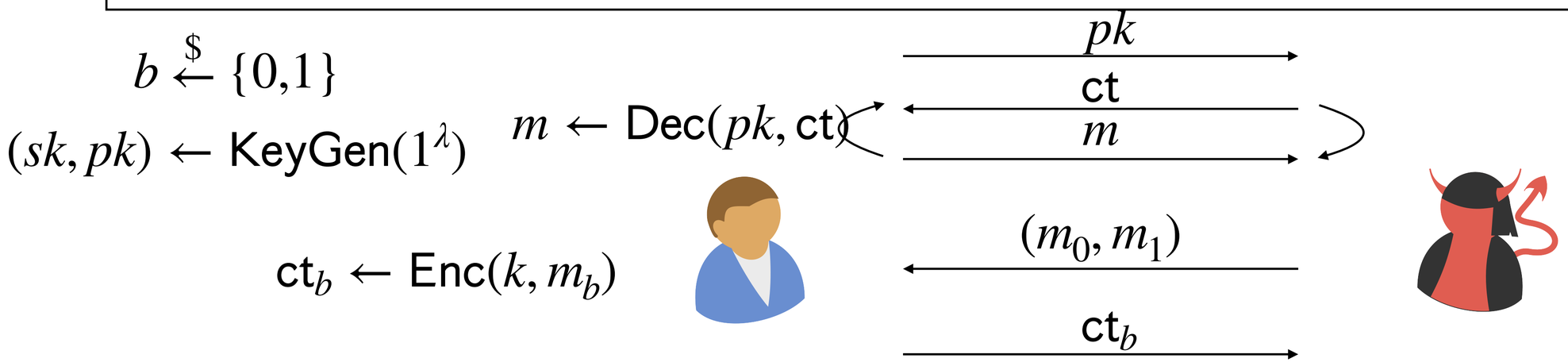

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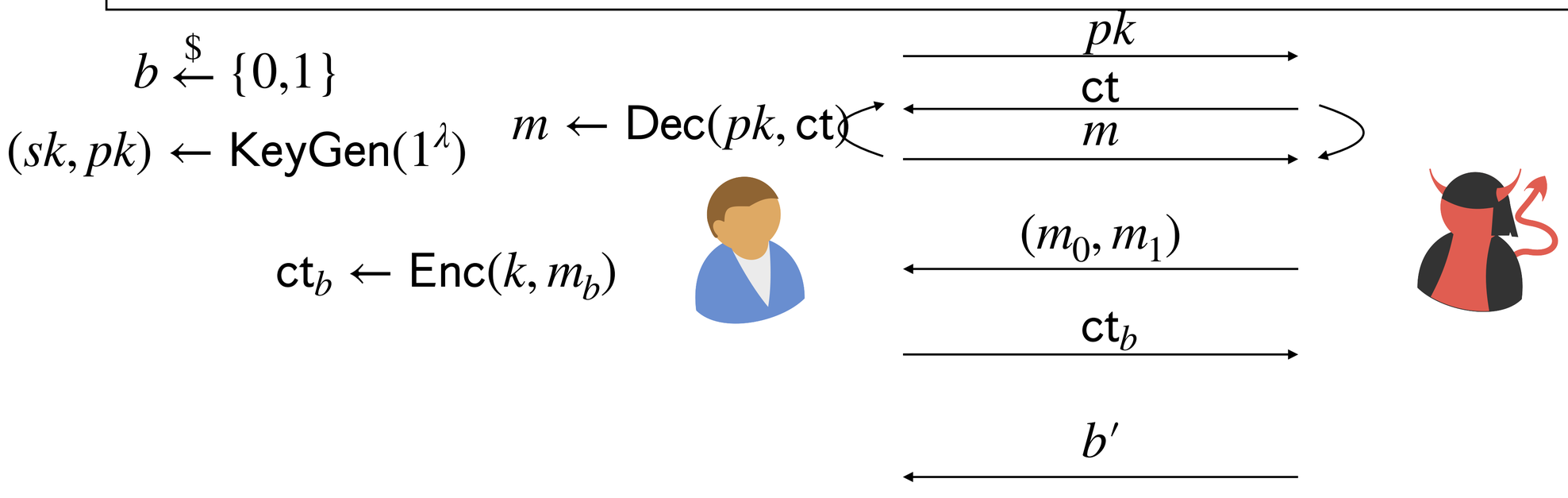

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ct

m

(m_0, m_1)

ct_b

b'

Wins if $b' = b$

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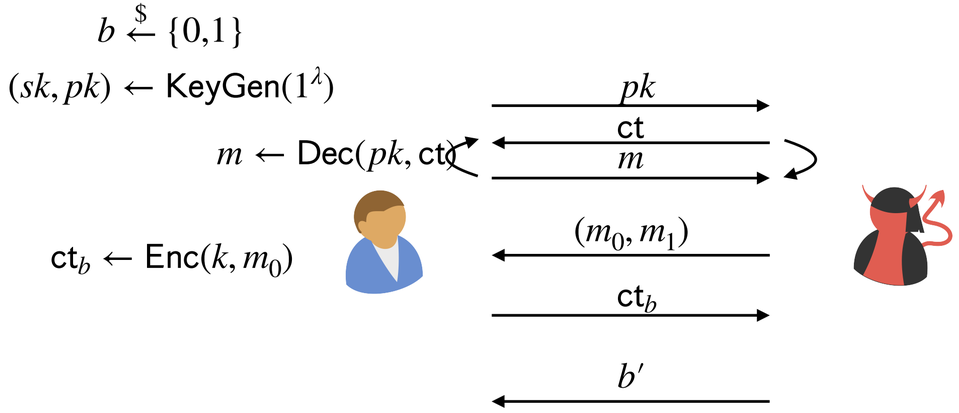
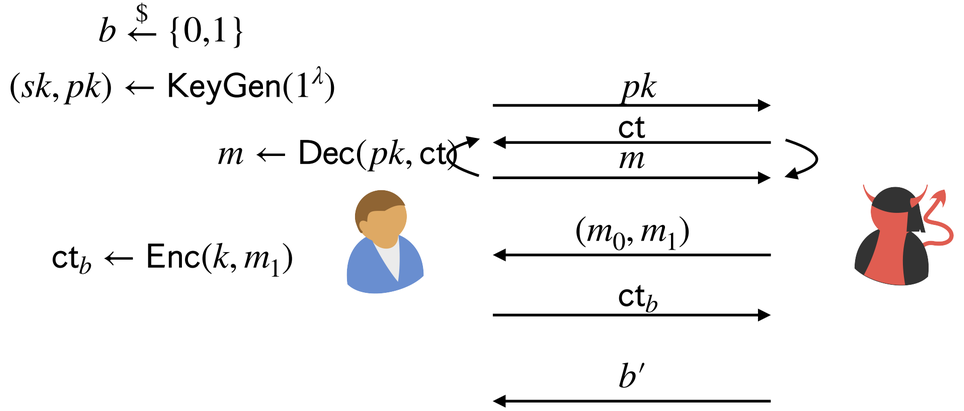
$$\left| \Pr[\mathcal{A} \text{ outputs } 1 \text{ in } \mathcal{G}_0] - \Pr[\mathcal{A} \text{ outputs } 1 \text{ in } \mathcal{G}_1] \right| \leq \nu(\lambda)$$





Attempted Proof: IND-CPA => IND-CCA-1

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Our CCA-1 Scheme

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Our CCA-1 Scheme

- $\mathsf{KeyGen}'(1^\lambda)$
  - return $\mathsf{KeyGen}(1^\lambda)$

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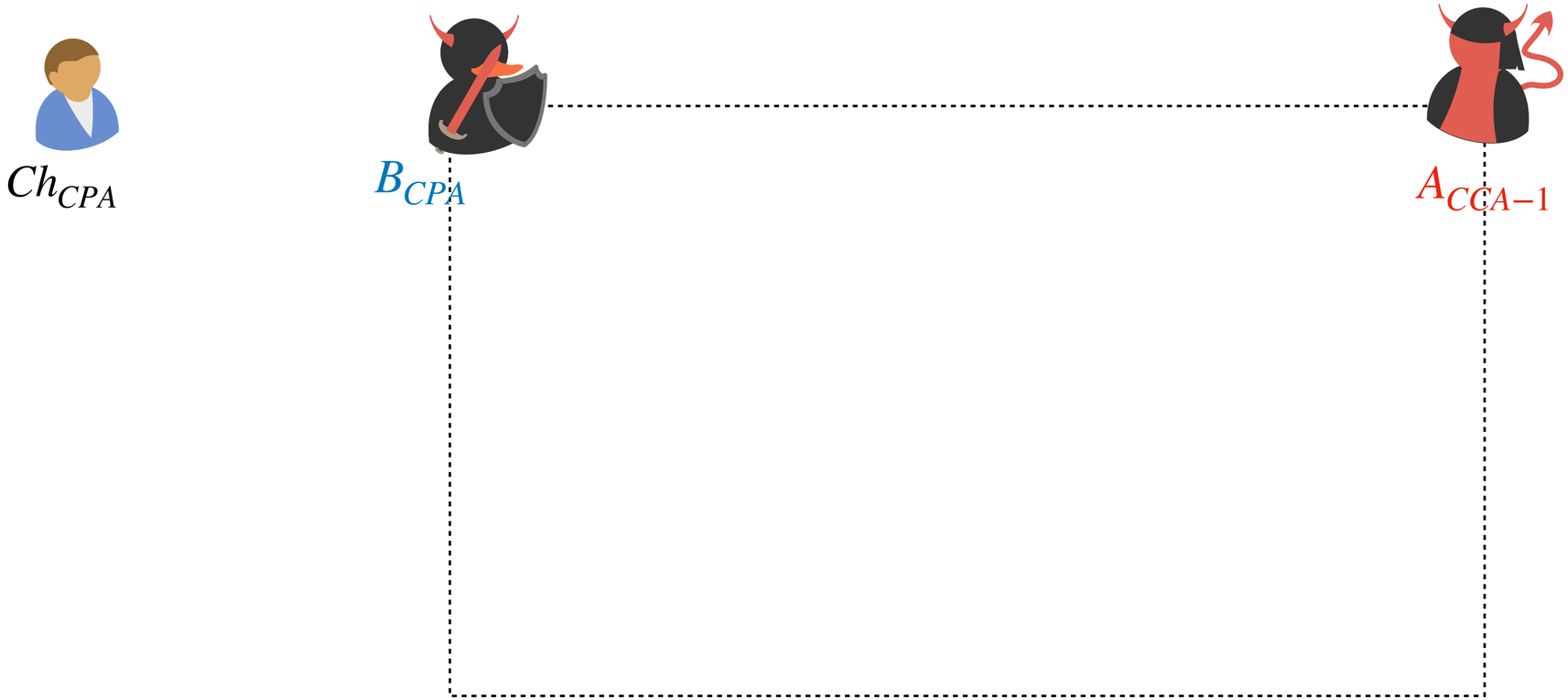

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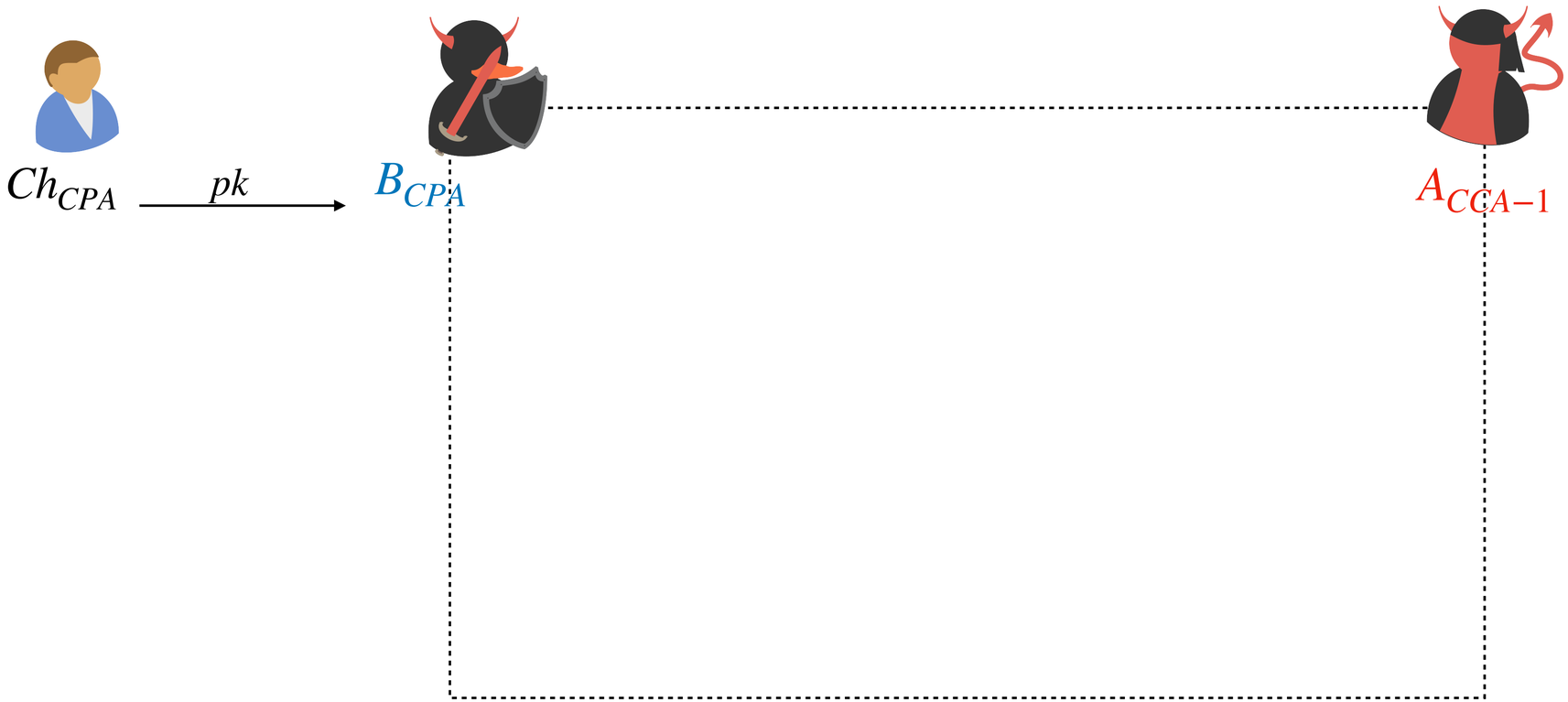

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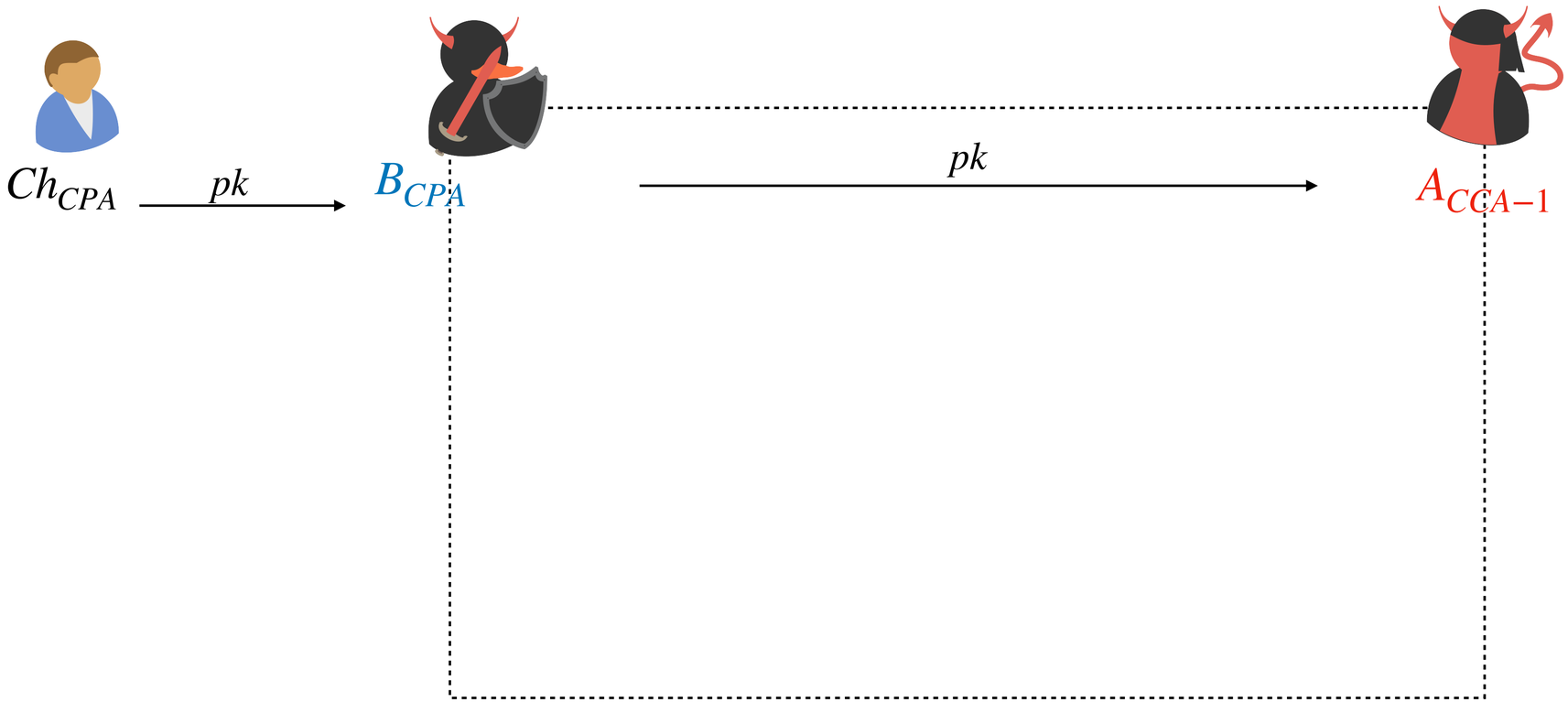

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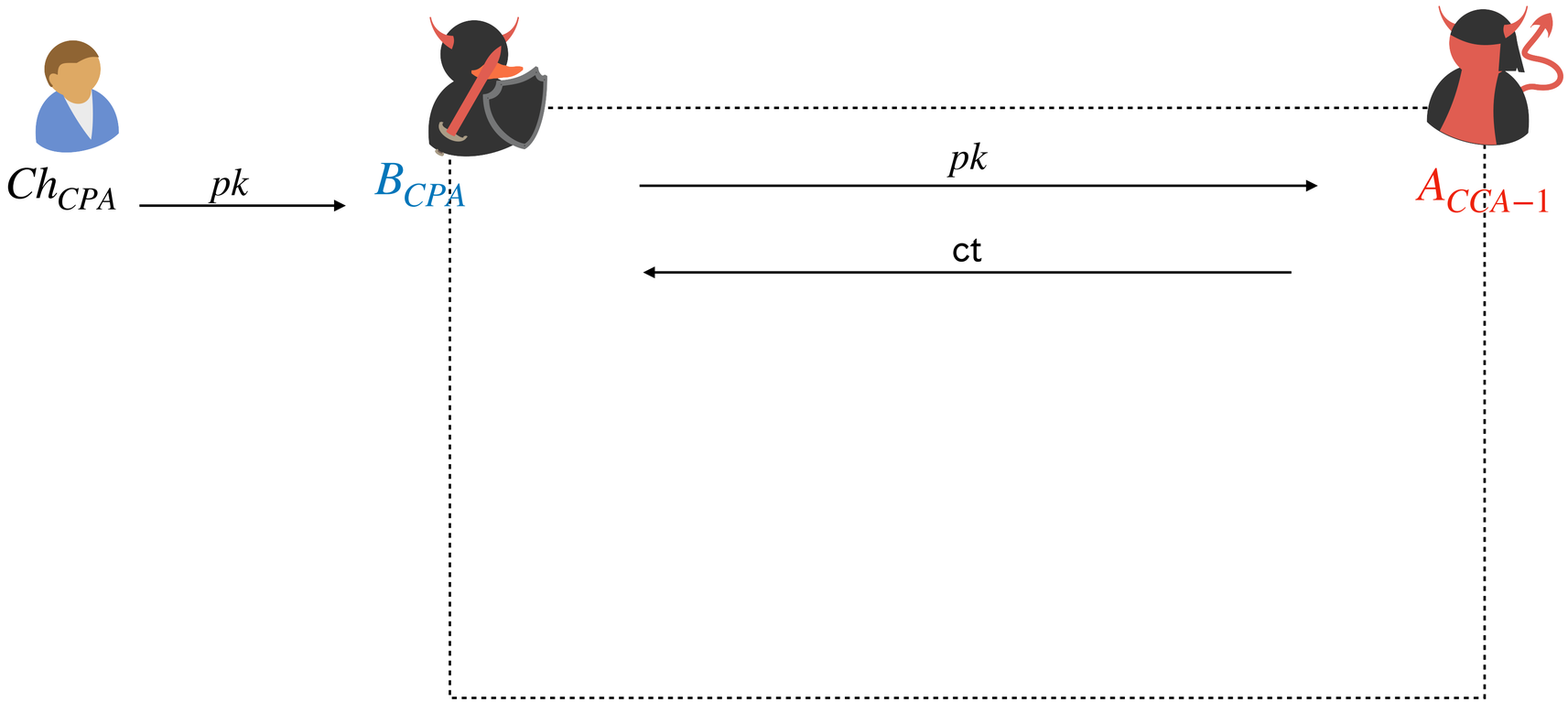

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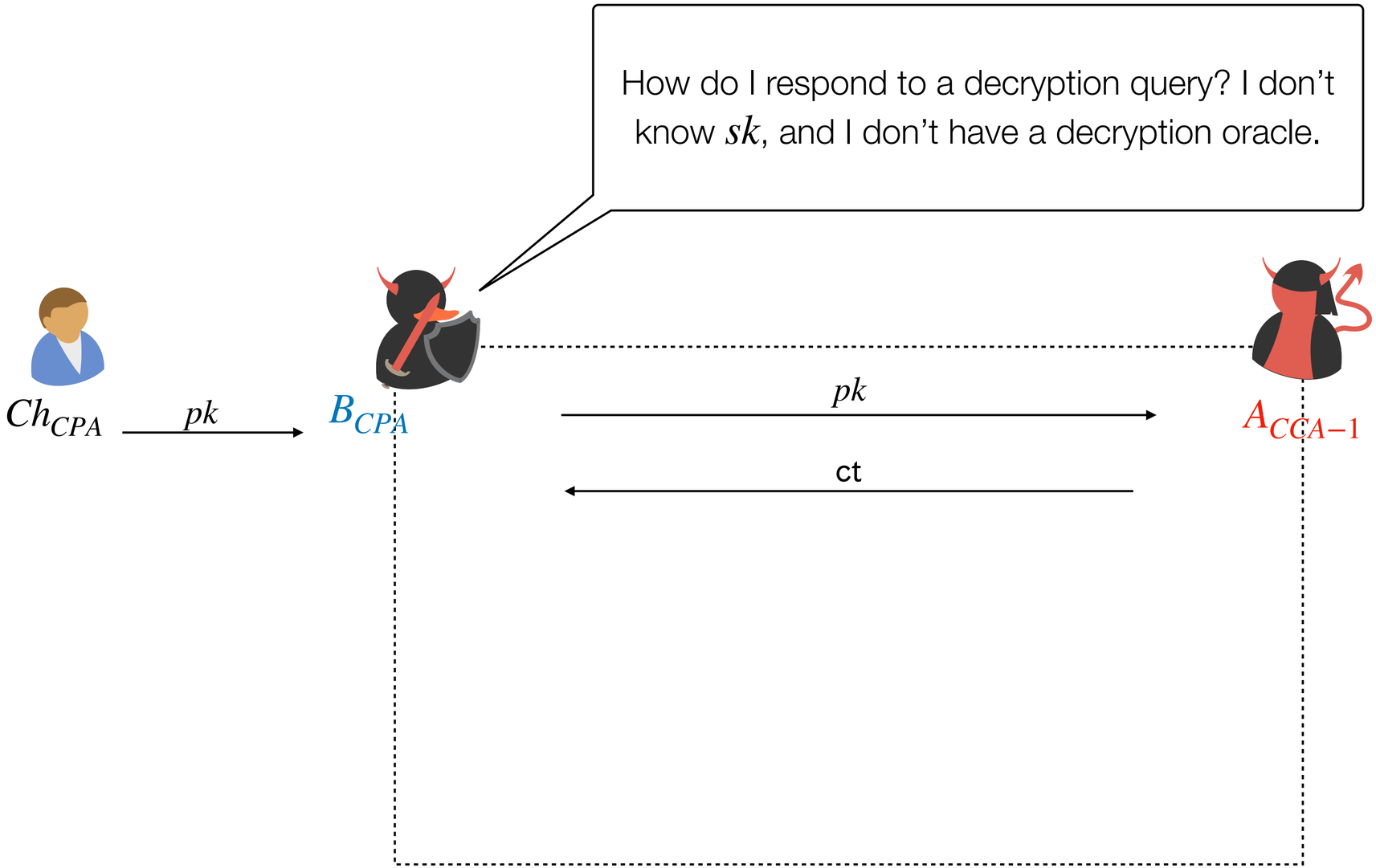

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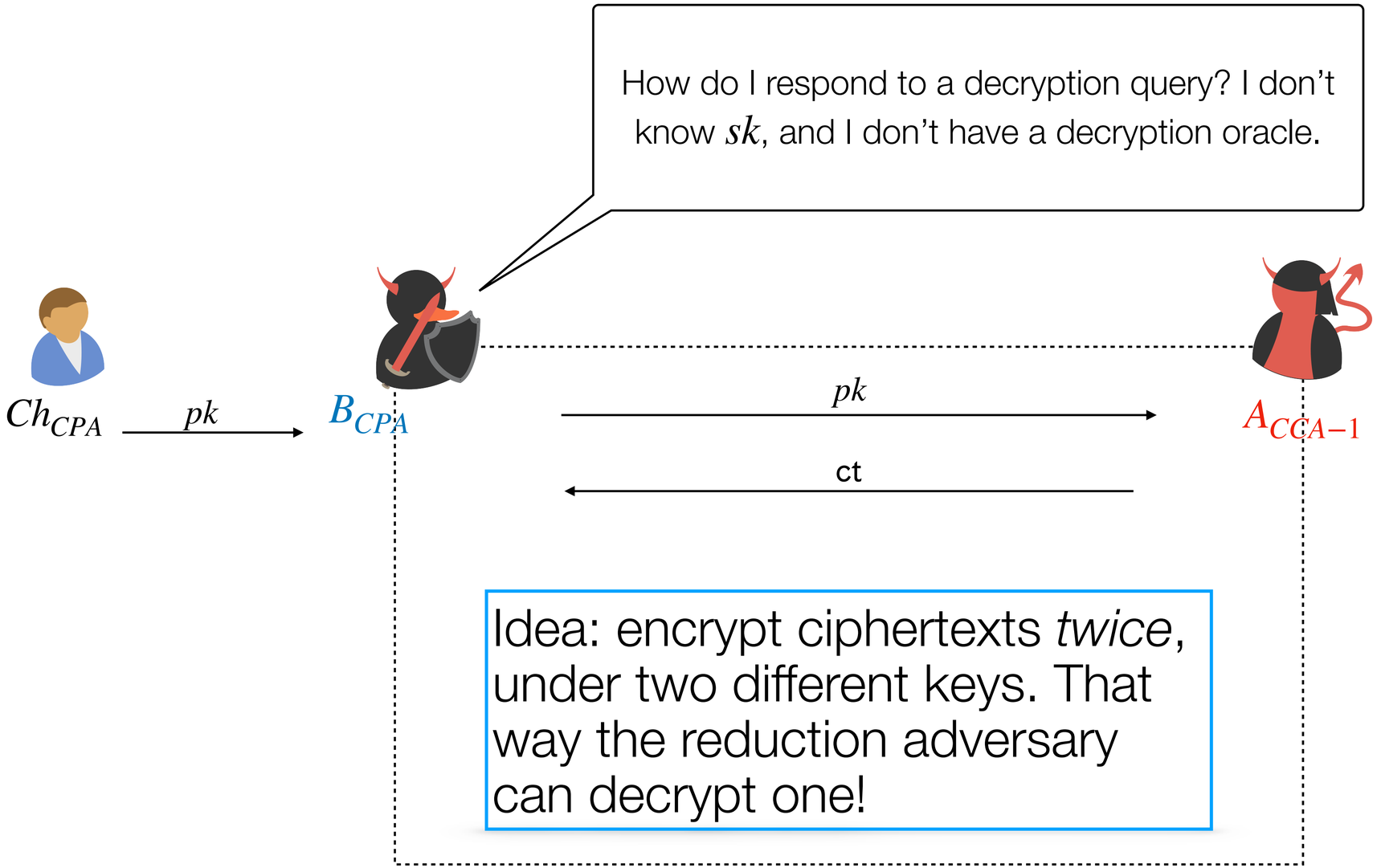

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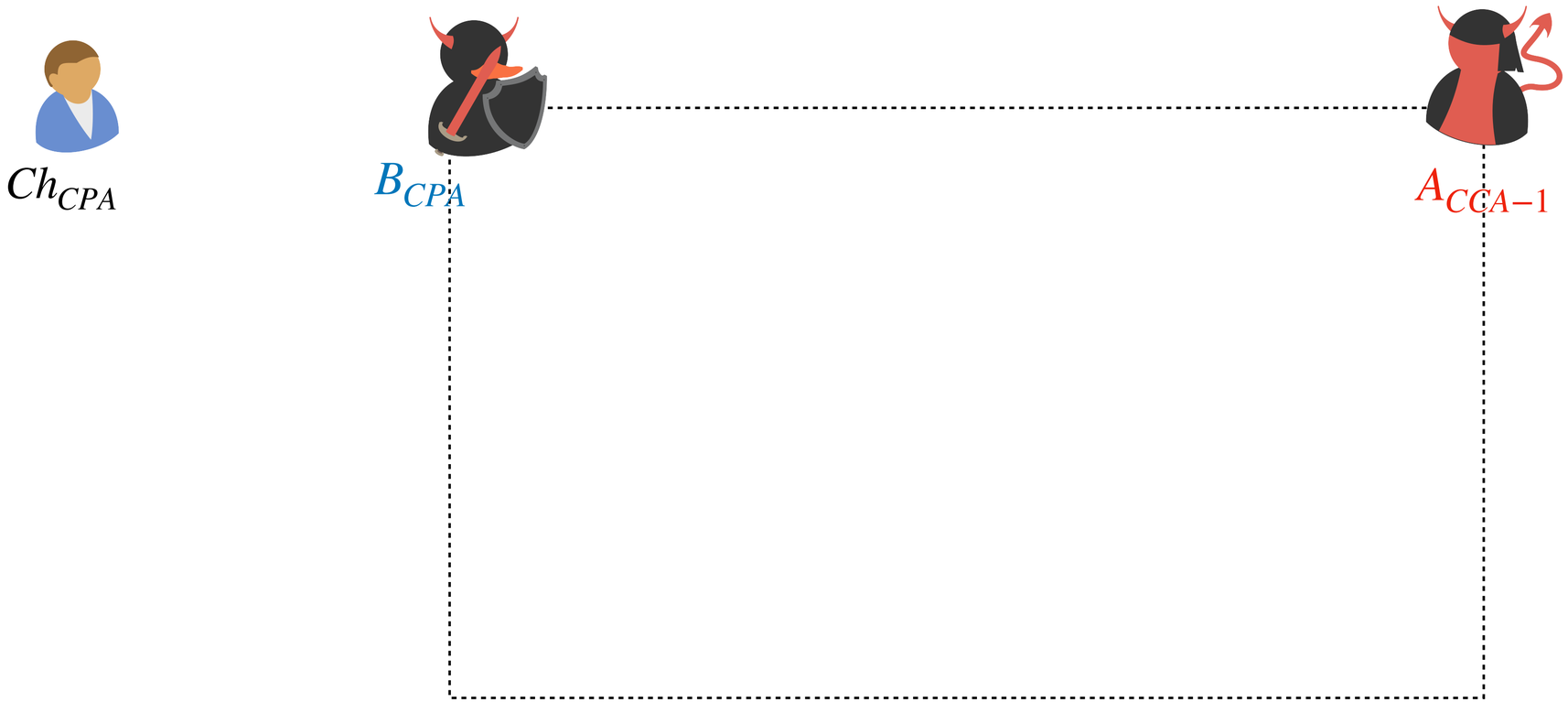
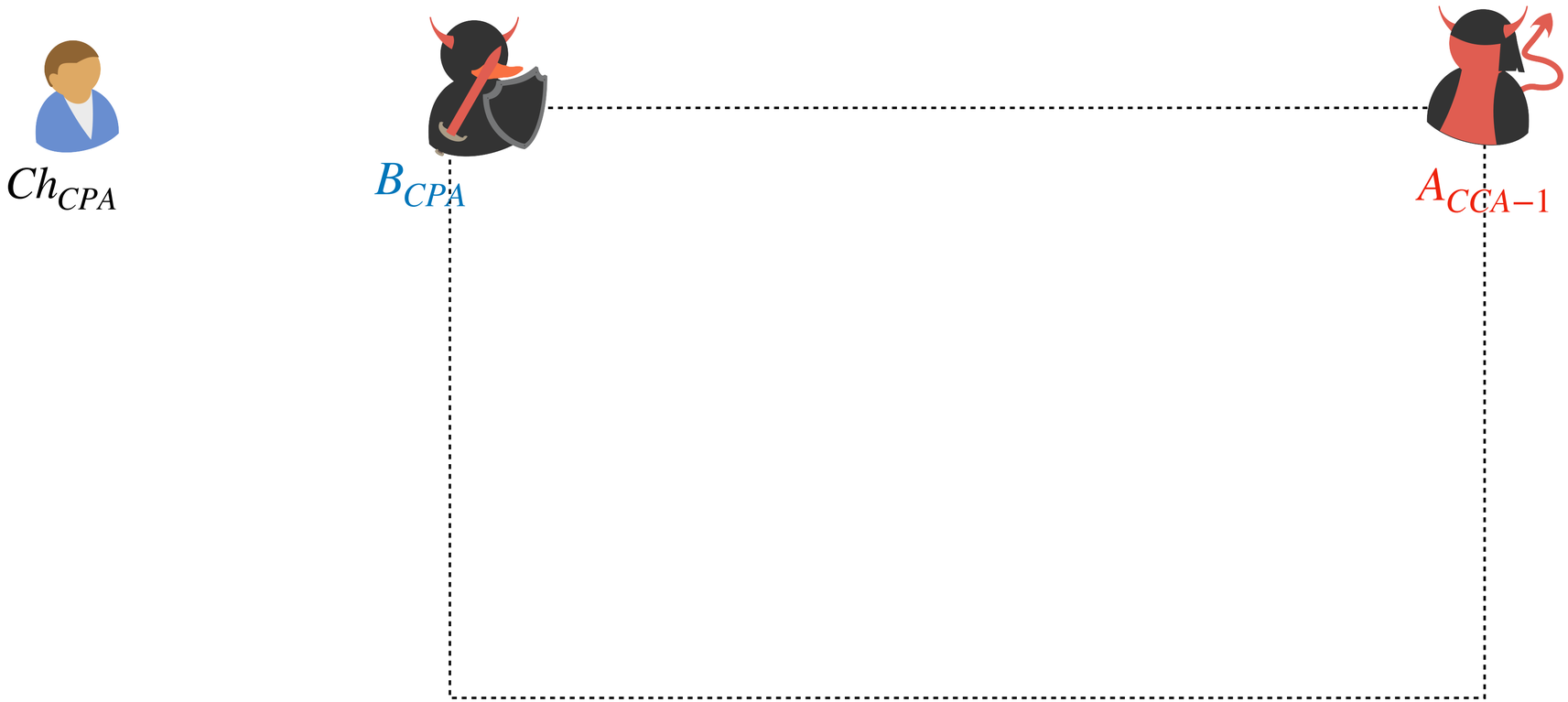


Attempted Proof: IND-CPA => IND-CCA-1

Our CCA-1 Scheme

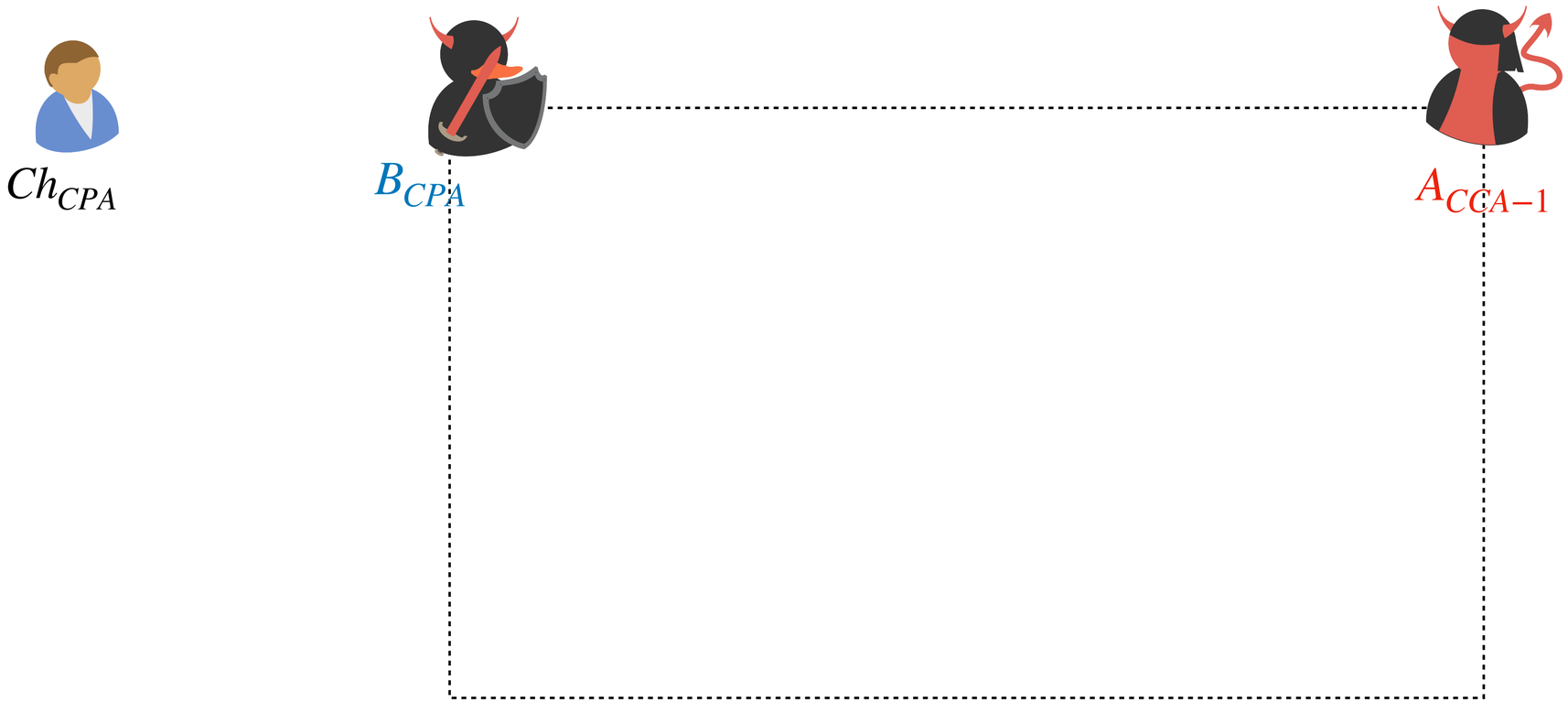
- KeyGen$'(1^{\lambda})$



Attempted Proof: IND-CPA => IND-CCA-1

Our CCA-1 Scheme

- KeyGen$'(1^\lambda)$
  - $pk_0 \leftarrow$ KeyGen(1^λ)



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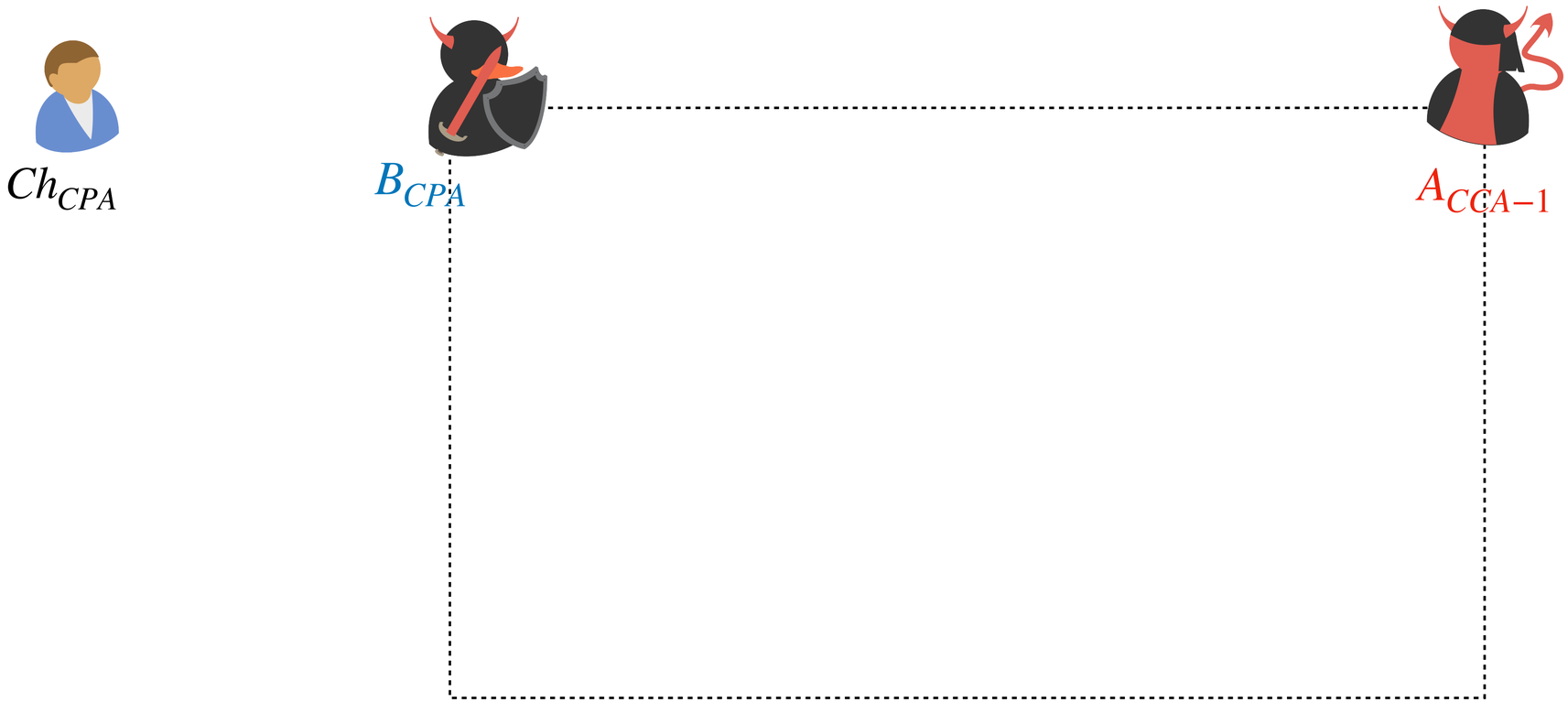

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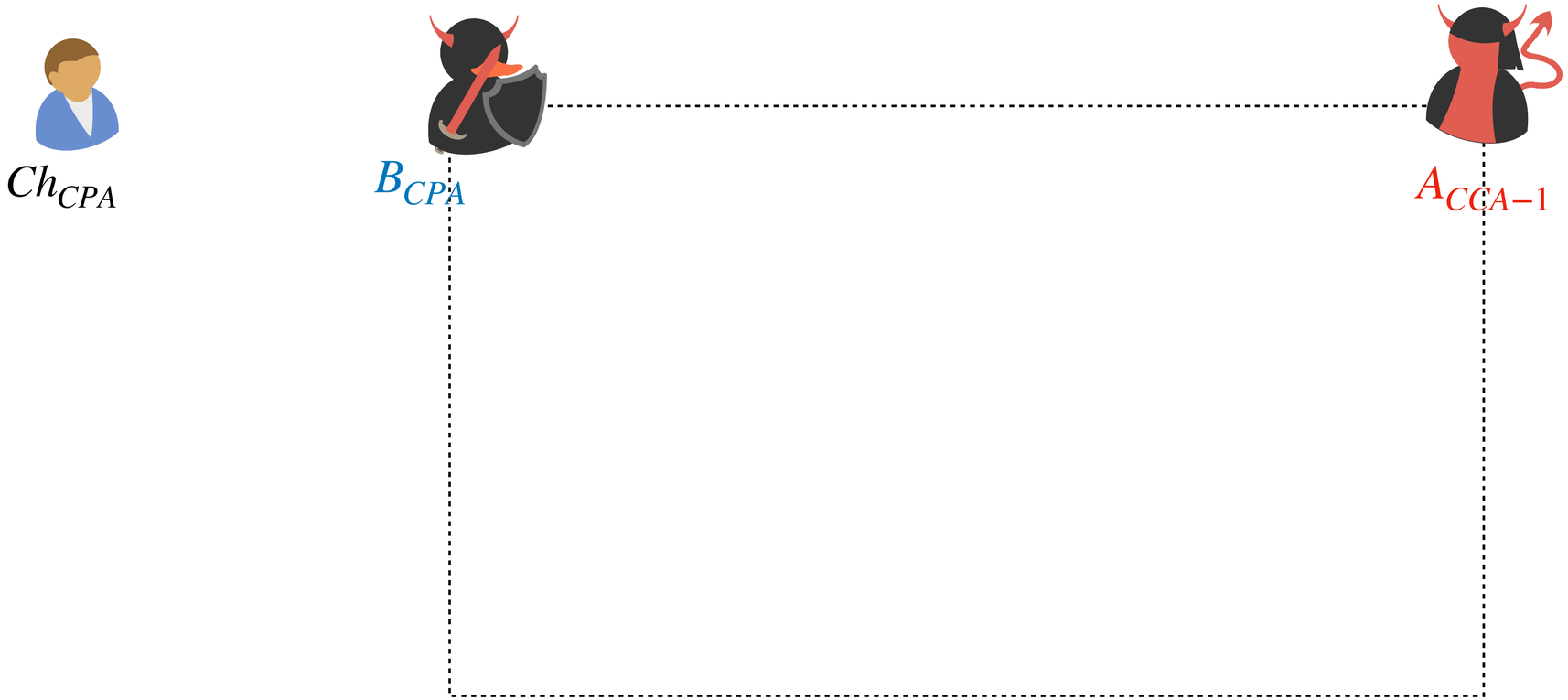

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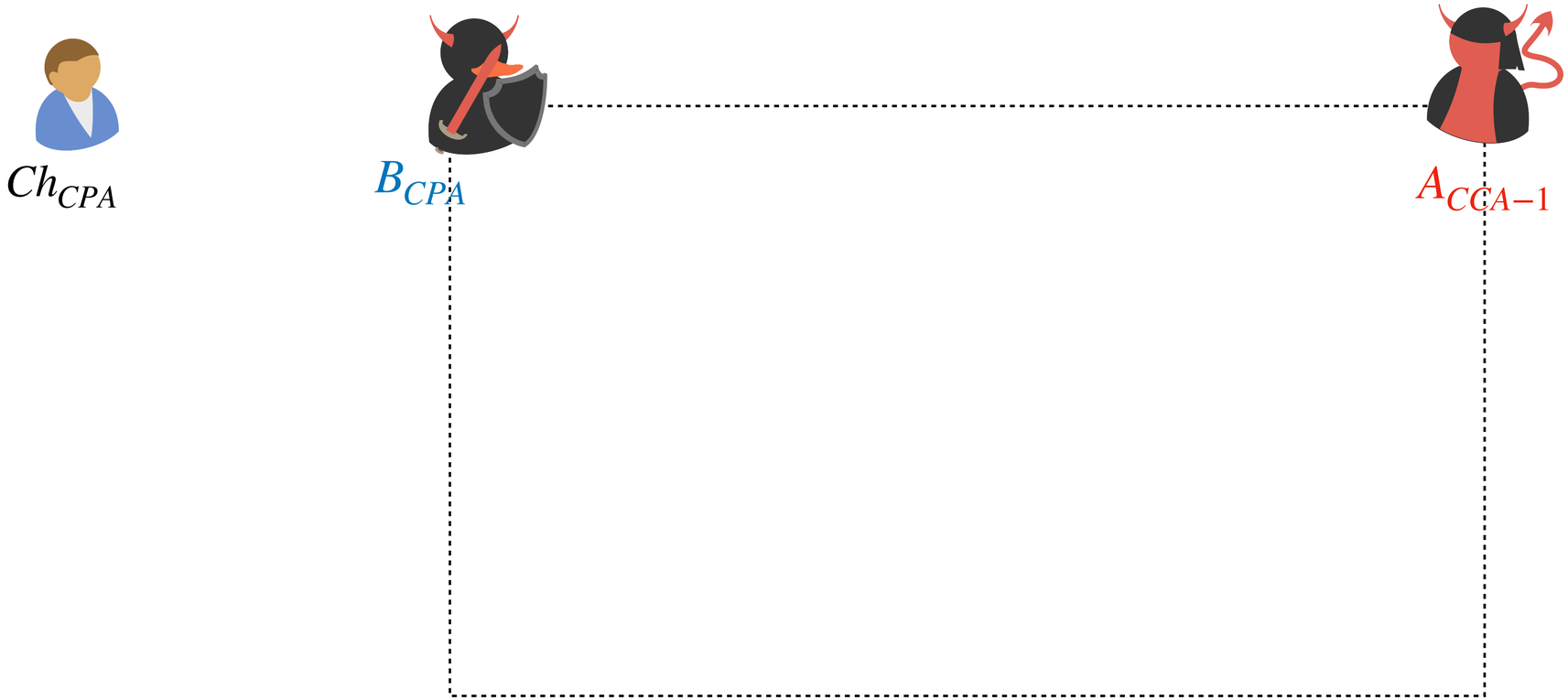
  - $\mathsf{ct}_0 \leftarrow \mathsf{Enc}(pk_0, m)$



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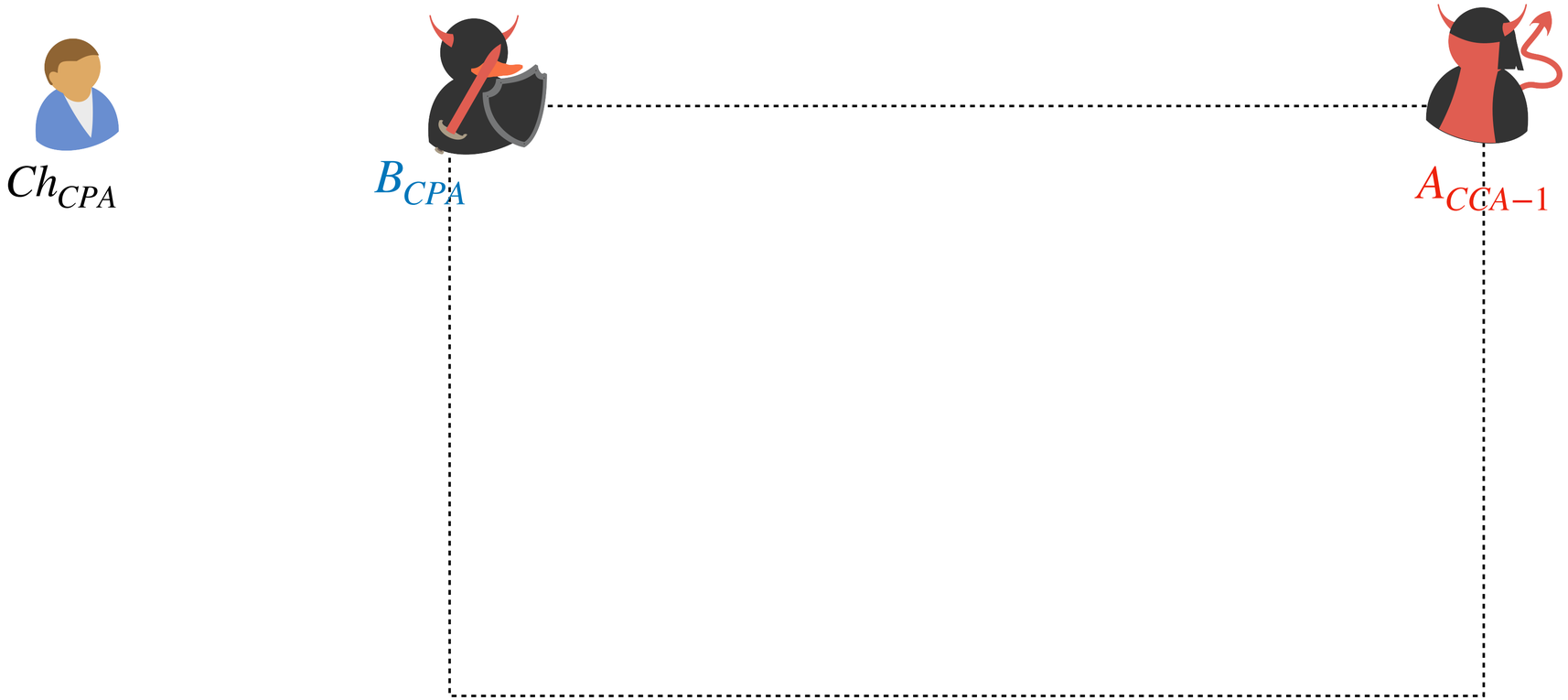

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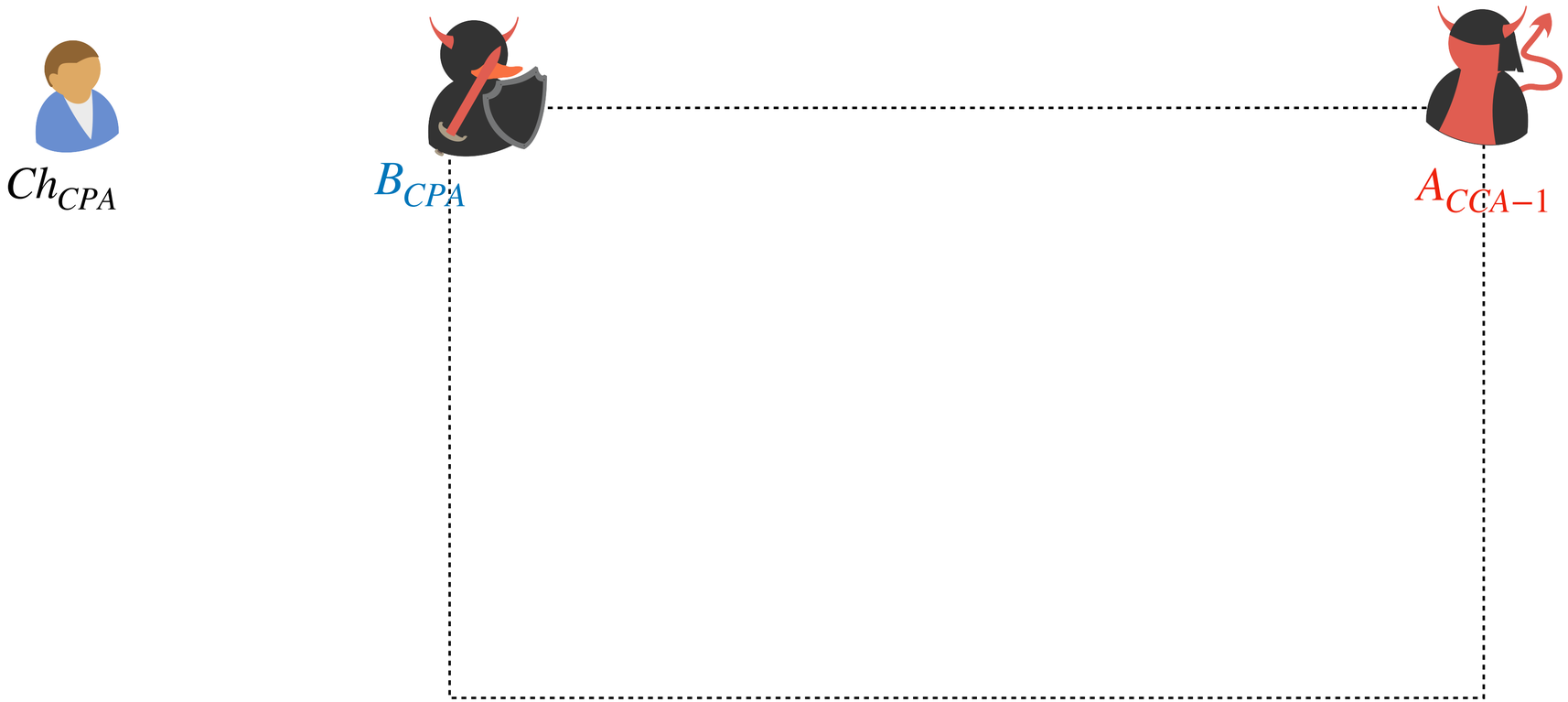

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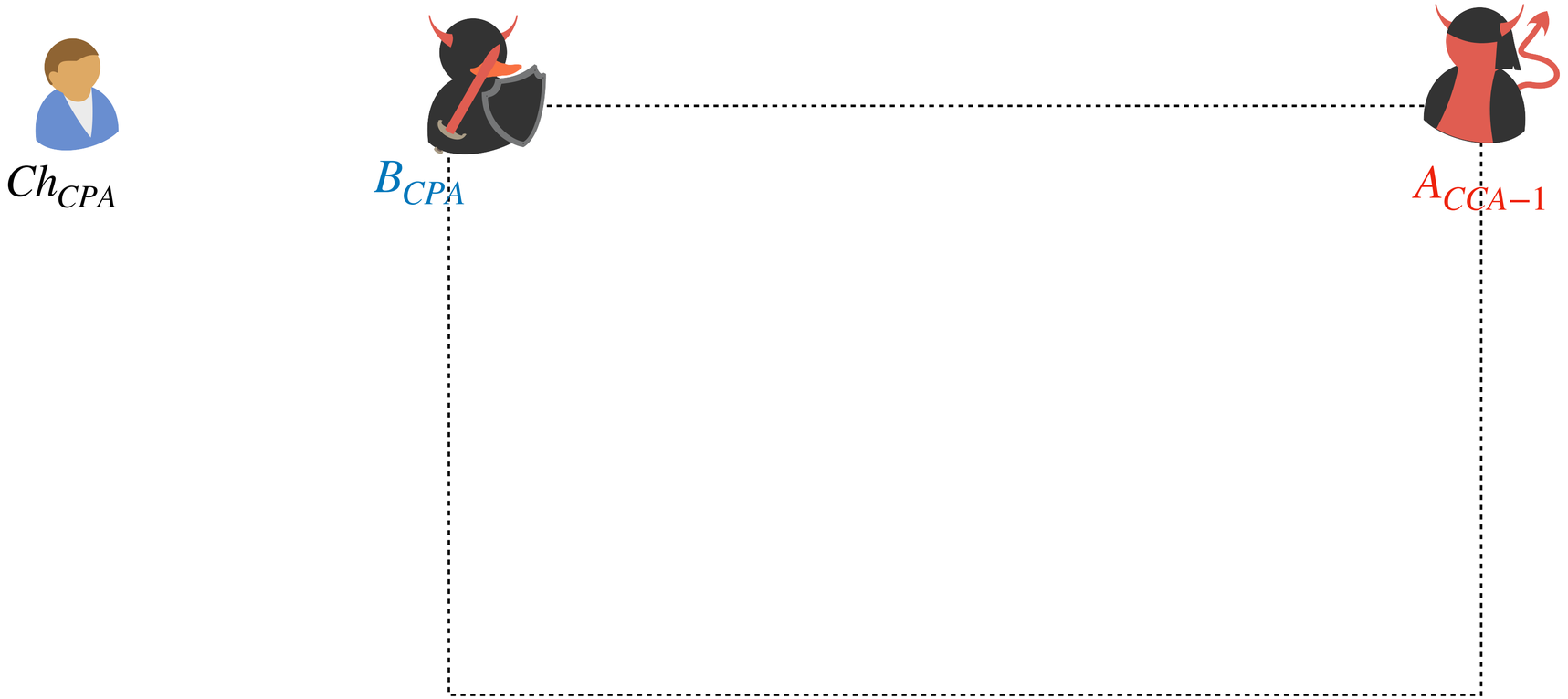

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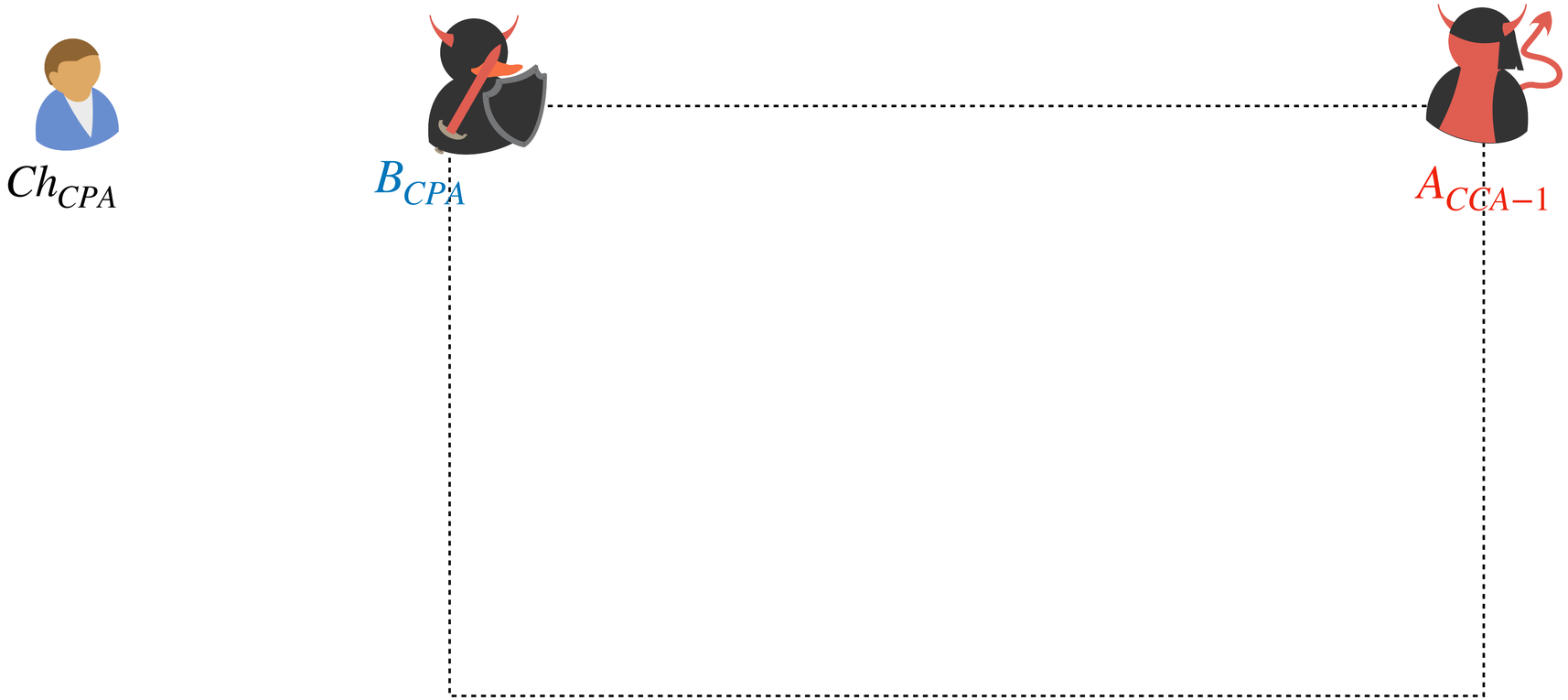

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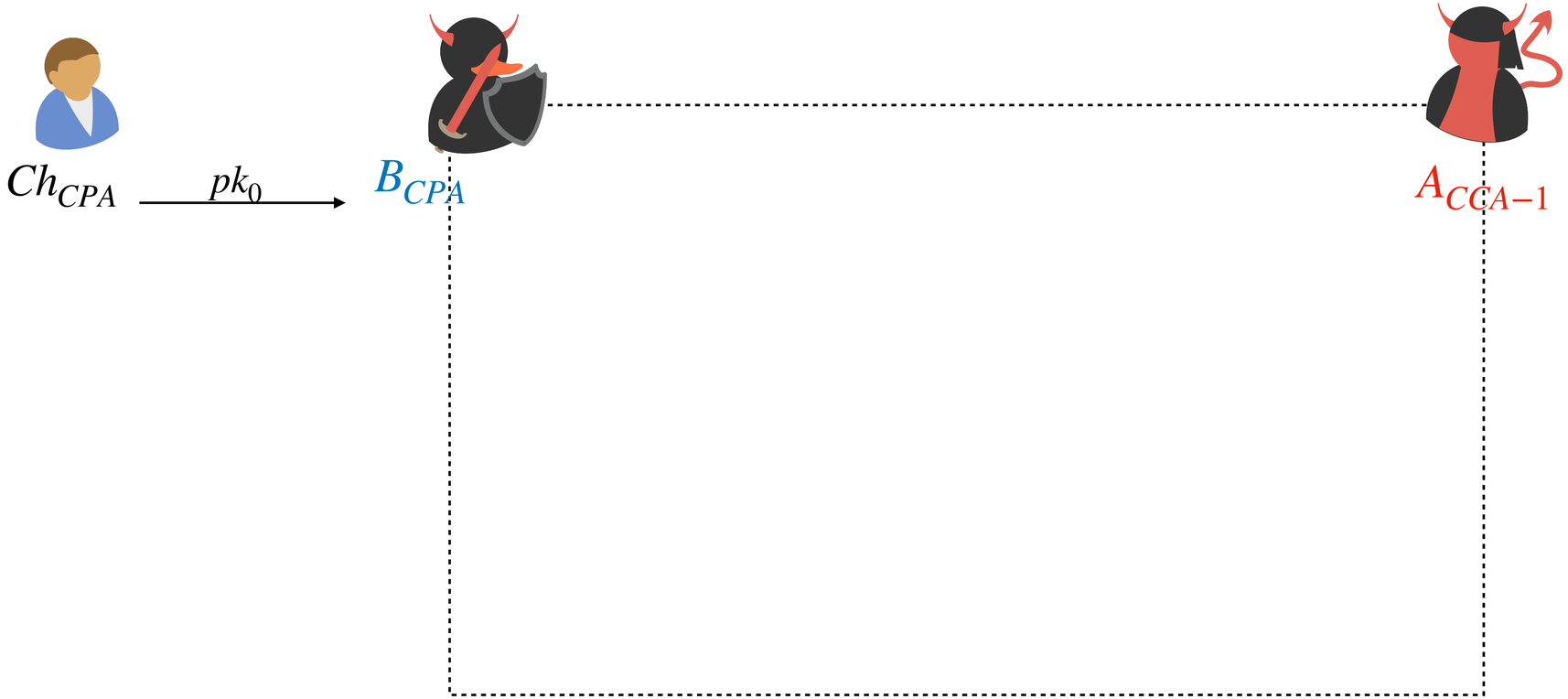

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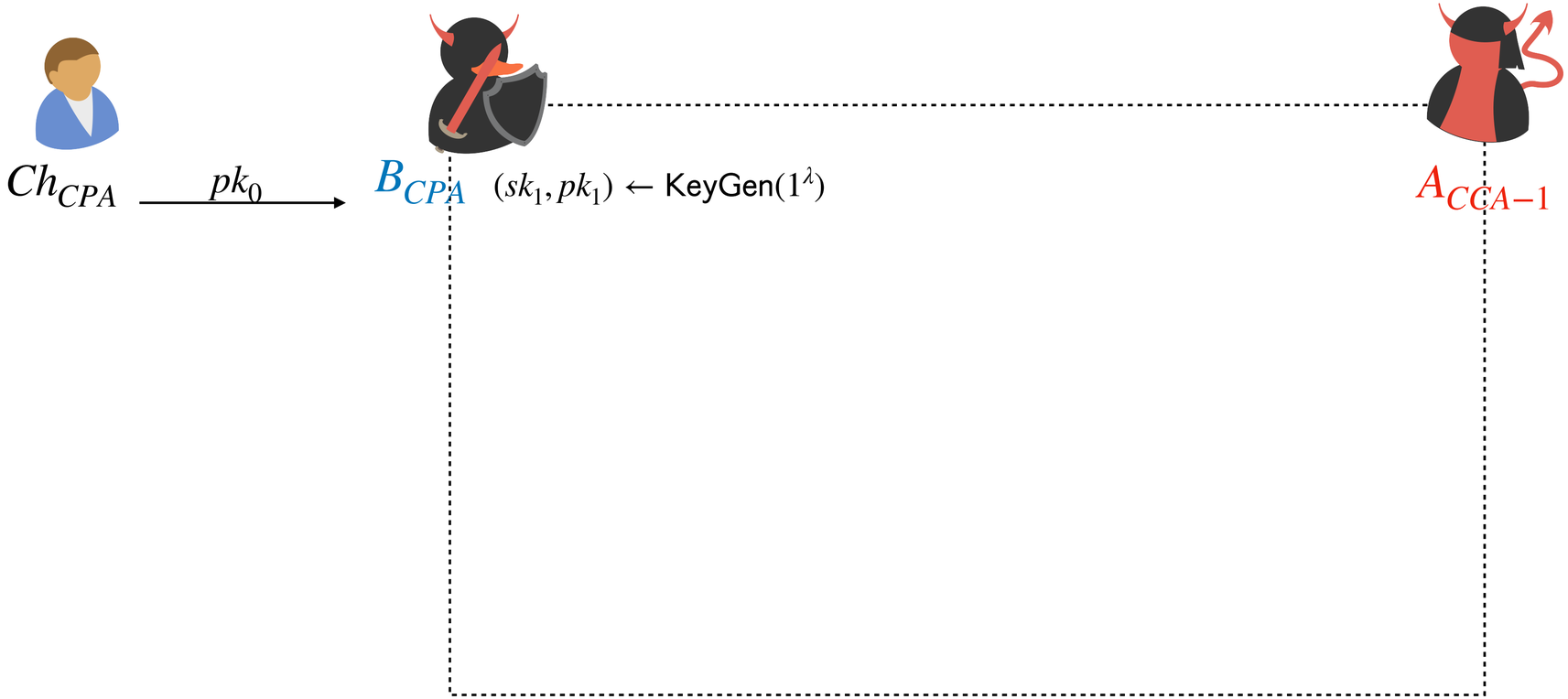

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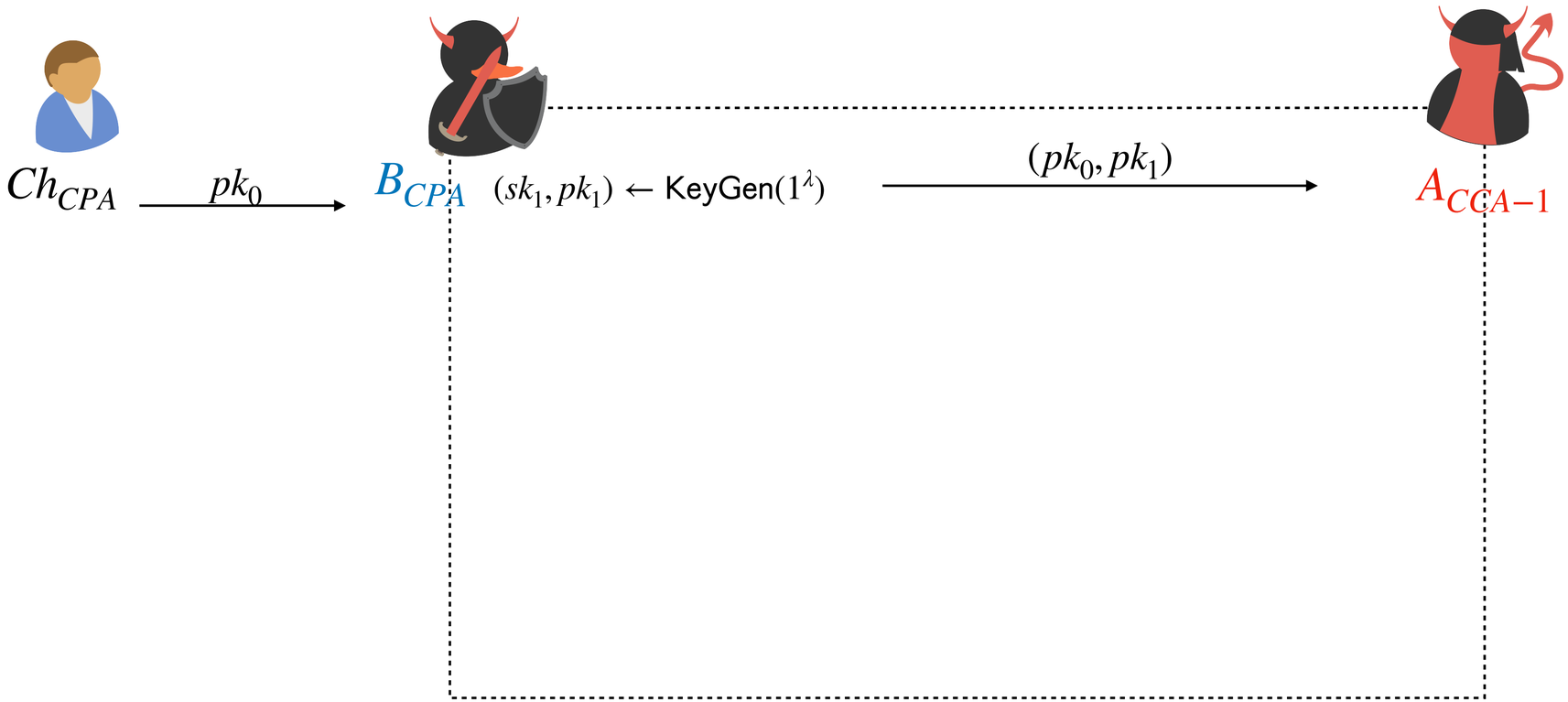

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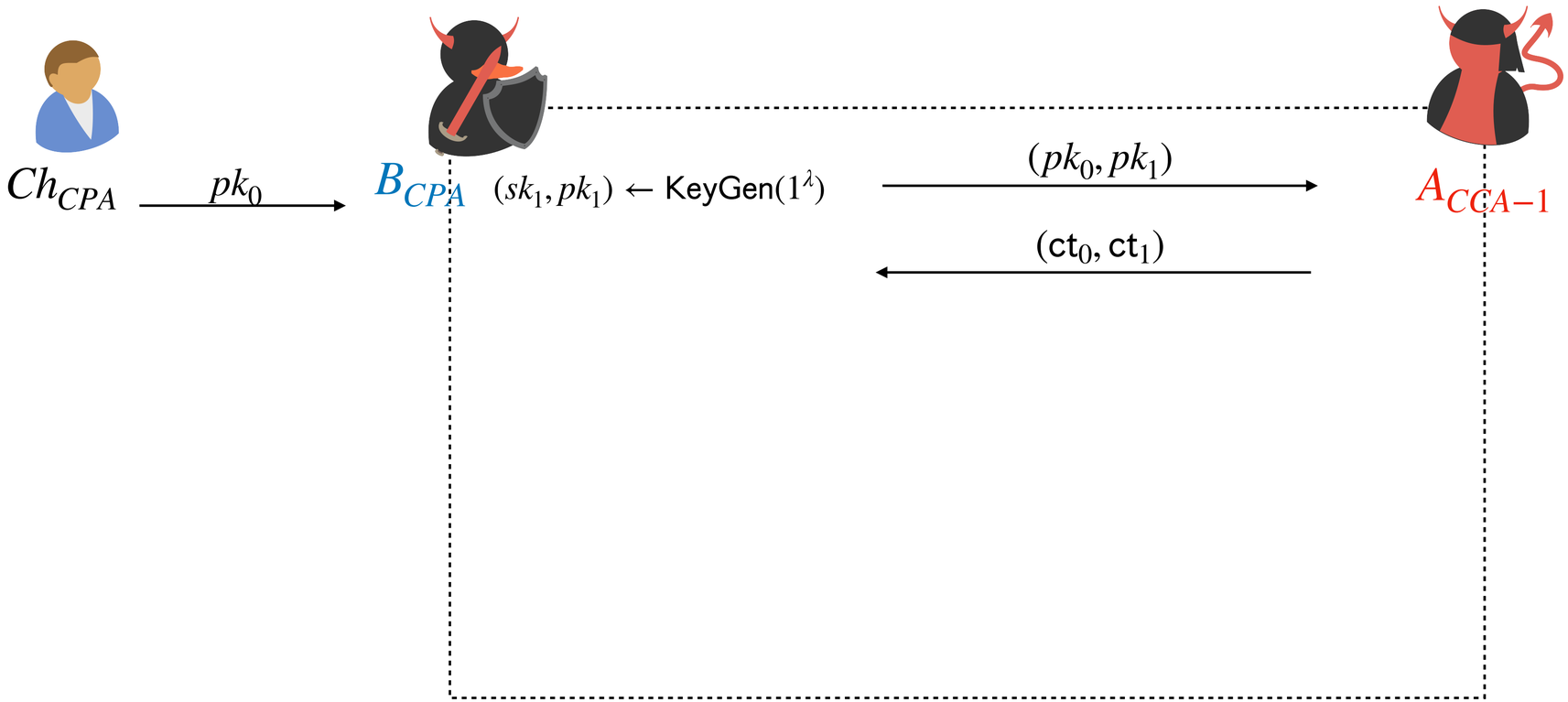

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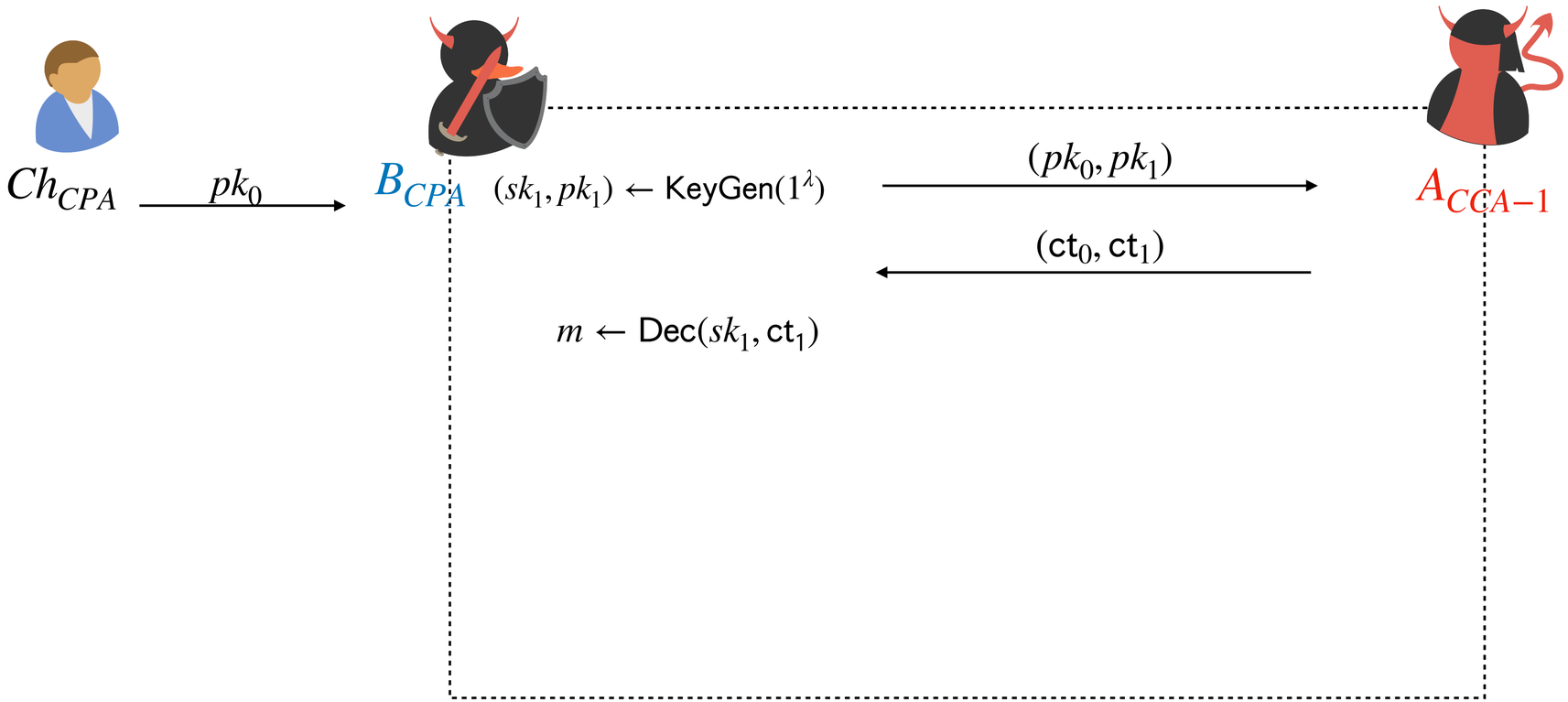

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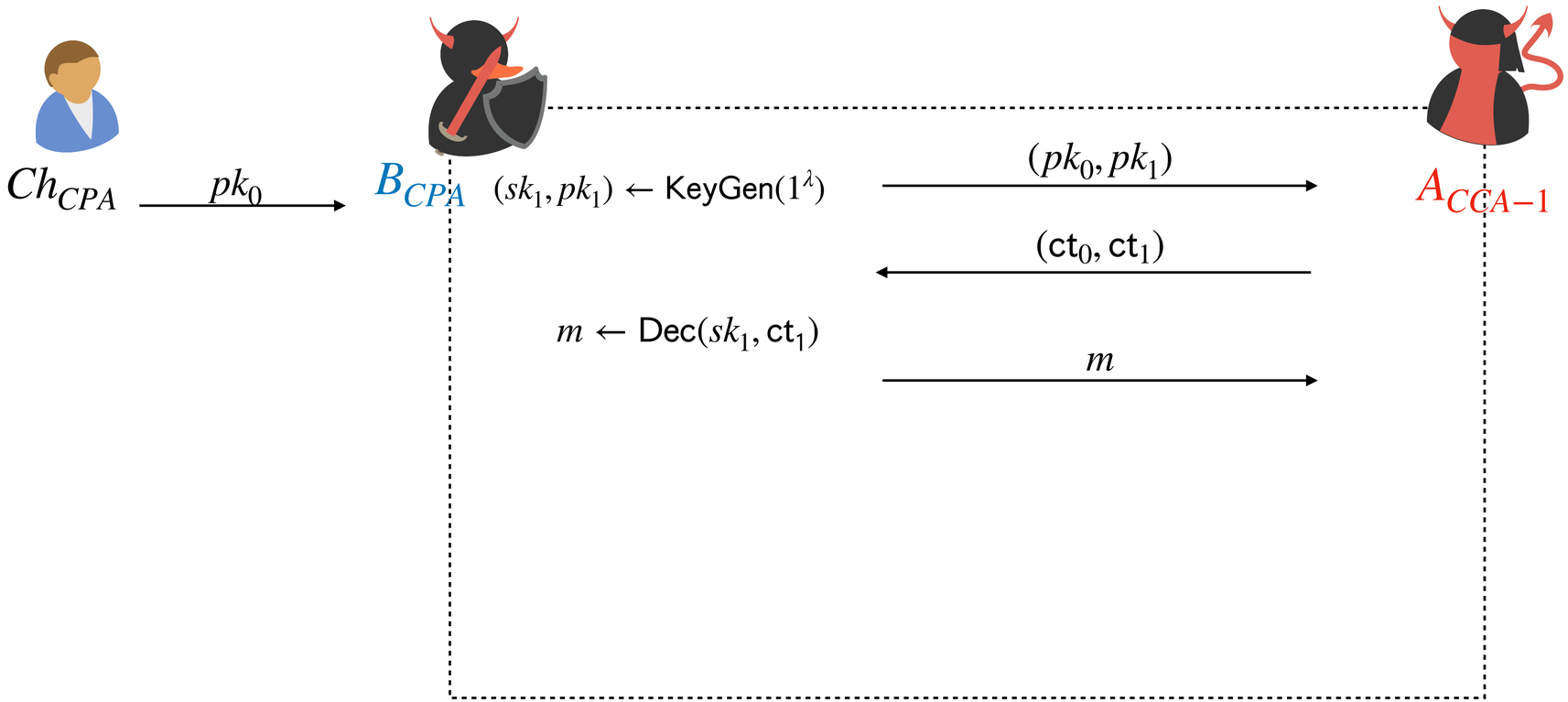

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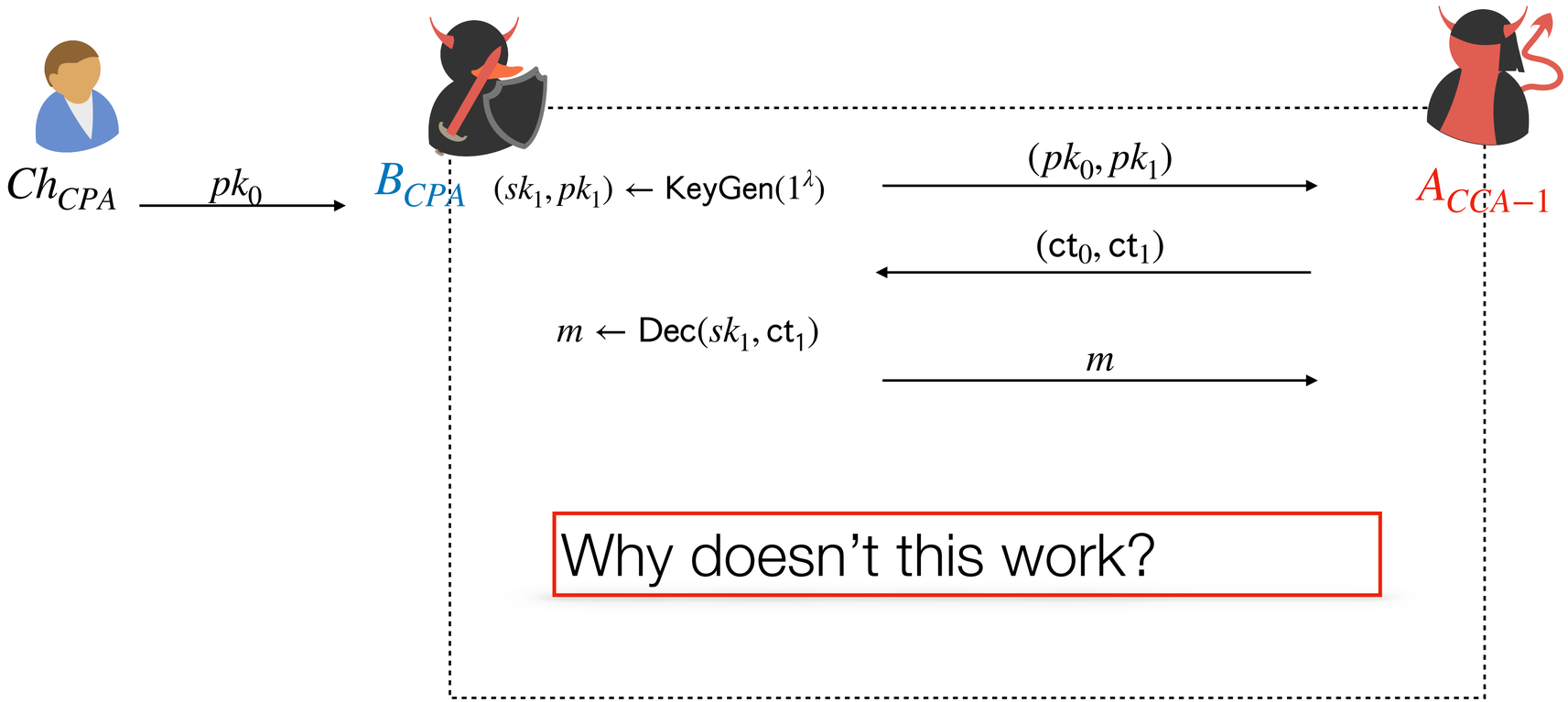

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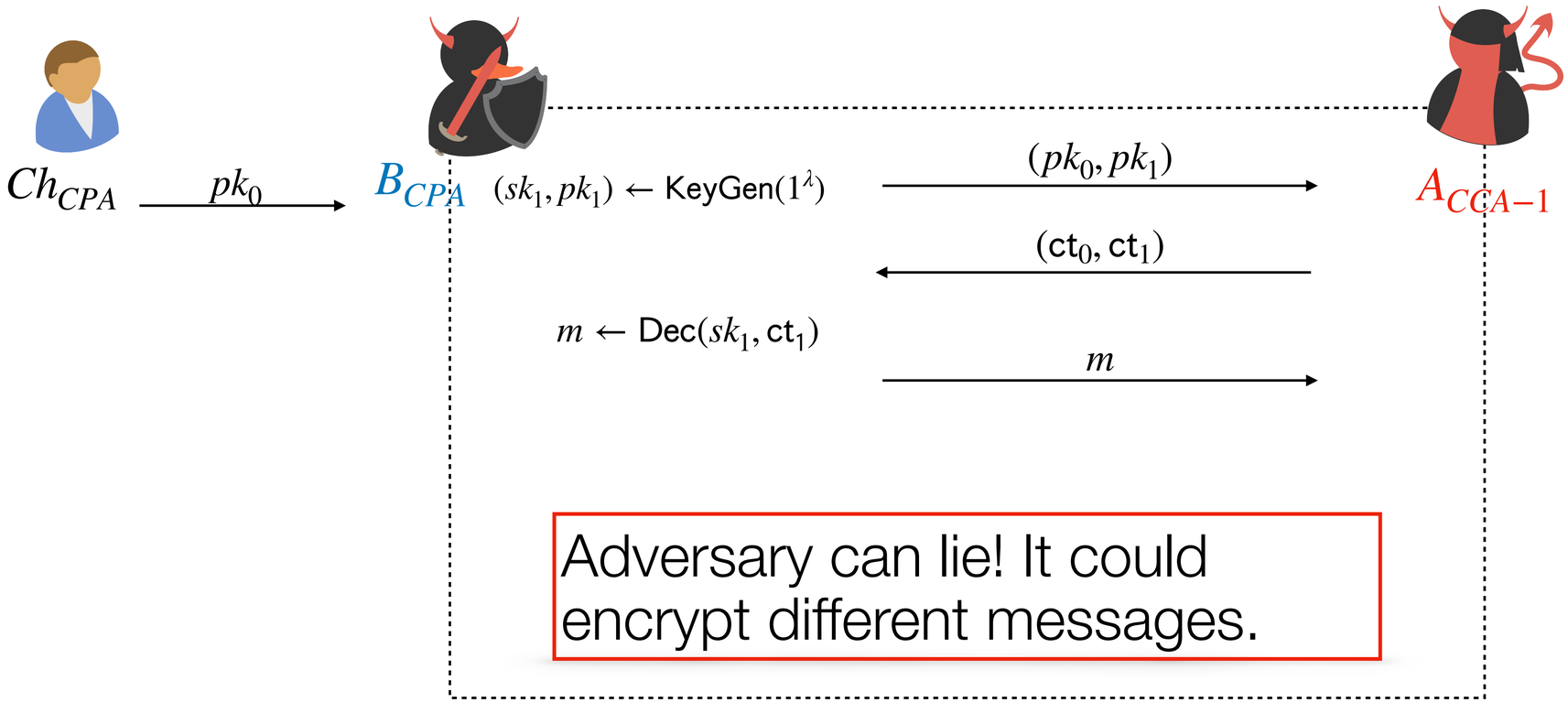

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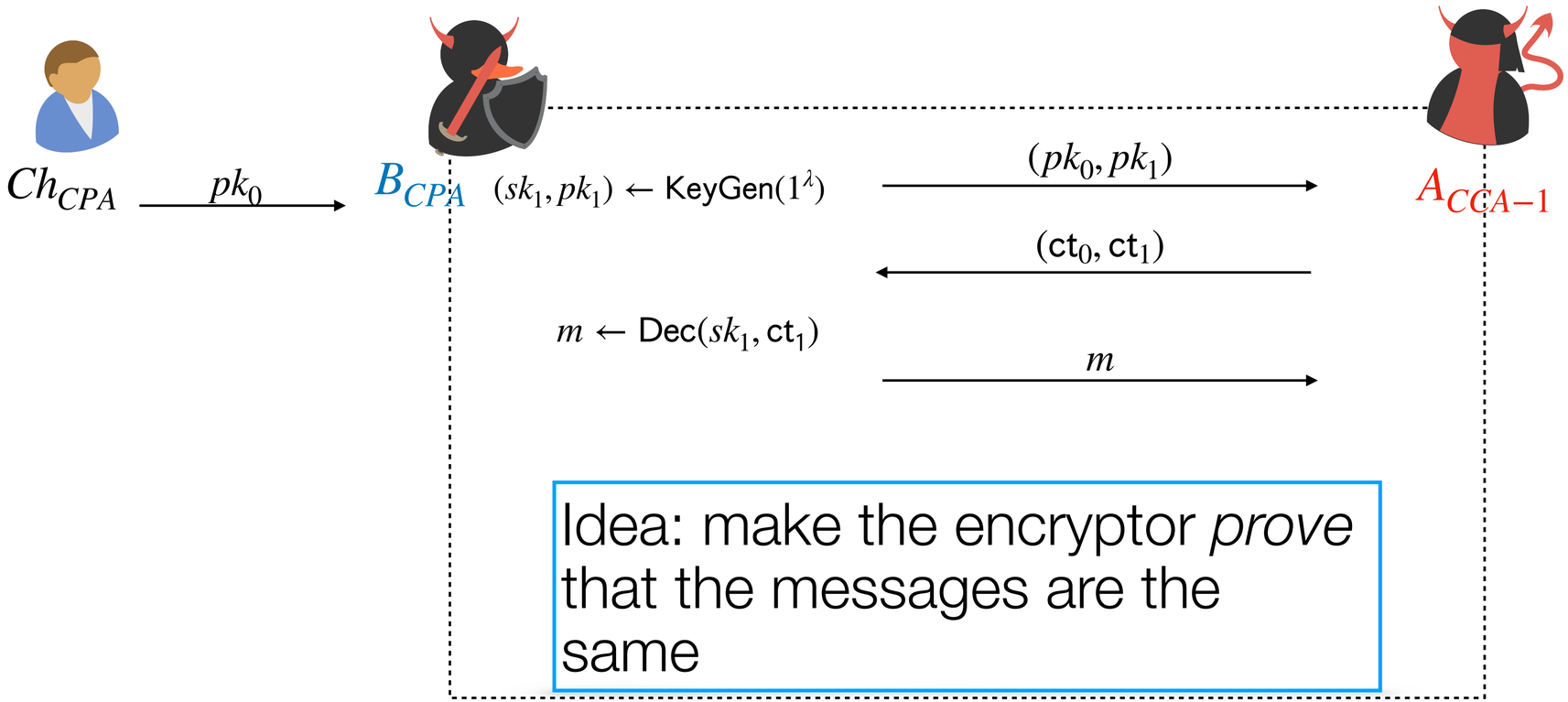

Idea: make the encryptor *prove* that the messages are the same

Naor-Yung

Naor-Yung

Naor-Yung

Naor-Yung

Let λ be the security parameter, $(\mathsf{KGen}, \mathsf{Enc}, \mathsf{Dec})$ be an IND-CPA secure public key encryption scheme, and let (P, V) be a NIZK for the following language:

Naor-Yung

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$$L = \left\{ (\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1) \ : \ \exists k, m, r_0, r_1 \ \text{ s.t. } \begin{array}{c} \mathsf{ct}_0 = \mathsf{Enc}(pk_0, m; r_0) \wedge \\ \mathsf{ct}_1 = \mathsf{Enc}(pk_1, m; r_1) \end{array} \right\}$$

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- $\mathsf{KGen}'(1^\lambda)$:

Naor-Yung

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- $\mathsf{KGen}'(1^\lambda)$:
  - $(sk_0, pk_0) \leftarrow \mathsf{KGen}(1^\lambda)$

Naor-Yung

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- $\mathsf{KGen}'(1^\lambda)$:
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Naor-Yung

Naor-Yung

Let λ be the security parameter, $(\mathsf{KGen}, \mathsf{Enc}, \mathsf{Dec})$ be an IND-CPA secure public key encryption scheme, and let (P, V) be a NIZK for the following language:

$$L = \left\{ (\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1) \ : \ \exists k, m, r_0, r_1 \ \text{ s.t. } \begin{array}{c} \mathsf{ct}_0 = \mathsf{Enc}(pk_0, m; r_0) \wedge \\ \mathsf{ct}_1 = \mathsf{Enc}(pk_1, m; r_1) \end{array} \right\}$$

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  - return $((sk_0, sk_1), (pk_0, pk_1))$

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  - return $((sk_0, sk_1), (pk_0, pk_1))$
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- $\mathsf{Enc}'((pk_0, pk_1), m)$:
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- $\mathsf{Enc}'((pk_0, pk_1), m)$:
  - $\mathsf{ct}_0 \leftarrow \mathsf{Enc}(pk_0, m)$
  - $\mathsf{ct}_1 \leftarrow \mathsf{Enc}(pk_1, m)$

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- $\mathsf{Enc}'((pk_0, pk_1), m)$:
  - $\mathsf{ct}_0 \leftarrow \mathsf{Enc}(pk_0, m)$
  - $\mathsf{ct}_1 \leftarrow \mathsf{Enc}(pk_1, m)$
  - $\pi \leftarrow P((ct_0, ct_1, pk_0, pk_1), \mathsf{crs})$

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  - return $(\mathsf{ct}_0, \mathsf{ct}_1, \pi)$

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  - return $((sk_0, sk_1), (pk_0, pk_1))$
- $\mathsf{Enc}'((pk_0, pk_1), m)$:
  - $\mathsf{ct}_0 \leftarrow \mathsf{Enc}(pk_0, m)$
  - $\mathsf{ct}_1 \leftarrow \mathsf{Enc}(pk_1, m)$
  - $\pi \leftarrow P((ct_0, ct_1, pk_0, pk_1), \mathsf{crs})$
  - return $(\mathsf{ct}_0, \mathsf{ct}_1, \pi)$
- $\mathsf{Dec}'((sk_0, sk_1), (\mathsf{ct}_0, \mathsf{ct}_1, \pi))$

Naor-Yung

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  - $\pi \leftarrow P((ct_0, ct_1, pk_0, pk_1), \mathsf{crs})$
  - return $(\mathsf{ct}_0, \mathsf{ct}_1, \pi)$
- $\mathsf{Dec}'((sk_0, sk_1), (\mathsf{ct}_0, \mathsf{ct}_1, \pi))$
  - if $V(\pi, \mathsf{crs}) = 0$ return $\perp$

Naor-Yung

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  - return $((sk_0, sk_1), (pk_0, pk_1))$
- $\mathsf{Enc}'((pk_0, pk_1), m)$:
  - $\mathsf{ct}_0 \leftarrow \mathsf{Enc}(pk_0, m)$
  - $\mathsf{ct}_1 \leftarrow \mathsf{Enc}(pk_1, m)$
  - $\pi \leftarrow P((ct_0, ct_1, pk_0, pk_1), \mathsf{crs})$
  - return $(\mathsf{ct}_0, \mathsf{ct}_1, \pi)$
- $\mathsf{Dec}'((sk_0, sk_1), (\mathsf{ct}_0, \mathsf{ct}_1, \pi))$
  - if $V(\pi, \mathsf{crs}) = 0$ return $\perp$
  - else return $\mathsf{Dec}(sk_0, \mathsf{ct}_0)$

Proof of Security

Proof of Security

$\mathcal{G}_0$

Proof of Security

$\mathcal{G}_0$

Proof of Security

$\mathrm{crs} \leftarrow \mathrm{CRSGen}(1^\lambda)$

$\mathcal{G}_0$

Proof of Security

$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$

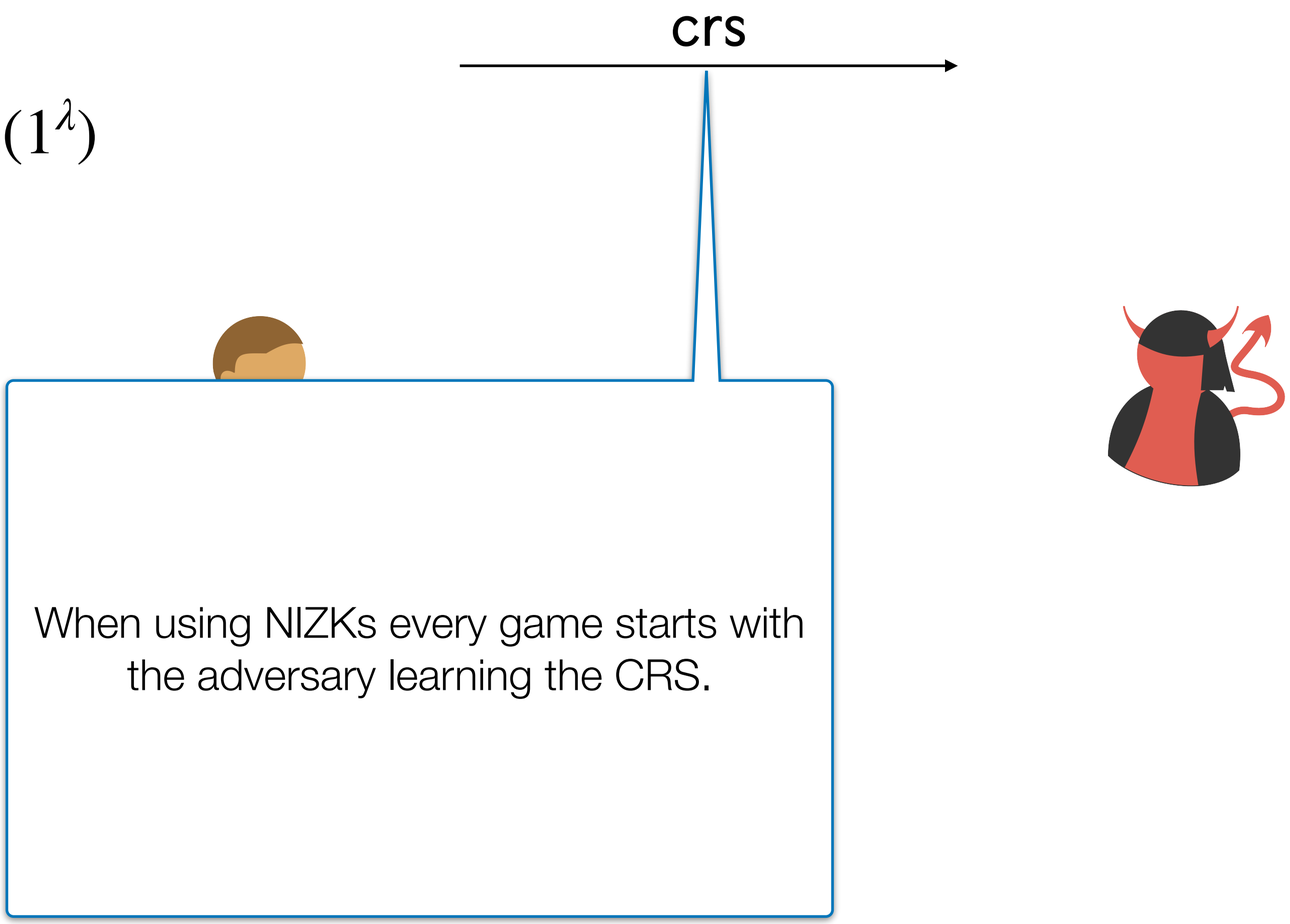
crs

$\mathcal{G}_0$

Proof of Security

$\mathrm{crs} \leftarrow \mathrm{CRSGen}(1^\lambda)$

$\mathcal{G}_0$



Proof of Security

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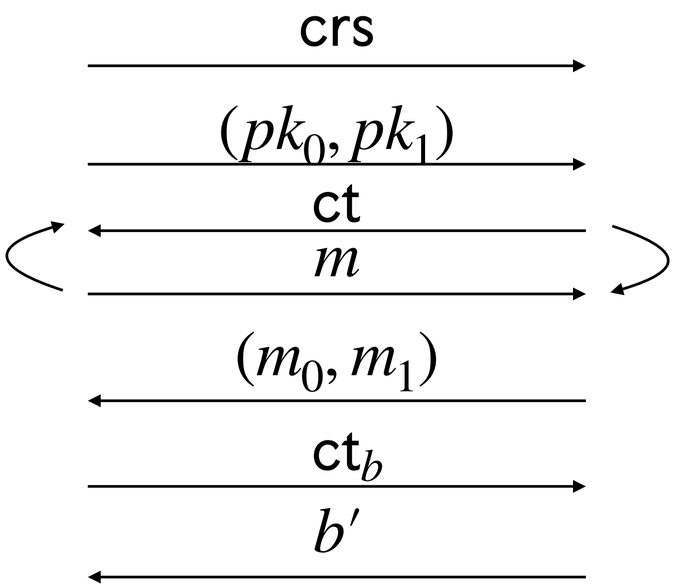
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow P((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



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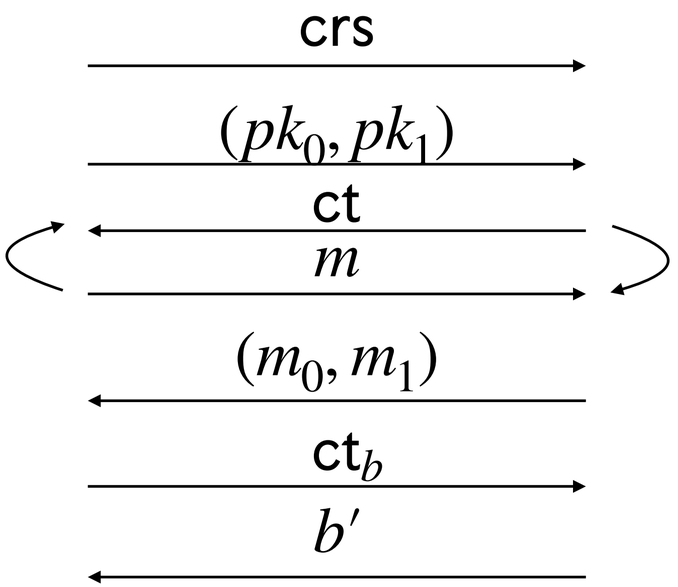

$\mathcal{G}_4$

Proof of Security

$\mathcal{G}_0$

$$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$$
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$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow P((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
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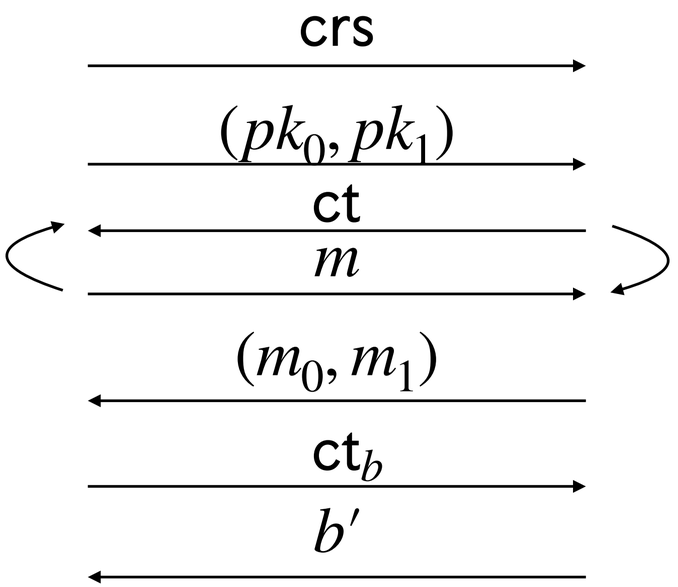

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$$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
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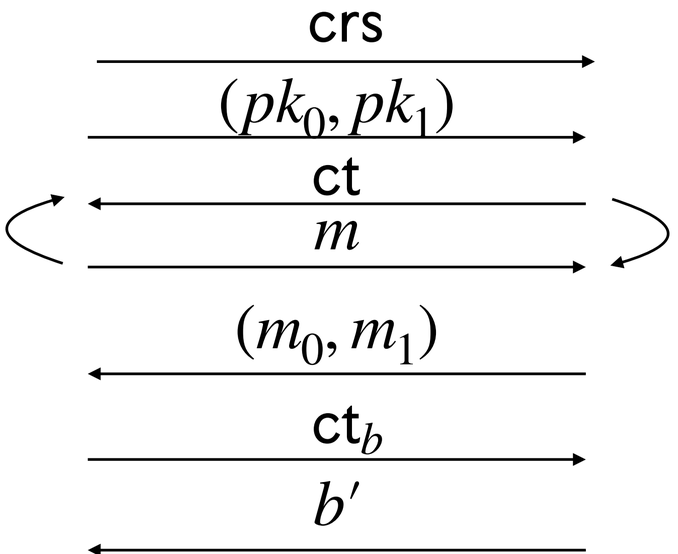
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



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$\mathcal{G}_0$

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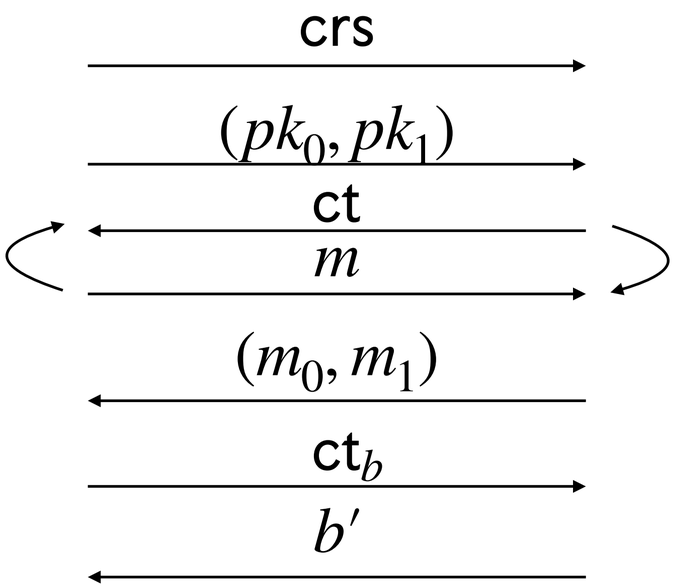

Goal:

$$\left|\Pr[\mathscr{A} \text{ outputs } 1 \text{ in } \mathcal{G}_0] - \Pr[\mathscr{A} \text{ outputs } 1 \text{ in } \mathcal{G}_4]\right| \leq \mathsf{negl}(\lambda)$$

$\mathcal{G}_4$

$$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_1)$$
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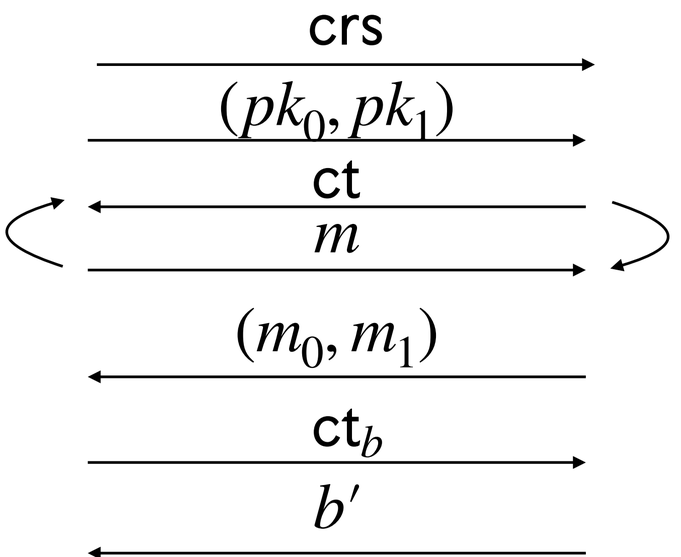
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



Proof of Security

$\mathcal{G}_0$

$$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$$
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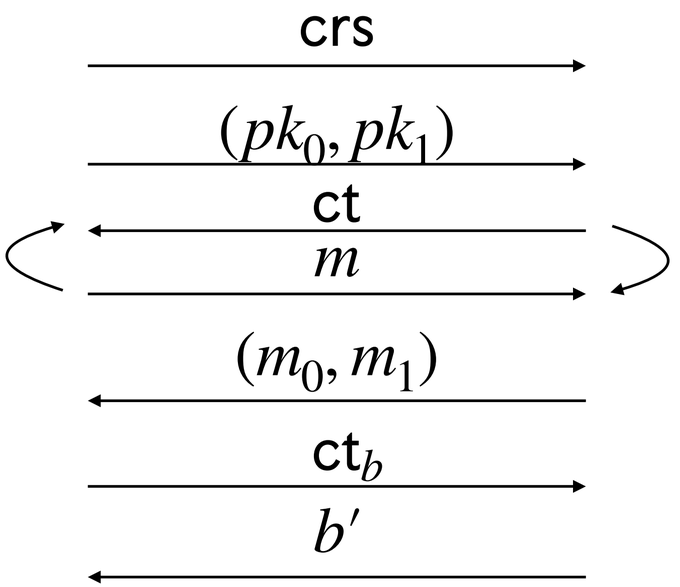

Goal: $\left|\Pr[W_0] - \Pr[W_4]\right| \leq \mathsf{negl}(\lambda)$

$\mathcal{G}_4$

$$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_1)$$
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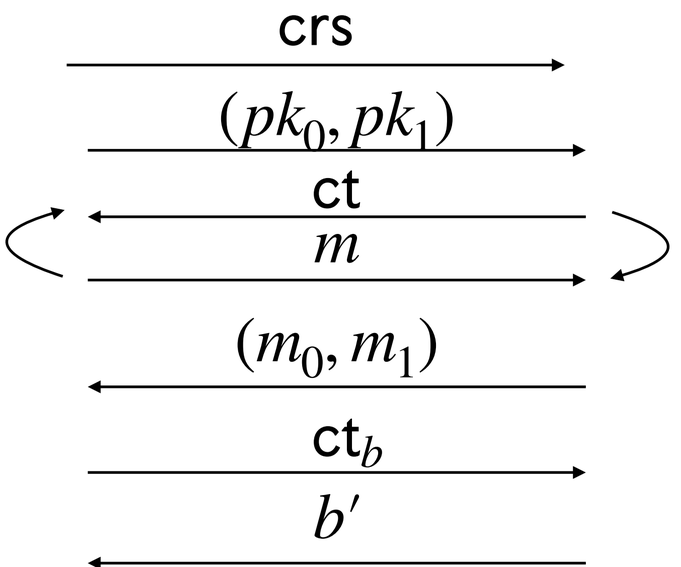

Proof of Security

$\mathcal{G}_0$

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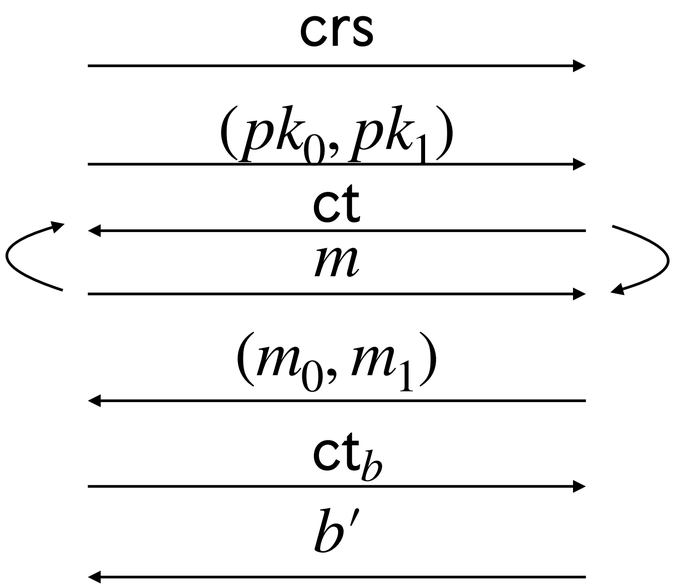

$\mathcal{G}_1$

Proof of Security

$\mathcal{G}_0$

$$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_0)$$
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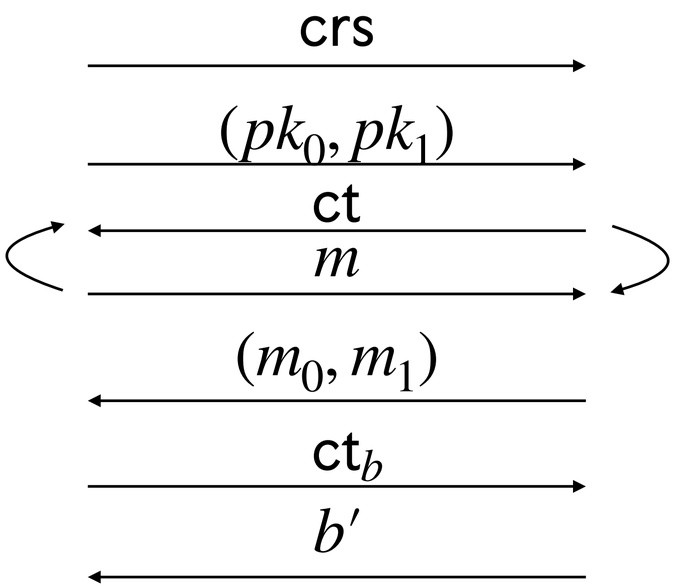
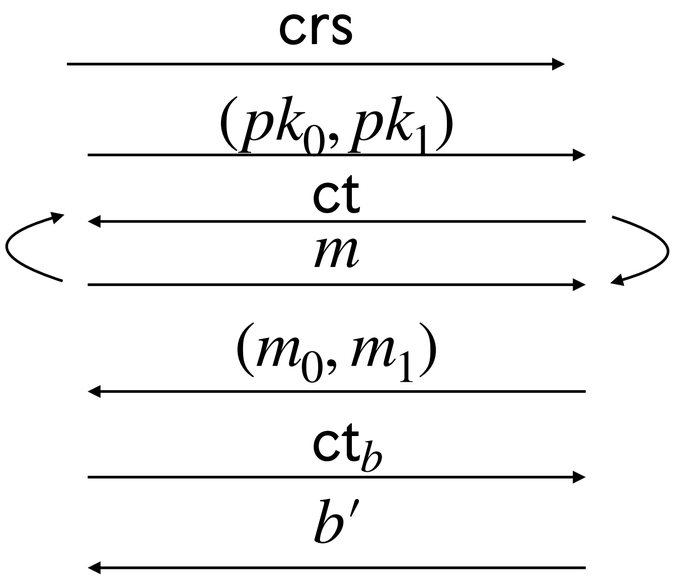


$\mathcal{G}_1$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_0)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



Proof of Security

$\mathcal{G}_0$

$$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$$
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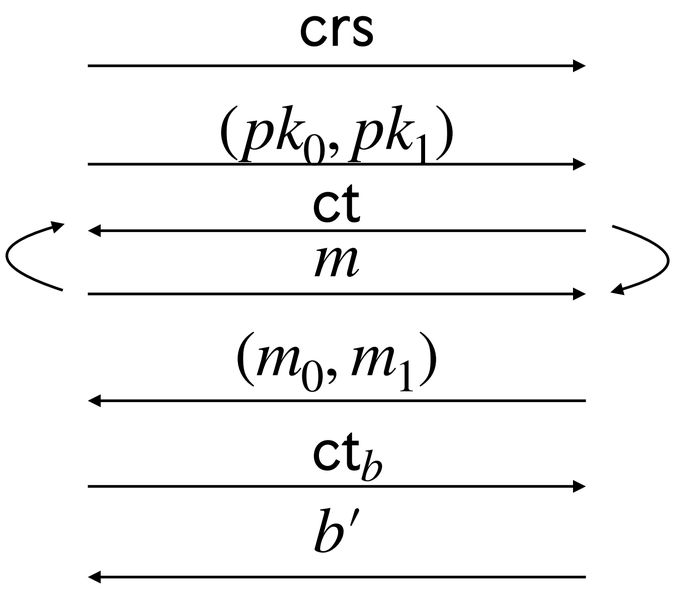

Claim: $\left|\Pr[W_0] - \Pr[W_1]\right| \leq \mathsf{negl}(\lambda)$ by the ZK of (P, V)

$\mathcal{G}_1$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
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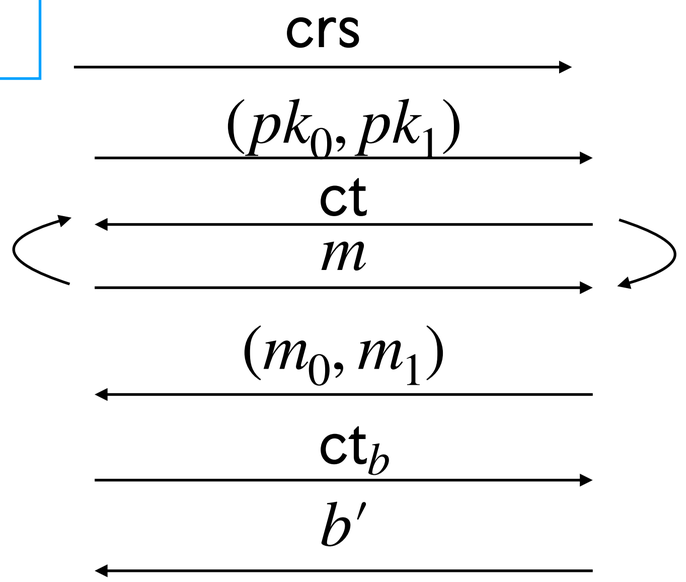

Proof of Security

$\mathcal{G}_1$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_0)$$
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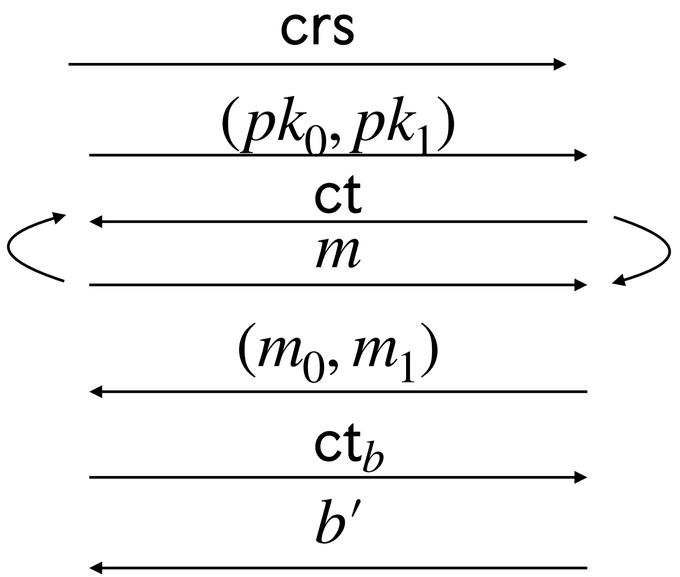
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$

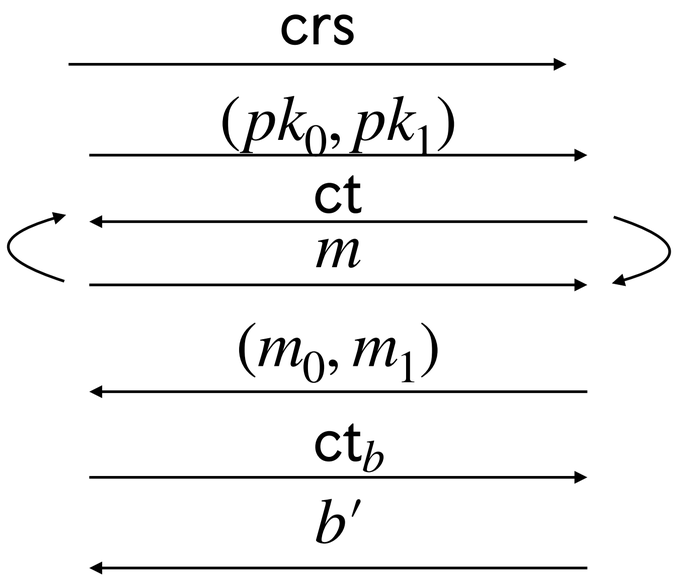


$\mathcal{G}_2$

Proof of Security

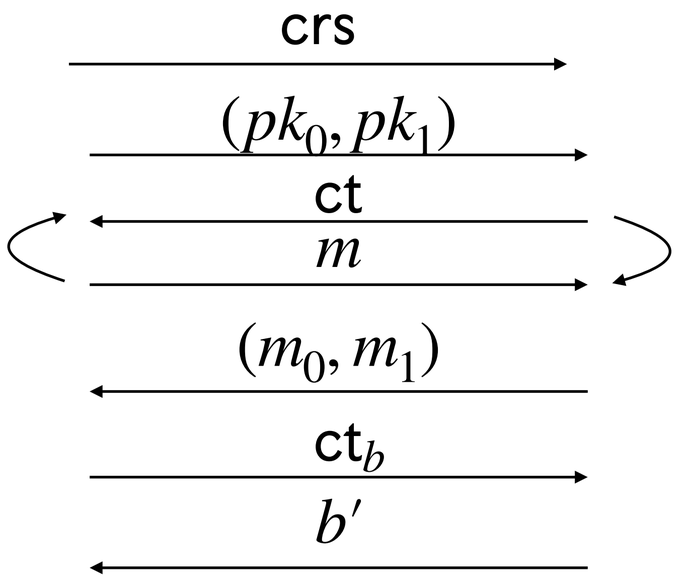
$\mathcal{G}_1$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_0)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



$\mathcal{G}_2$

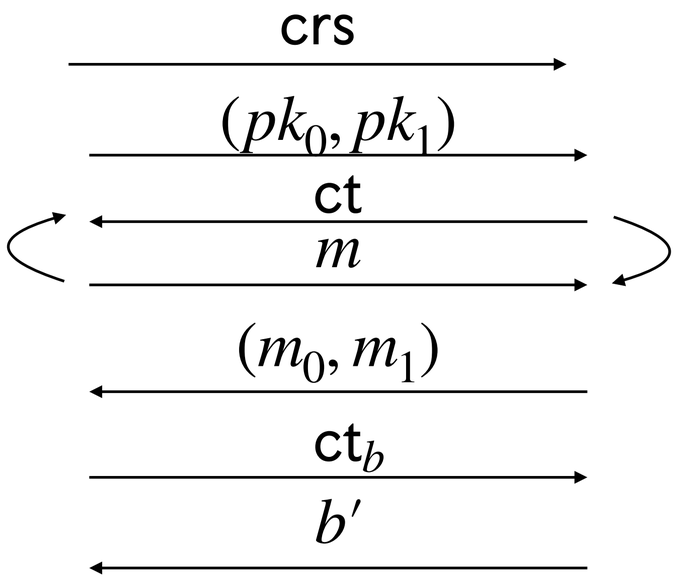
$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



Proof of Security

$\mathcal{G}_1$

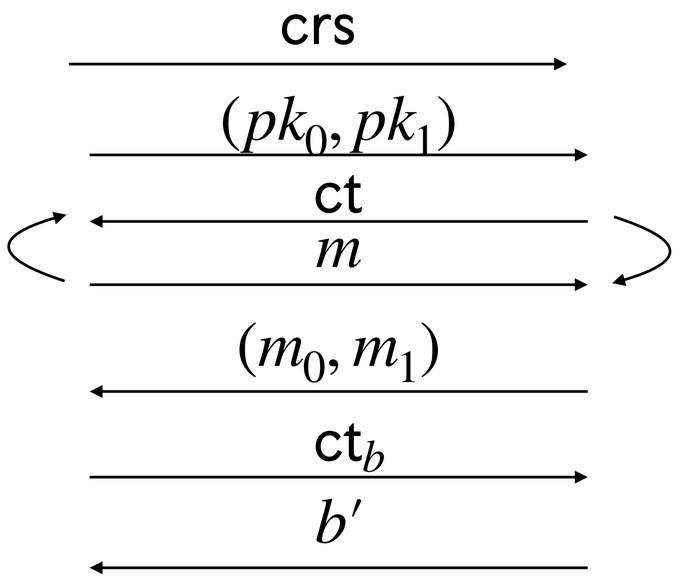
$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_0)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



Claim: $\left|\Pr[W_1] - \Pr[W_2]\right| \leq \mathsf{negl}(\lambda)$ by IND-CPA

$\mathcal{G}_2$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



Proof of Security

Proof of Security

$\mathsf{crs} \leftarrow S_0(1^\lambda)$

$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_0)$

$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$

$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$

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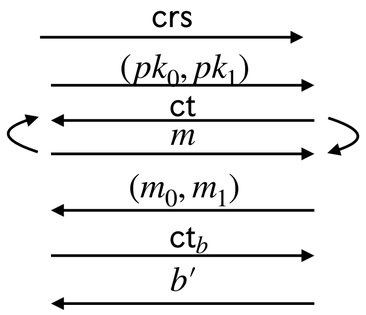
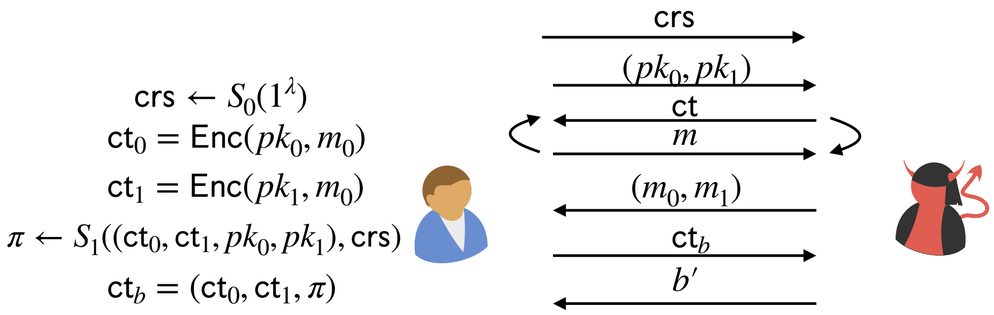
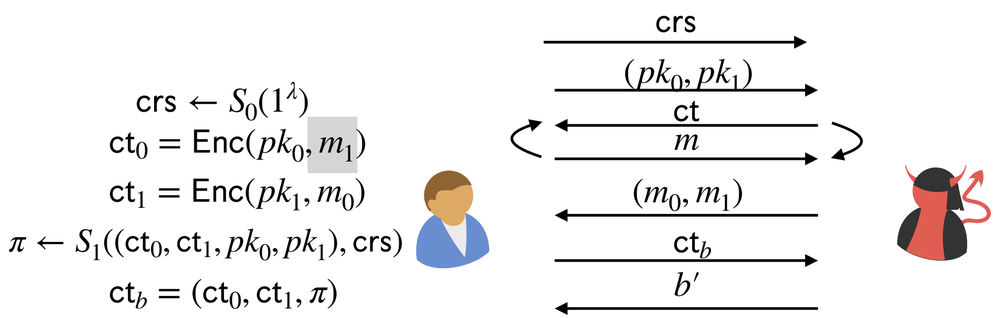


Proof of Security





Proof of Security

Claim: $\left|\Pr[W_1] - \Pr[W_2]\right| \leq \mathbf{negl}(\lambda)$ by IND-CPA

$\mathsf{crs} \leftarrow S_0(1^\lambda)$
$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_0)$
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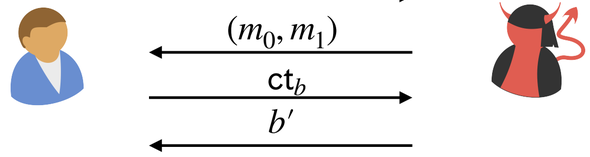
$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$



$\mathsf{crs} \leftarrow S_0(1^\lambda)$
$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$
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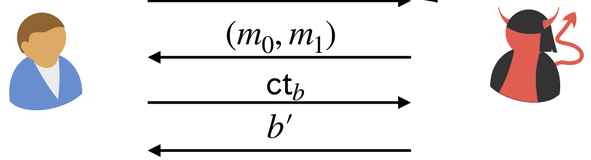
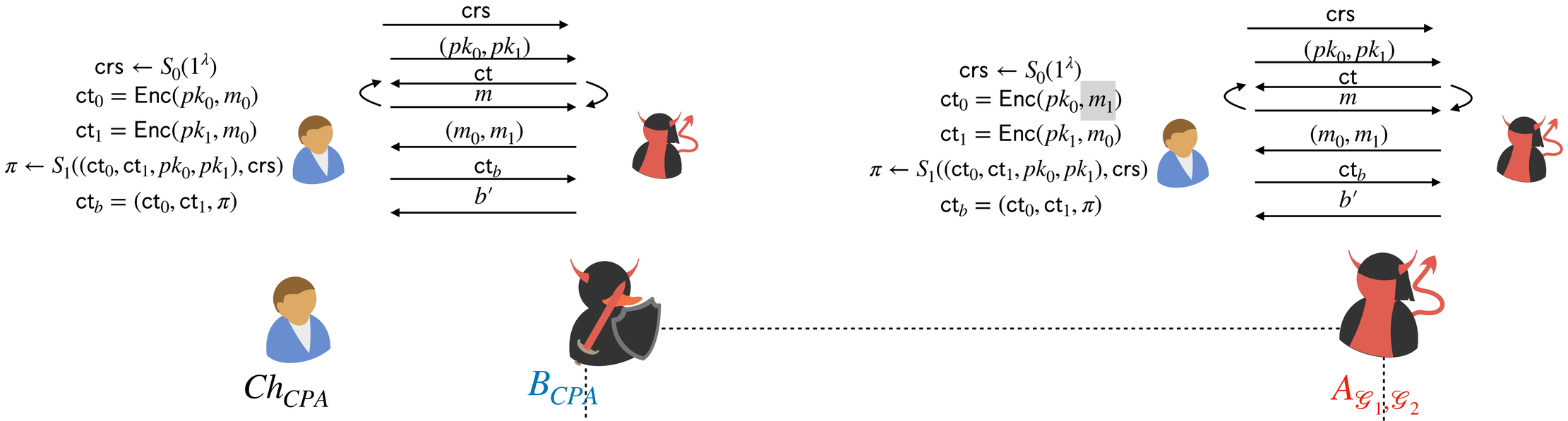


Proof of Security

Claim: $\left|\mathsf{Pr}[W_1] - \mathsf{Pr}[W_2]\right| \leq \mathbf{negl}(\lambda)$ by IND-CPA



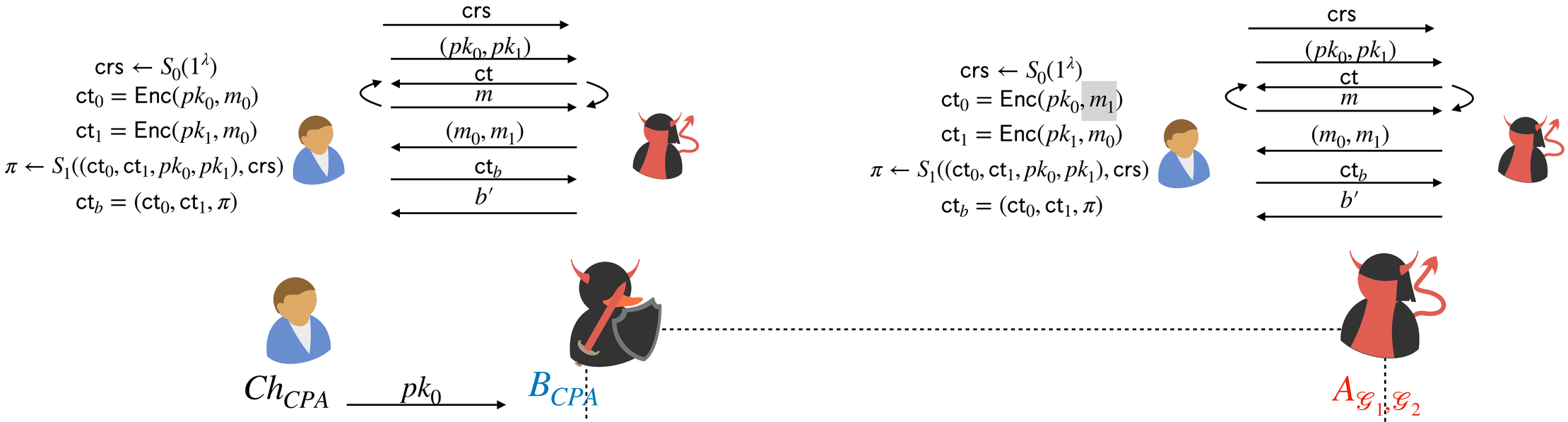
Ch_{CPA}

B_{CPA}

$A_{\mathcal{G}_1,\mathcal{G}_2}$

Proof of Security

Claim: $\left|\Pr[W_1] - \Pr[W_2]\right| \leq \mathsf{negl}(\lambda)$ by IND-CPA



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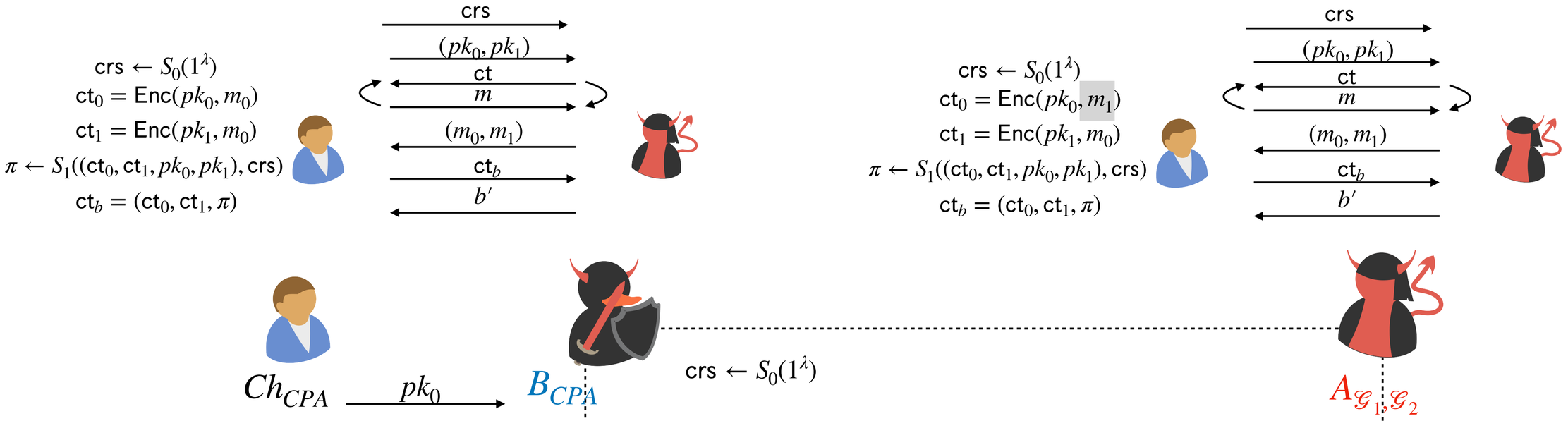

Proof of Security

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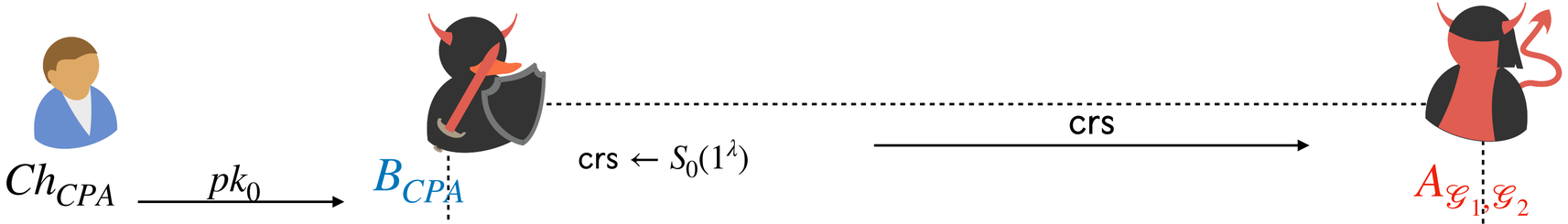

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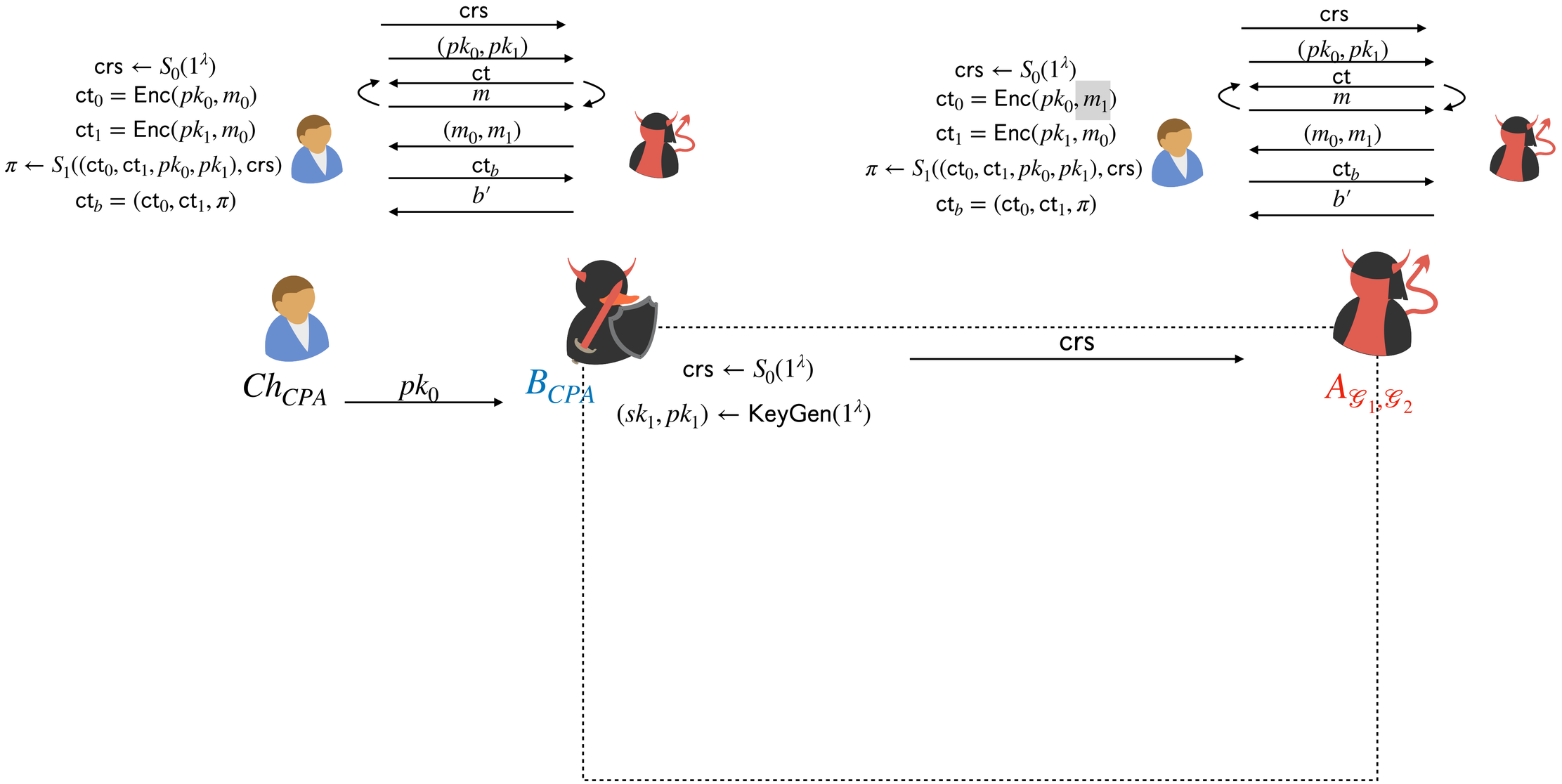

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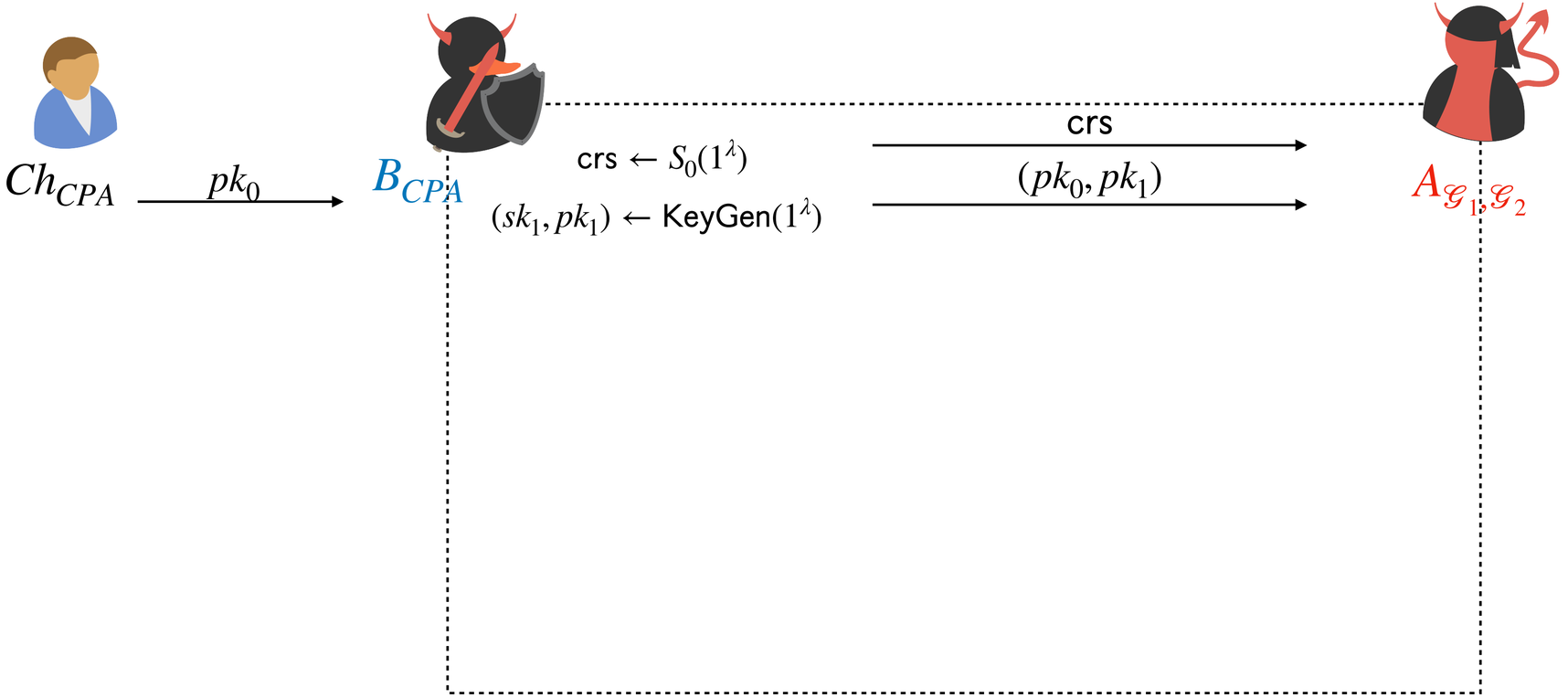

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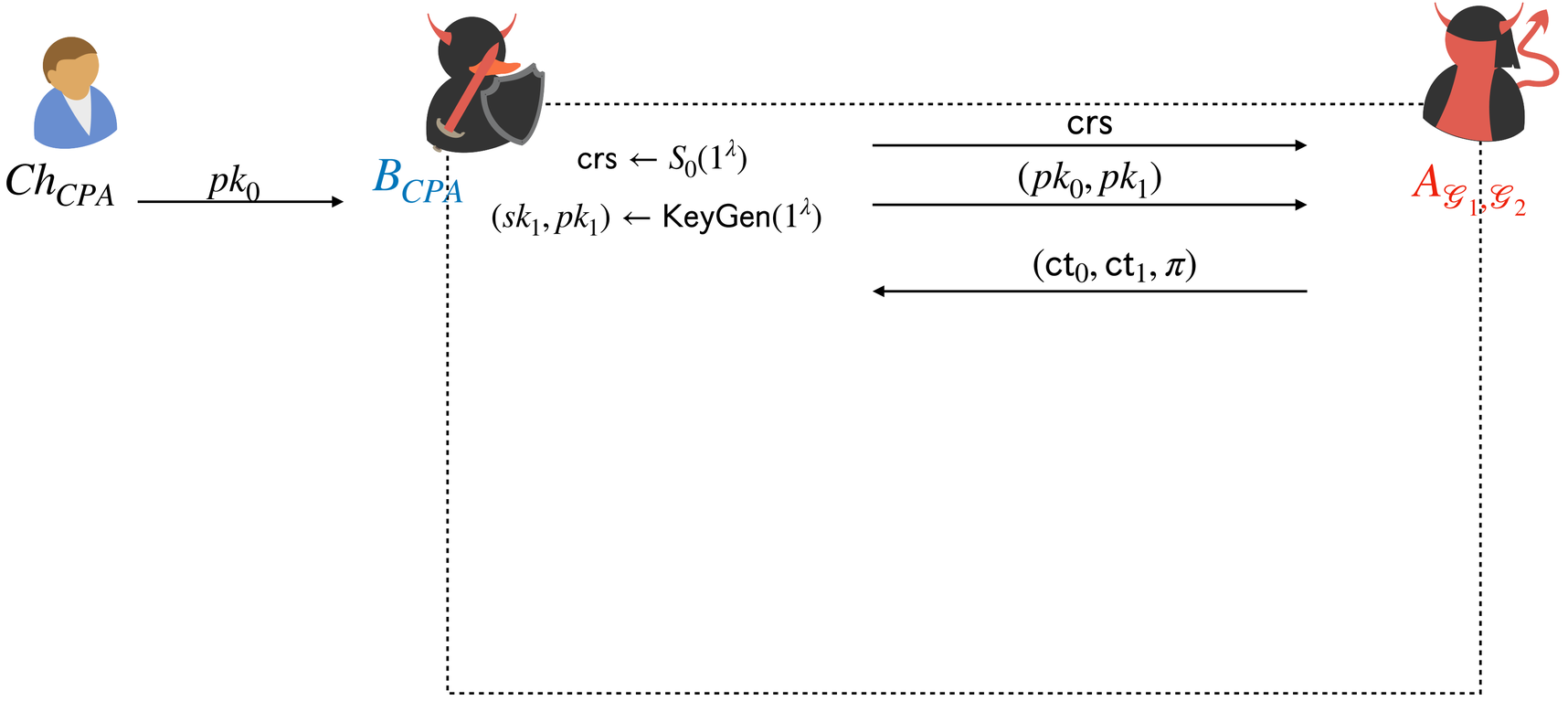

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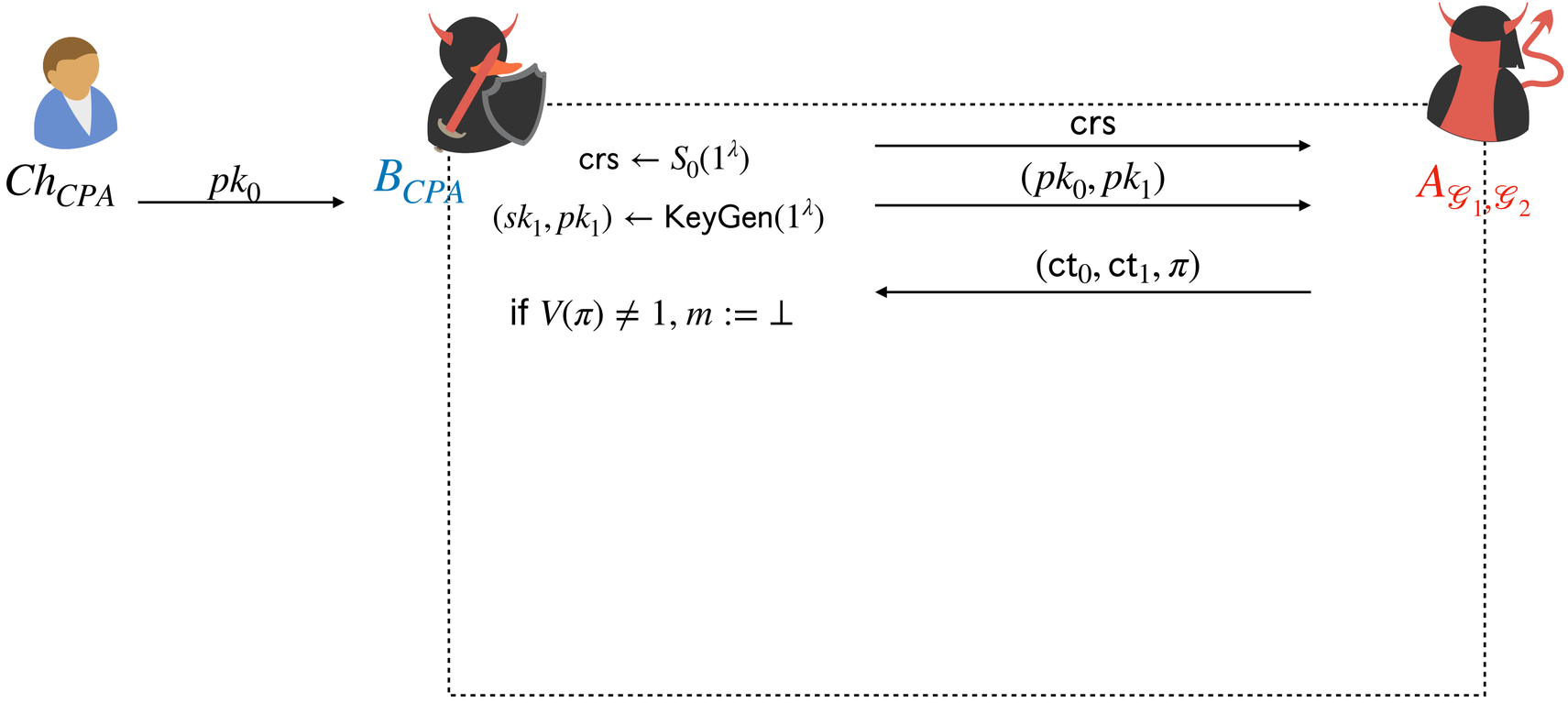

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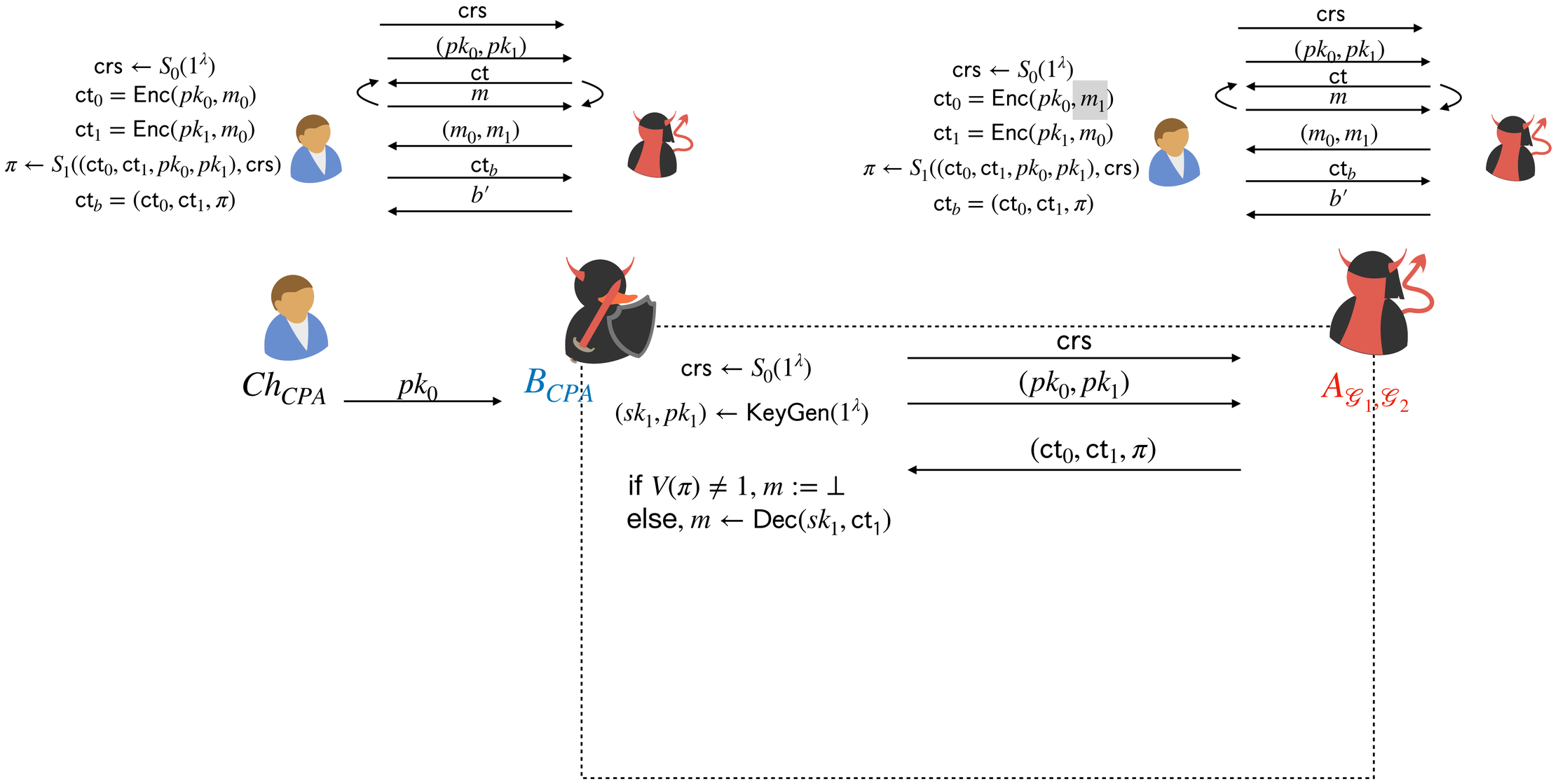

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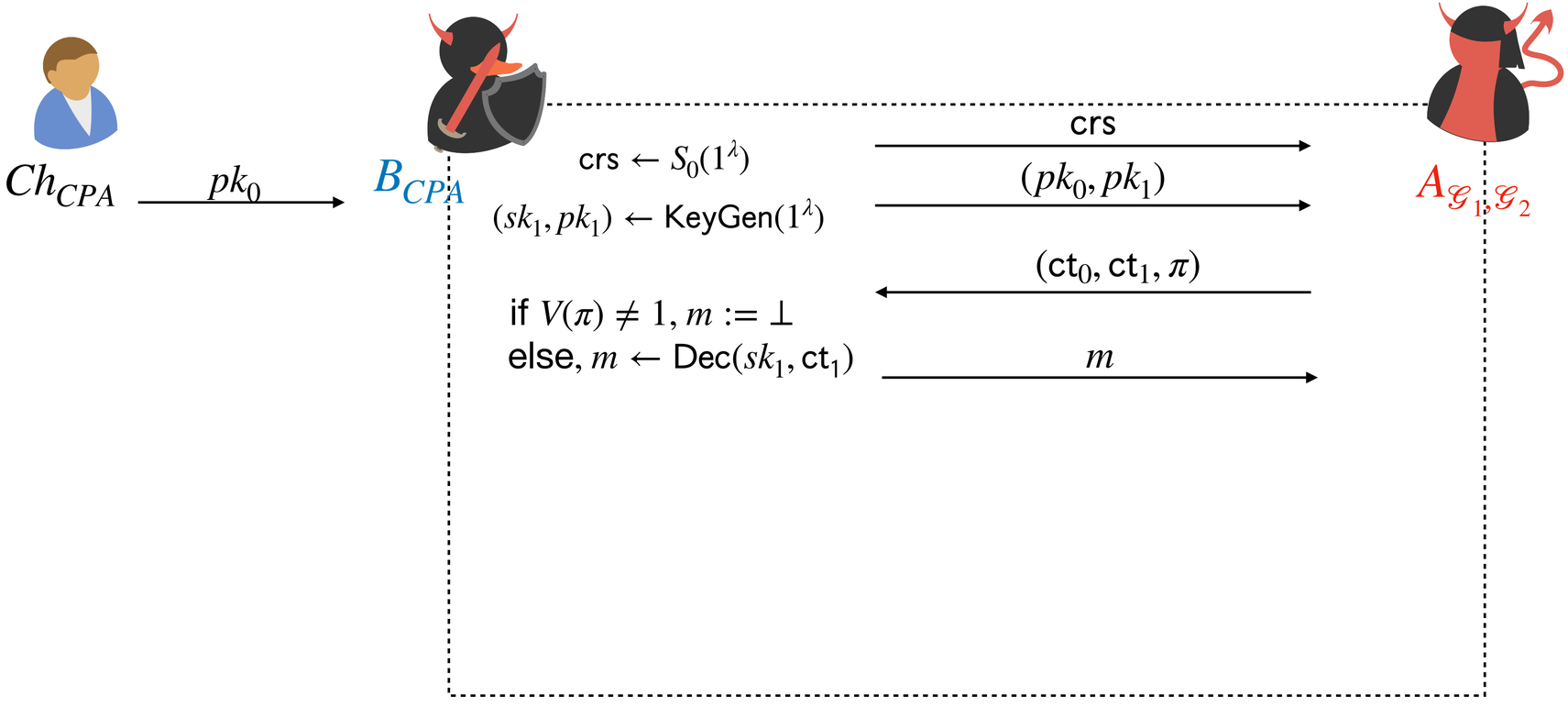

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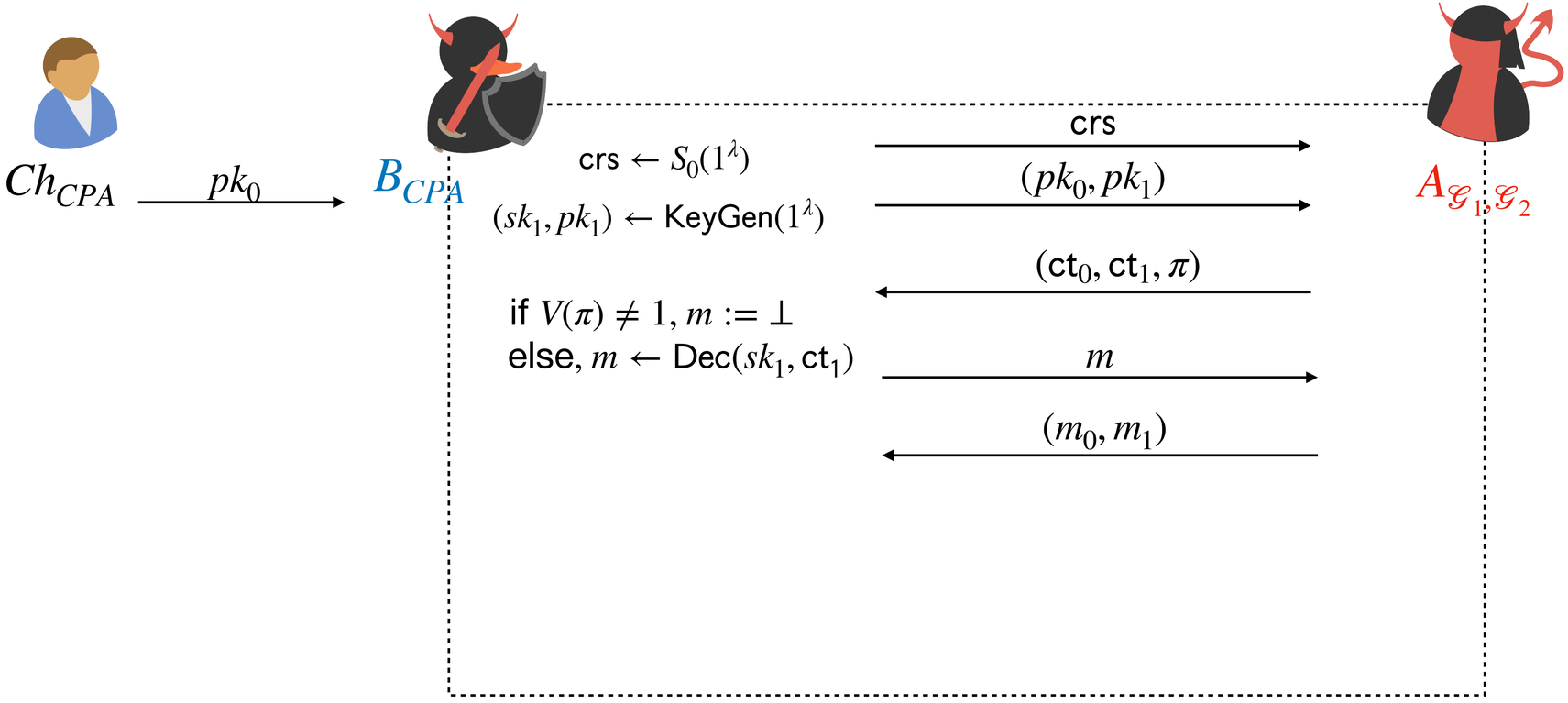

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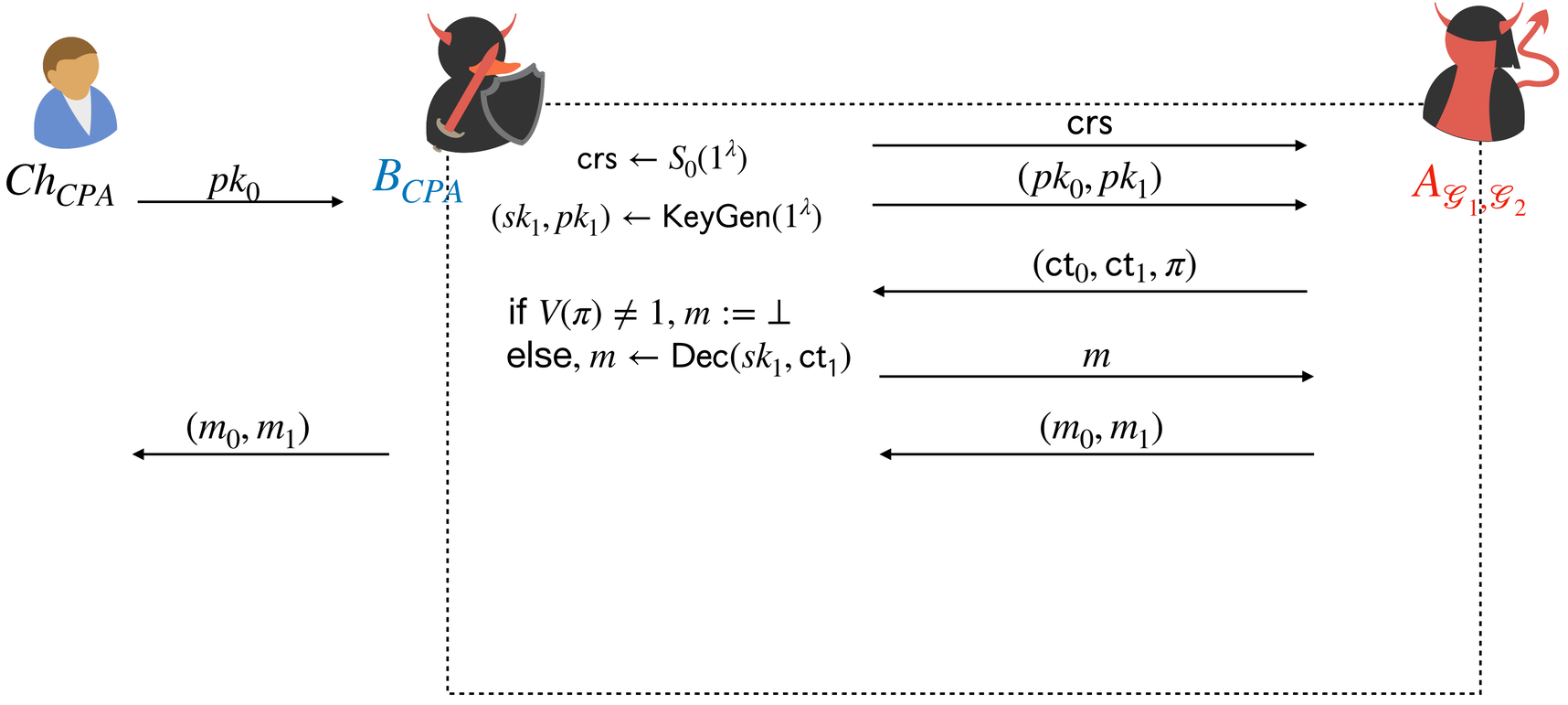

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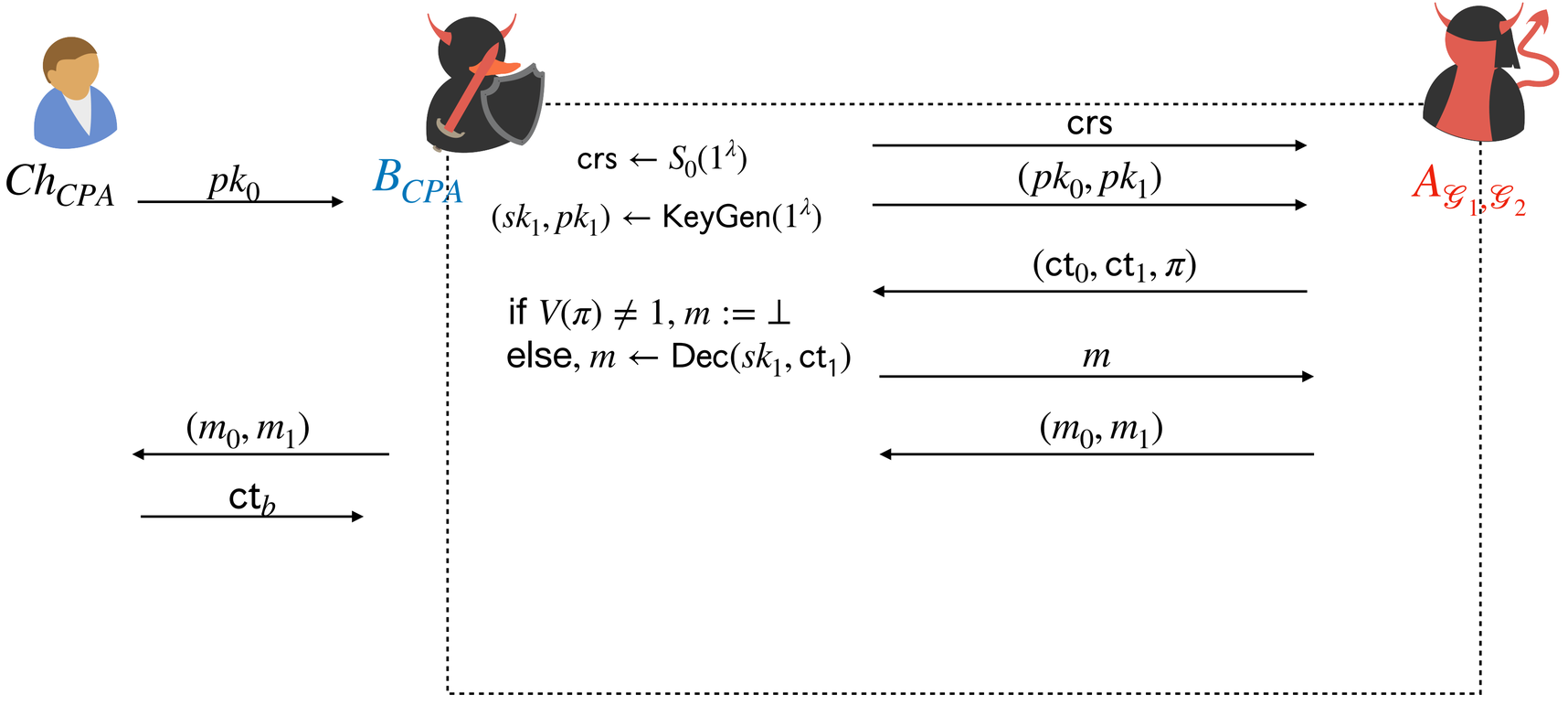

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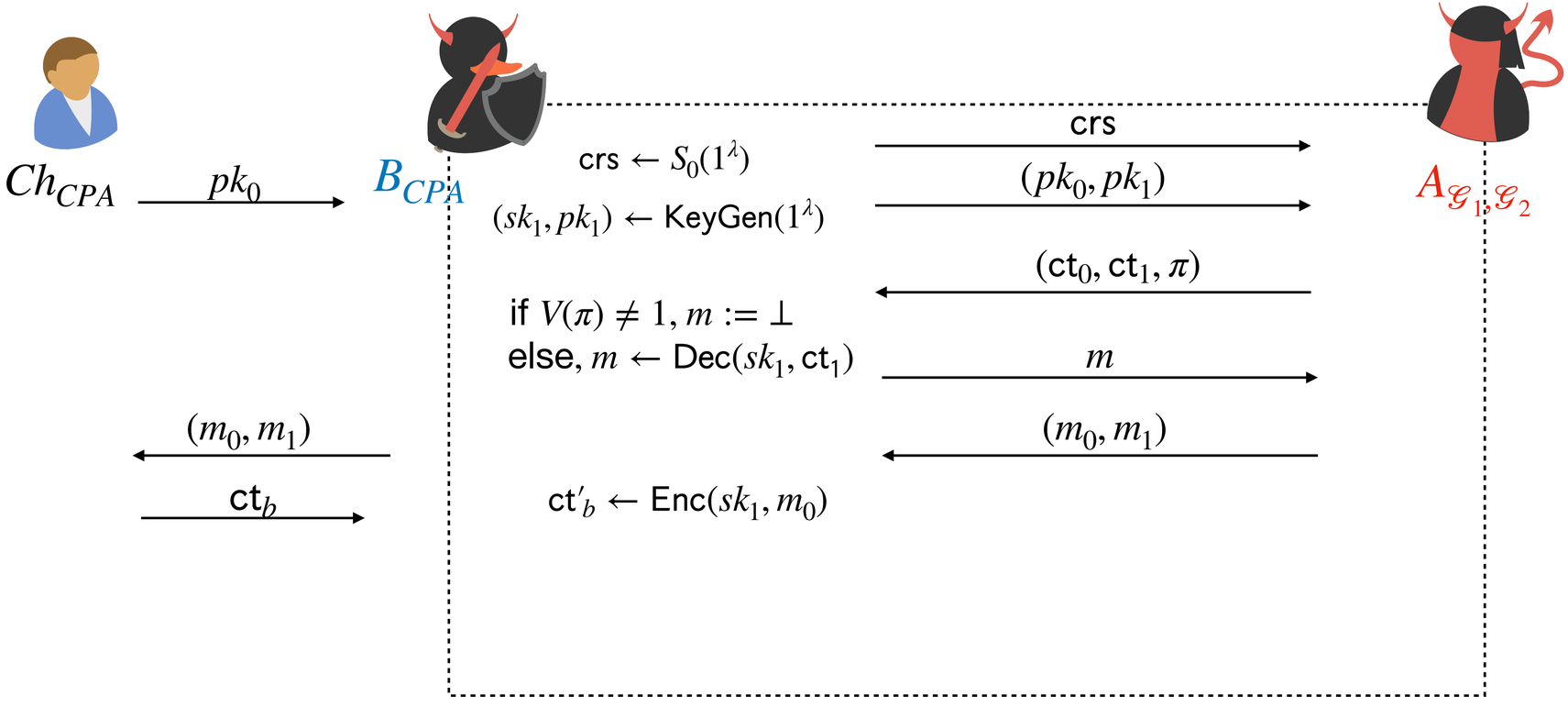

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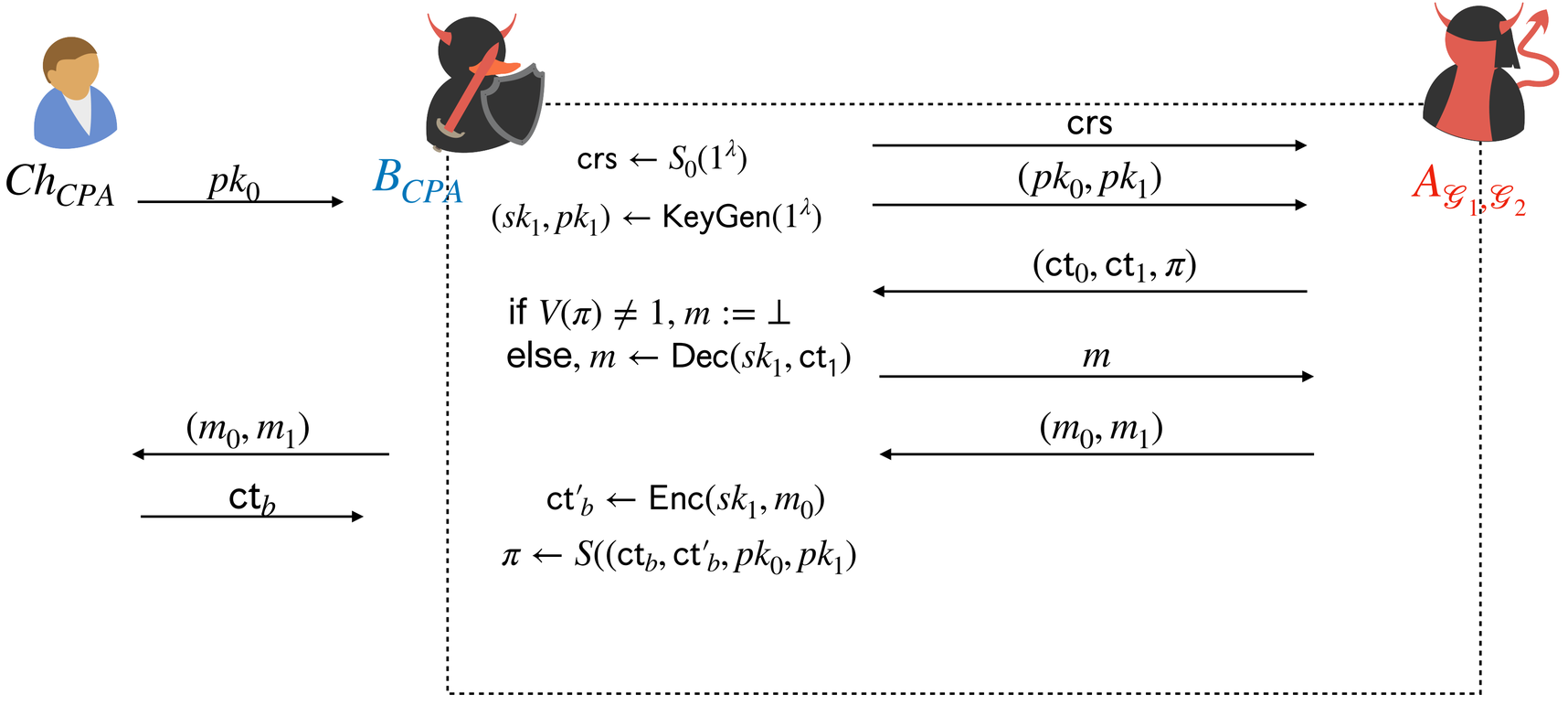

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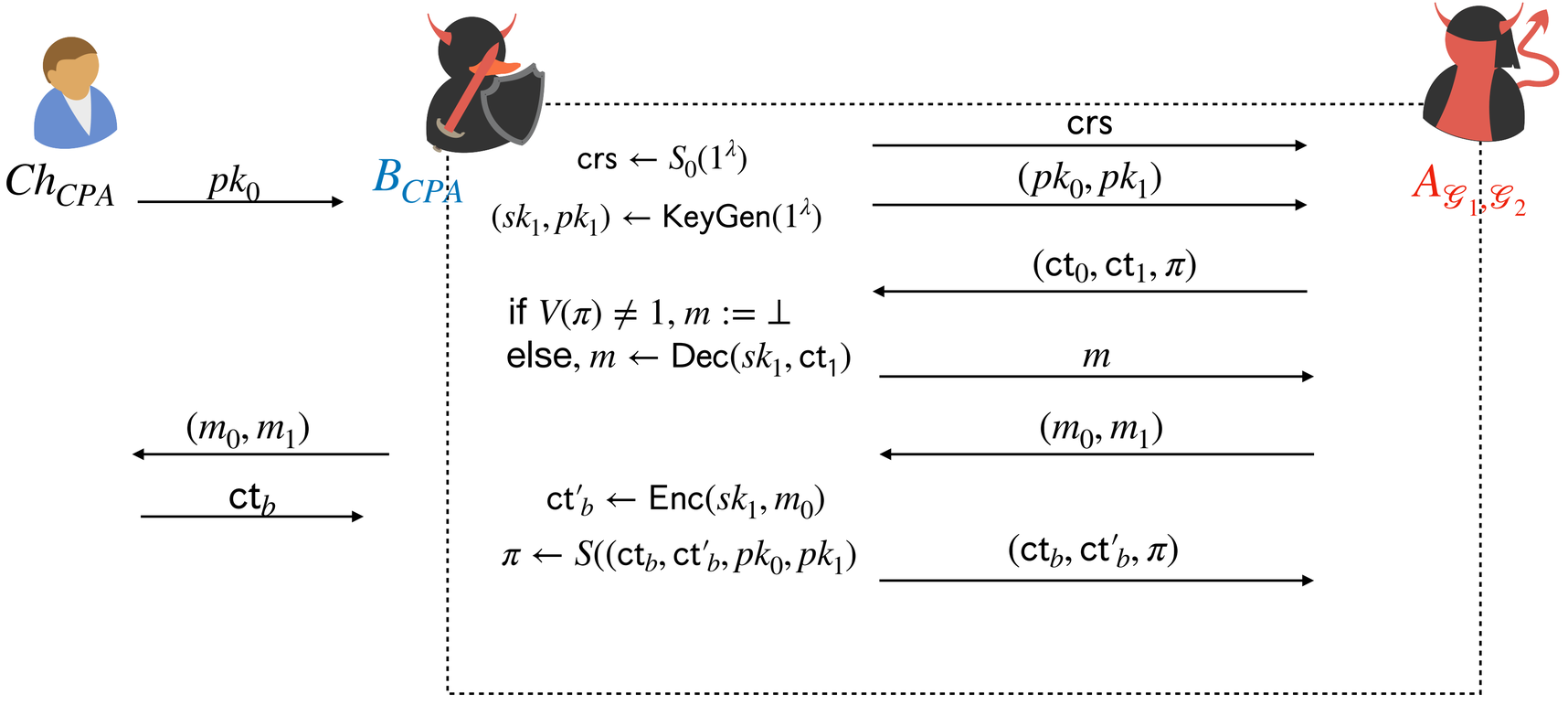
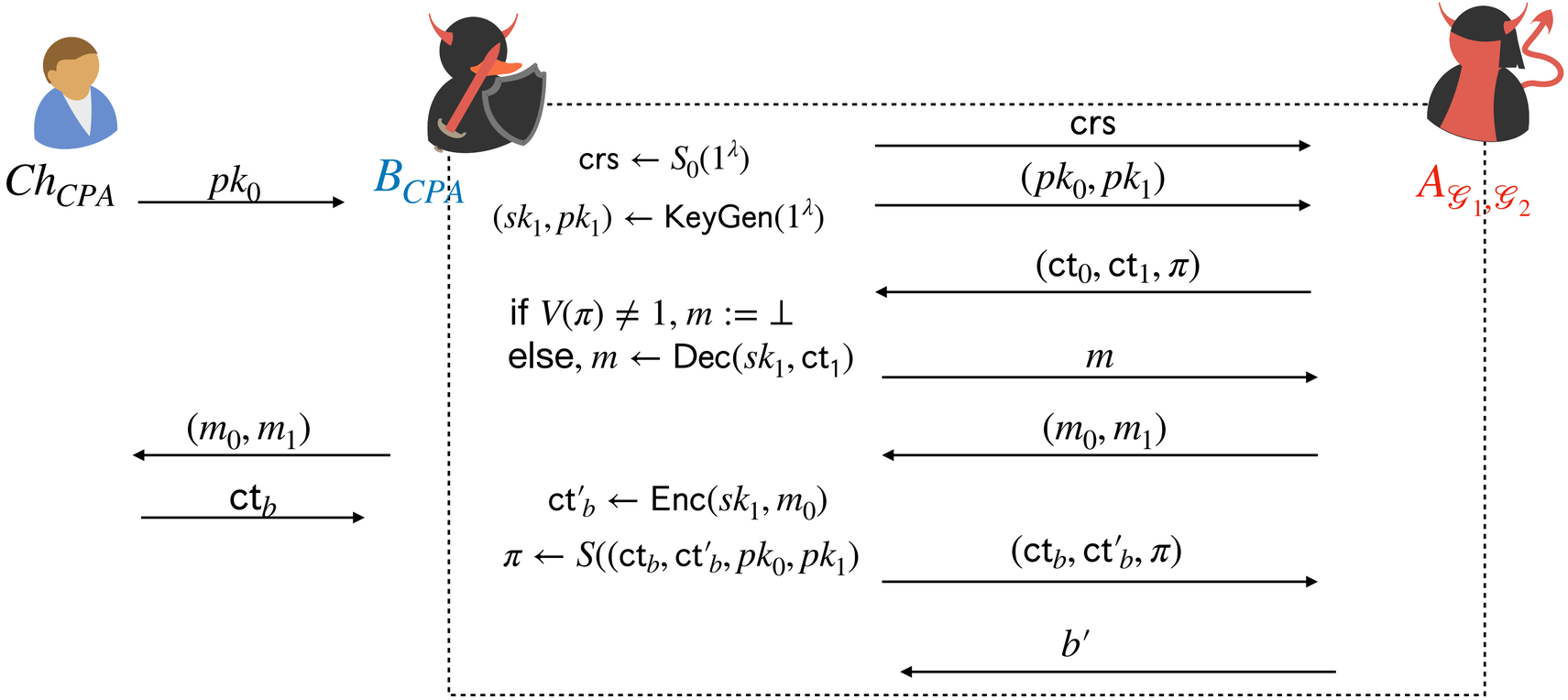
Claim: $\left|\Pr[W_1] - \Pr[W_2]\right| \leq \mathsf{negl}(\lambda)$ by IND-CPA



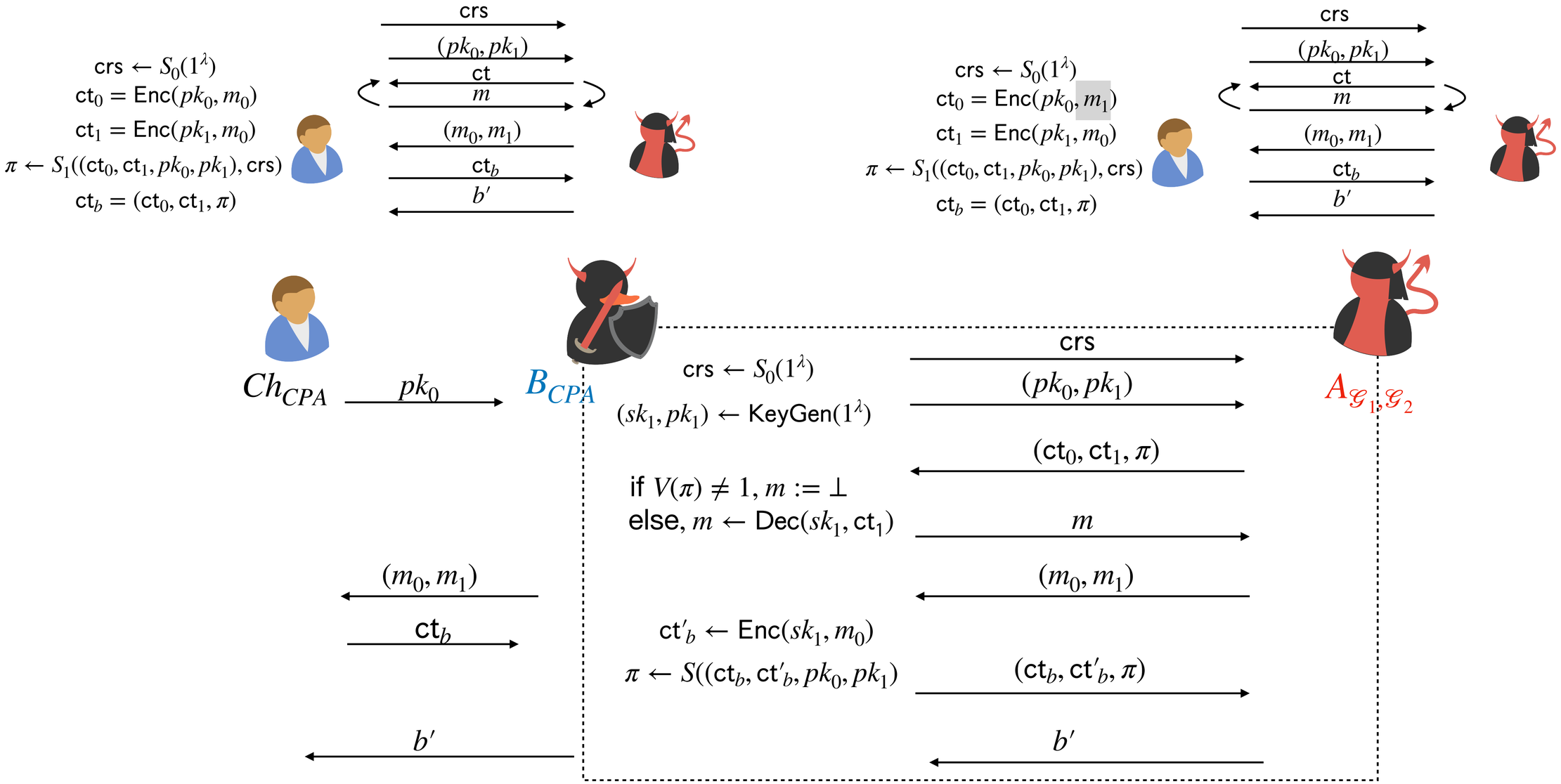
Proof of Security

Claim: $\left|\Pr[W_1] - \Pr[W_2]\right| \leq \mathsf{negl}(\lambda)$ by IND-CPA



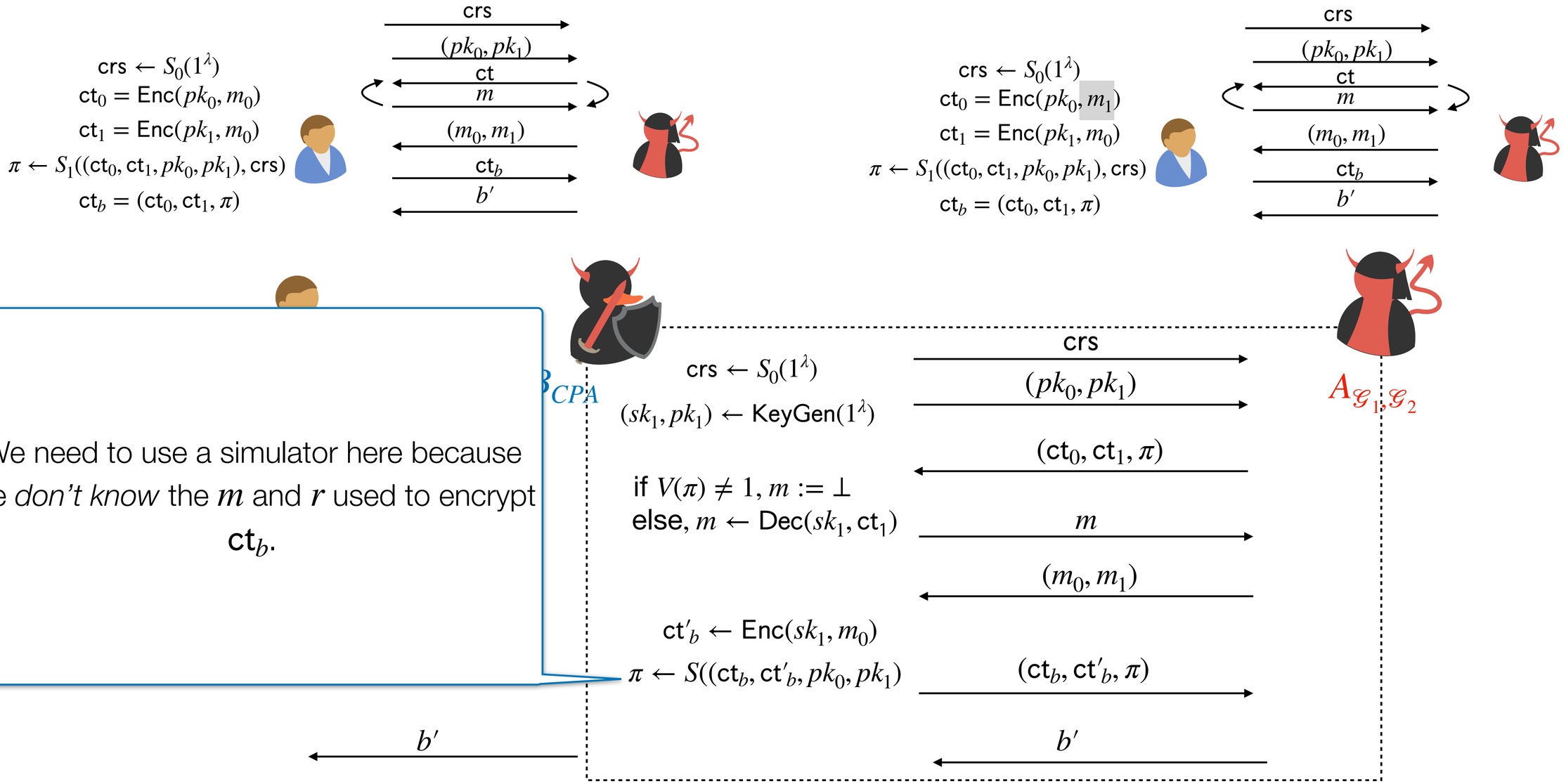
Proof of Security

Claim: $\left|\Pr[W_1] - \Pr[W_2]\right| \leq \mathsf{negl}(\lambda)$ by IND-CPA



Proof of Security

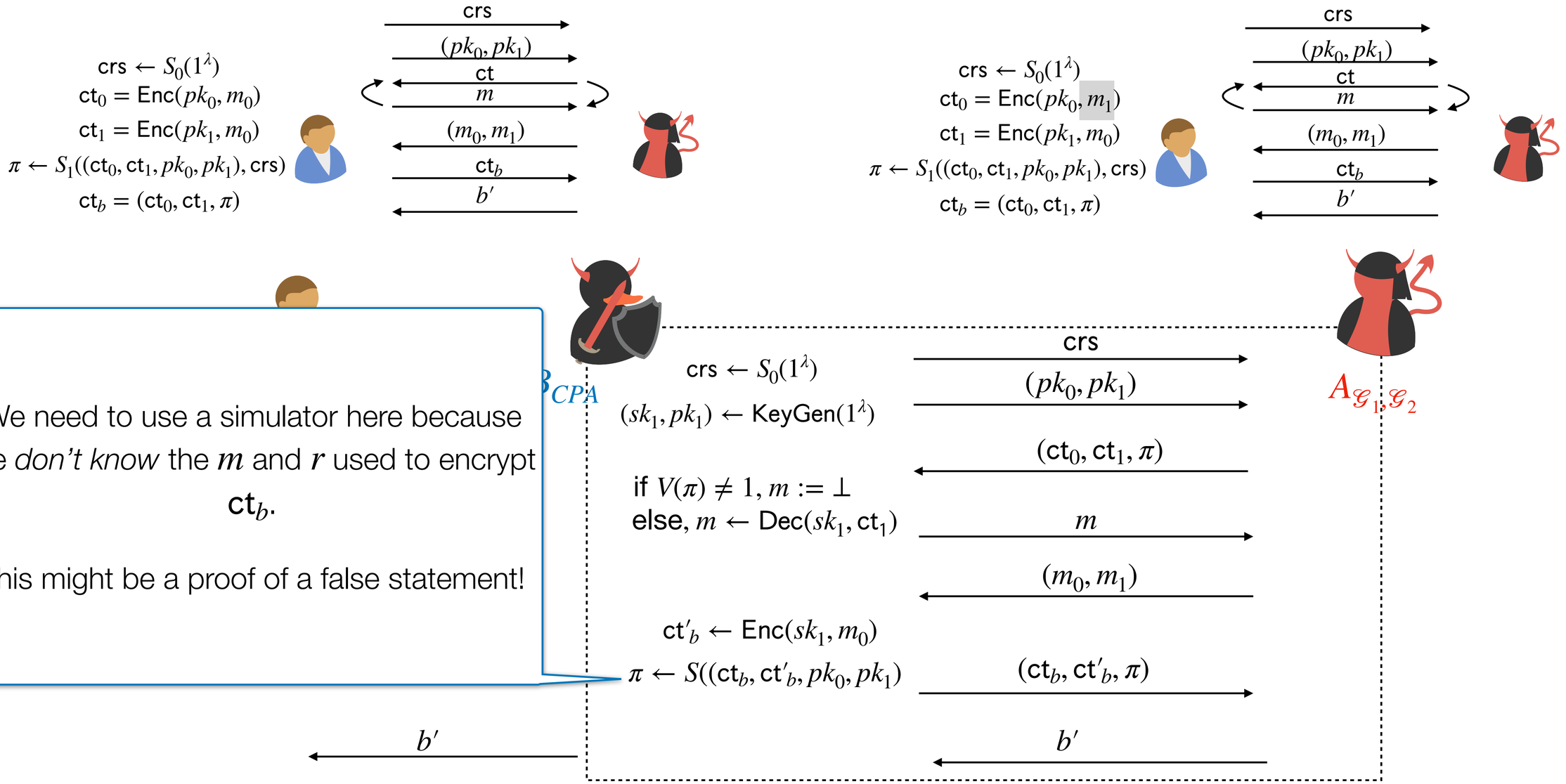
Claim: $\left|\Pr[W_1] - \Pr[W_2]\right| \leq \mathsf{negl}(\lambda)$ by IND-CPA



We need to use a simulator here because we *don't know* the m and r used to encrypt ct_b.

Proof of Security

Claim: $\left|\Pr[W_1] - \Pr[W_2]\right| \leq \mathsf{negl}(\lambda)$ by IND-CPA



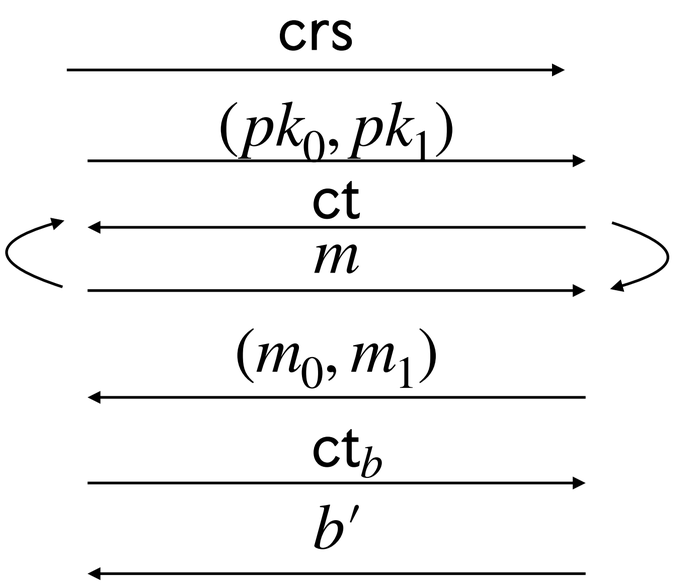
We need to use a simulator here because we *don't know* the m and r used to encrypt ct_b.

This might be a proof of a false statement!

Proof of Security

$\mathcal{G}_2$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



Proof of Security

$\mathcal{G}_2$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
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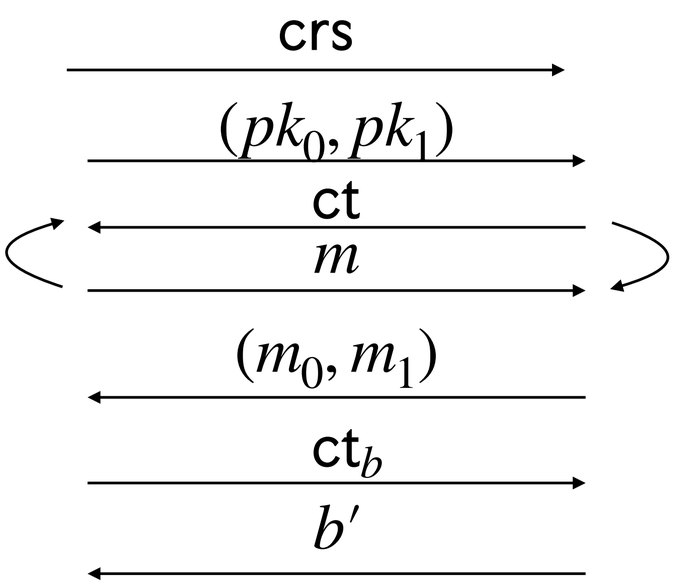
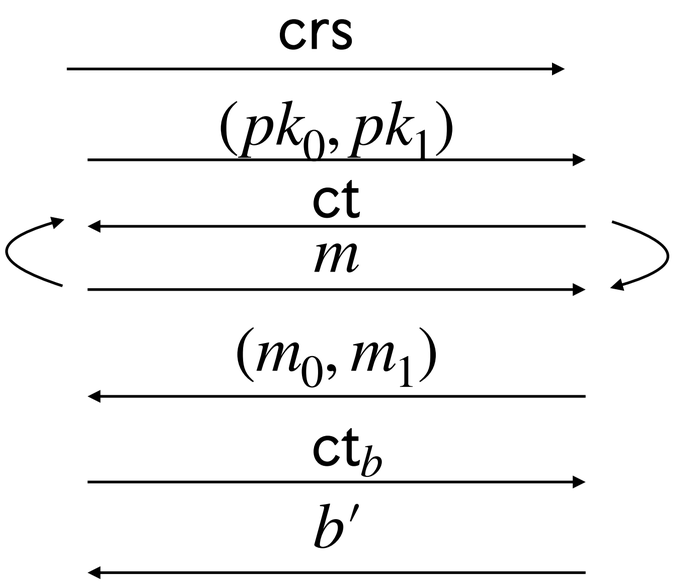


$\mathcal{G}_3$

Proof of Security

$\mathcal{G}_2$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



$\mathcal{G}_3$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_1)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
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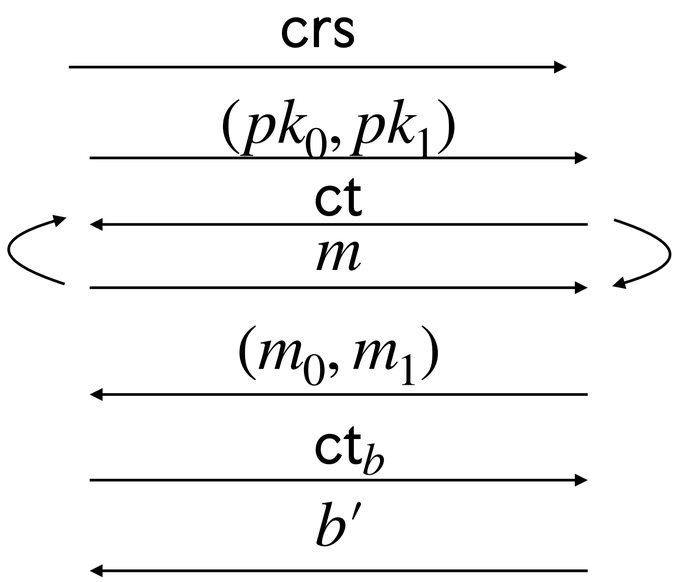

Proof of Security

$\mathcal{G}_2$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_0)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
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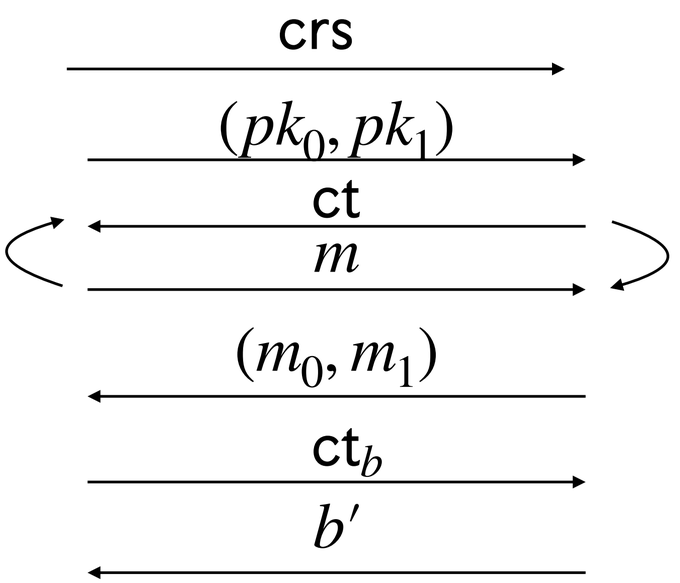

$\left|\Pr[W_2] - \Pr[W_3]\right| \leq \mathsf{negl}(\lambda)$ by IND-CPA

$\mathcal{G}_3$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_1)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
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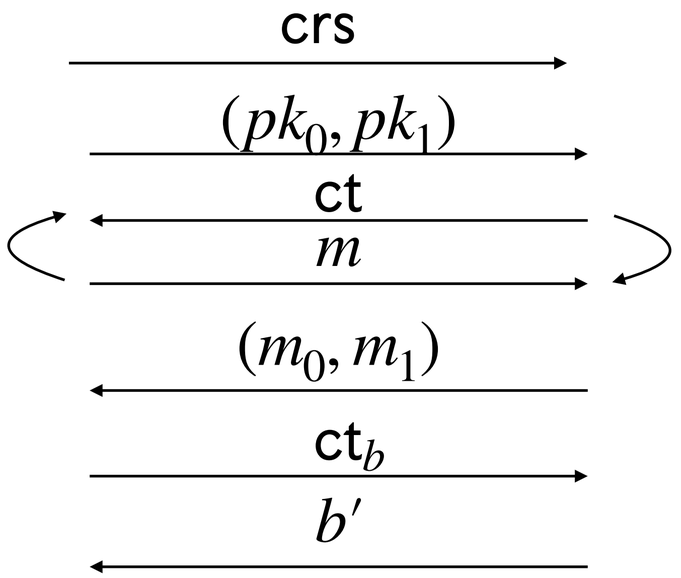

Proof of Security

$\mathcal{G}_3$

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$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
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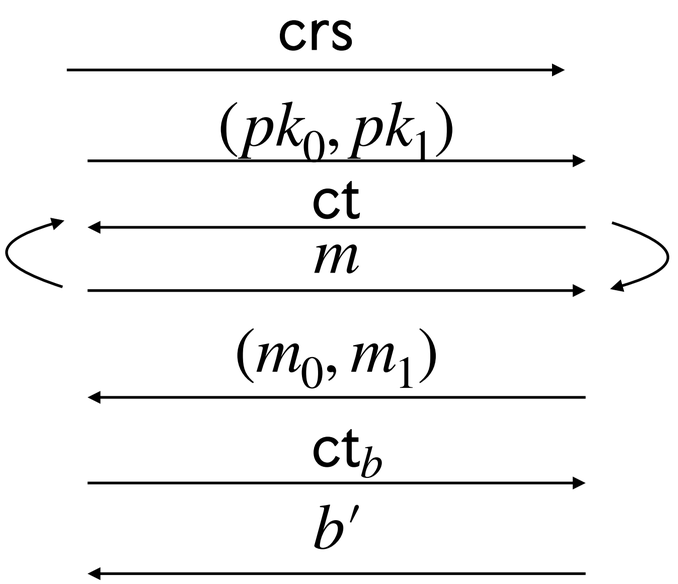

Proof of Security

$\mathcal{G}_3$

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$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
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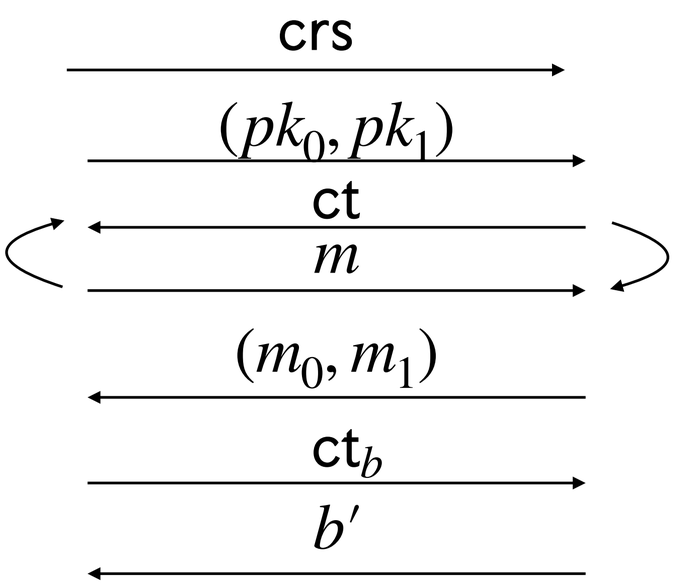
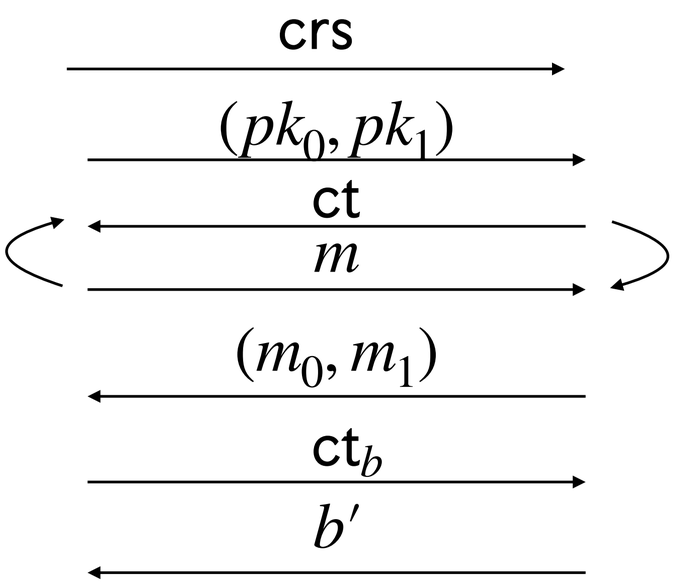


$\mathcal{G}_4$

Proof of Security

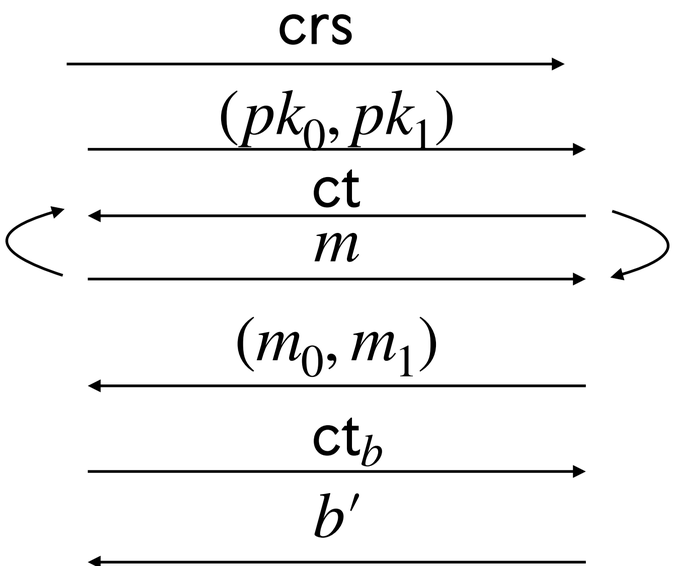
$\mathcal{G}_3$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_1)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



$\mathcal{G}_4$

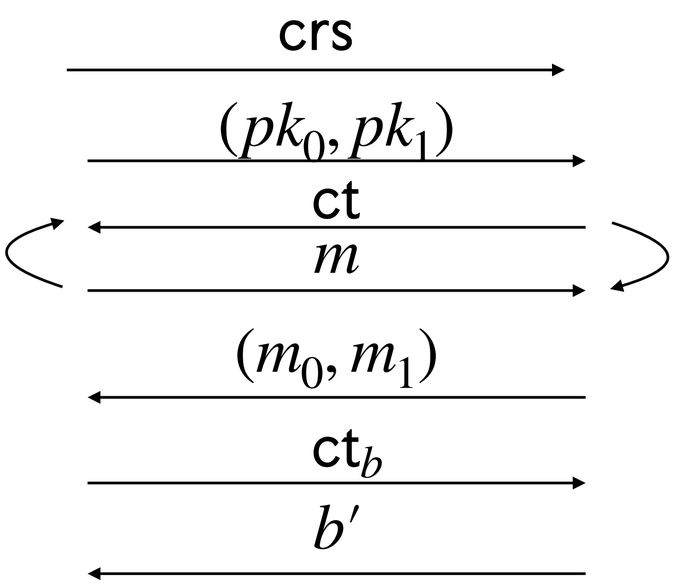
$$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_1)$$
$$\pi \leftarrow P((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



Proof of Security

$\mathcal{G}_3$

$$\mathsf{crs} \leftarrow S_0(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_1)$$
$$\pi \leftarrow S_1((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$



$$\left|\Pr[W_3] - \Pr[W_4]\right| \leq \mathsf{negl}(\lambda) \text{ by ZK}$$

$\mathcal{G}_4$

$$\mathsf{crs} \leftarrow \mathsf{CRSGen}(1^\lambda)$$
$$\mathsf{ct}_0 = \mathsf{Enc}(pk_0, m_1)$$
$$\mathsf{ct}_1 = \mathsf{Enc}(pk_1, m_1)$$
$$\pi \leftarrow P((\mathsf{ct}_0, \mathsf{ct}_1, pk_0, pk_1), \mathsf{crs})$$
$$\mathsf{ct}_b = (\mathsf{ct}_0, \mathsf{ct}_1, \pi)$$