Secure Computation

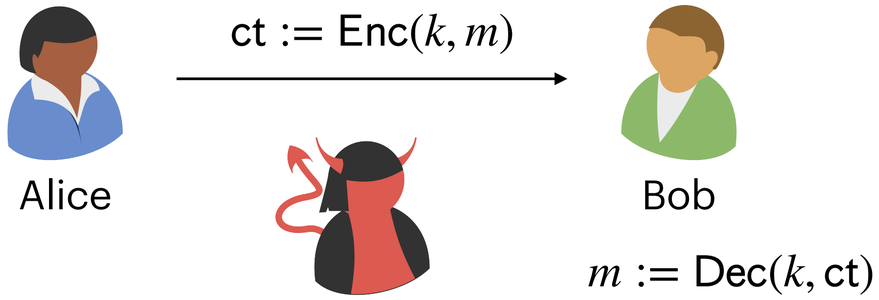
601.442/642 Modern Cryptography

16th April 2026

Recap

Recap

Encryption



Private communication in the presence of an external passive eavesdropper.

Recap

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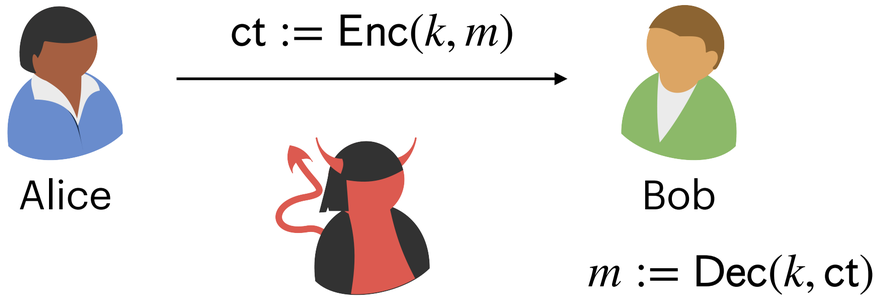

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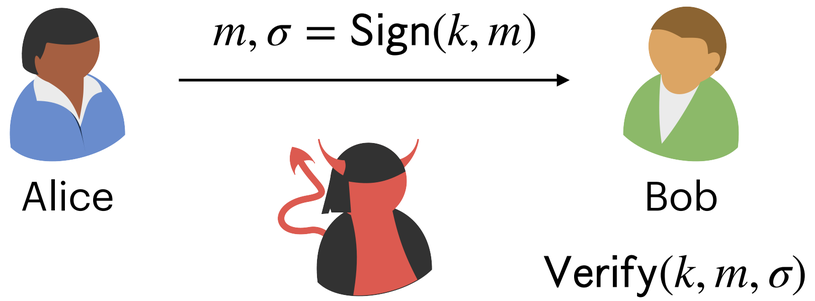

Authentic communication in the presence of an external impersonator.

Recap

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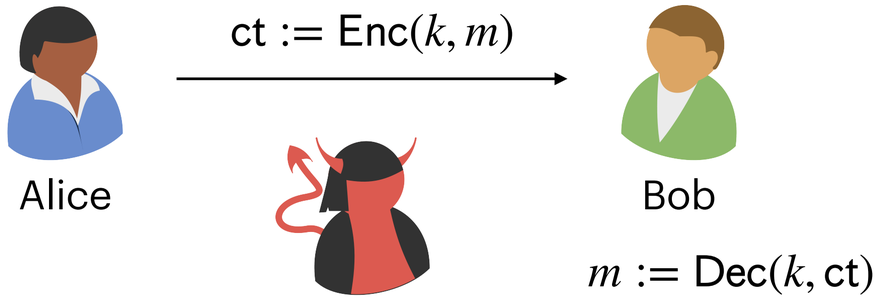

Private communication in the presence of an external active eavesdropper.

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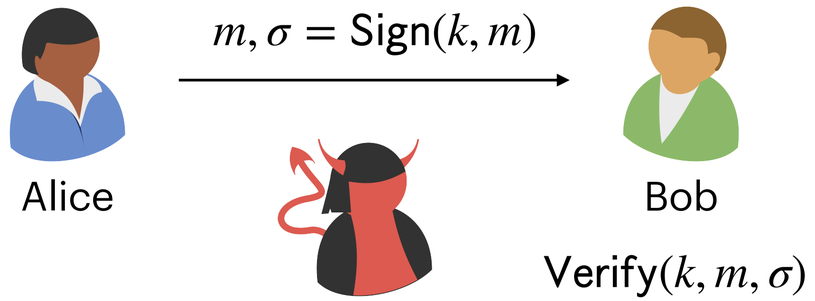
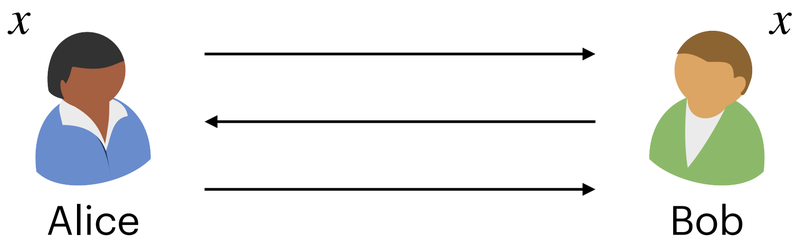


Authentic communication in the presence of an external impersonator.

Recap

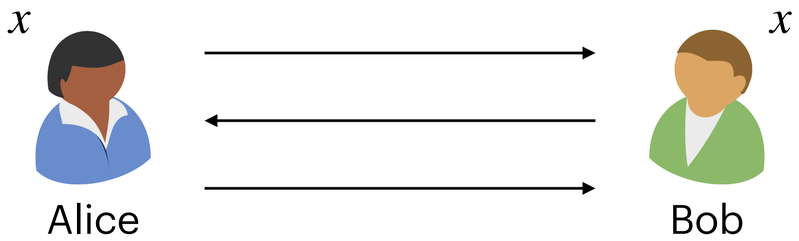
Zero Knowledge Proofs



Mutually distrusting parties. Alice wants to convince Bob of her claim.

Recap

Zero Knowledge Proofs



Mutually distrusting parties. Alice wants to convince Bob of her claim.

What else do mutually distrusting parties want to achieve?

Examples

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- **Contact Discovery:** Alice registers on Signal and wants to find who among her contacts is also on Signal.
  - Alice does not wish to share her list of contacts with the server.
  - The server doesn’t want to reveal the contacts of all registered users.

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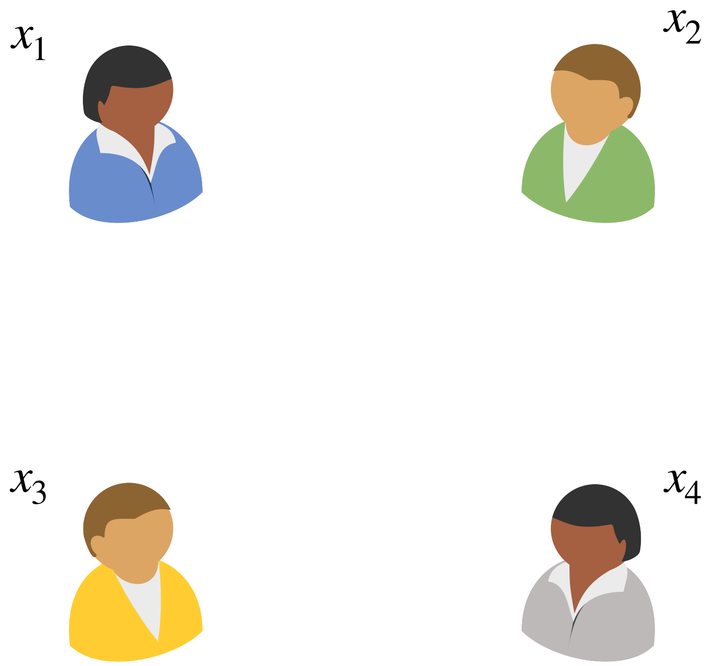
- **Genome-Wide Association Studies:** A researcher wants to compute statistics across genomic data held by multiple hospitals, but regulations like HIPAA and GDPR prohibit the hospitals from sharing data with the researcher or with each other.
- **Threshold Signing:** A signing key stored on one server is a single point of failure. Splitting the key across multiple servers is safer. But then how do you sign?

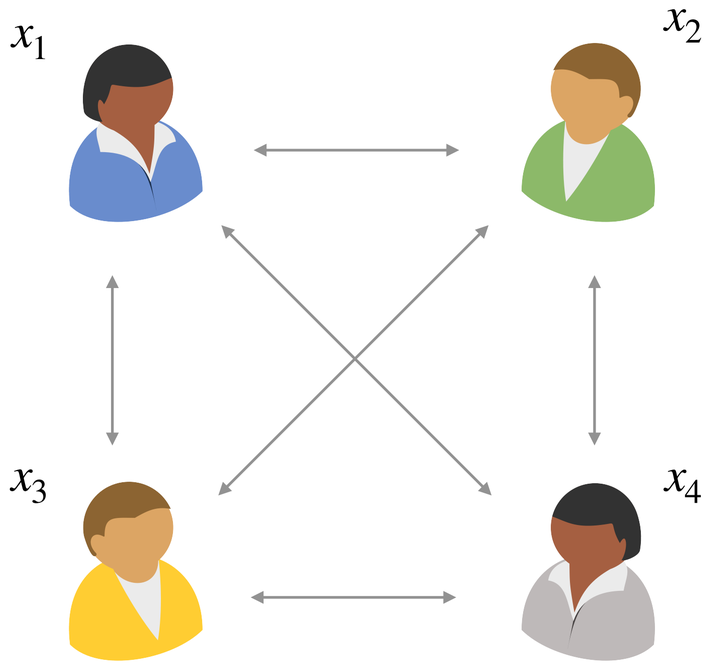
Secure Computation

Secure Computation



- Each party has its own private input x_i.

Secure Computation



$$z = P(x_1, x_2, x_3, x_4)$$

- Each party has its own private input x_i.
- They communicate with each other and compute the output of a program P evaluated on their joint inputs.

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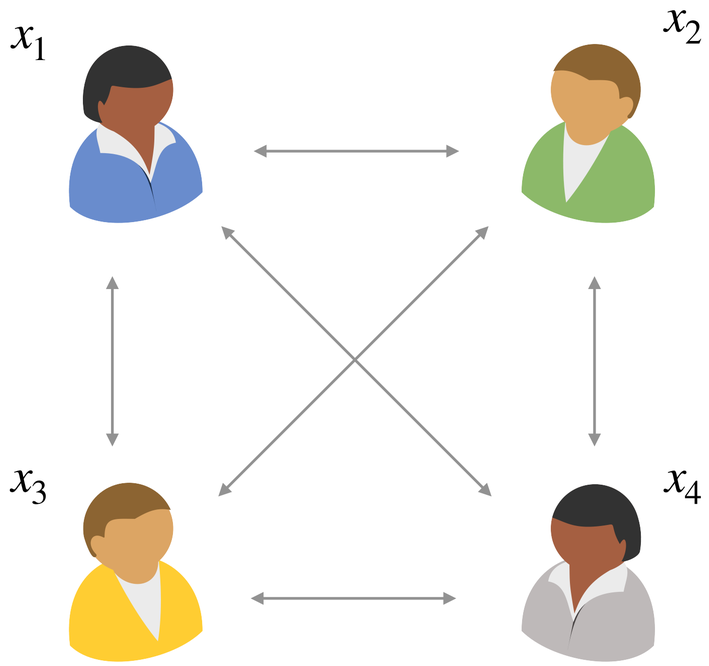

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- Each party has its own private input x_i.
- They communicate with each other and compute the output of a program P evaluated on their joint inputs.
- Each party should learn nothing beyond its input x_i and output z.

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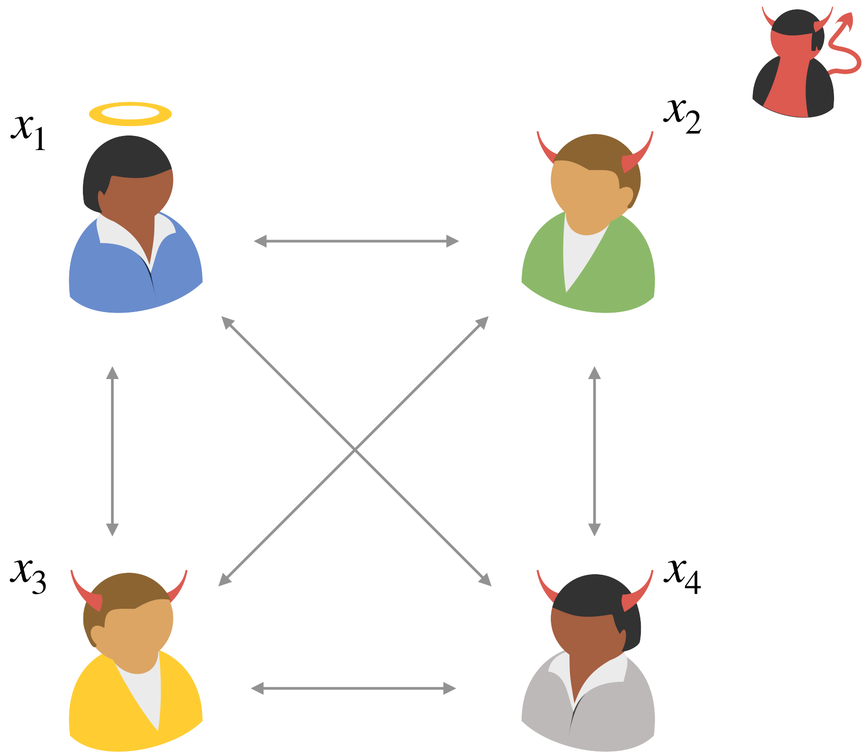

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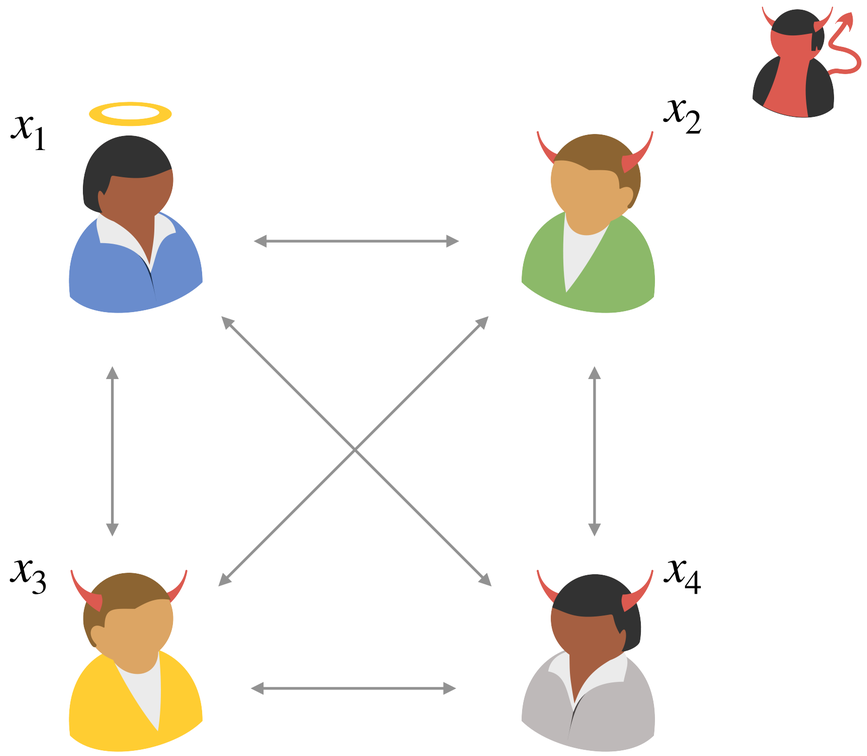

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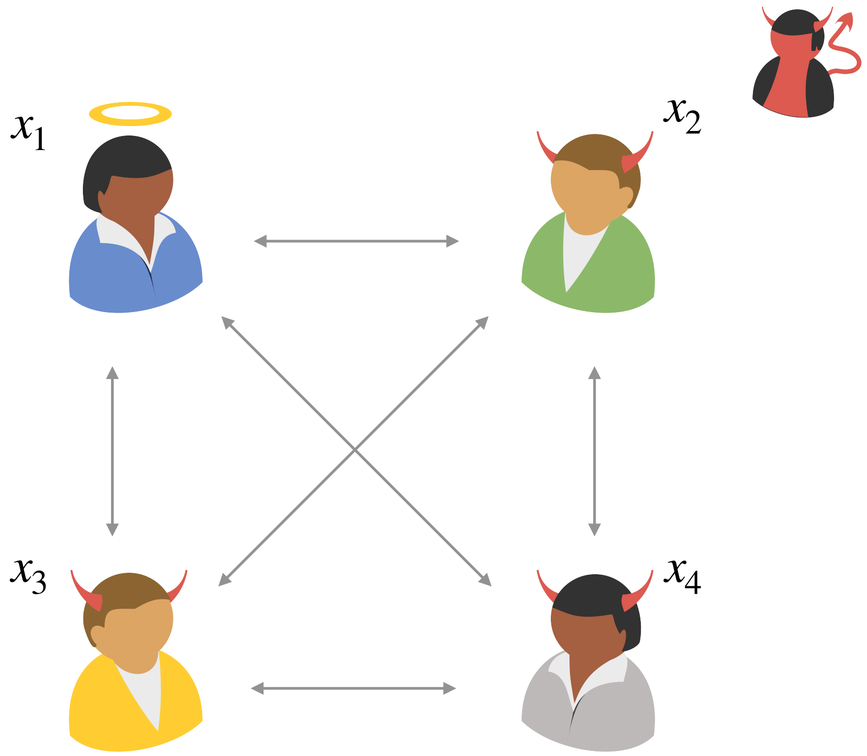

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- **Adversarial Model:** A subset of parties are corrupt by a single adversary.
- Two types of adversaries
  - Semi-honest
    - Corrupt parties follow the protocol description.
    - Adversary tries to learn as much as possible from the protocol transcript of corrupt parties.

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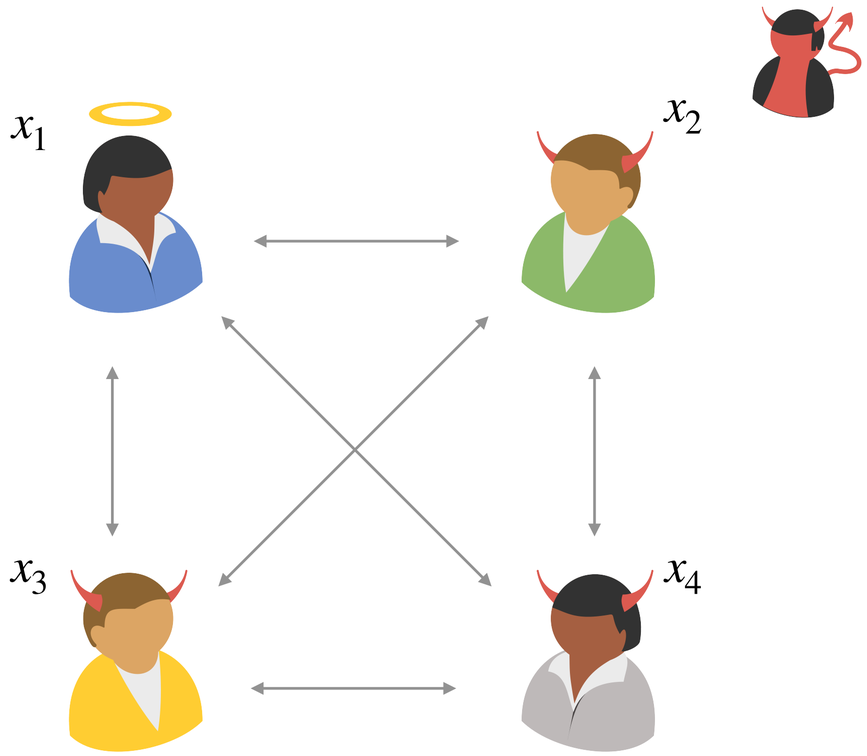

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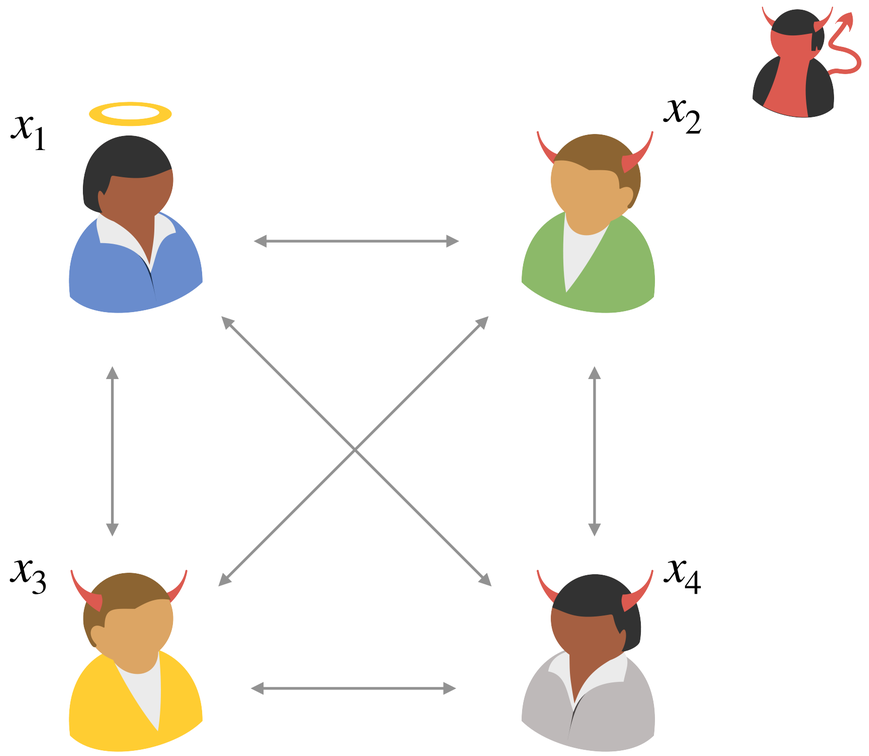

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We will only consider semi-honest adversaries.

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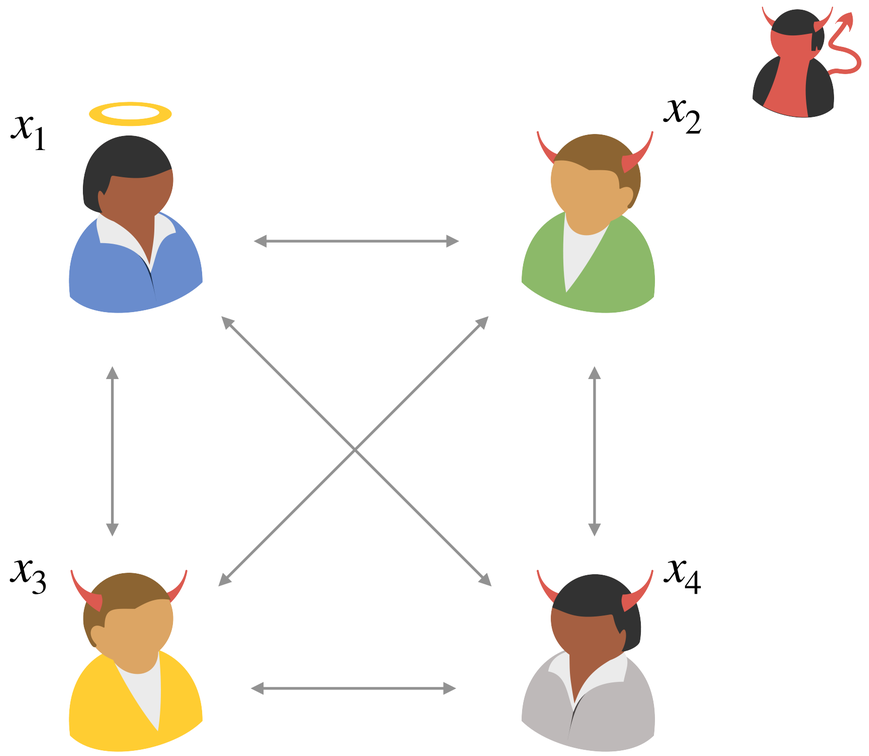

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We will primarily focus on 2-party secure computation.

We will only consider semi-honest adversaries.

Defining Secure Computation

- How to define secure computation?

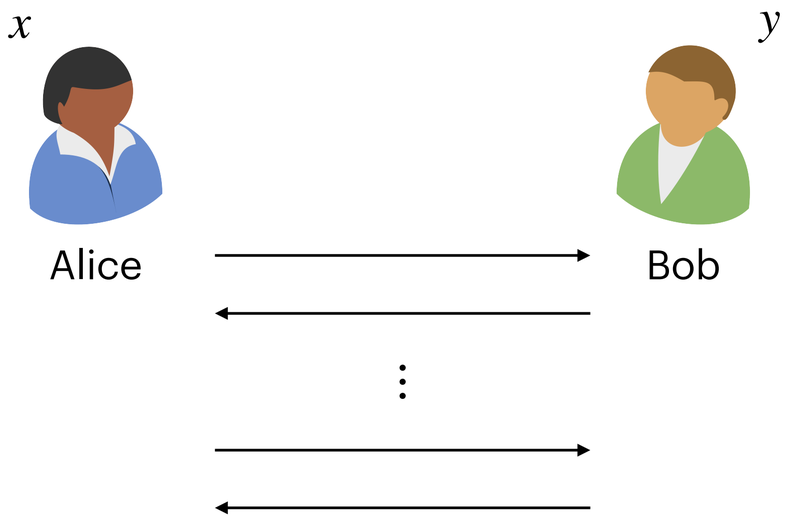
x

Alice

y

Bob

Defining Secure Computation



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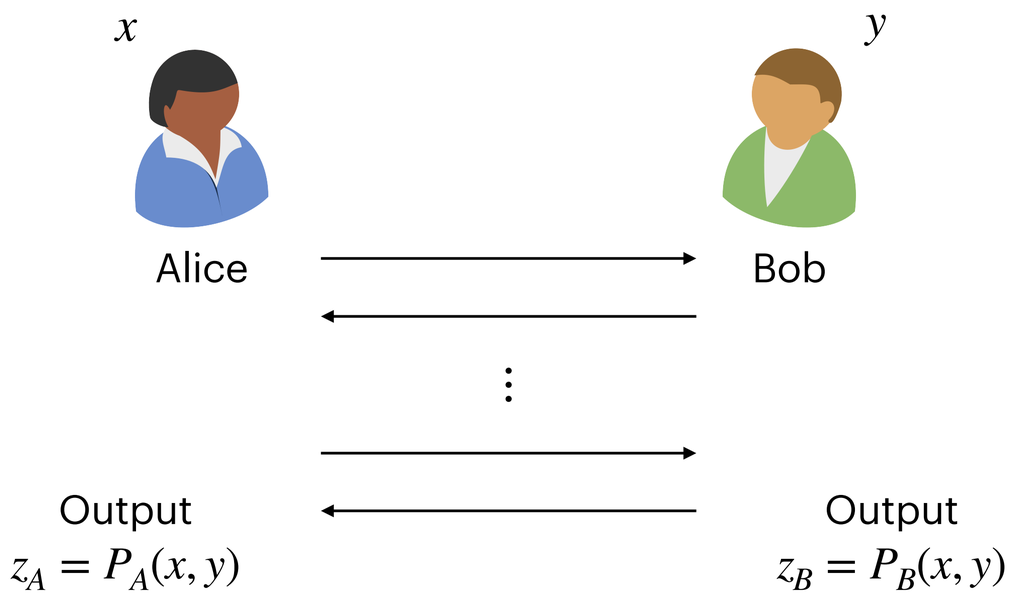

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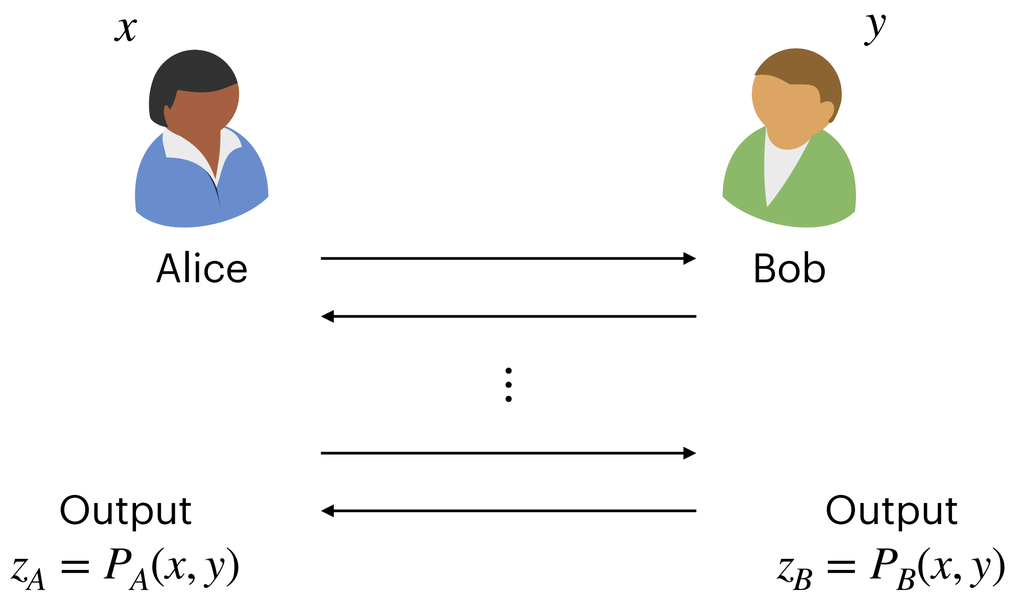

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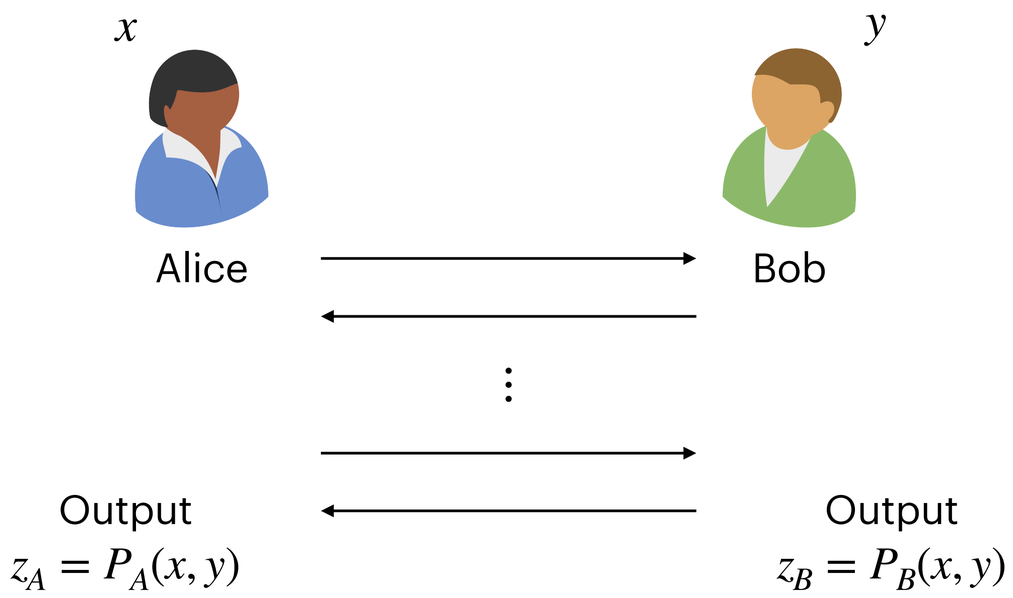

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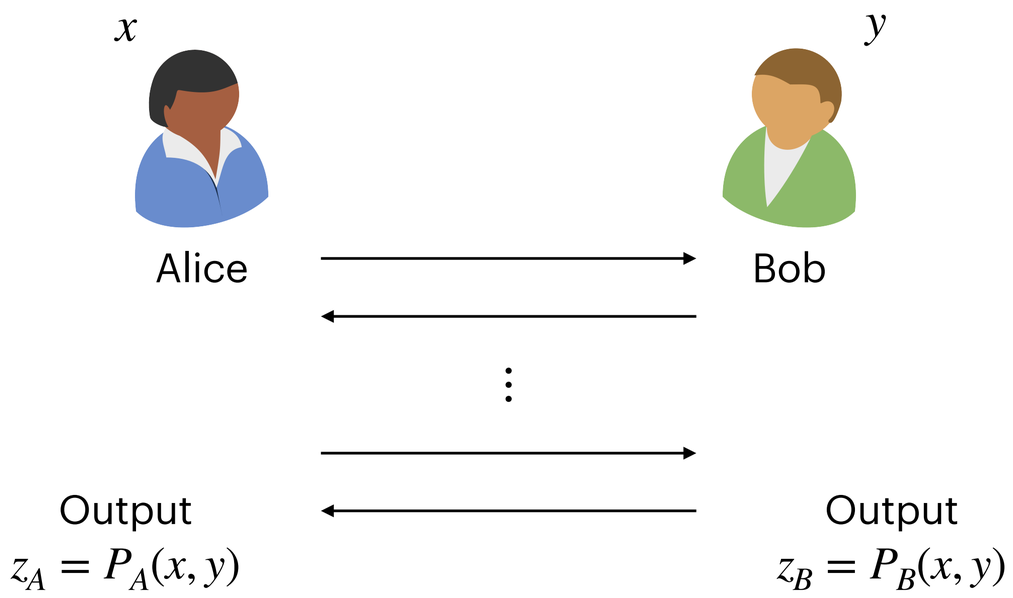

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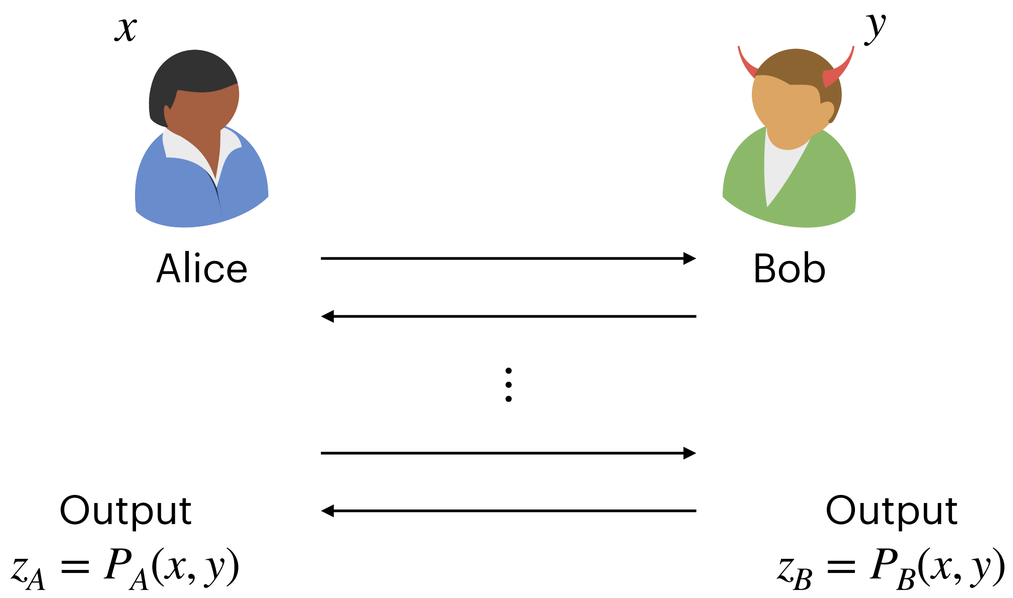

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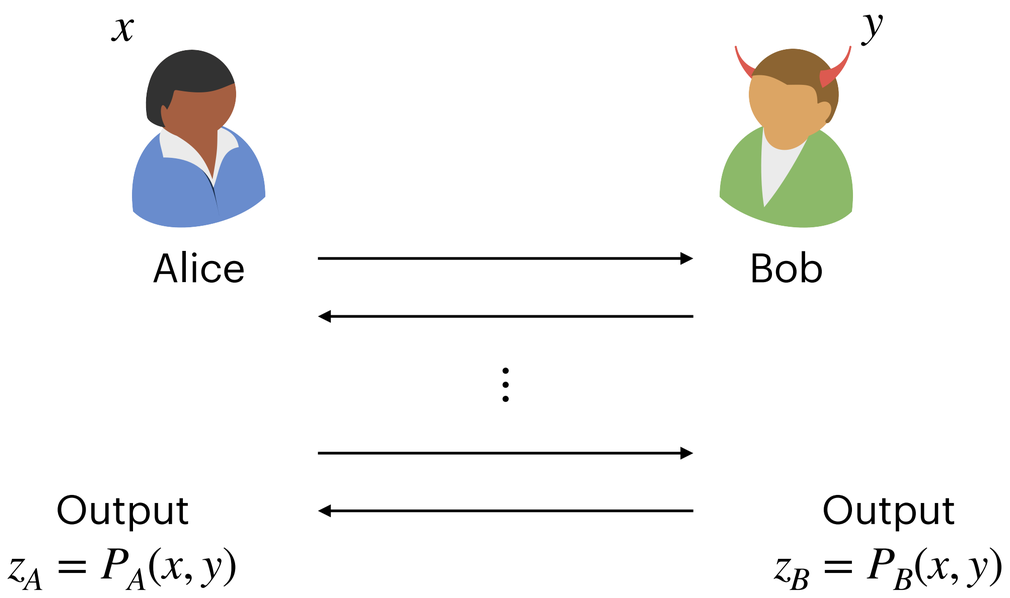

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  - The adversary is allowed to learn the corrupt party's input y and final output z_B.

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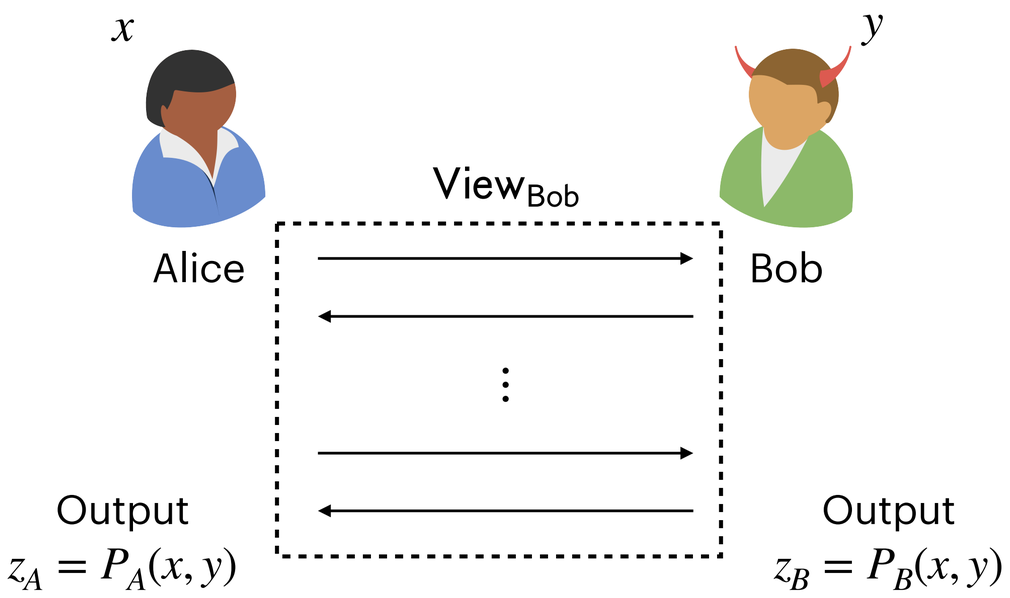

$\text{View}_{\text{Bob}} \equiv$ Bob's random tape, Alice's messages.

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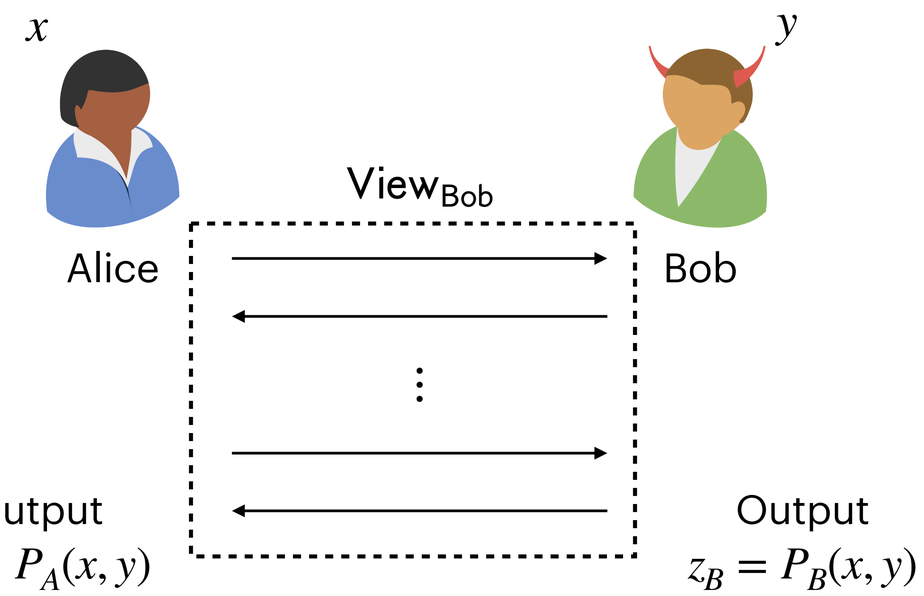

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$$\text{View}_{\text{Bob}} \overset{c}{\approx} S(y, P_B(x, y))$$

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- **Simulation paradigm:** Anything a party could compute from its own inputs is not knowledge it gains by interacting with an external entity.
  - The adversary is allowed to learn the corrupt party's input y and final output z_B.
  - There is an efficient simulator that can generate the corrupt party's view only using y and z_B, without the honest party's input x.

Semi-Honest Secure Two-Party Computation

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A protocol $\Pi = (A, B)$ securely computes a program $P = (P_A, P_B)$ in the presence of semi-honest adversaries if there exists PPT algorithms S_A and S_B such that for all $x, y \in \{0,1\}^*$ we have

where $z_A \leftarrow P_A(x, y)$, $z_B \leftarrow P_B(x, y)$, and $z = (z_A, z_B)$.

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$$\{S_A(1^\lambda, x, z_A) \} \overset{c}{\approx} \{\text{View}_A[A(x) \leftrightarrow B(y)] \}, \text{ and}$$

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$$\{S_A(1^\lambda, x, z_A), z\} \overset{c}{\approx} \{\text{View}_A[A(x) \leftrightarrow B(y)], \text{Output}[A(x) \leftrightarrow B(y)]\}, \text{ and}$$

Output of both parties at the end of the protocol.

where $z_A \leftarrow P_A(x, y)$, $z_B \leftarrow P_B(x, y)$, and $z = (z_A, z_B)$.

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where $z_A \leftarrow P_A(x, y)$, $z_B \leftarrow P_B(x, y)$, and $z = (z_A, z_B)$.

- **Correctness:** Alice and Bob compute $P_A(x, y)$ and $P_B(x, y)$ respectively upon running the protocol.
  - Captured by the definition since we require $z \overset{c}{\approx} \text{Output}[A(x) \leftrightarrow B(y)]$.

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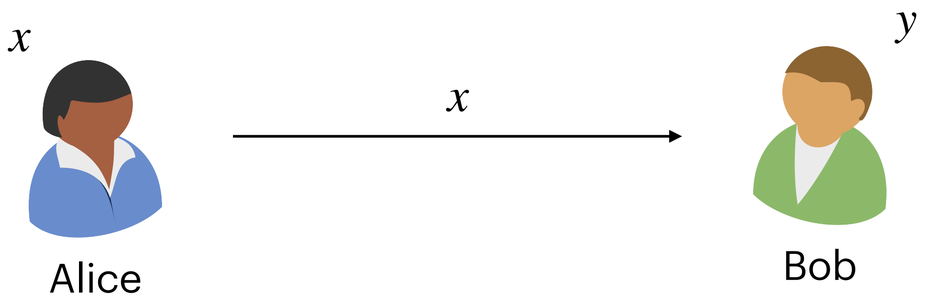
x

Alice

y

Bob

Semi-Honest Secure Two-Party Computation



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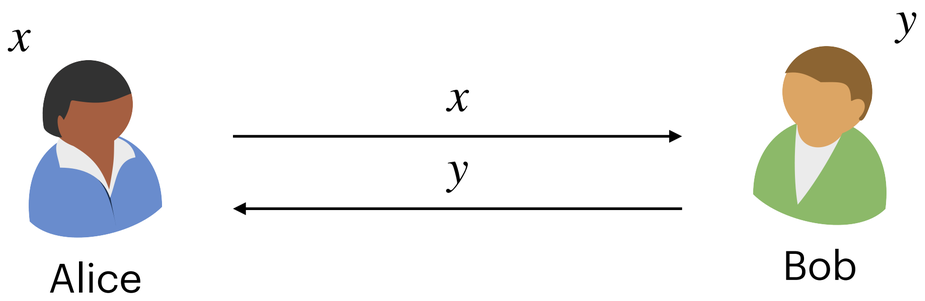

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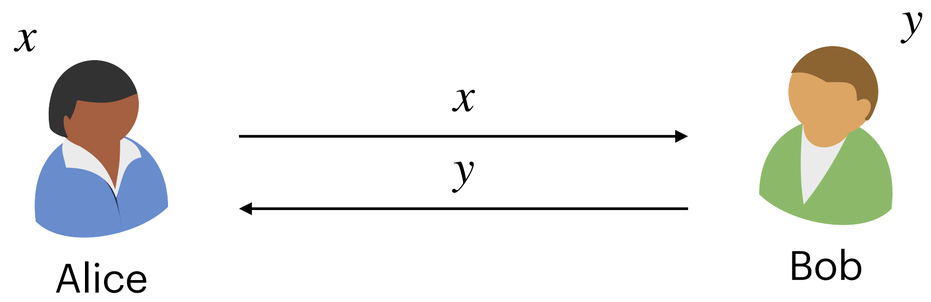

Output $x + y$

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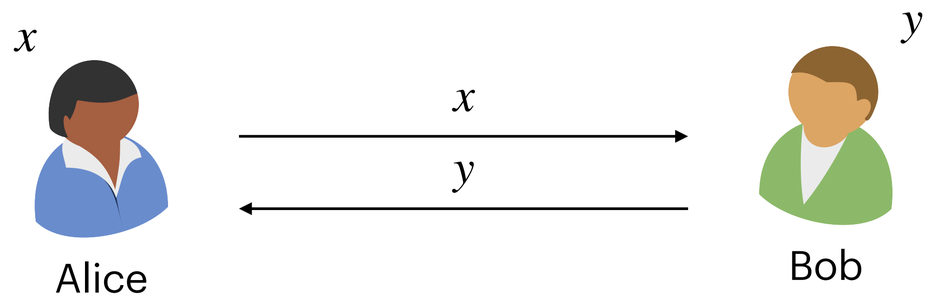

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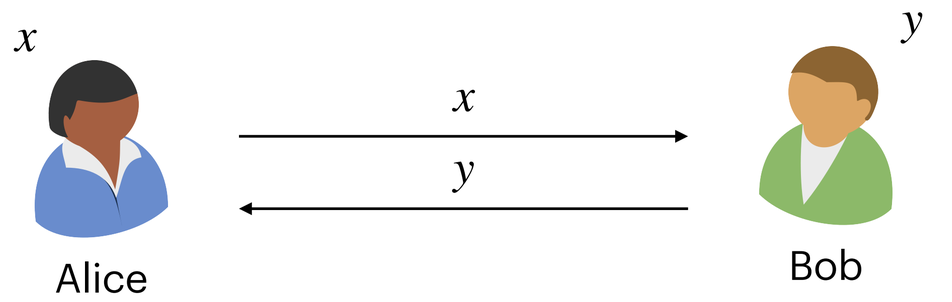

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Is this protocol secure?

- Yes! Adversary can learn honest party's input only from the corrupt party's input and the sum.
- Formally, $S_A(x, x + y)$ can simply compute y and generate Alice's view.

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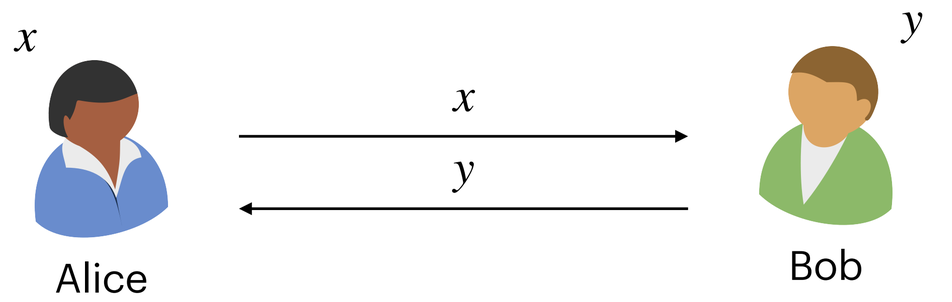

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- The definition only says the adversary does not learn anything beyond what it could compute using the corrupt party's input and function output.

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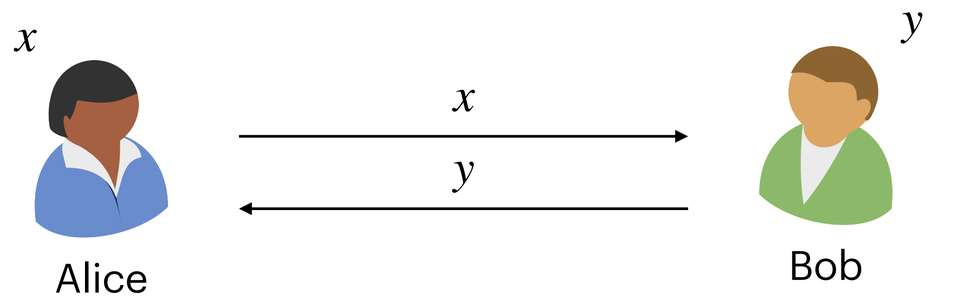

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  - Privacy of the honest party's input depends on the function being computed.

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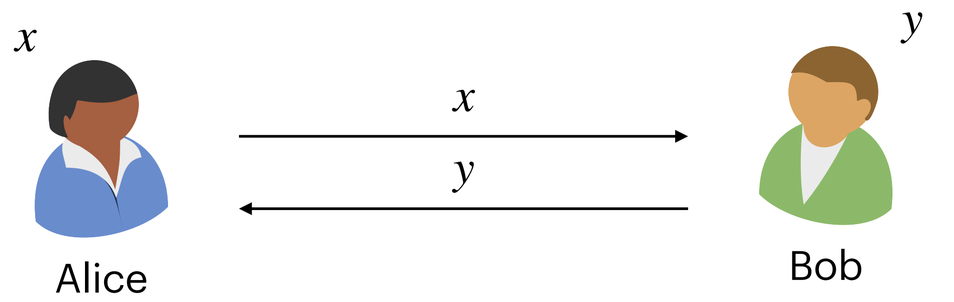

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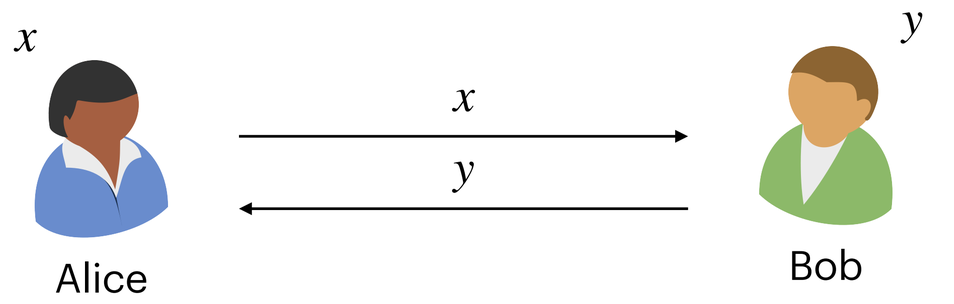
- Formally, $S_A(x, x + y)$ can simply compute y and generate Alice's view.

- The definition only says the adversary does not learn anything beyond what it could compute using the corrupt party's input and function output.
  - Privacy of the honest party's input depends on the function being computed.
- **Another Example:** Say parties use a secure two-party protocol to compute the AND function $z = x \wedge y$.

Semi-Honest Secure Two-Party Computation



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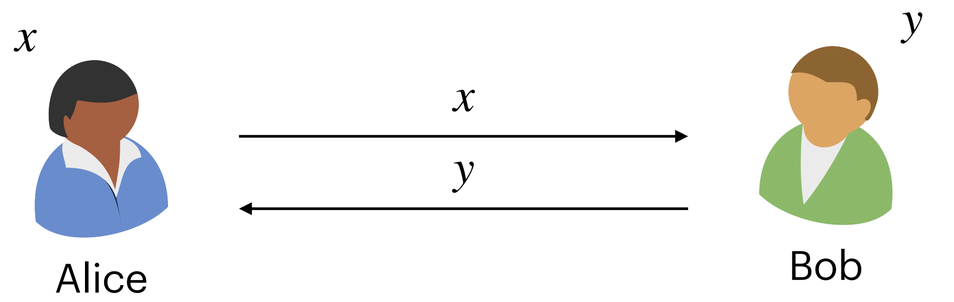

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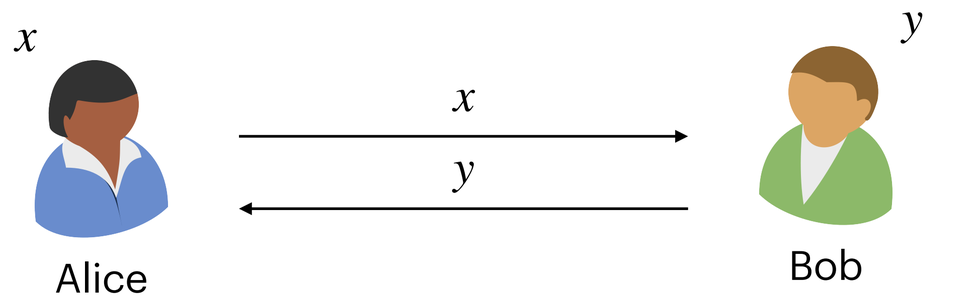

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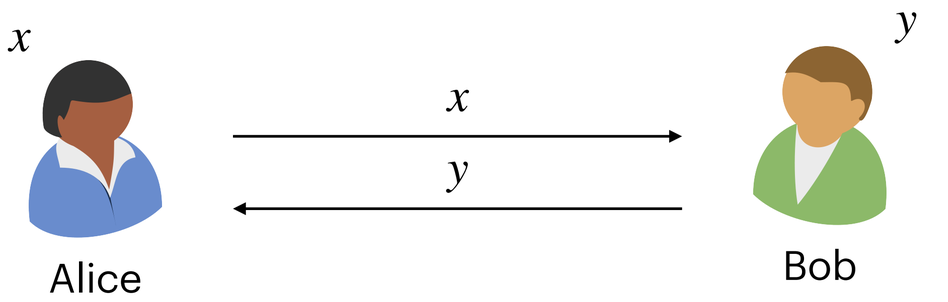

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A protocol $\Pi = (A, B)$ securely computes a program $P = (P_A, P_B)$ in the presence of semi-honest adversaries if there exists PPT algorithms S_A and S_B such that for all $x, y \in \{0,1\}^*$ we have

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    - A program that samples a uniformly random bitstring for Alice and outputs nothing to Bob.

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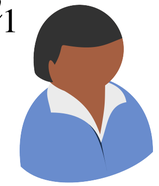
  - Extending the definition to capture security against malicious adversaries is non-trivial.
- This definition naturally extends to more than two parties.

Oblivious Transfer

Oblivious Transfer (OT)

m_0, m_1



Alice

c

Bob

- **Inputs**
  - Alice has two messages m_0 and m_1.
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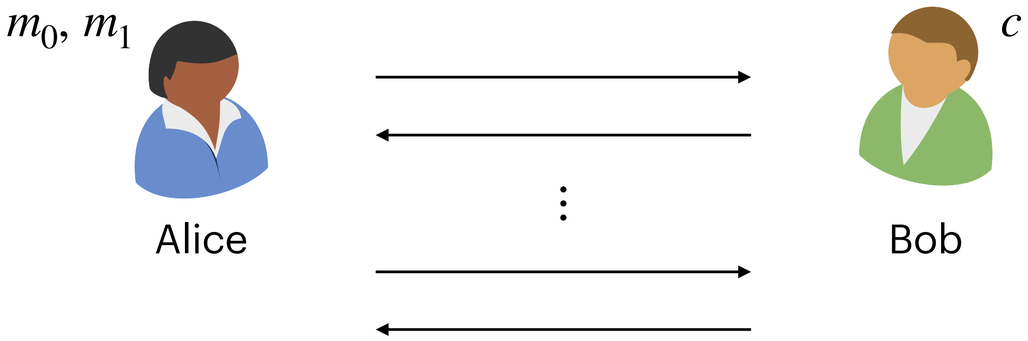
Oblivious Transfer (OT)



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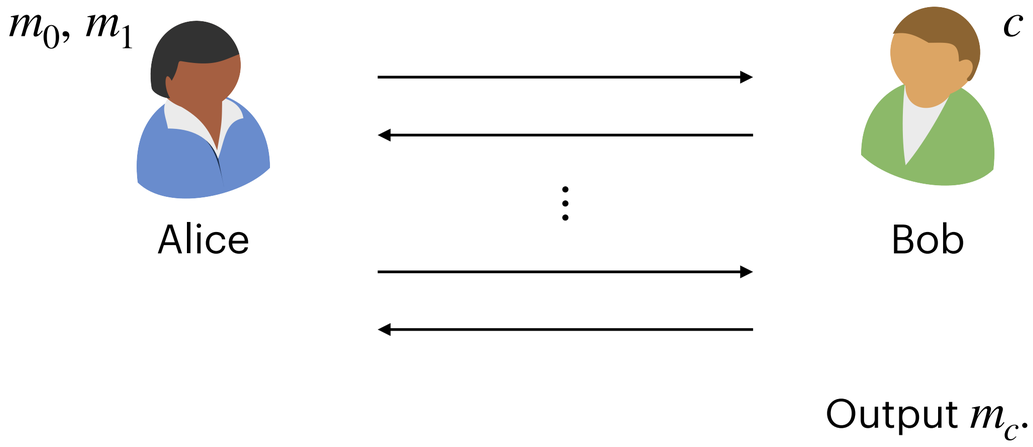

- **Inputs**
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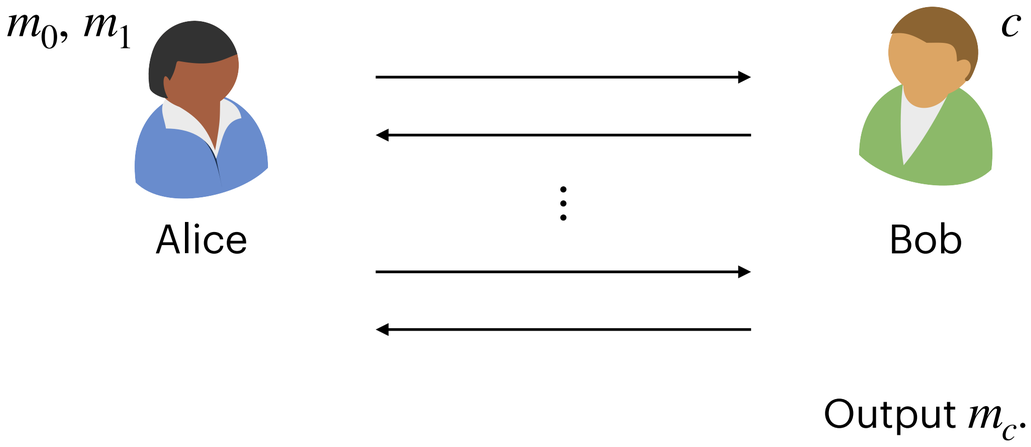
Oblivious Transfer (OT)



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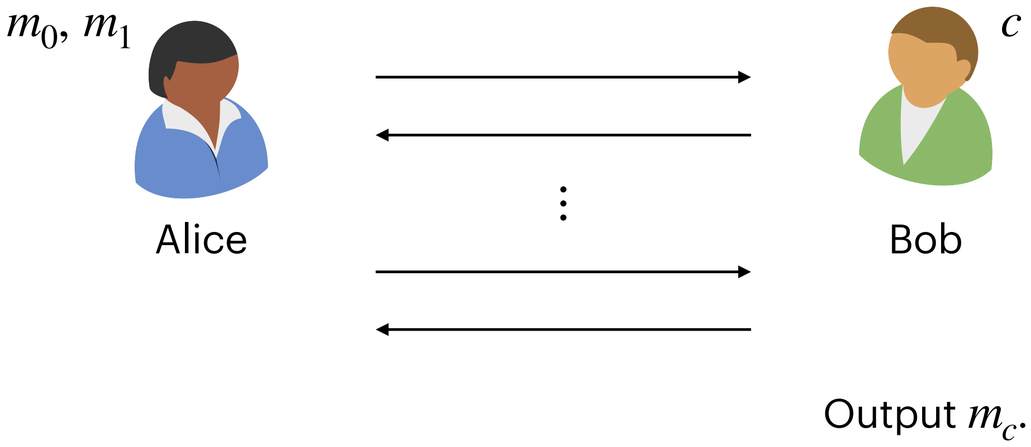

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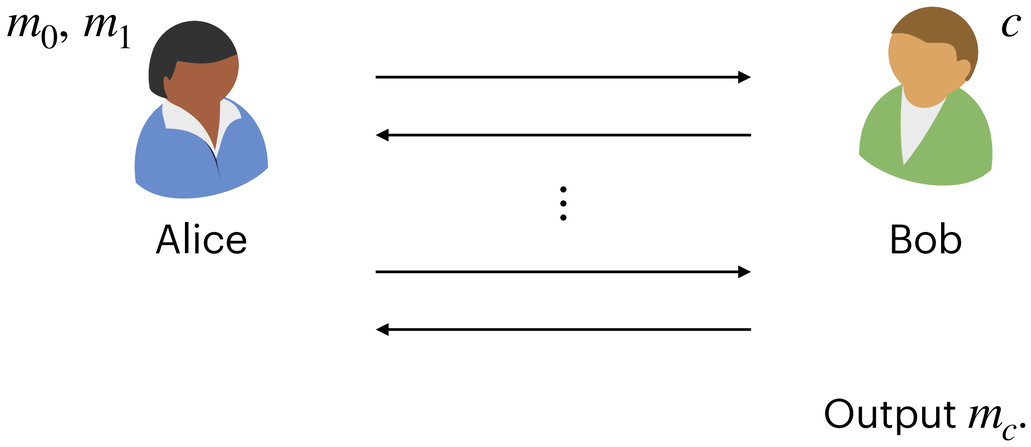

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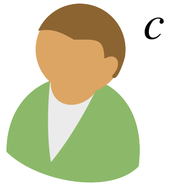
OT is necessary and complete for secure multi-party computation!

Constructing Oblivious Transfer Protocol

Constructing Oblivious Transfer Protocol

m_0, m_1

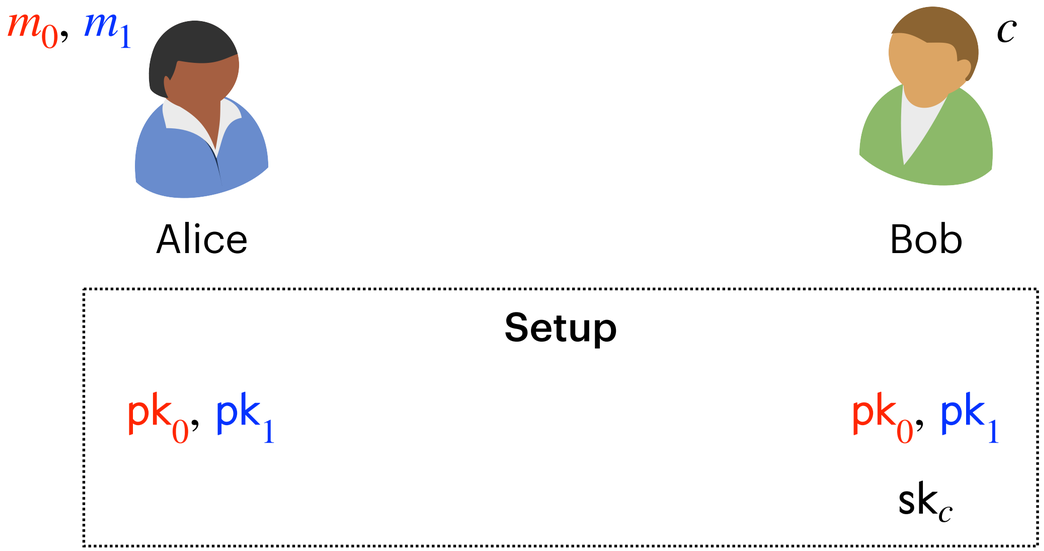
Alice



c

Bob

Constructing Oblivious Transfer Protocol



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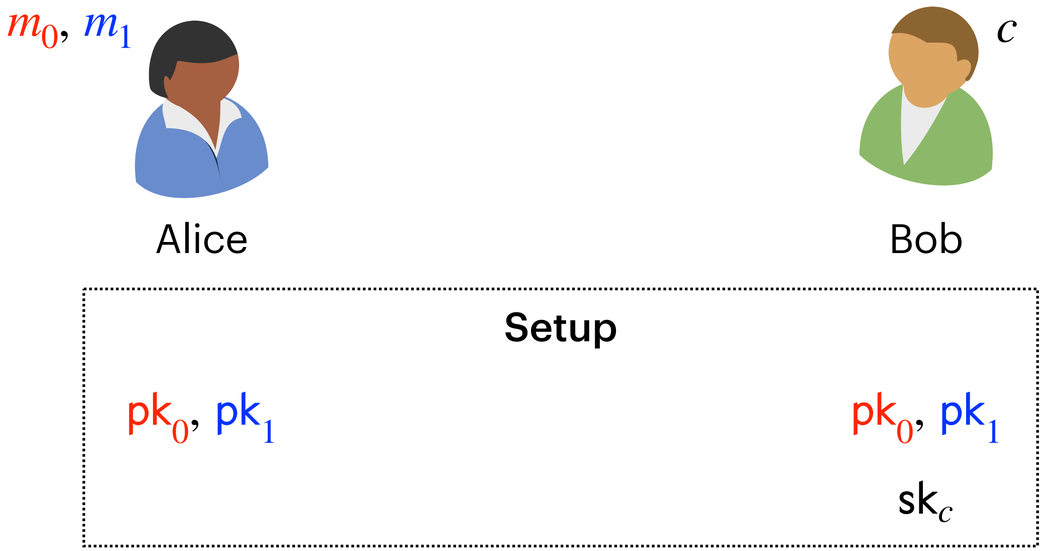

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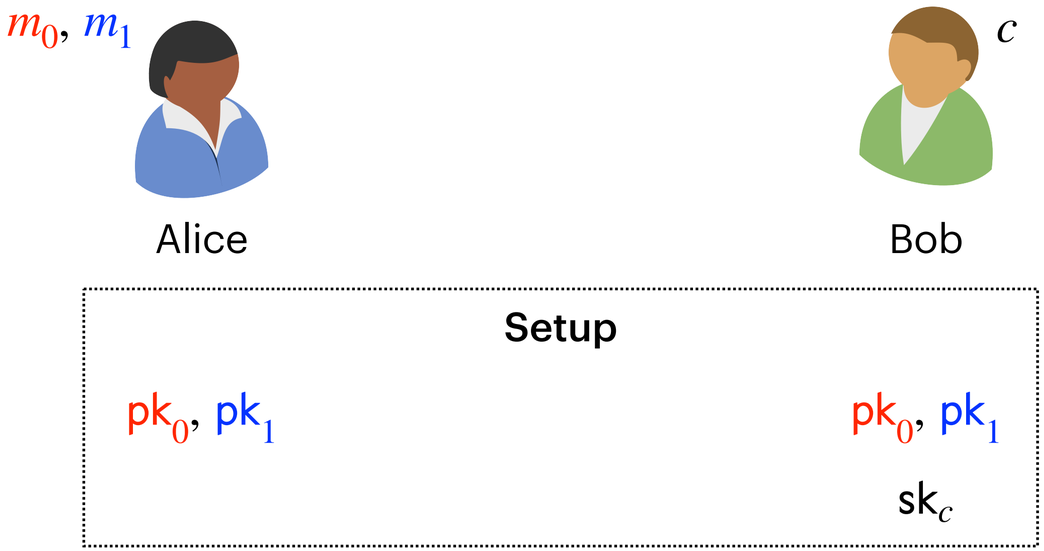

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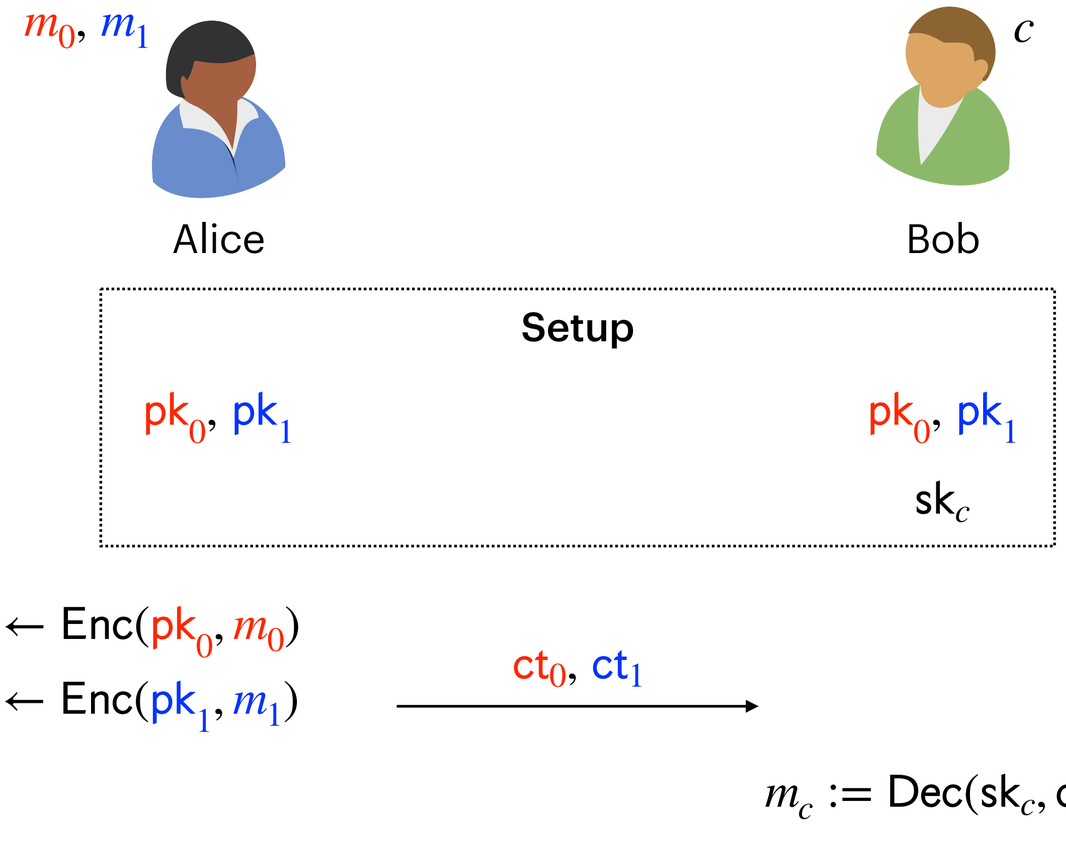

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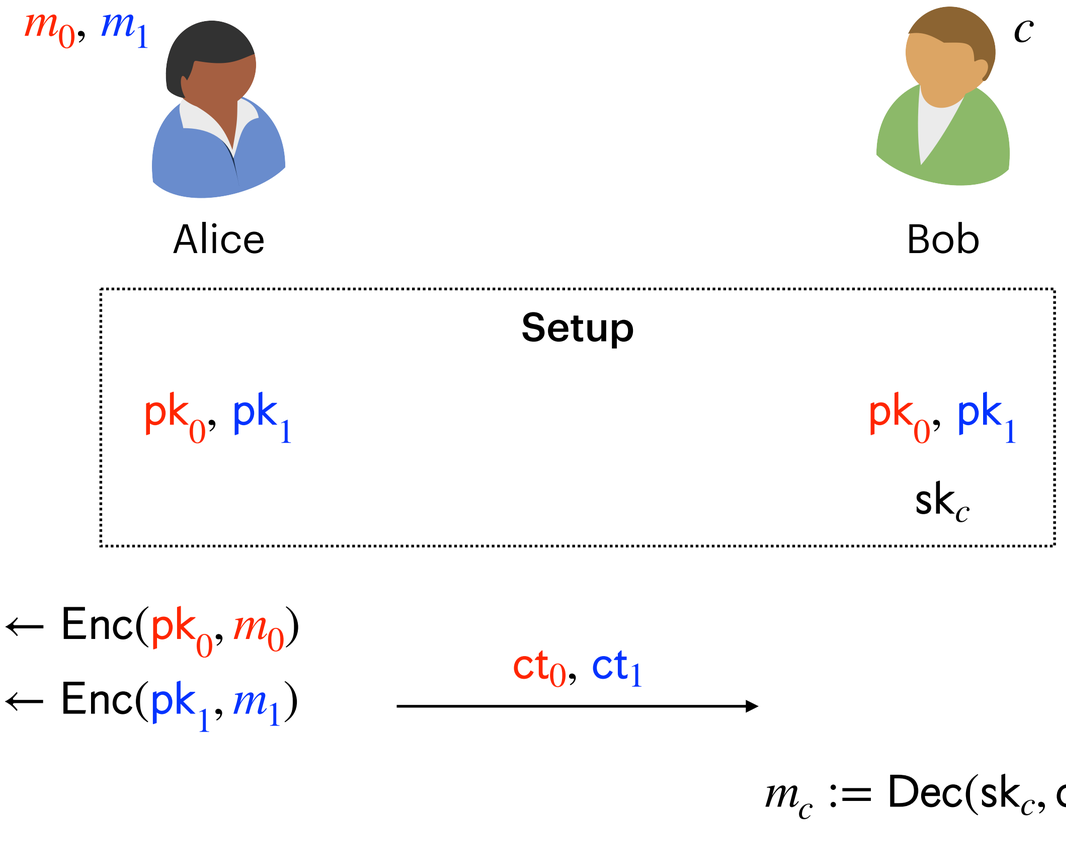

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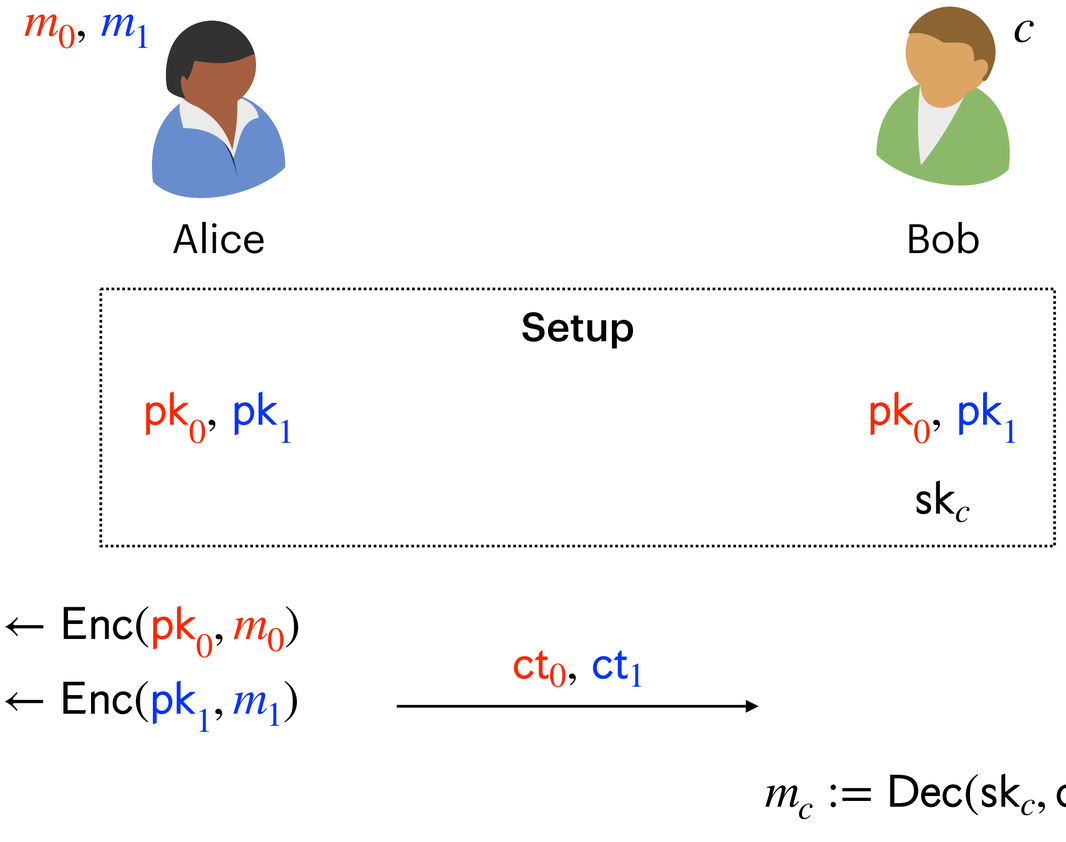

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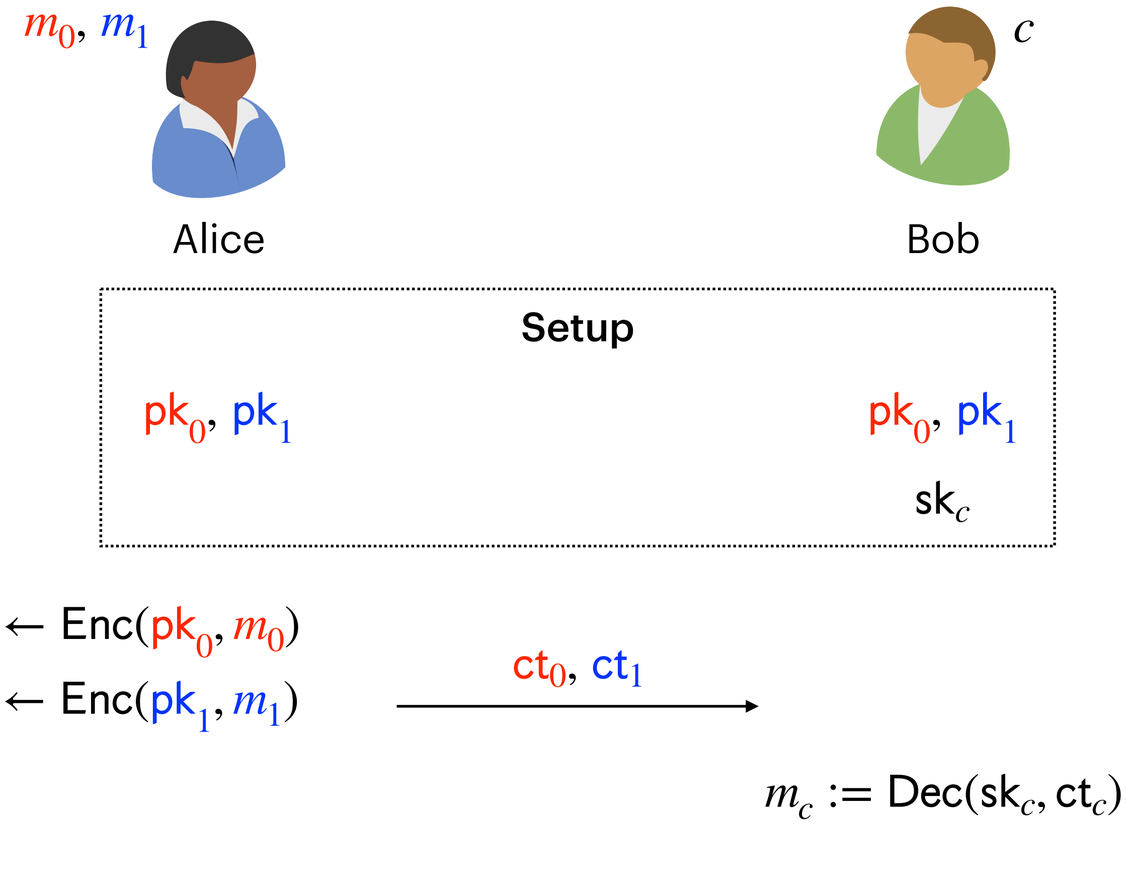
Constructing Oblivious Transfer Protocol



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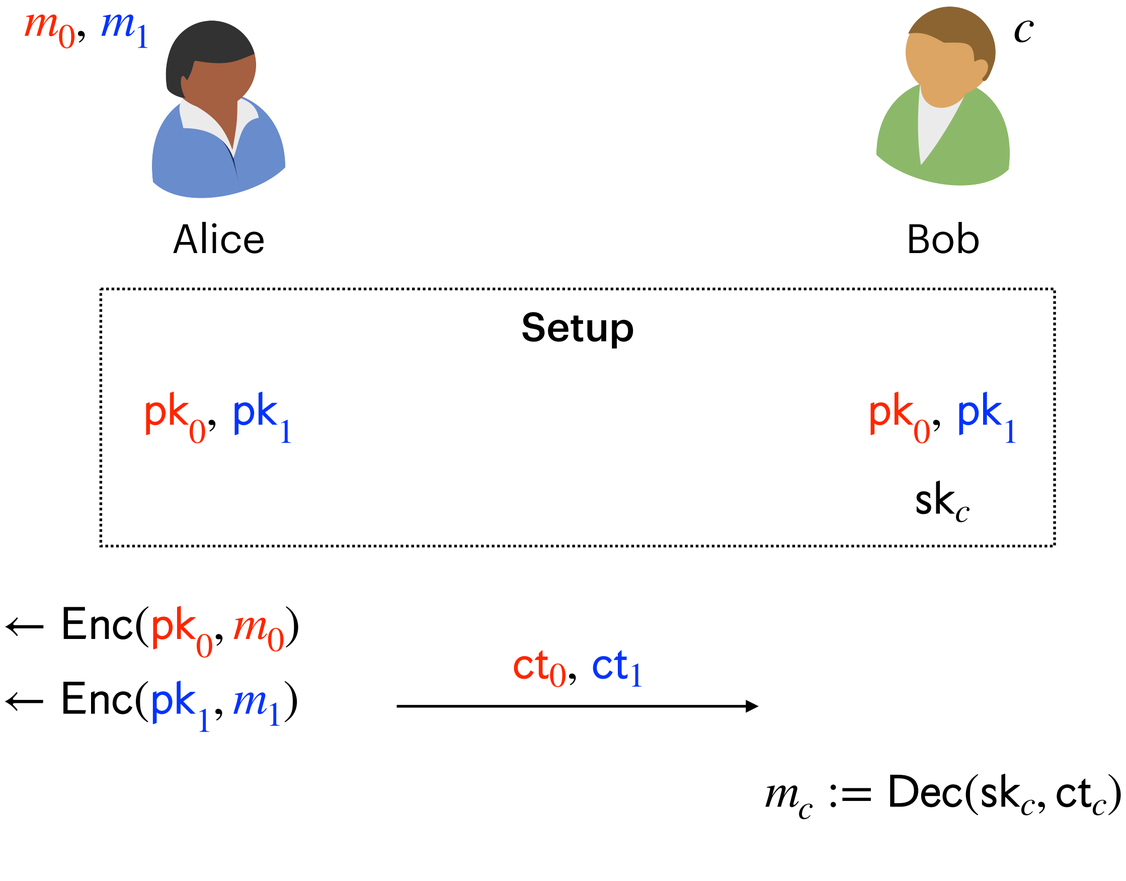
Constructing Oblivious Transfer Protocol



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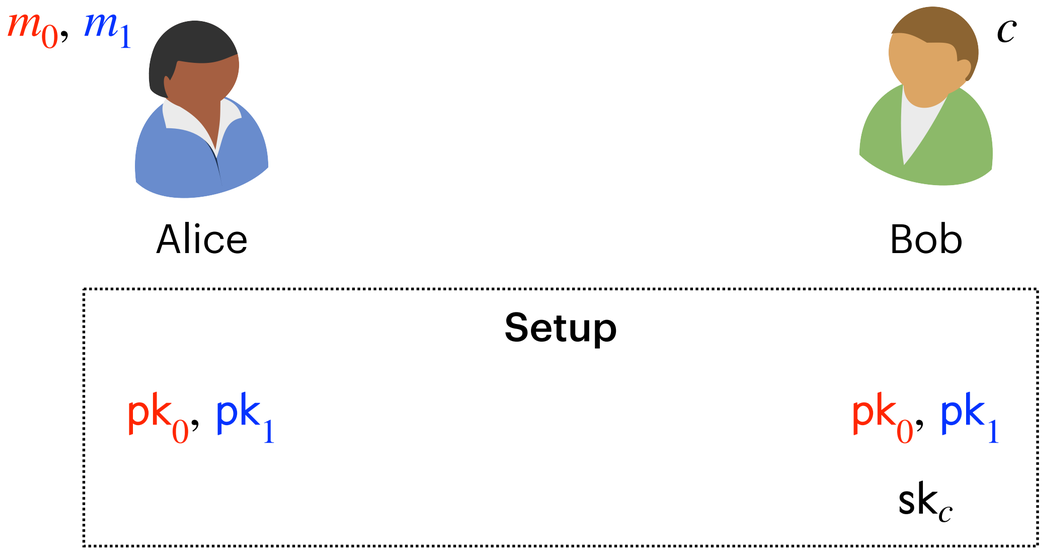
Constructing Oblivious Transfer Protocol



How can parties compute the setup?

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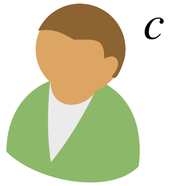
Constructing Oblivious Transfer Protocol

m_0, m_1

Alice

c



Bob

Constructing Oblivious Transfer Protocol

m_0, m_1

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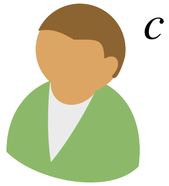


c

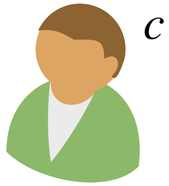
Bob

$(\mathbb{G}, g, p)$

Constructing Oblivious Transfer Protocol

m_0, m_1

Alice



c

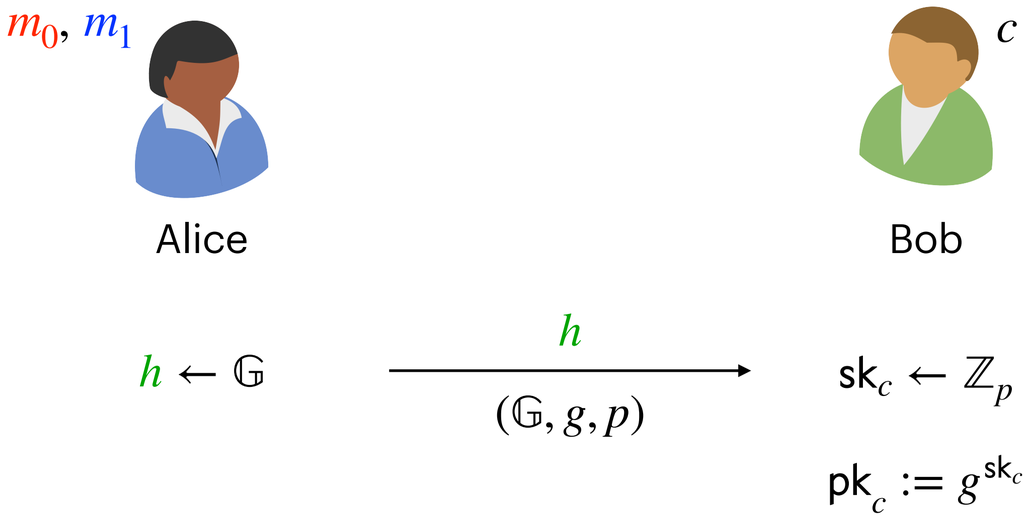
Bob

$\xrightarrow{(\mathbb{G}, g, p)}$

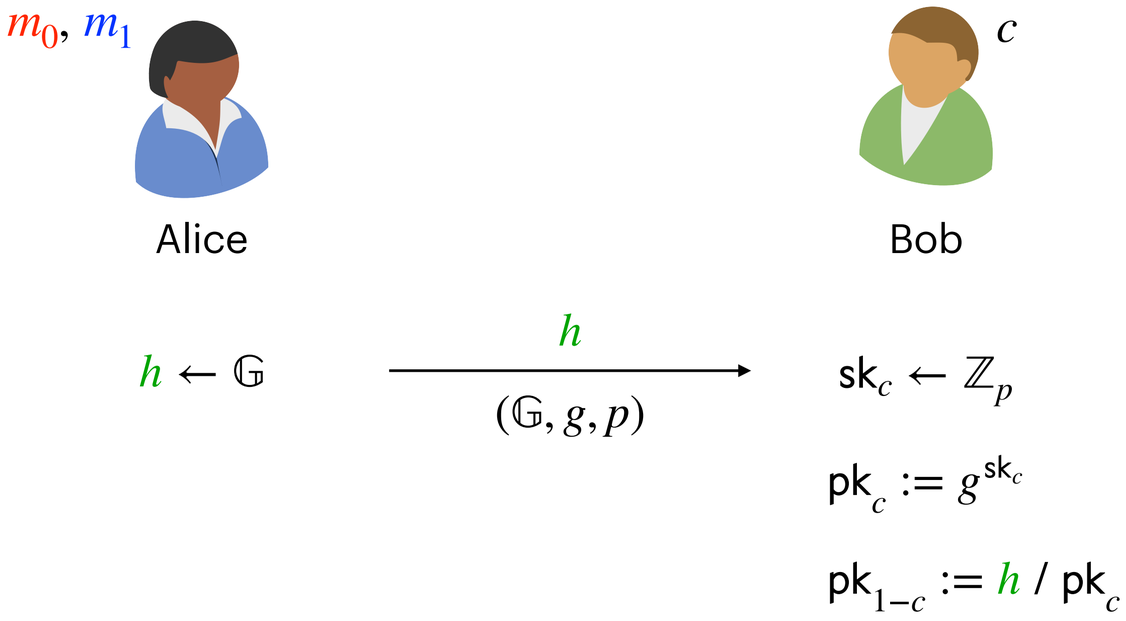
$\mathsf{sk}_c \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_c := g^{\mathsf{sk}_c}$

Constructing Oblivious Transfer Protocol



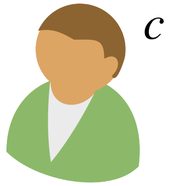
Constructing Oblivious Transfer Protocol



Constructing Oblivious Transfer Protocol

m_0, m_1

Alice



c

Bob

$h \leftarrow \mathbb{G}$

h

$(\mathbb{G}, g, p)$

$\mathsf{sk}_c \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_c := g^{\mathsf{sk}_c}$

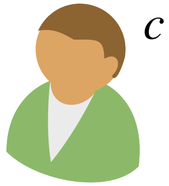
$\mathsf{pk}_{1-c} := h \,/\, \mathsf{pk}_c$

Does Bob know the secret key sk_{1-c} for pk_{1-c}?

Constructing Oblivious Transfer Protocol

m_0, m_1

Alice



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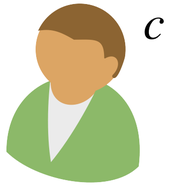
Constructing Oblivious Transfer Protocol



m_0, m_1

c

Alice

Bob

$h \leftarrow \mathbb{G}$

h

$(\mathbb{G}, g, p)$

$\mathsf{sk}_c \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_c := g^{\mathsf{sk}_c}$

$\mathsf{pk}_1 := h \,/\, \mathsf{pk}_0$

pk_0

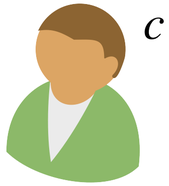
$\mathsf{pk}_{1-c} := h \,/\, \mathsf{pk}_c$

Does Bob know the secret key sk_{1-c} for pk_{1-c}?

Constructing Oblivious Transfer Protocol

m_0, m_1

Alice



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$h \leftarrow \mathbb{G}$

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$\mathsf{sk}_c \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_c := g^{\mathsf{sk}_c}$

$\mathsf{pk}_1 := h \,/\, \mathsf{pk}_0$

pk_0

$\mathsf{pk}_{1-c} := h \,/\, \mathsf{pk}_c$

$\mathsf{pk}_0, \mathsf{pk}_1$

$\mathsf{pk}_0, \mathsf{pk}_1$ sk_c

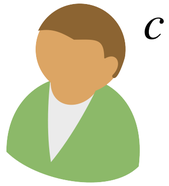
Does Bob know the secret key sk_{1-c} for pk_{1-c}?

Alice and Bob have computed the setup!

Constructing Oblivious Transfer Protocol

m_0, m_1

Alice



c

Bob

$h \leftarrow \mathbb{G}$

h

$(\mathbb{G}, g, p)$

$\mathsf{sk}_c \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_c := g^{\mathsf{sk}_c}$

$\mathsf{pk}_1 := h \,/\, \mathsf{pk}_0$

pk_0

$\mathsf{pk}_{1-c} := h \,/\, \mathsf{pk}_c$

Does Bob know the secret key sk_{1-c} for pk_{1-c}?

$\mathsf{pk}_0, \mathsf{pk}_1$

$\mathsf{pk}_0, \mathsf{pk}_1$ sk_c

Alice and Bob have computed the setup!

$\mathsf{ct}_0 \leftarrow \mathsf{Enc}(\mathsf{pk}_0, m_0)$

$\mathsf{ct}_1 \leftarrow \mathsf{Enc}(\mathsf{pk}_1, m_1)$

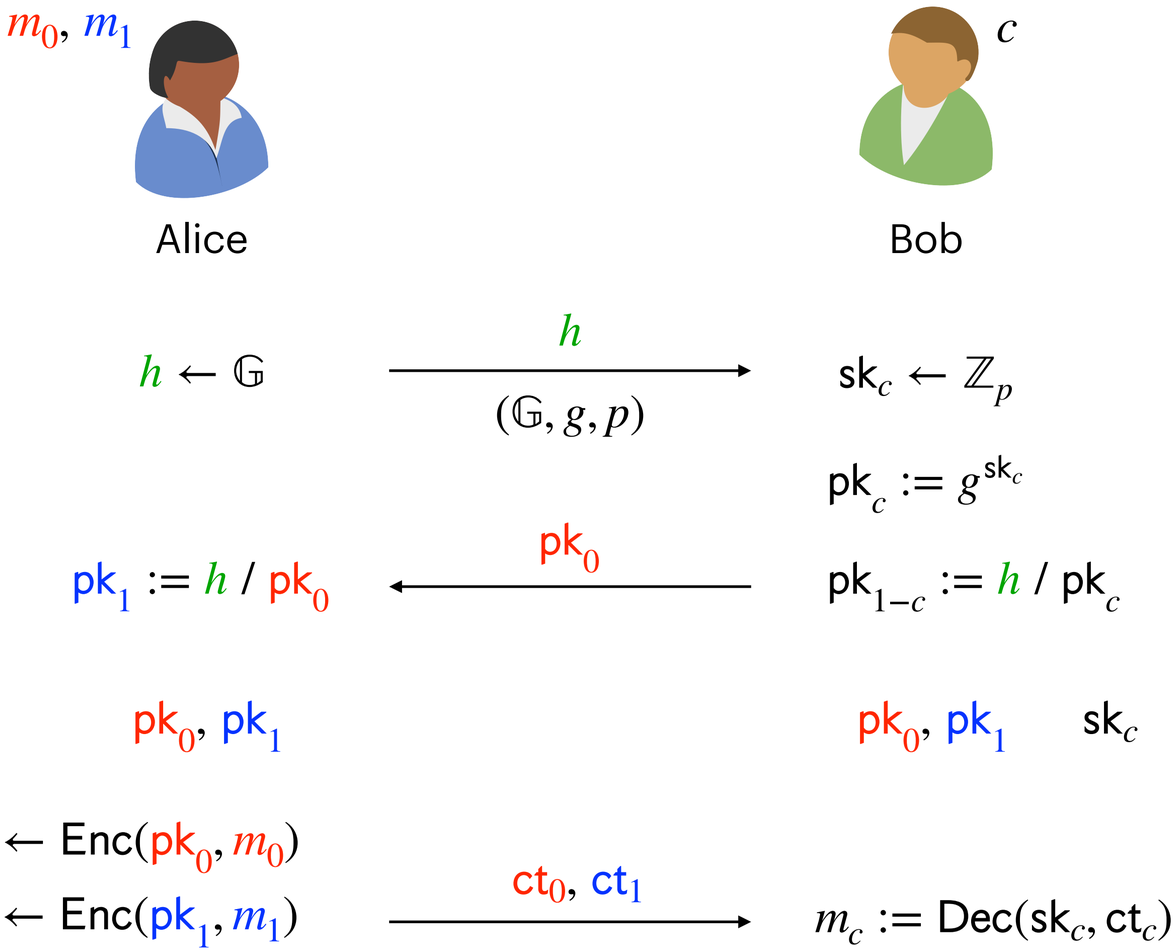
$\mathsf{ct}_0, \mathsf{ct}_1$

$m_c := \mathsf{Dec}(\mathsf{sk}_c, \mathsf{ct}_c)$

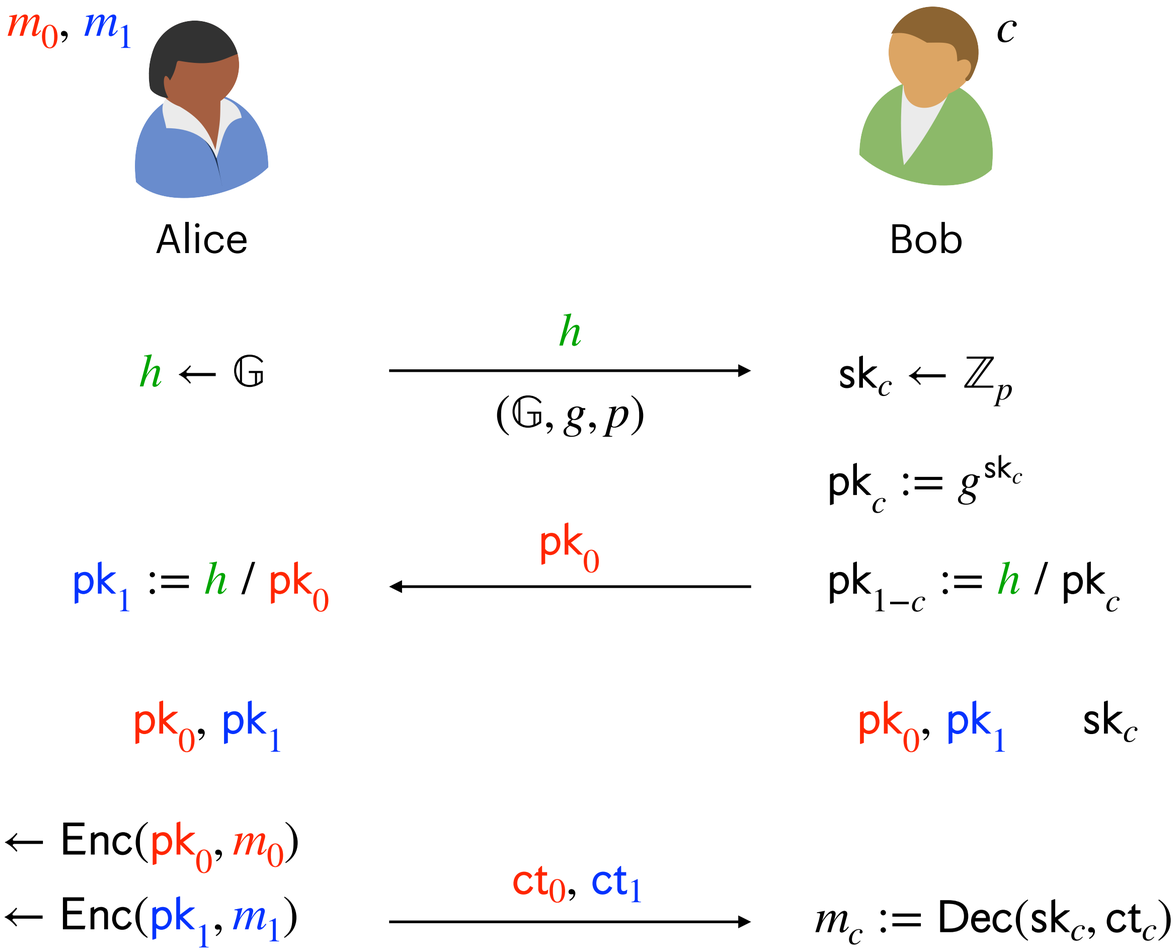
Alice uses ElGamal encryption to encrypt the messages.

Constructing Oblivious Transfer Protocol

- **Correctness:**

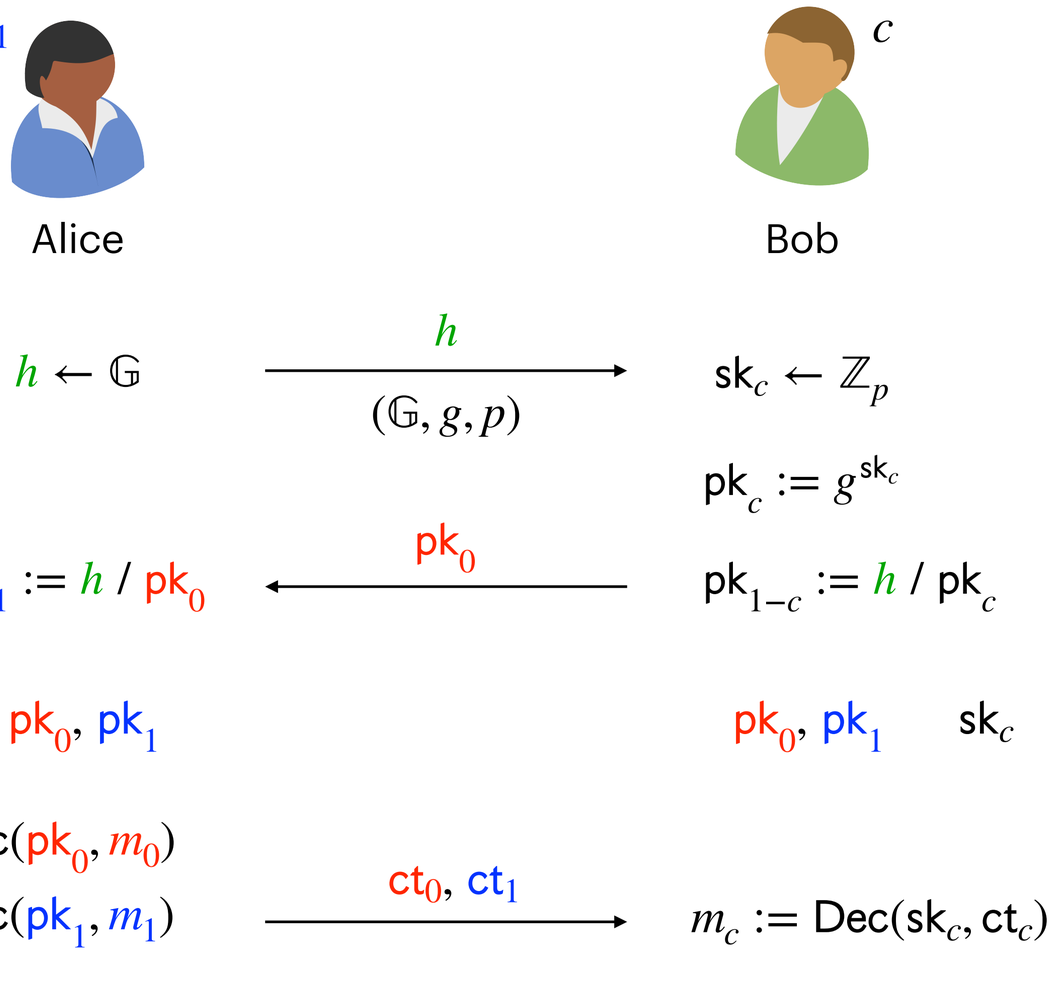


Constructing Oblivious Transfer Protocol



- **Correctness:**
  - Bob ensures that

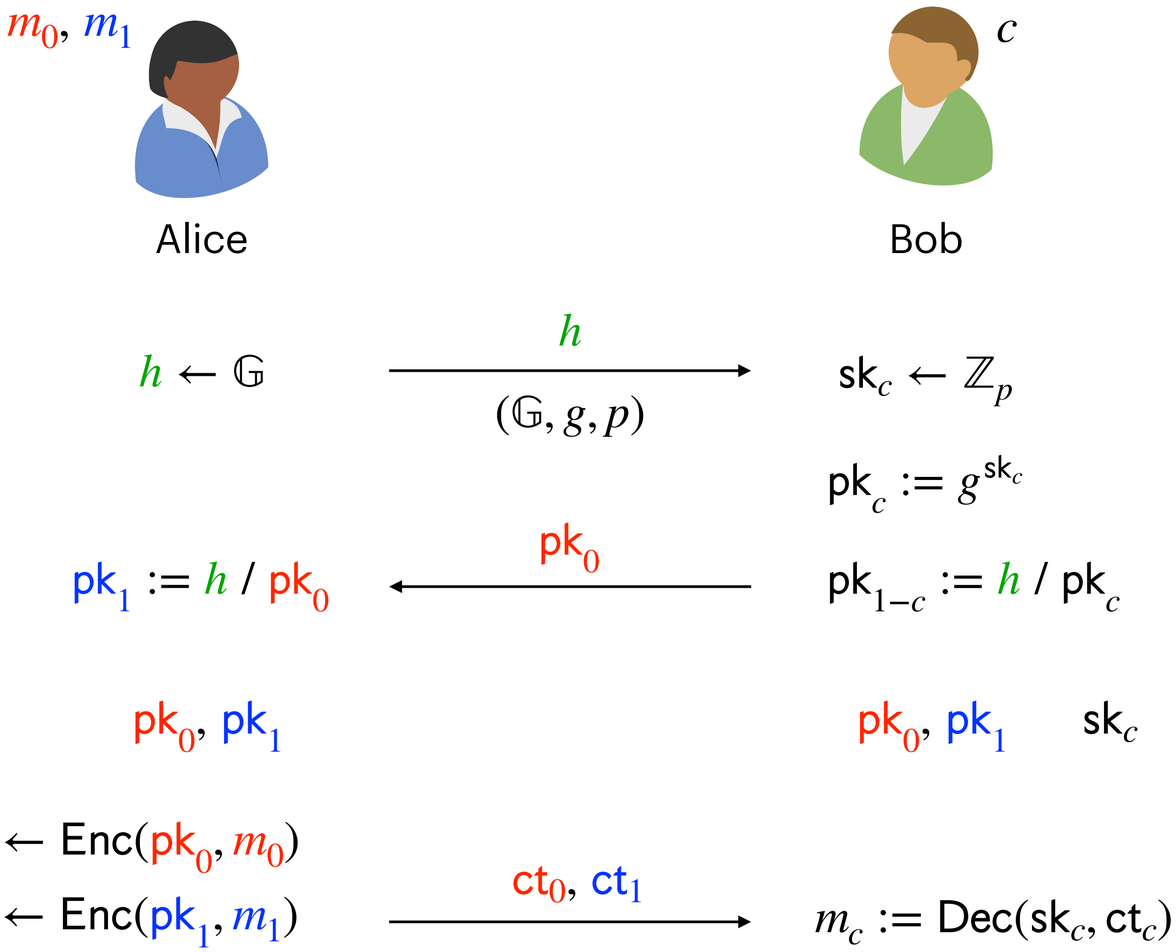
Constructing Oblivious Transfer Protocol



- **Correctness:**
  - Bob ensures that

$$\mathsf{pk}_c \cdot \mathsf{pk}_{1-c} = \mathsf{pk}_0 \cdot \mathsf{pk}_1 = h.$$

Constructing Oblivious Transfer Protocol

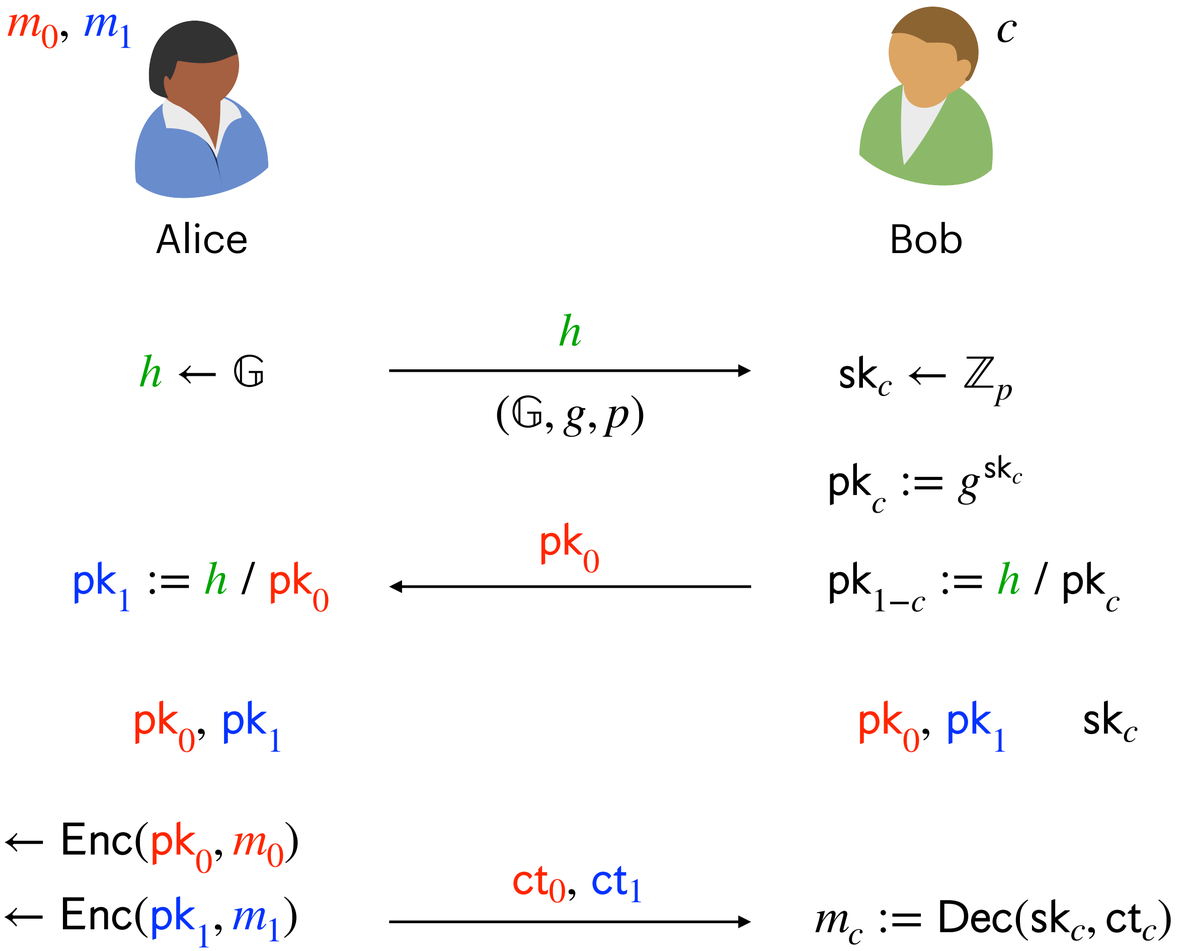


- **Correctness:**
  - Bob ensures that

$$\mathsf{pk}_c \cdot \mathsf{pk}_{1-c} = \mathsf{pk}_0 \cdot \mathsf{pk}_1 = h.$$

  - Therefore, Alice computes the same public keys as Bob.

Constructing Oblivious Transfer Protocol

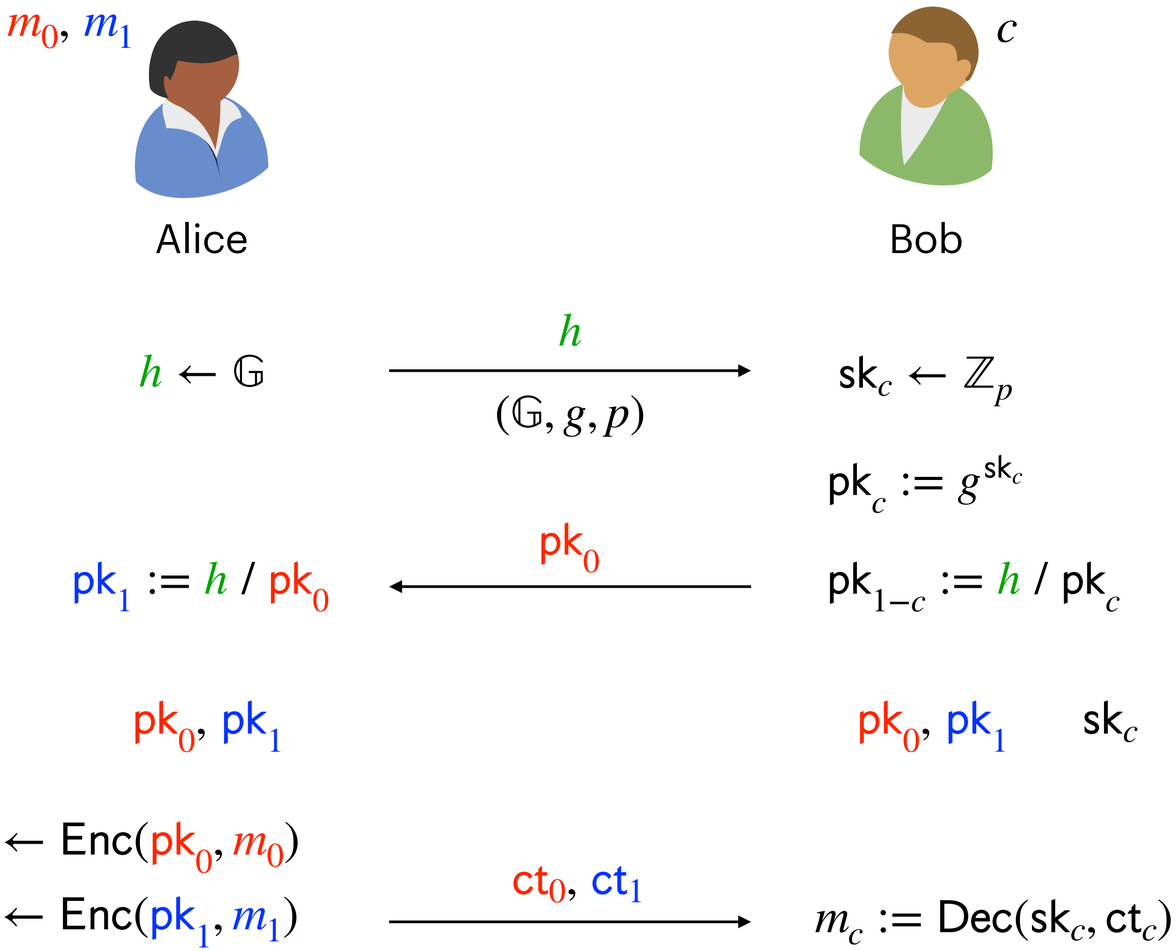


- **Correctness:**
  - Bob ensures that

$$\mathsf{pk}_c \cdot \mathsf{pk}_{1-c} = \mathsf{pk}_0 \cdot \mathsf{pk}_1 = h.$$

  - Therefore, Alice computes the same public keys as Bob.
  - Bob outputs correct m_c due to correctness of decryption.

Constructing Oblivious Transfer Protocol



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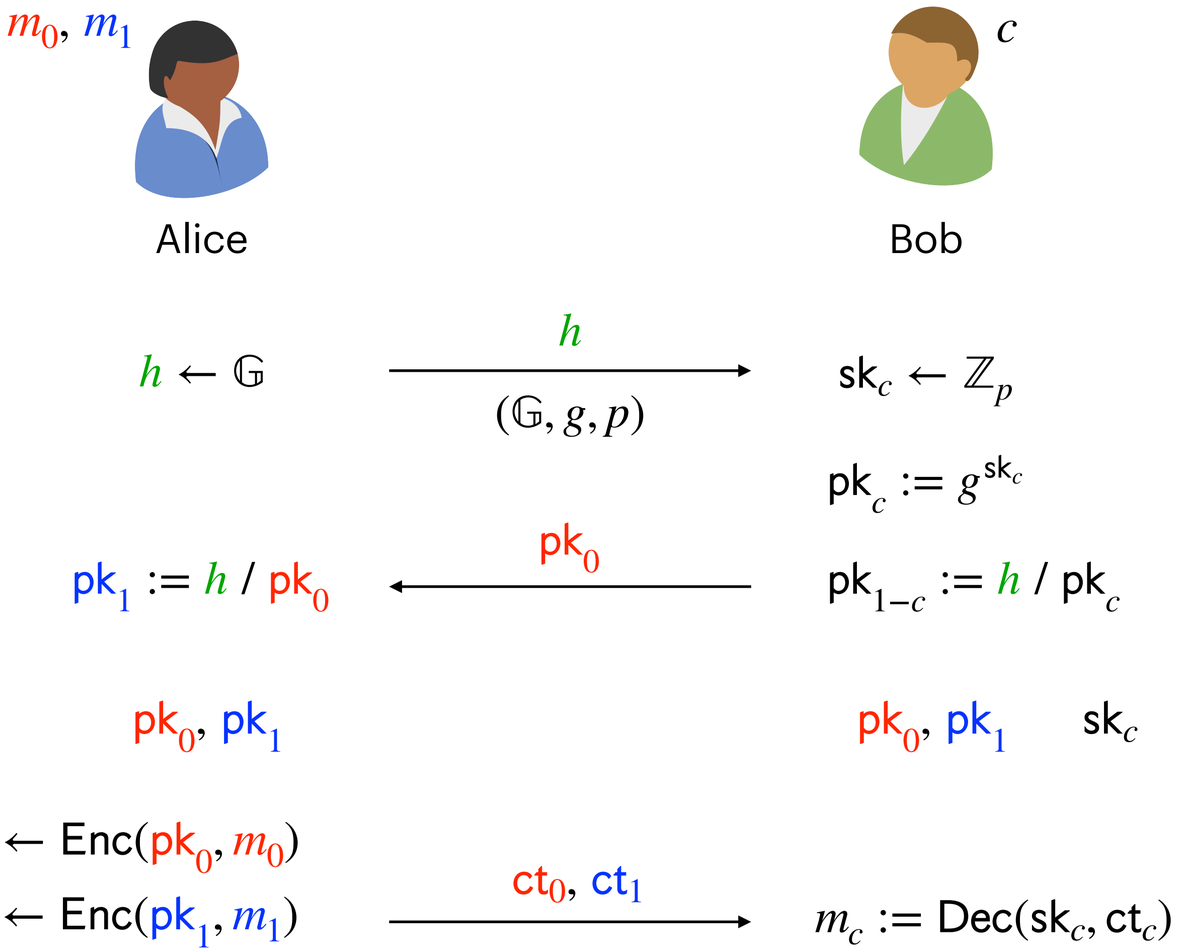
- **Security:**

Constructing Oblivious Transfer Protocol

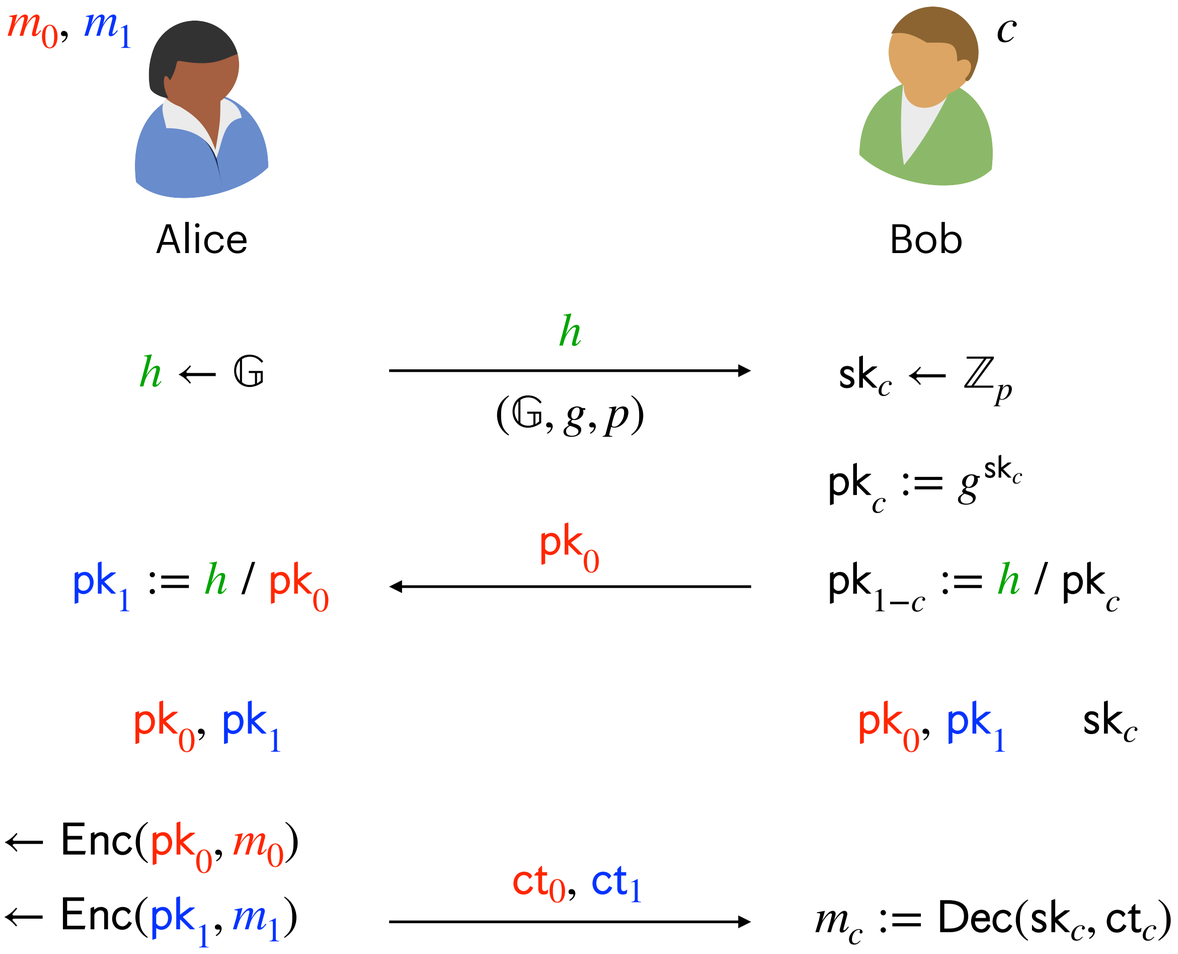


- **Correctness:**
  - Bob ensures that

$$\mathsf{pk}_c \cdot \mathsf{pk}_{1-c} = \mathsf{pk}_0 \cdot \mathsf{pk}_1 = h.$$

  - Therefore, Alice computes the same public keys as Bob.
  - Bob outputs correct m_c due to correctness of decryption.
- **Security:**
  - h is a random group element chosen by Alice. Bob cannot efficiently compute its discrete log.

Constructing Oblivious Transfer Protocol



- **Correctness:**
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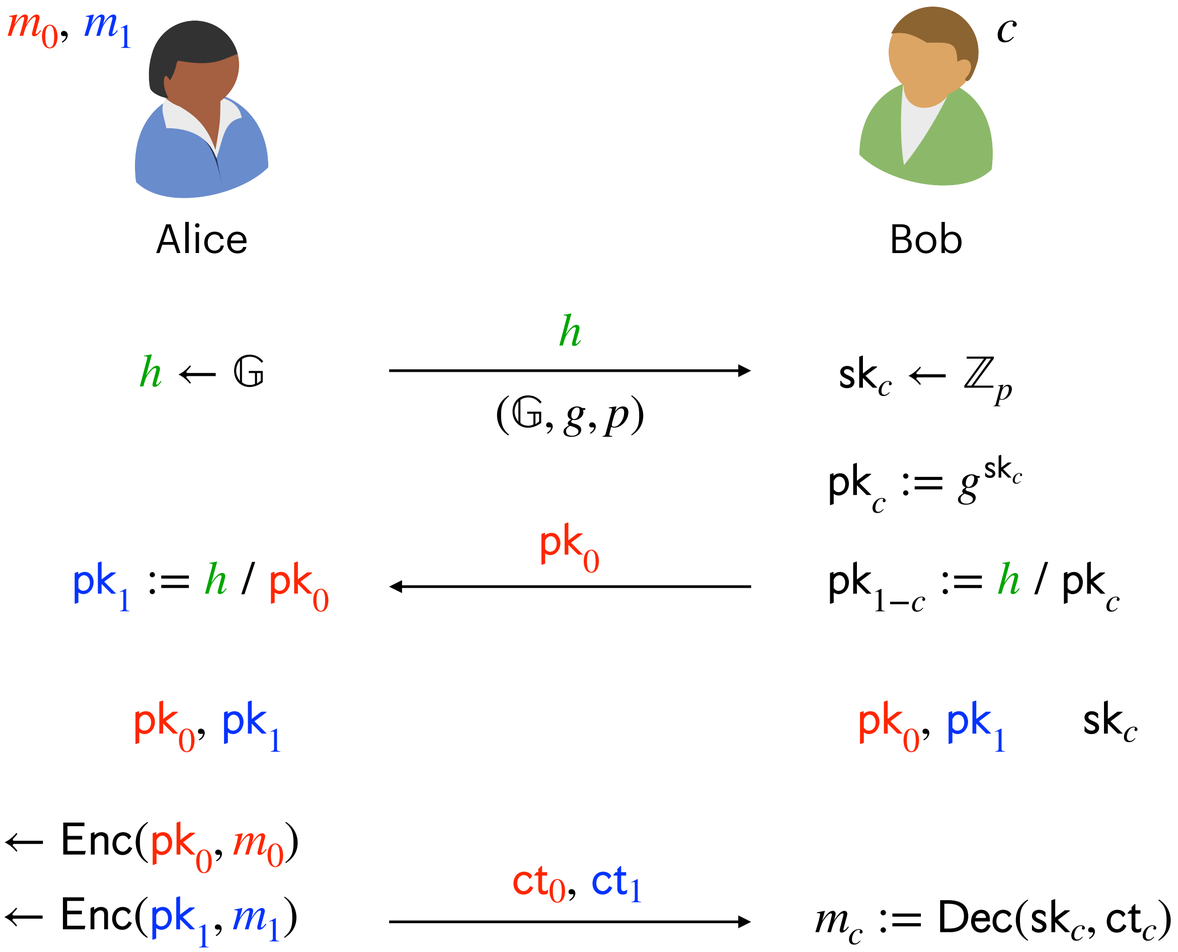
Constructing Oblivious Transfer Protocol



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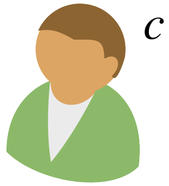
  - Bob outputs correct m_c due to correctness of decryption.
- **Security:**
  - h is a random group element chosen by Alice. Bob cannot efficiently compute its discrete log.
  - Bob cannot efficiently compute the discrete log of

$$\mathsf{pk}_{1-c} := h \,/\, \mathsf{pk}_c.$$

Constructing Oblivious Transfer Protocol

- **Simulator** $S_A(m_0, m_1)$

Alice



Bob

m_0, m_1 c

$h \leftarrow \mathbb{G}$

h →

$(\mathbb{G}, g, p)$

$\mathsf{sk}_c \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_c := g^{\mathsf{sk}_c}$

$\mathsf{pk}_1 := h \,/\, \mathsf{pk}_0$

← pk_0

$\mathsf{pk}_{1-c} := h \,/\, \mathsf{pk}_c$

$\mathsf{pk}_0, \mathsf{pk}_1$ $\mathsf{pk}_0, \mathsf{pk}_1$ sk_c

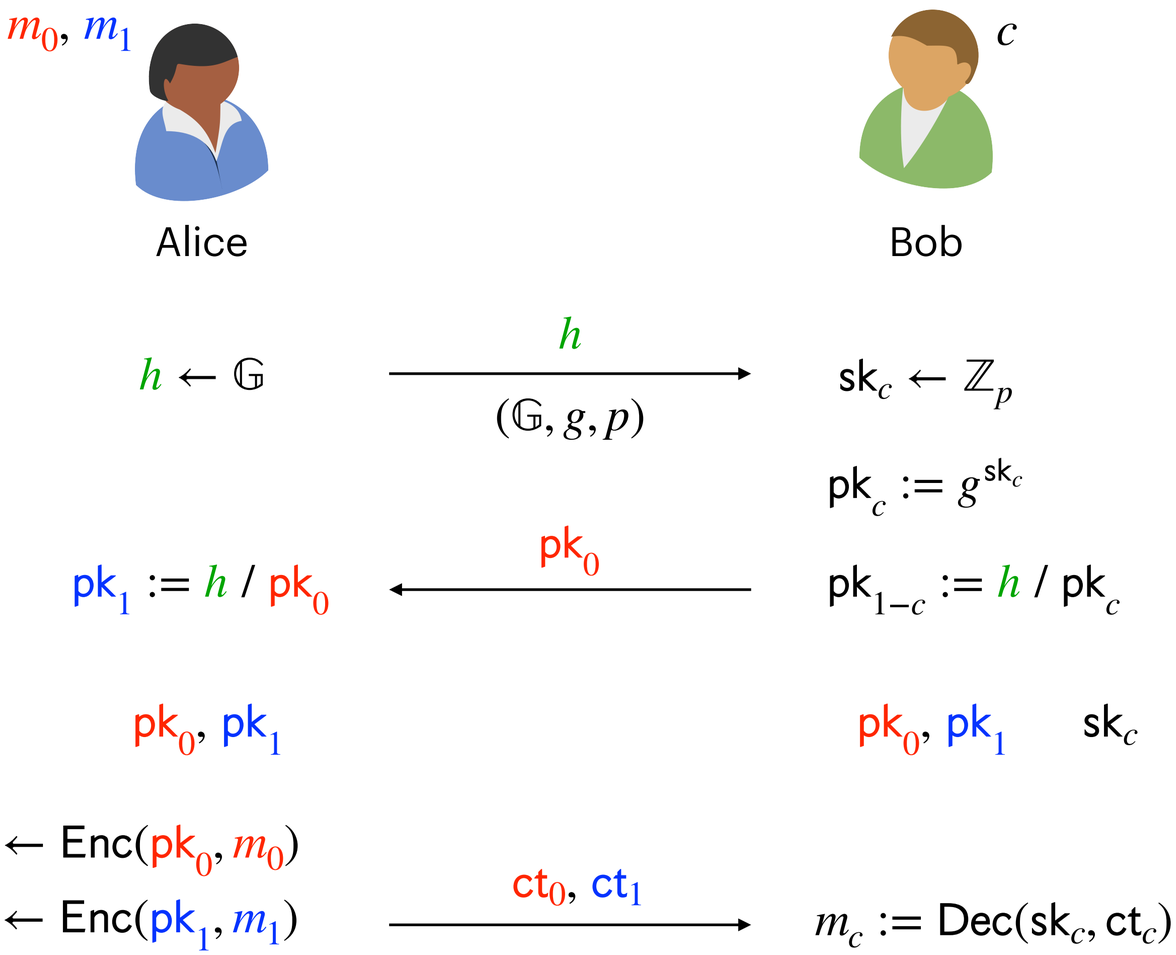
$\mathsf{ct}_0 \leftarrow \mathsf{Enc}(\mathsf{pk}_0, m_0)$

$\mathsf{ct}_1 \leftarrow \mathsf{Enc}(\mathsf{pk}_1, m_1)$

$\mathsf{ct}_0, \mathsf{ct}_1$ →

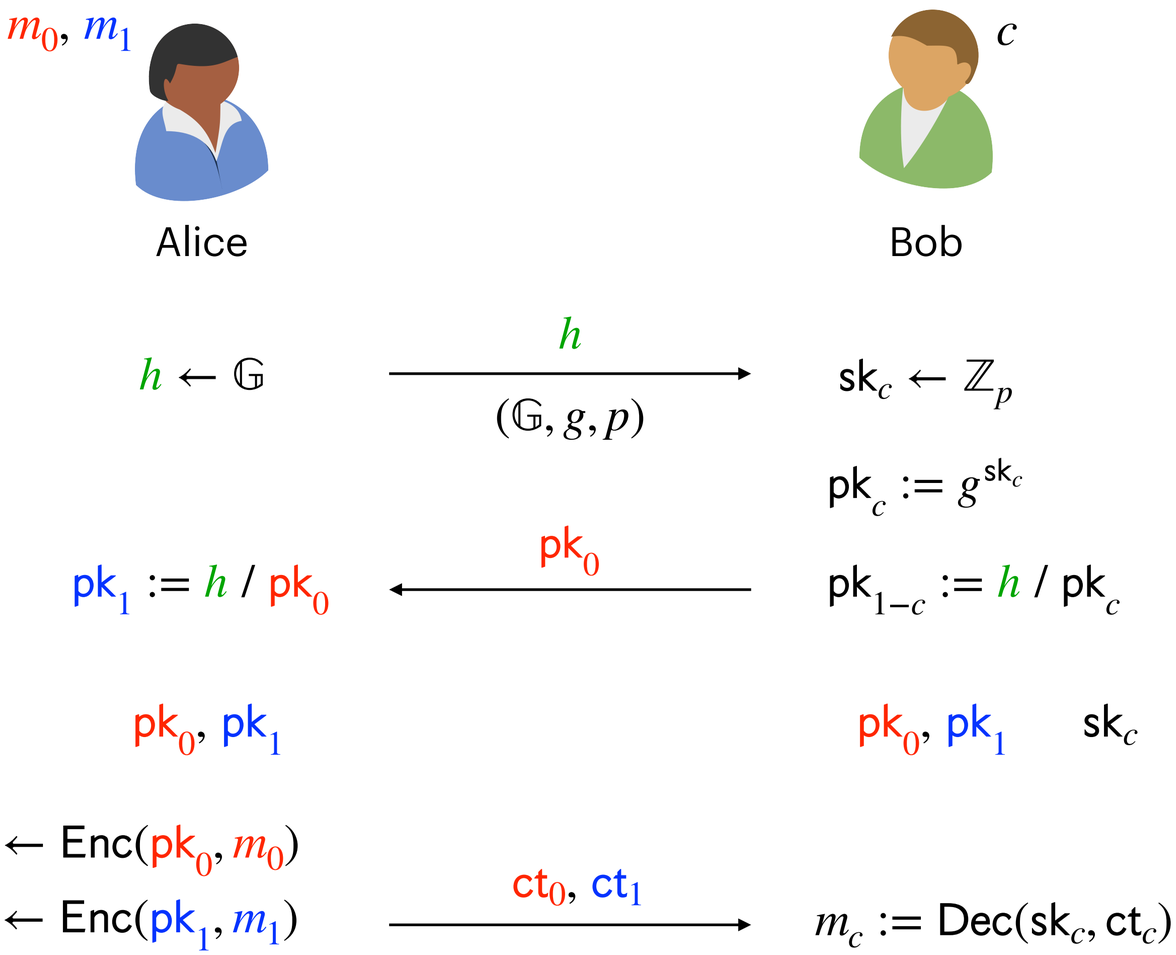
$m_c := \mathsf{Dec}(\mathsf{sk}_c, \mathsf{ct}_c)$

Constructing Oblivious Transfer Protocol



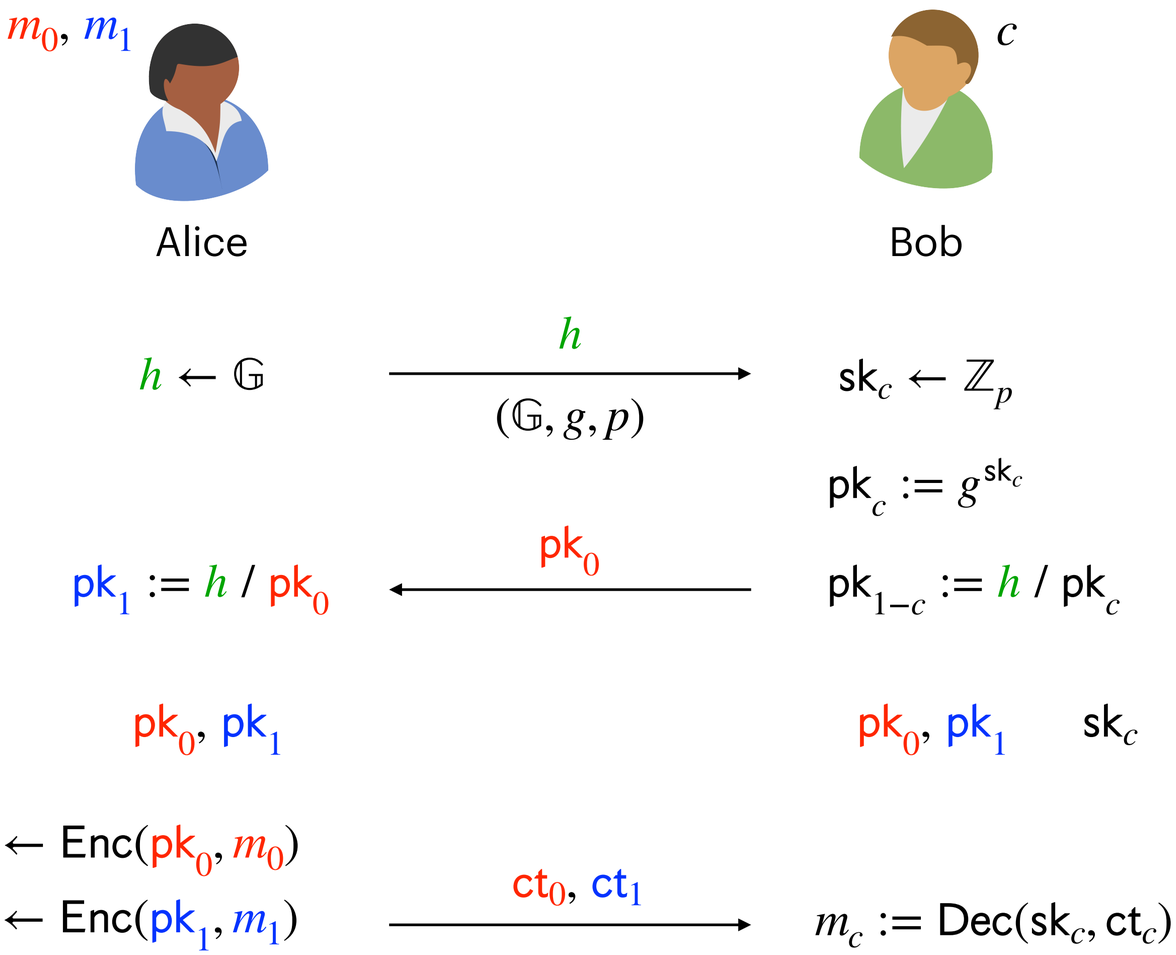
- **Simulator** $S_A(m_0, m_1)$
  - Run $A(m_0, m_1)$ and receive the first message h.

Constructing Oblivious Transfer Protocol



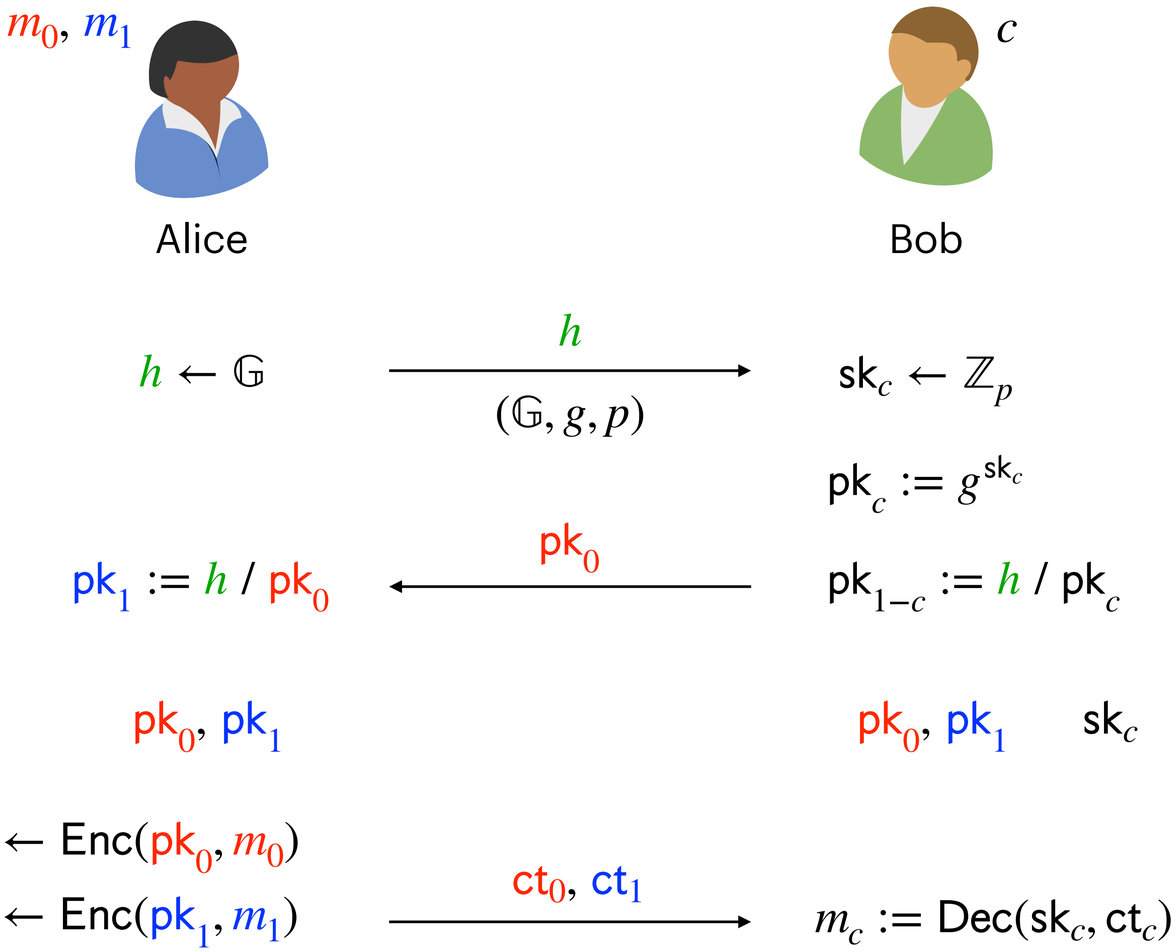
- **Simulator** $S_A(m_0, m_1)$
  - Run $A(m_0, m_1)$ and receive the first message h.
  - Sample $\mathsf{pk}_0 \xleftarrow{\$} \mathbb{G}$ as B's message.

Constructing Oblivious Transfer Protocol



- **Simulator** $S_A(m_0, m_1)$
  - Run $A(m_0, m_1)$ and receive the first message h.
  - Sample $\mathsf{pk}_0 \xleftarrow{\$} \mathbb{G}$ as B's message.
  - Receive ct_0, ct_1.

Constructing Oblivious Transfer Protocol

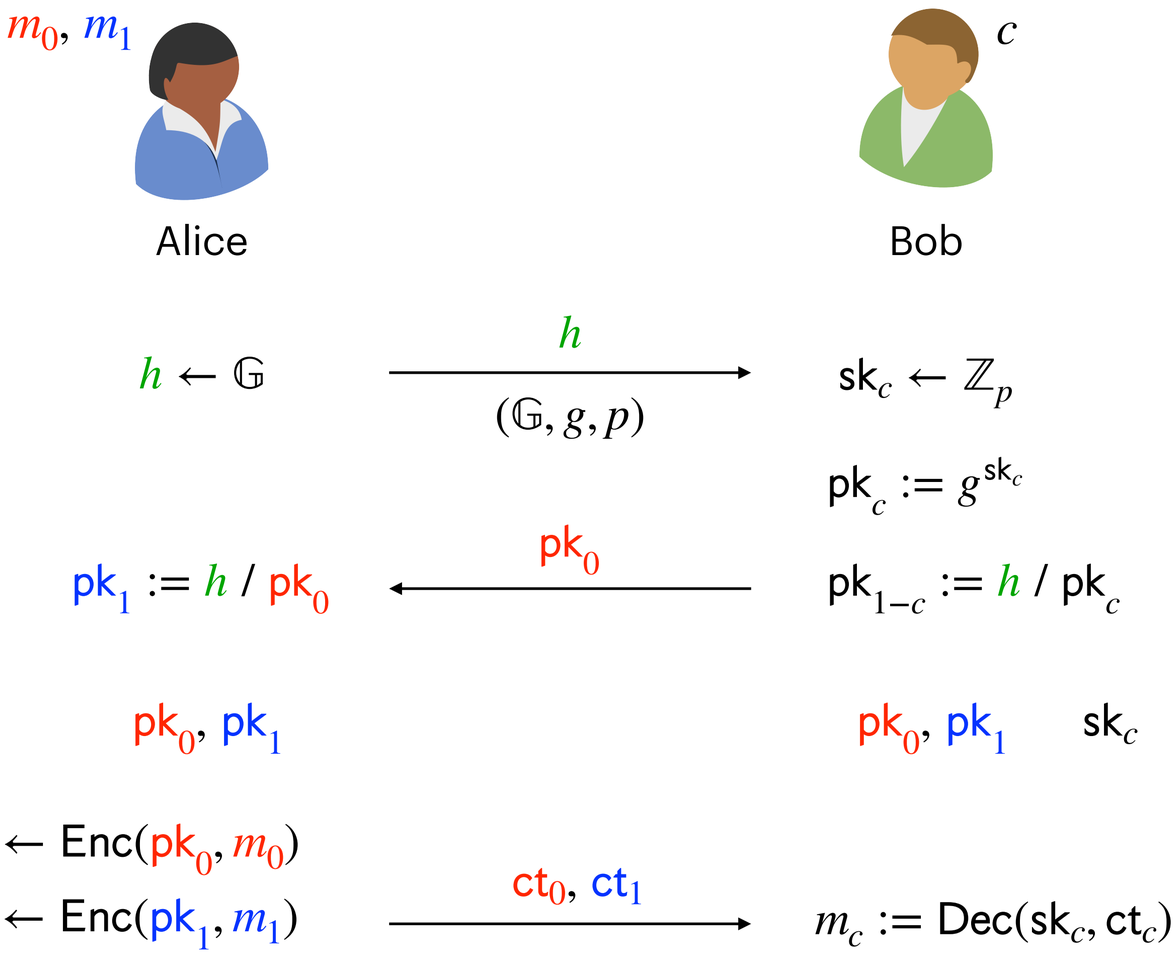


- **Simulator** $S_A(m_0, m_1)$
  - Run $A(m_0, m_1)$ and receive the first message h.
  - Sample $\mathsf{pk}_0 \xleftarrow{\$} \mathbb{G}$ as B's message.
  - Receive ct_0, ct_1.
- **Claim:**

$$\{S_A(m_0, m_1), m_c\} \overset{c}{\approx}$$

$$\{\mathsf{View}_A[A(m_0, m_1) \leftrightarrow B(c)], \mathsf{Output}[A(m_0, m_1) \leftrightarrow B(c)]\}$$

Constructing Oblivious Transfer Protocol

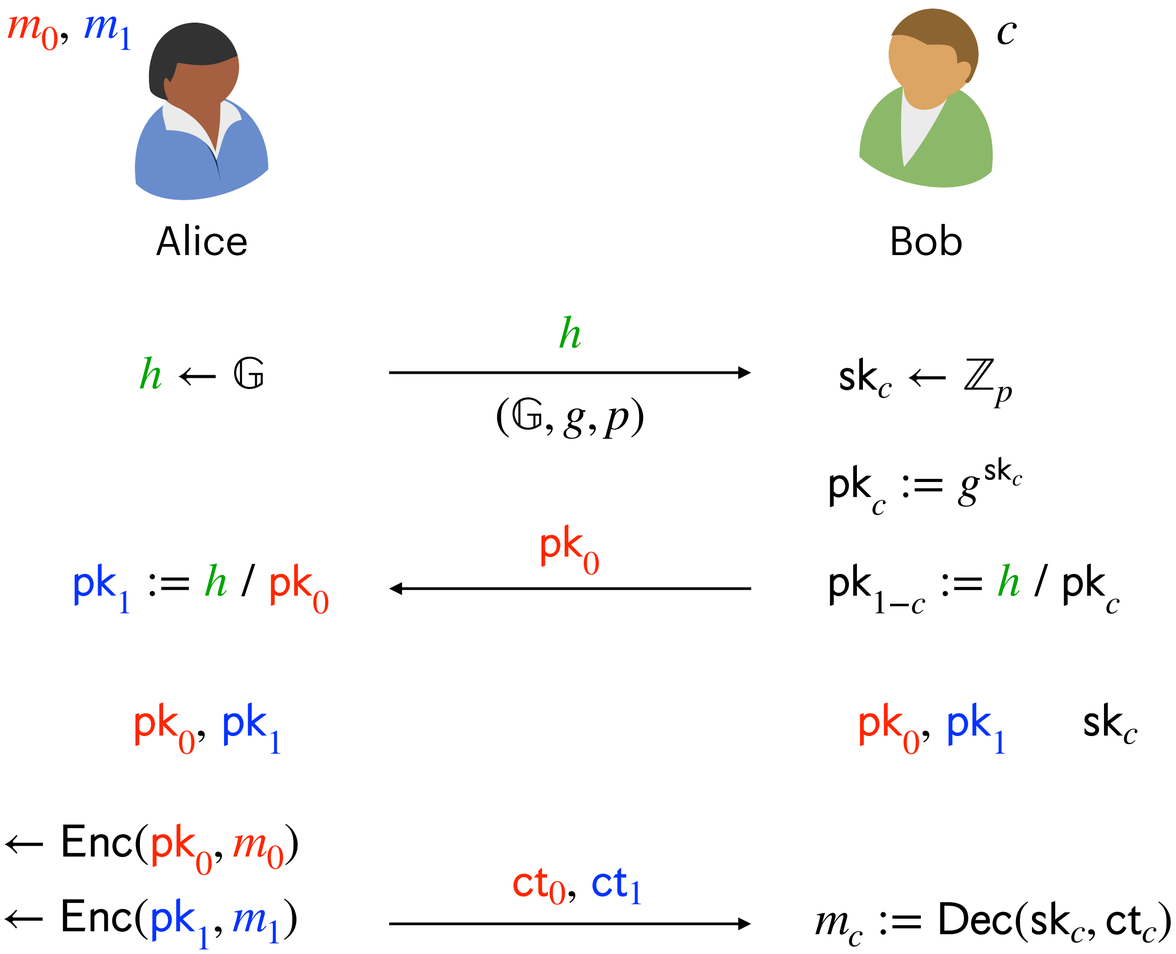


- **Simulator** $S_A(m_0, m_1)$
  - Run $A(m_0, m_1)$ and receive the first message h.
  - Sample $\mathsf{pk}_0 \xleftarrow{\$} \mathbb{G}$ as B's message.
  - Receive ct_0, ct_1.
- **Claim:**

$$\{S_A(m_0, m_1)\} \overset{c}{\approx} \{\mathsf{View}_A[A(m_0, m_1) \leftrightarrow B(c)]\}.$$

Equivalent definitions for deterministic programs.

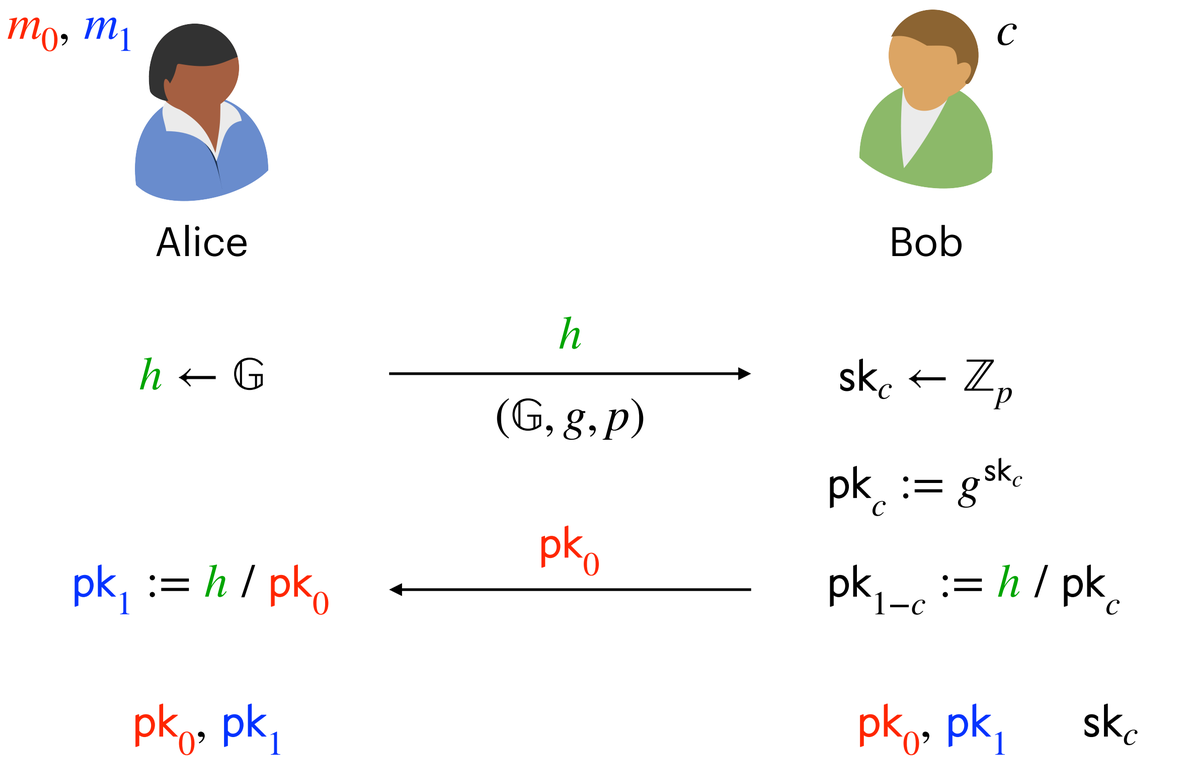
Constructing Oblivious Transfer Protocol



- **Simulator** $S_A(m_0, m_1)$
  - Run $A(m_0, m_1)$ and receive the first message h.
  - Sample $\mathsf{pk}_0 \xleftarrow{\$} \mathbb{G}$ as B's message.
  - Receive ct_0, ct_1.
- **Claim:**

$$\{S_A(m_0, m_1)\} \overset{c}{\approx} \{\mathsf{View}_A[A(m_0, m_1) \leftrightarrow B(c)]\}.$$

Constructing Oblivious Transfer Protocol

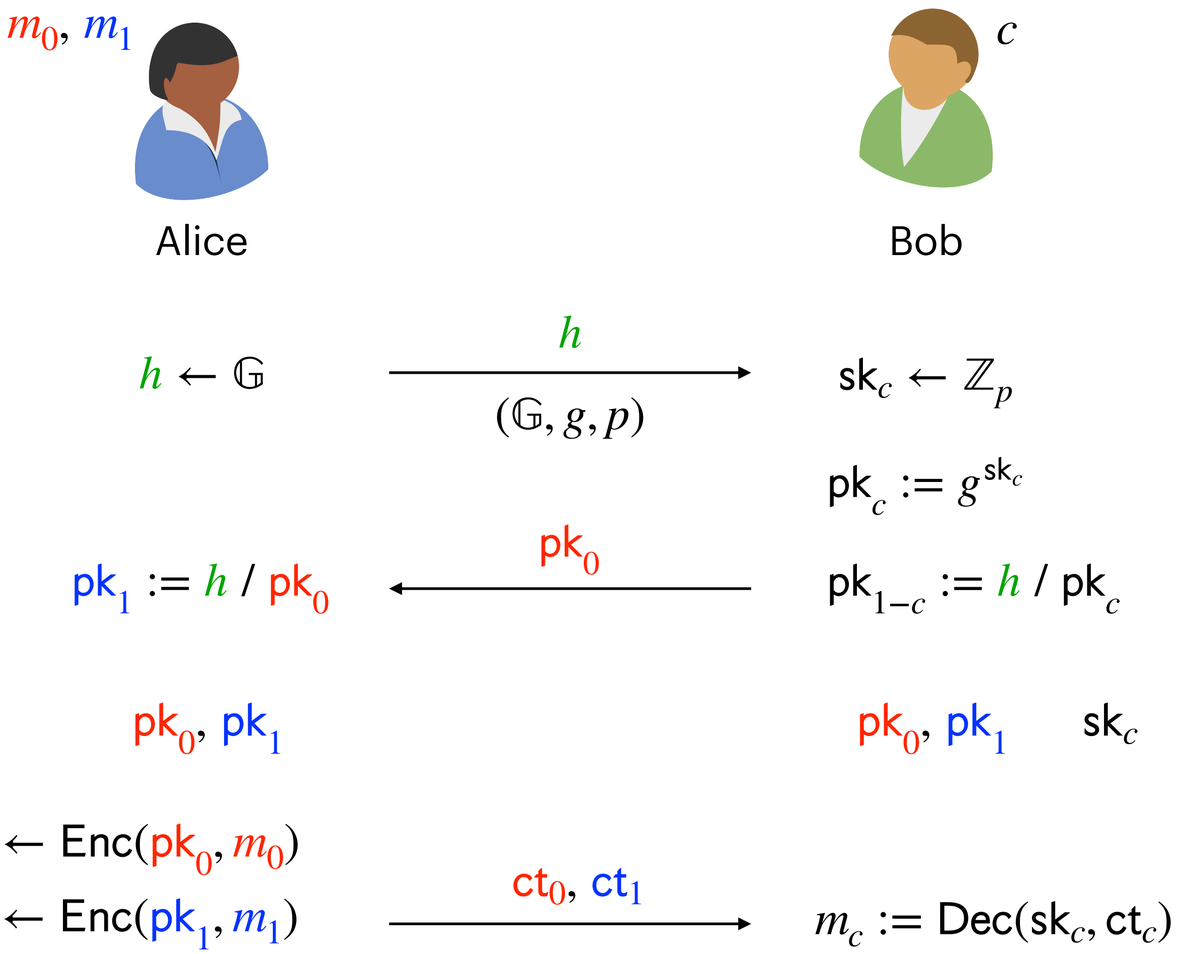


- **Simulator** $S_A(m_0, m_1)$
  - Run $A(m_0, m_1)$ and receive the first message h.
  - Sample $\mathsf{pk}_0 \xleftarrow{\$} \mathbb{G}$ as B's message.
  - Receive ct_0, ct_1.
- **Claim:**

  $\{S_A(m_0, m_1)\} \overset{c}{\approx} \{\mathsf{View}_A[A(m_0, m_1) \leftrightarrow B(c)]\}.$

  - The only difference is in computation of pk_0.

Constructing Oblivious Transfer Protocol

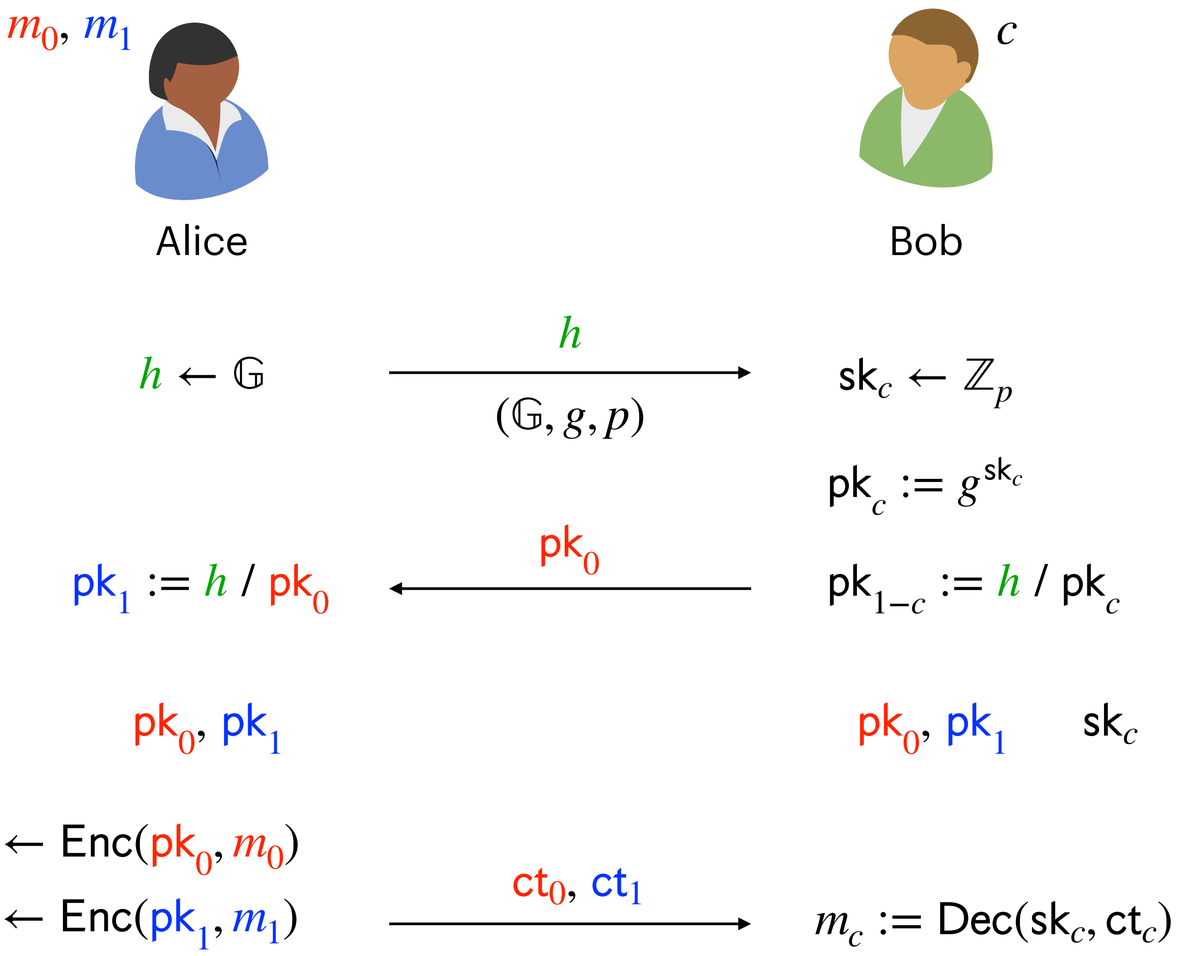


- **Simulator** $S_A(m_0, m_1)$
  - Run $A(m_0, m_1)$ and receive the first message h.
  - Sample $\mathsf{pk}_0 \xleftarrow{\$} \mathbb{G}$ as B's message.
  - Receive ct_0, ct_1.
- **Claim:**

  $\{S_A(m_0, m_1)\} \overset{c}{\approx} \{\mathsf{View}_A[A(m_0, m_1) \leftrightarrow B(c)]\}.$

  - The only difference is in computation of pk_0.
  - If $c = 0$, then Bob sends uniformly random pk_0 since sk_c is uniformly random and $\mathsf{pk}_0 := g^{\mathsf{sk}_c}$.

Constructing Oblivious Transfer Protocol



- **Simulator** $S_A(m_0, m_1)$
  - Run $A(m_0, m_1)$ and receive the first message h.
  - Sample $\mathsf{pk}_0 \xleftarrow{\$} \mathbb{G}$ as B's message.
  - Receive ct_0, ct_1.
- **Claim:**

$$\{S_A(m_0, m_1)\} \overset{c}{\approx} \{\mathsf{View}_A[A(m_0, m_1) \leftrightarrow B(c)]\}.$$

  - The only difference is in computation of pk_0.
  - If $c = 0$, then Bob sends uniformly random pk_0 since sk_c is uniformly random and $\mathsf{pk}_0 := g^{\mathsf{sk}_c}$.
  - If $c = 1$, pk_1 is uniformly random, which means that $\mathsf{pk}_0 := h \,/\, \mathsf{pk}_1$ is uniformly random.

Constructing Oblivious Transfer Protocol

m_0, m_1

Alice

c

Bob

$h \leftarrow \mathbb{G}$

h

$(\mathbb{G}, g, p)$

$\mathsf{sk}_c \leftarrow \mathbb{Z}_p$

$\mathsf{pk}_c := g^{\mathsf{sk}_c}$

$\mathsf{pk}_1 := h \,/\, \mathsf{pk}_0$

pk_0

$\mathsf{pk}_{1-c} := h \,/\, \mathsf{pk}_c$

$\mathsf{pk}_0, \mathsf{pk}_1$

$\mathsf{pk}_0, \mathsf{pk}_1 \quad \mathsf{sk}_c$

$\mathsf{ct}_0 \leftarrow \mathsf{Enc}(\mathsf{pk}_0, m_0)$

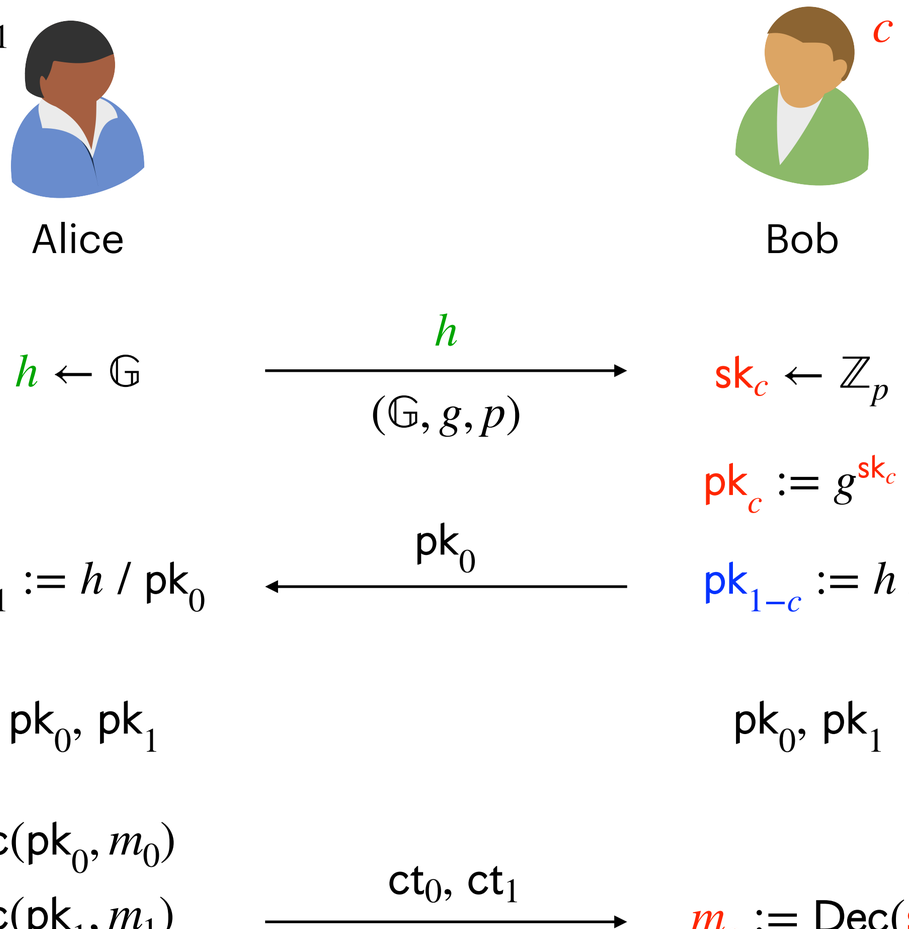
$\mathsf{ct}_1 \leftarrow \mathsf{Enc}(\mathsf{pk}_1, m_1)$

$\mathsf{ct}_0, \mathsf{ct}_1$

$m_c := \mathsf{Dec}(\mathsf{sk}_c, \mathsf{ct}_c)$

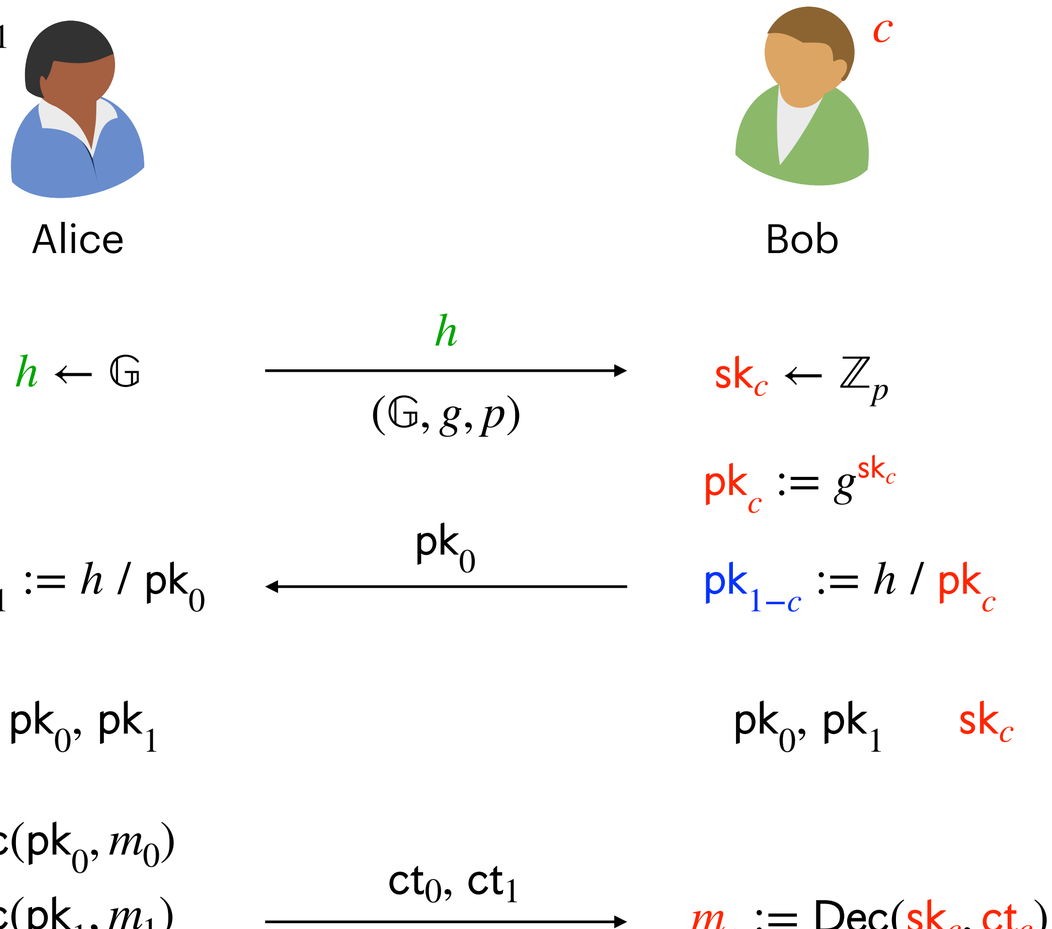
Constructing Oblivious Transfer Protocol

- **Simulator** $S_B(c, m_c)$

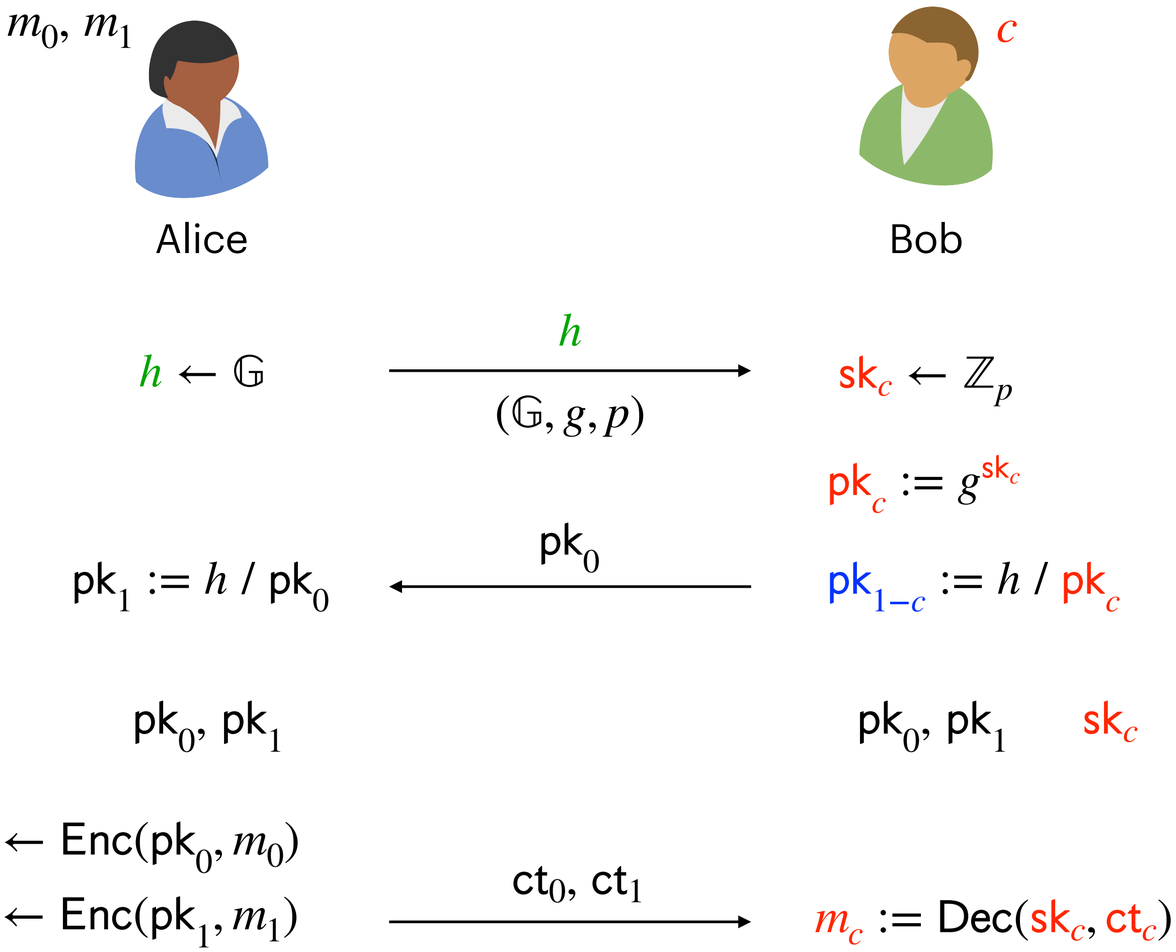


Constructing Oblivious Transfer Protocol

- **Simulator** $S_B(c, m_c)$
  - Run $B(c)$.

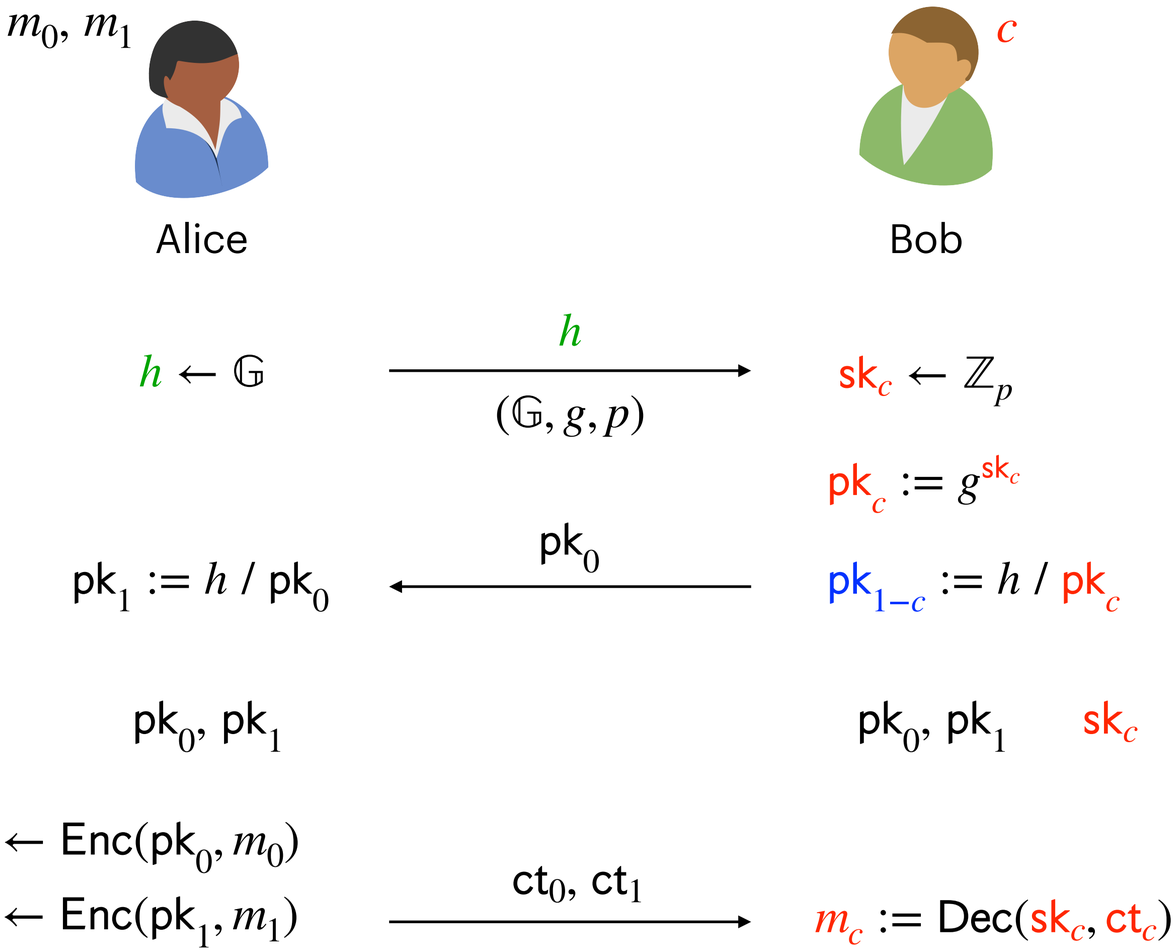


Constructing Oblivious Transfer Protocol



- **Simulator** $S_B(c, m_c)$
  - Run $B(c)$.
  - Sample $h \xleftarrow{\$} \mathbb{G}$ uniformly at random as A's message.

Constructing Oblivious Transfer Protocol



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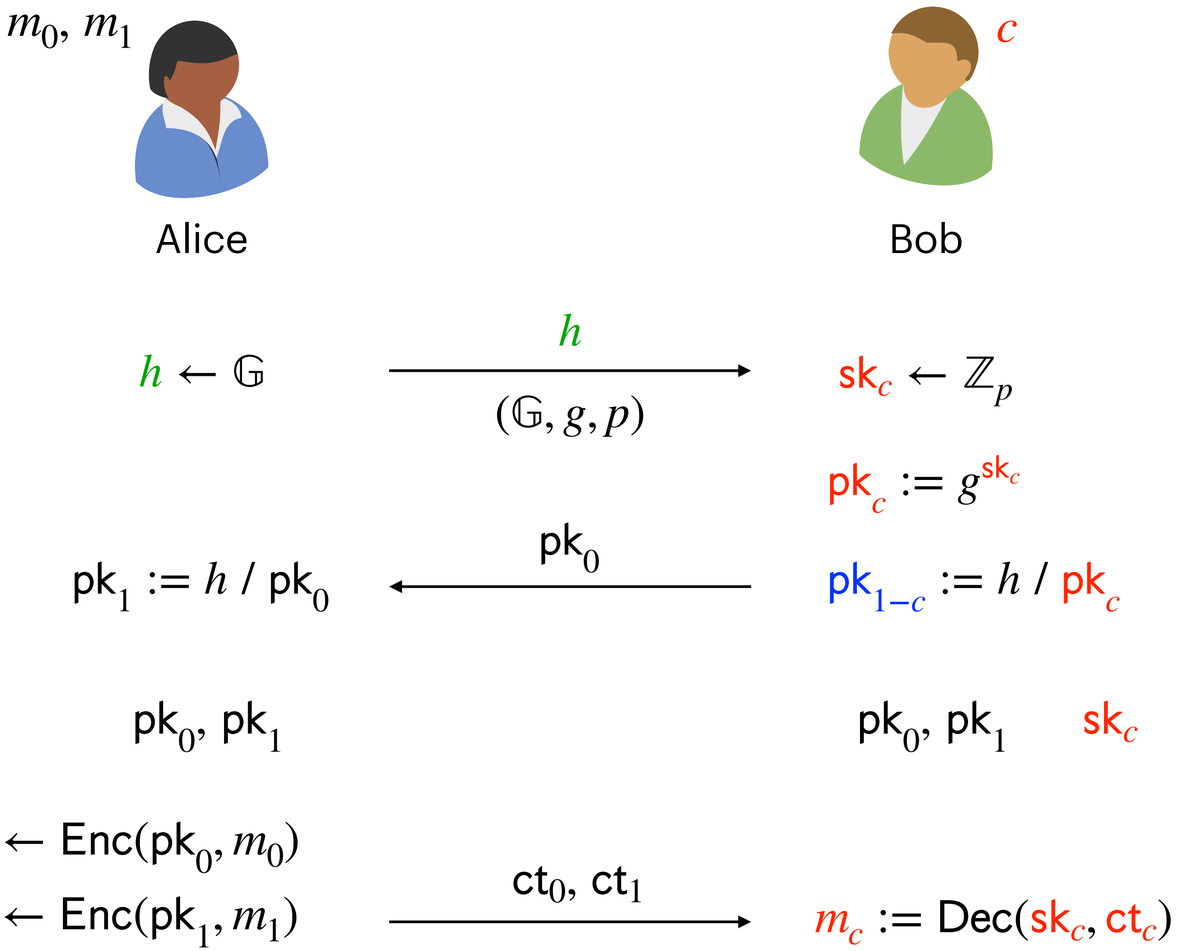
  - Run $B(c)$.
  - Sample $h \xleftarrow{\$} \mathbb{G}$ uniformly at random as A's message.
  - Receive pk_0.

Constructing Oblivious Transfer Protocol



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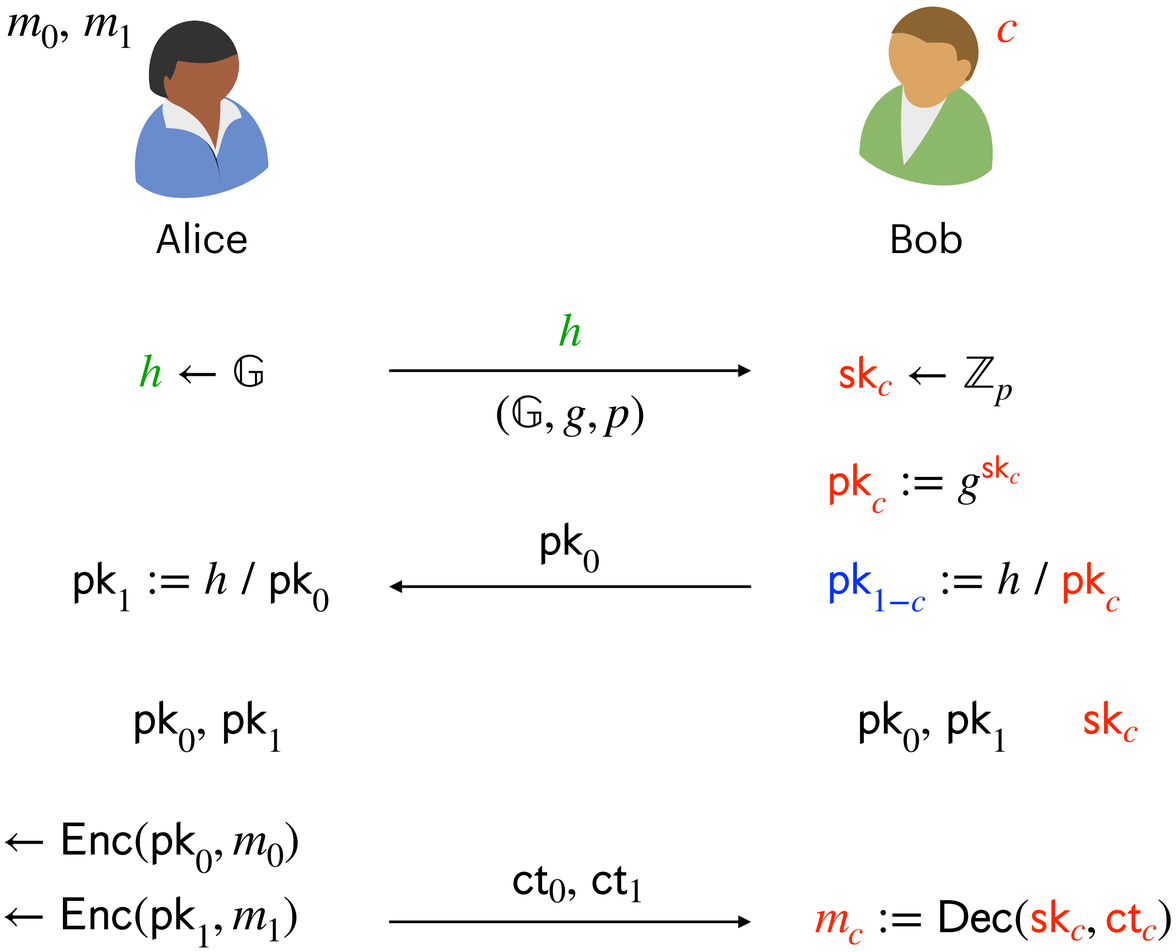
Constructing Oblivious Transfer Protocol



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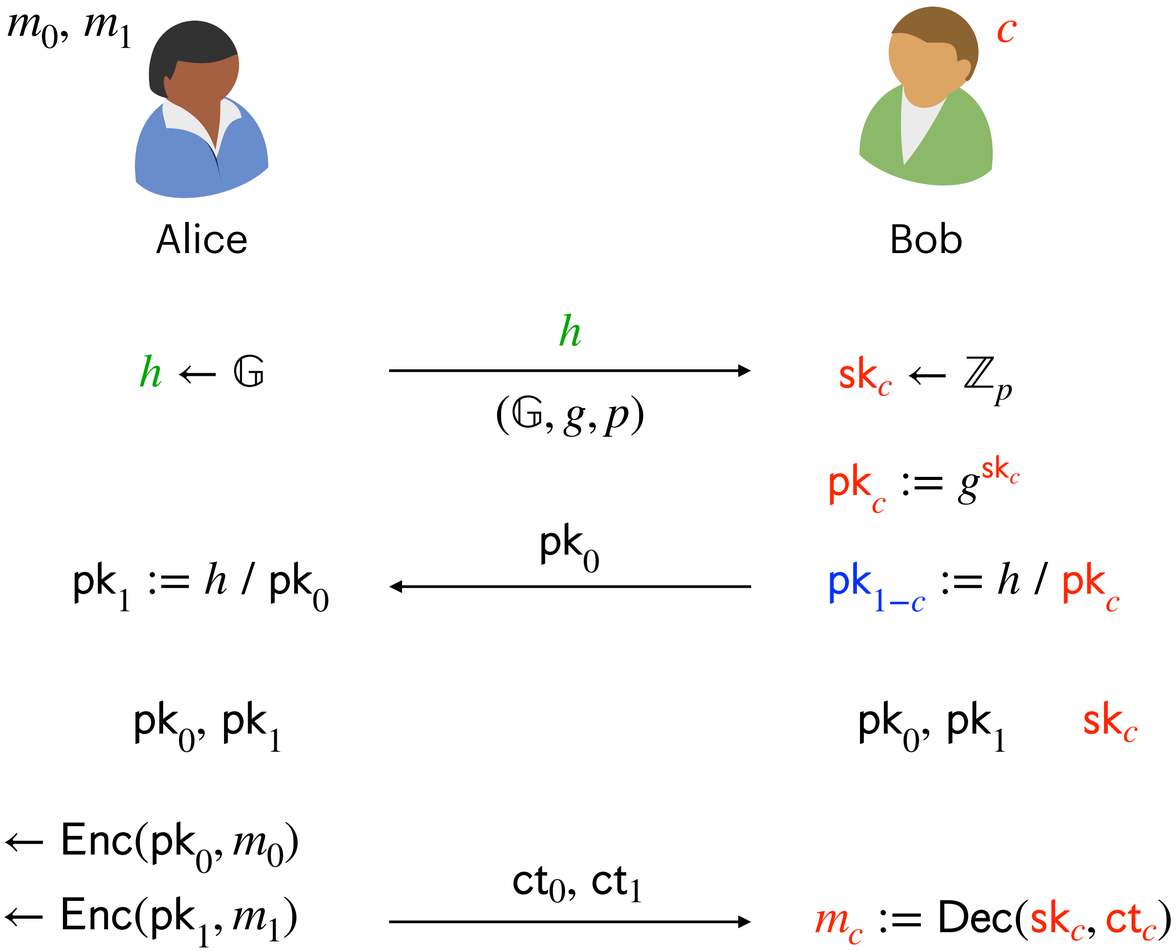

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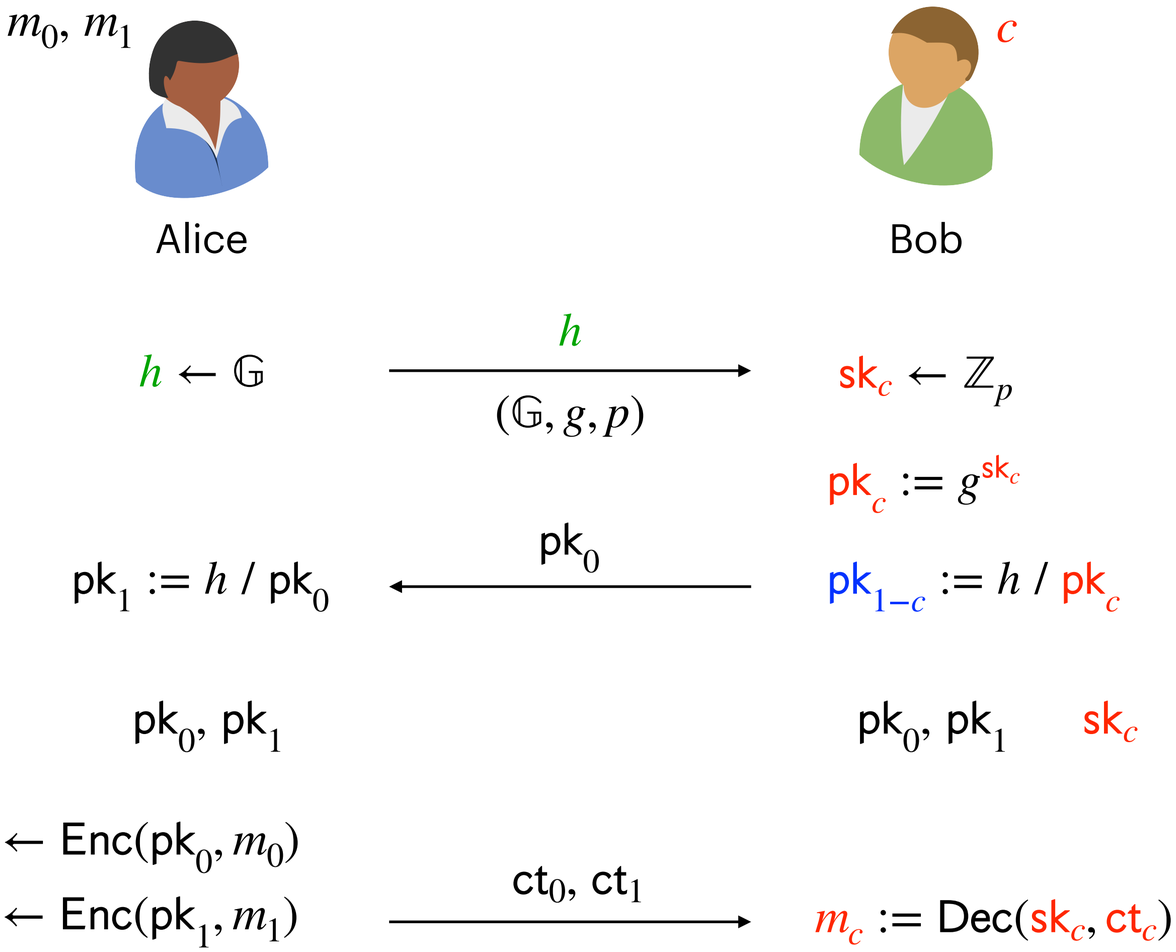

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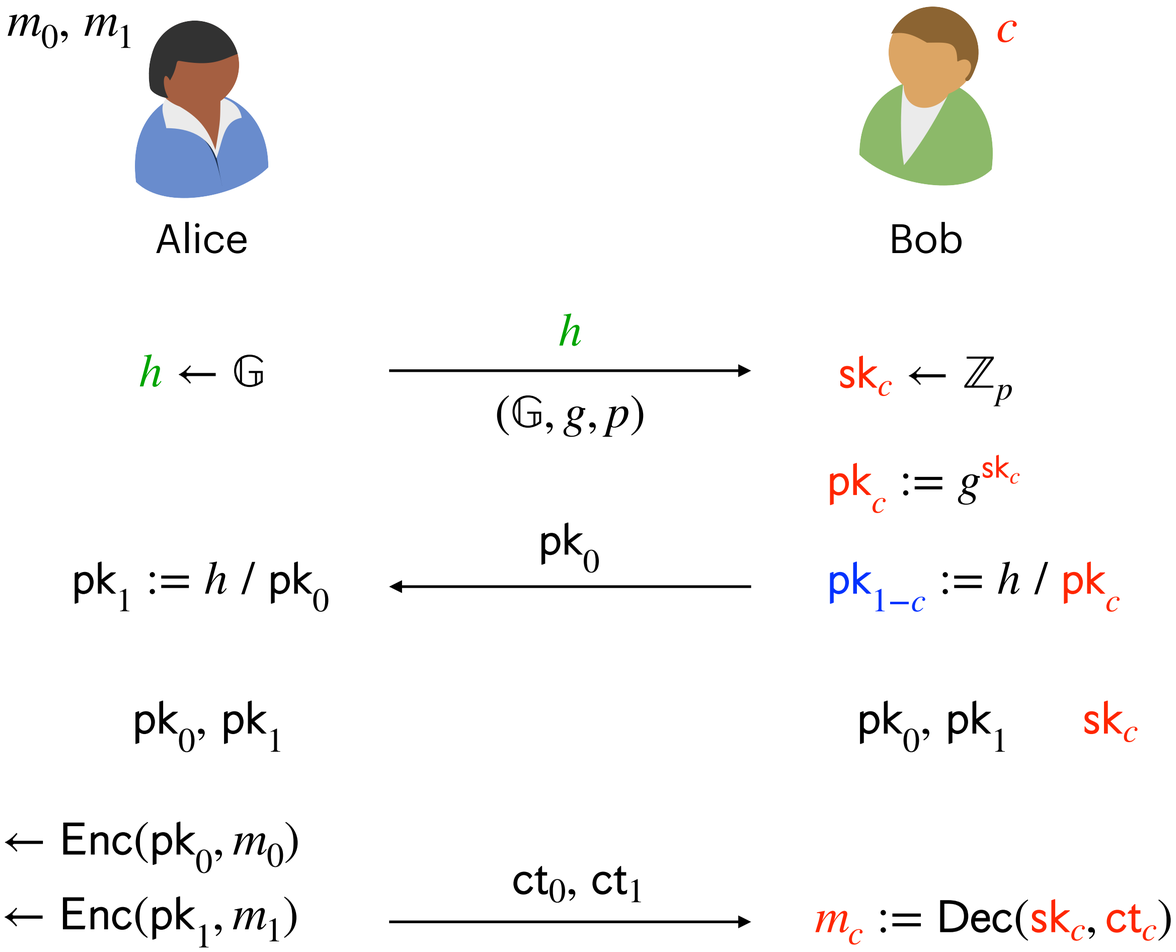

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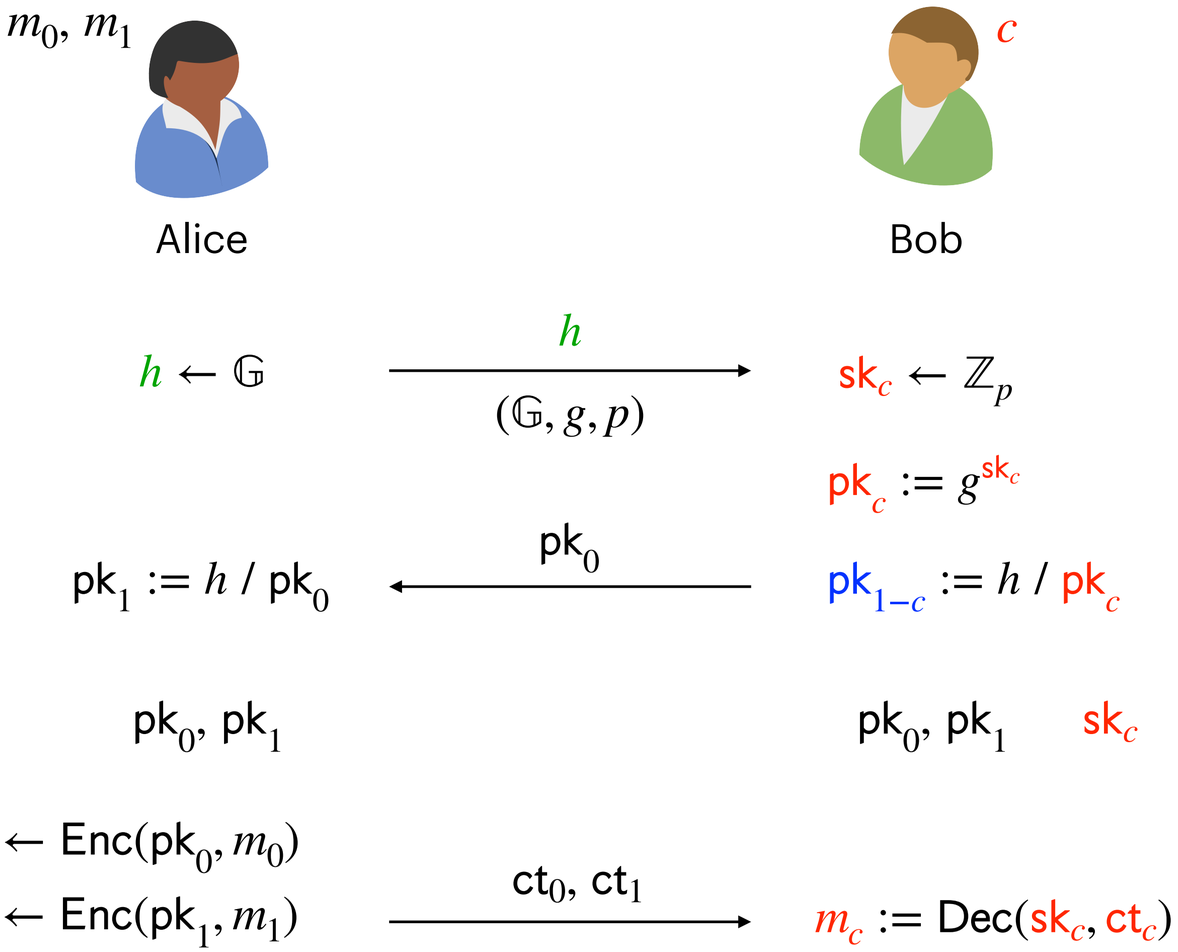

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  - The transformation uses a coin-flipping protocol (to make sure that the adversary's random tape is truly random) and zero-knowledge proofs (to make sure the adversary is following the protocol description).