Pseudorandomness

601.442/642 Modern Cryptography

3rd February 2026

Announcement

- Homework 2 is due on **5th Feb**.

Computational Indistinguishability

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Ensures A is polynomial in λ.

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Two probability ensembles $X = \{X_i\}_{i \in \mathbb{N}}$ and $Y = \{Y_i\}_{i \in \mathbb{N}}$ are computationally indistinguishable if every non-uniform PPT adversary A, there exists a negligible function $\mu(\lambda)$ such that for all $\lambda \in \mathbb{N}$

$$\left| \Pr_{x \leftarrow X_\lambda}\left[A(1^\lambda, x) = 1\right] - \Pr_{y \leftarrow Y_\lambda}\left[A(1^\lambda, y) = 1\right] \right| \leq \nu(\lambda),$$

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No efficient test can distinguish between the ensembles X and Y.

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- We use $X \overset{c}{\approx} Y$ as a shorthand to denote that the two ensembles are computationally indistinguishable.

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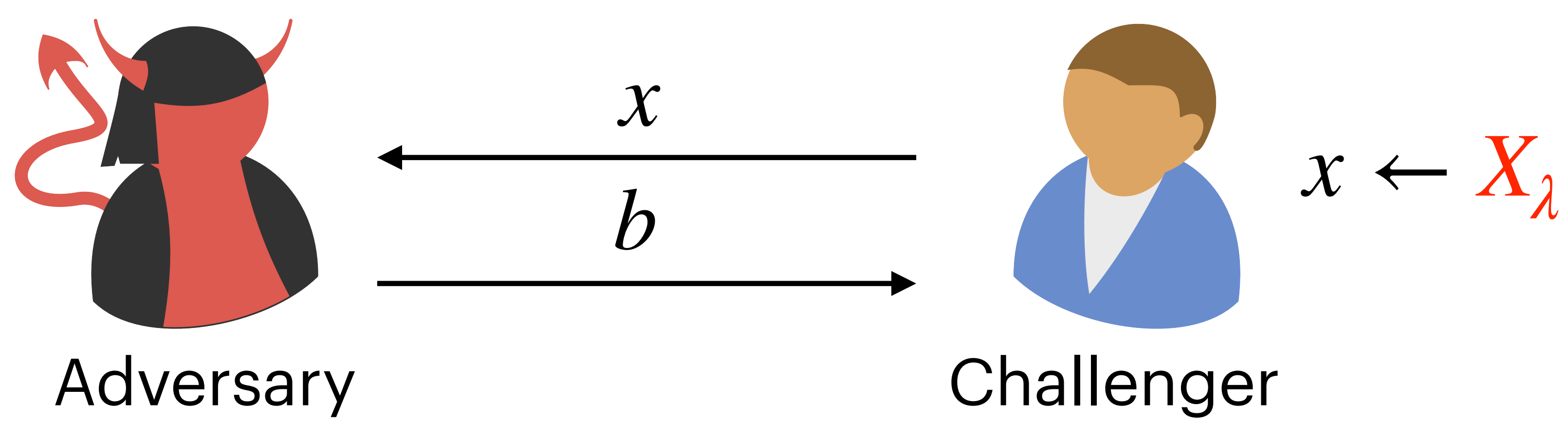
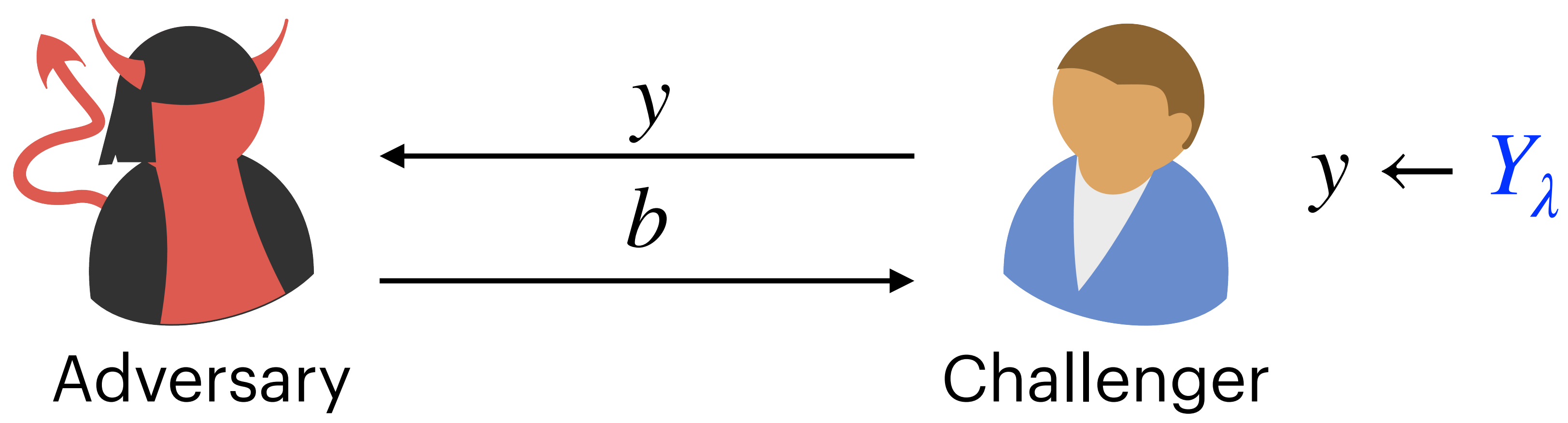
- $X \overset{c}{\approx} Y$ if all non-uniform PPT adversaries have negligible advantage in distinguishing between the two ensembles.

Computational Indistinguishability





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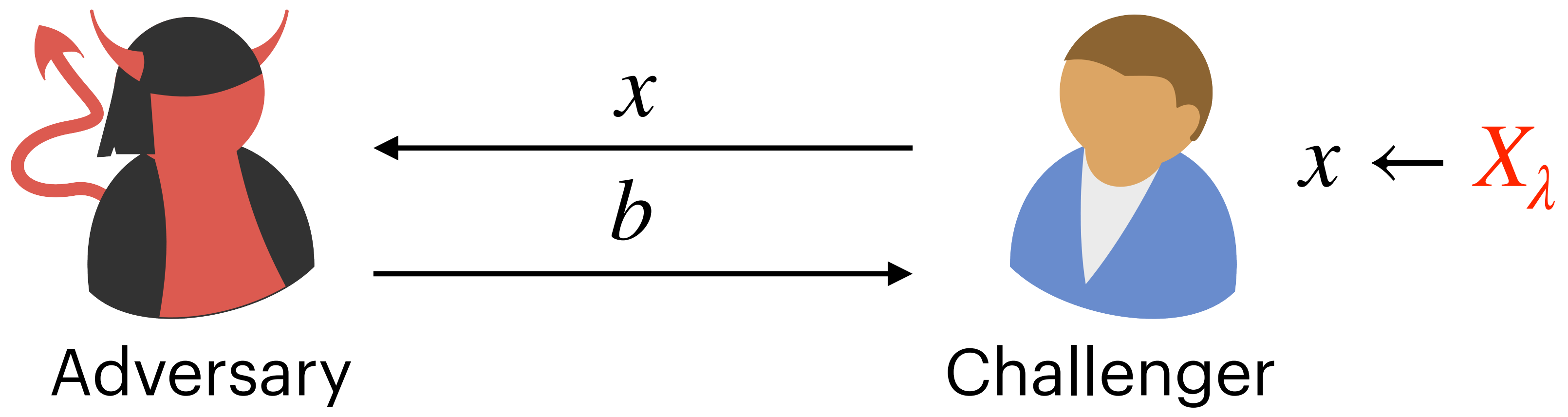

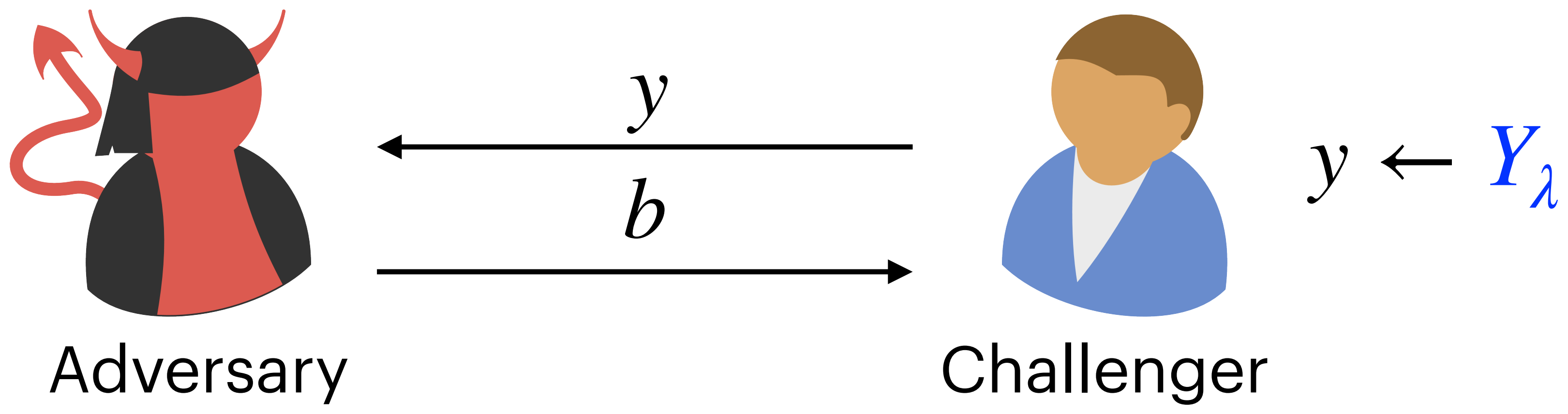

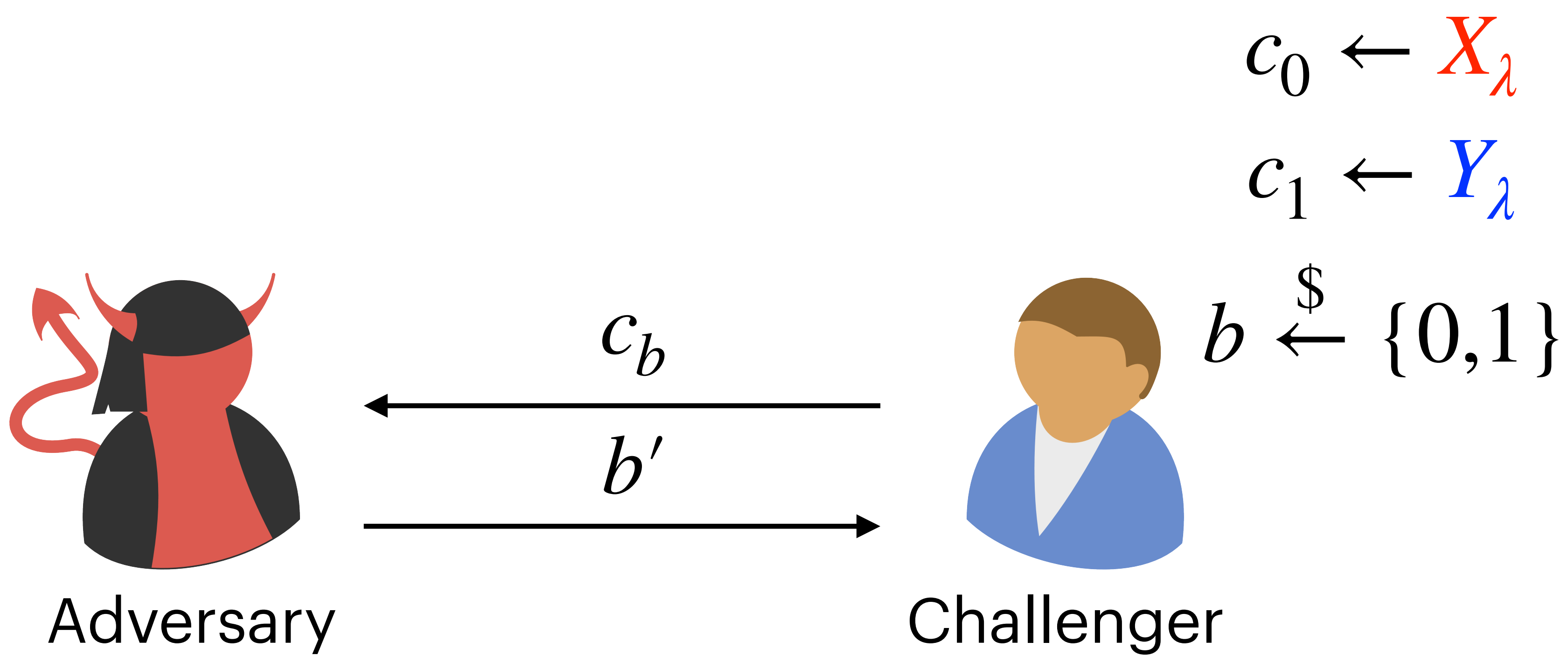

A wins if $b = b'$.

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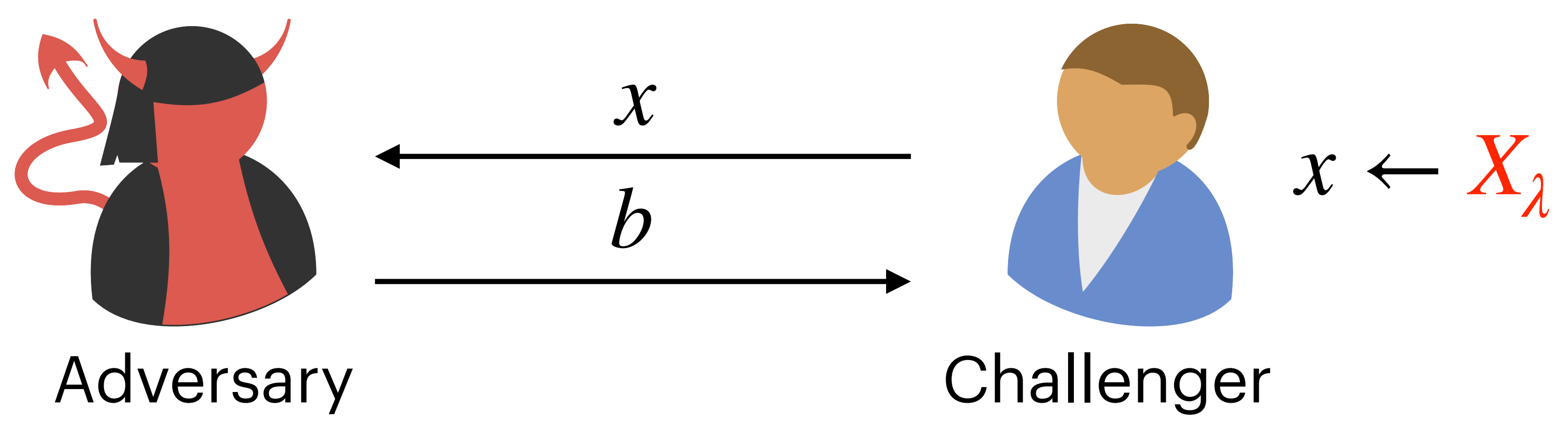

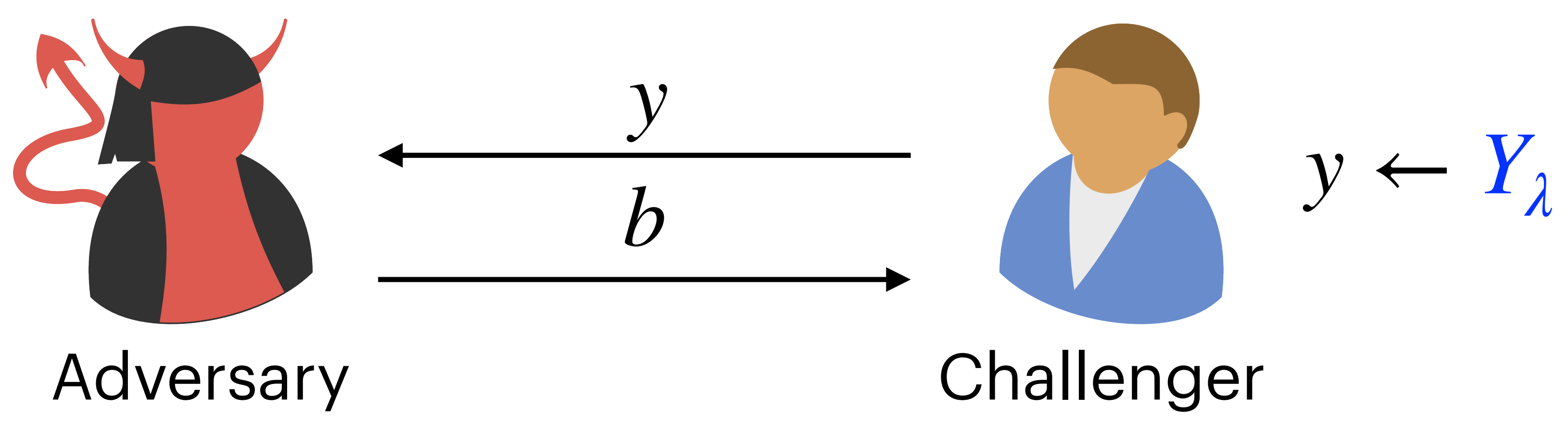

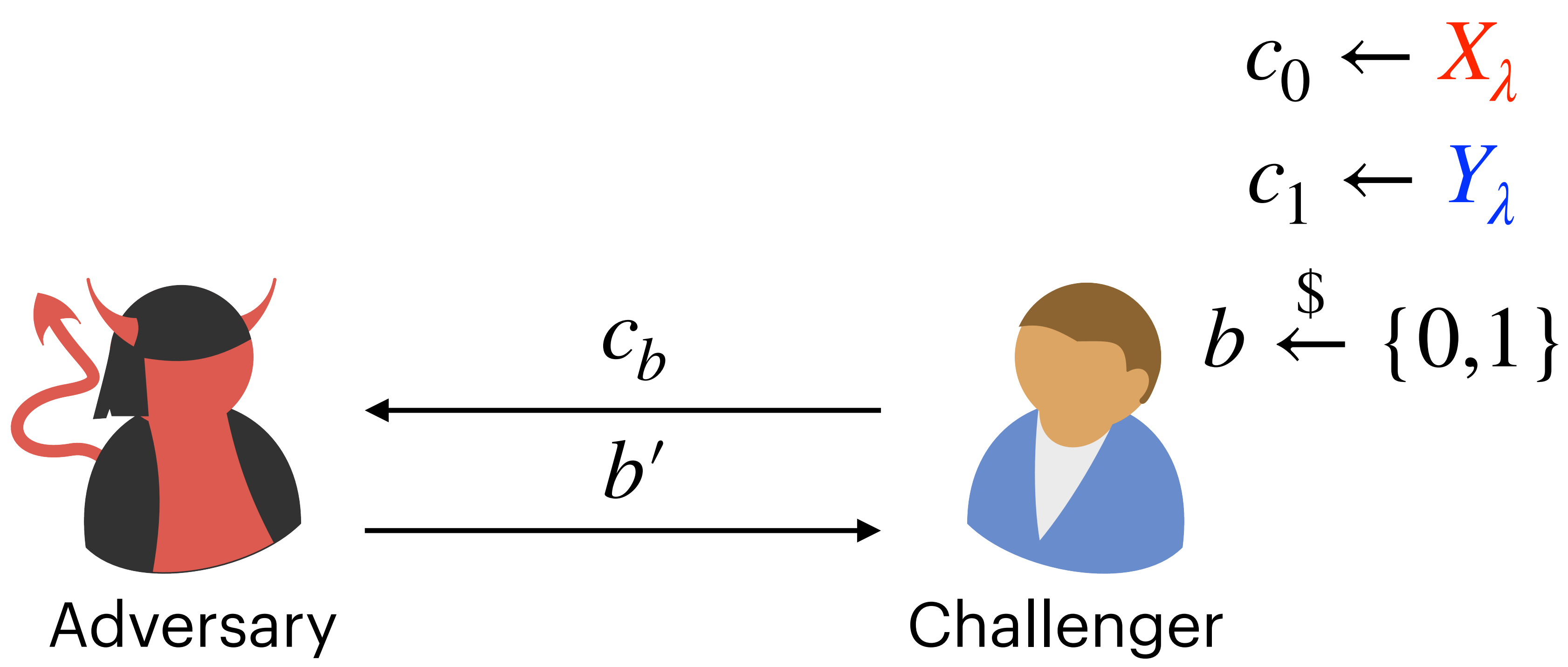

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Advantage:

$$\left| \Pr\left[A(1^\lambda, c_b) = 1 \mid b = 0\right] - \Pr\left[A(1^\lambda, c_b) = 1 \mid b = 1\right] \right| \leq \mathsf{negl}(\lambda)$$

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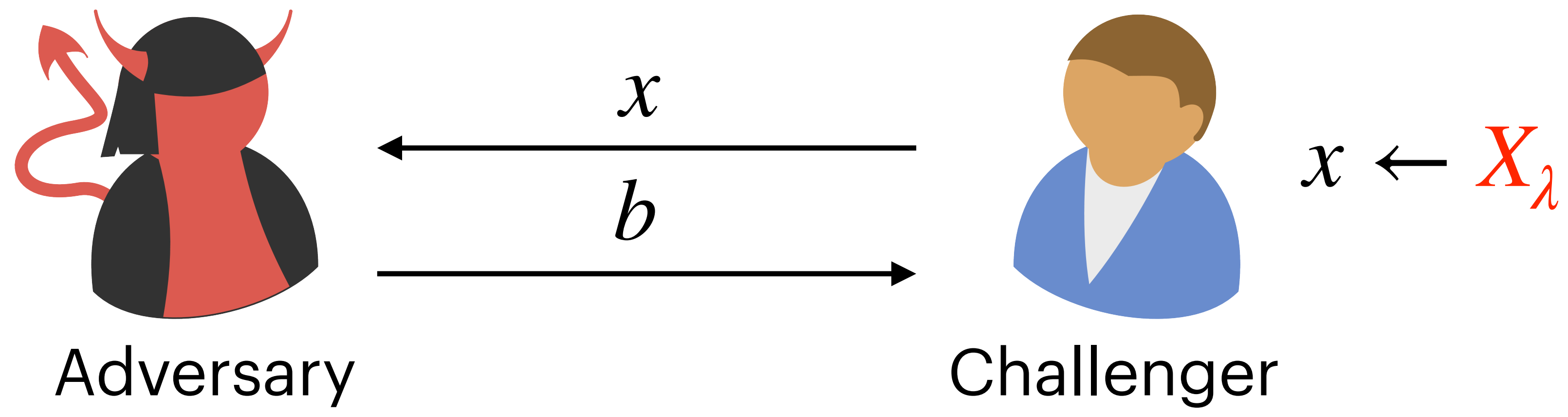

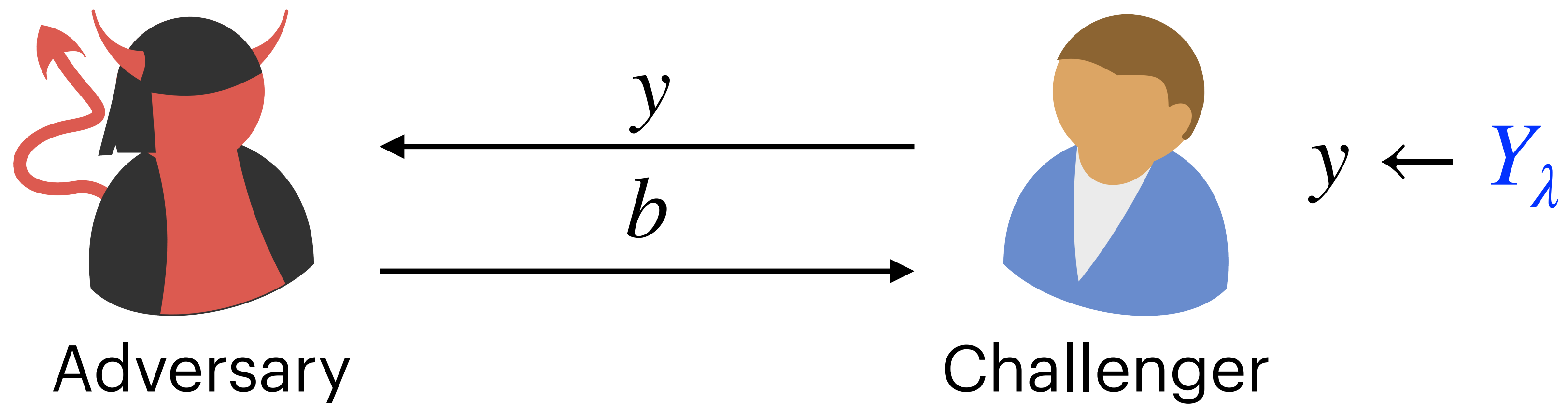

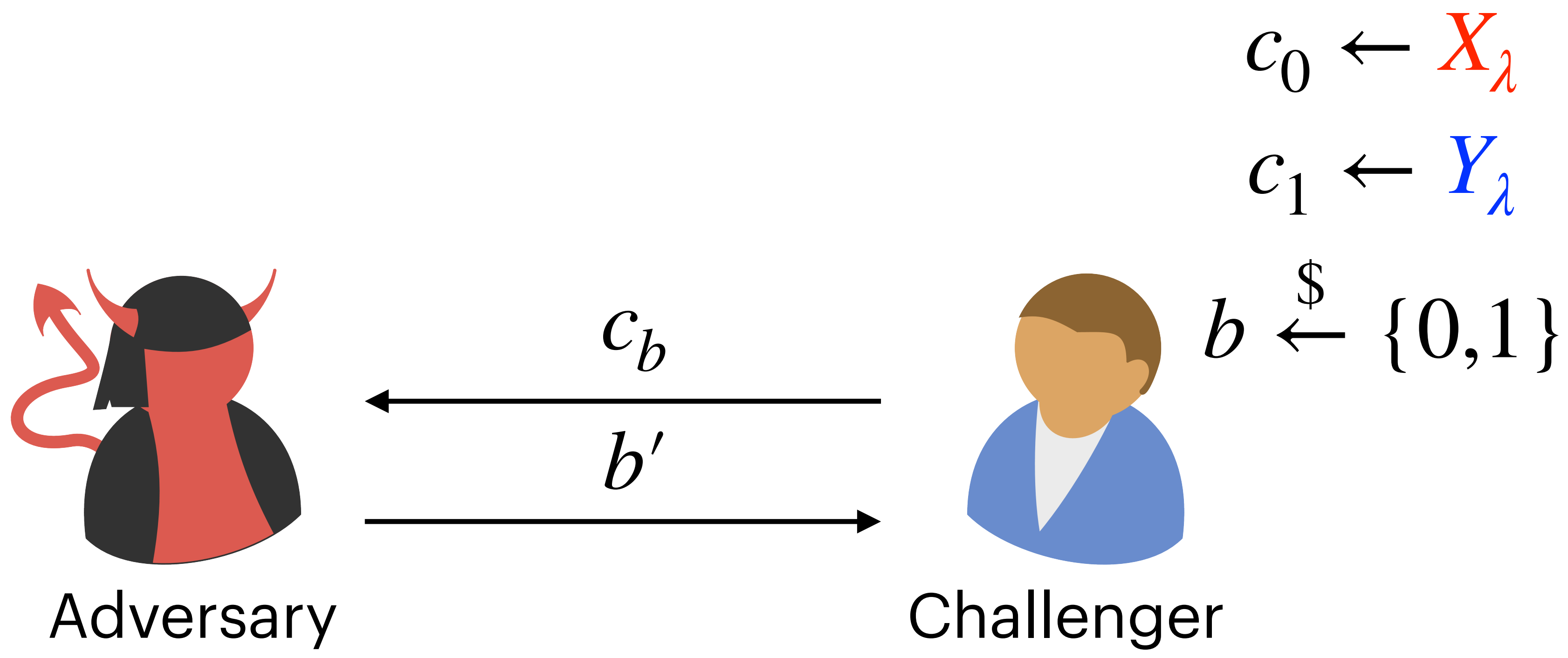

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Bit-Guessing:

$$\Pr\left[b' = b : \begin{array}{c} c_0 \leftarrow X_\lambda \\ c_1 \leftarrow Y_\lambda \\ b \xleftarrow{\$} \{0,1\} \\ b' \leftarrow A(1^\lambda, c_b) \end{array} \right] \leq \frac{1}{2} + \mathsf{negl}(\lambda)\,.$$

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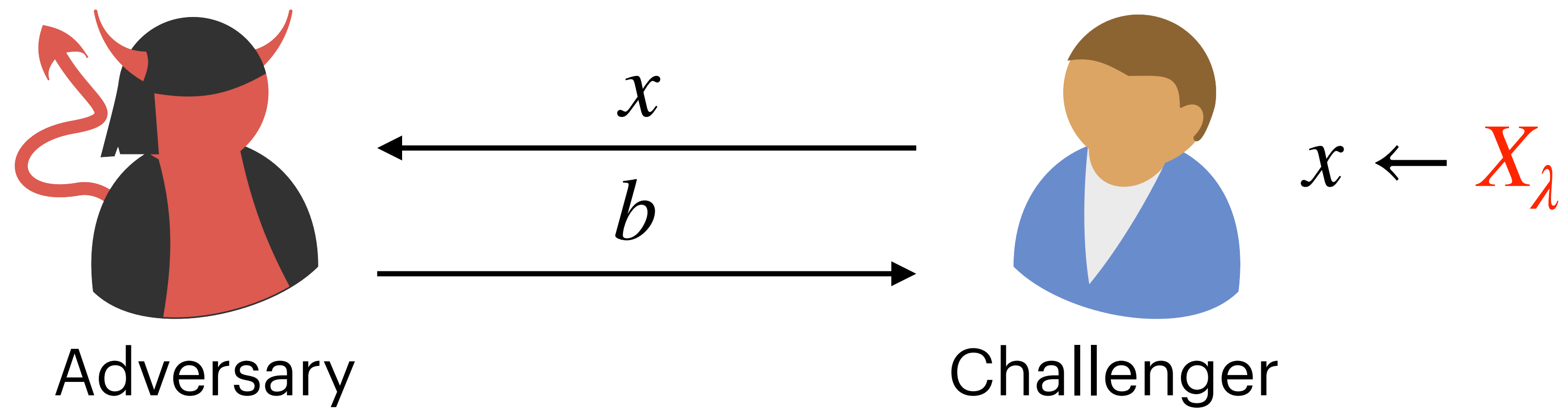

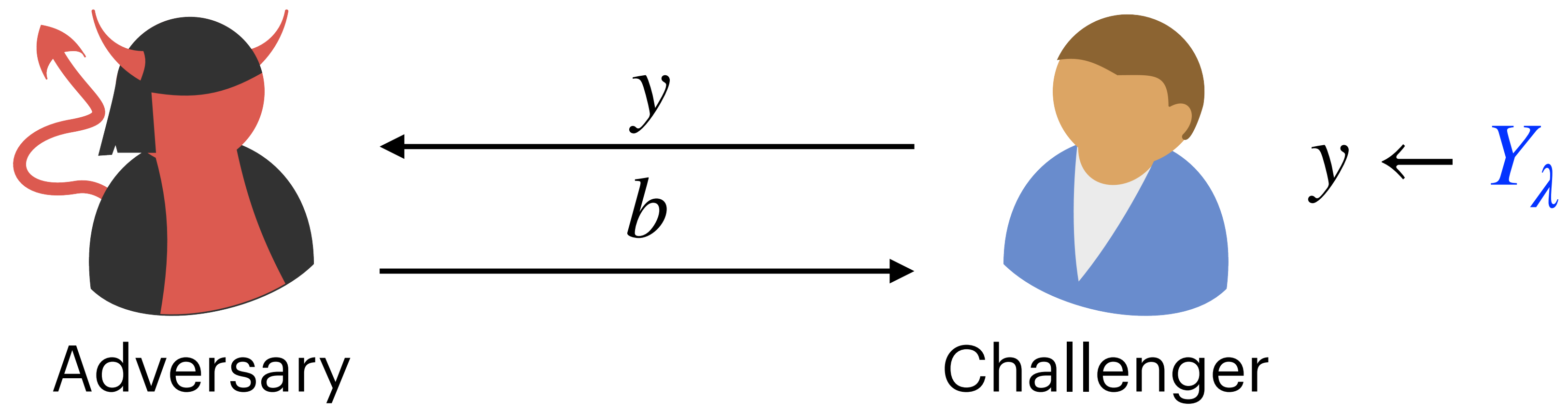

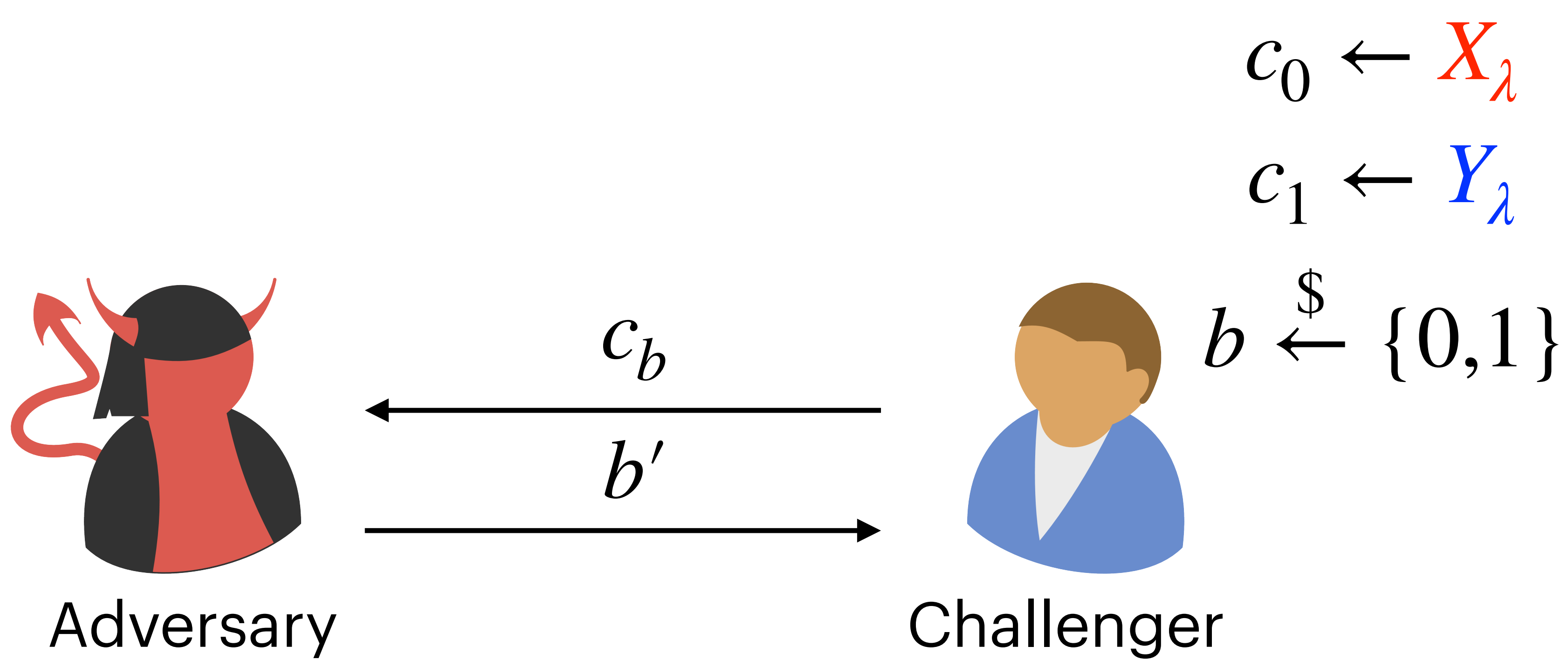

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Exercise: Prove both definitions are equivalent.

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$$Y \overset{c}{\approx} Z \quad \Longrightarrow \quad \left| \Pr_{y \leftarrow Y_\lambda}\left[A(1^\lambda, y) = 1\right] - \Pr_{z \leftarrow Z_\lambda}\left[A(1^\lambda, z) = 1\right] \right| \leq \nu_2(\lambda)\,.$$

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Thus $\left| \Pr_{x \leftarrow X_\lambda}\left[A(1^\lambda, x) = 1\right] - \Pr_{z \leftarrow Z_\lambda}\left[A(1^\lambda, z) = 1\right] \right|$

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> Thus $$\left| \Pr_{x \leftarrow X_\lambda}\left[A(1^\lambda, x) = 1\right] - \Pr_{z \leftarrow Z_\lambda}\left[A(1^\lambda, z) = 1\right] \right| \leq \nu_1(\lambda) + \nu_2(\lambda) \leq \nu(\lambda)\,.$$

Generalizing Transitivity: Hybrid Lemma

Lemma (Hybrid Lemma): Let λ be the security parameter and $n := n(\lambda)$ be a polynomial. If $X_1, \ldots, X_n$ be probability ensembles such that for all $i \in \{0, \ldots, n-1\}$ $X_i \overset{c}{\approx} X_{i+1}$, then $X_1 \overset{c}{\approx} X_n$.

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This is the hybrid technique, stated more generally, in the computational setting.

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Used in most crypto proofs!

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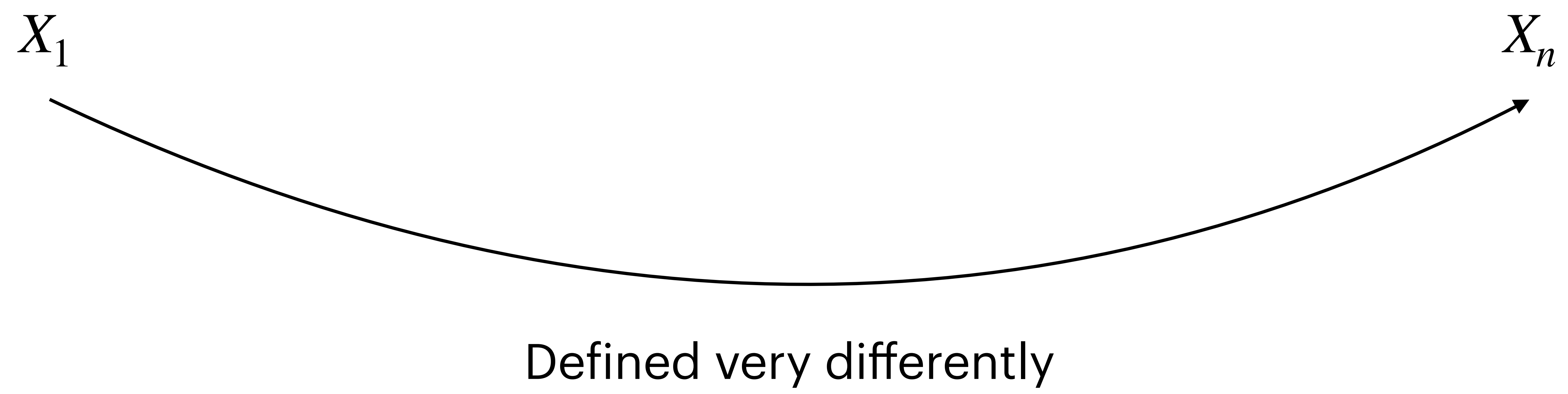

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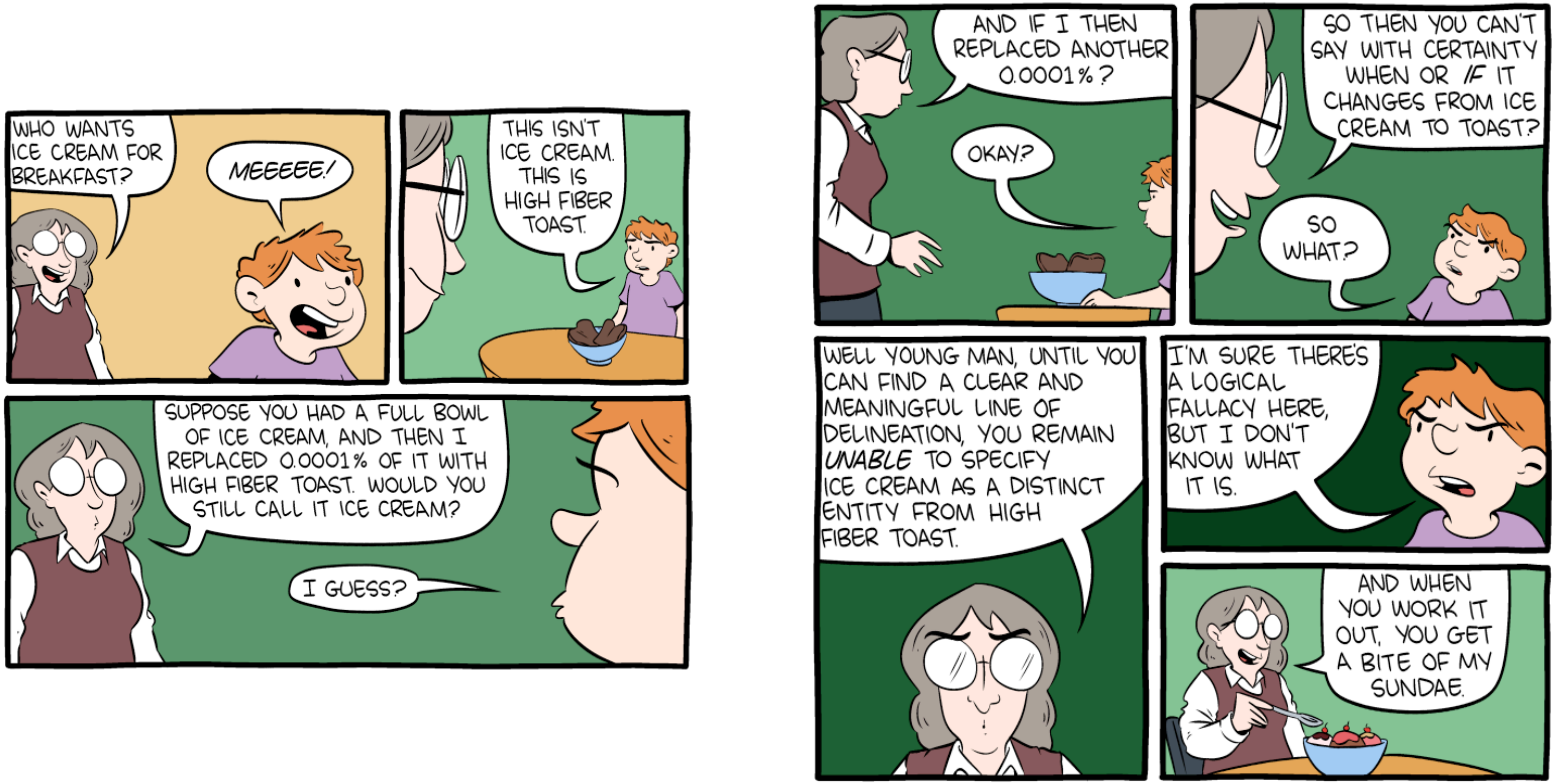
Generalizing Transitivity: Hybrid Lemma



Overcoming Shannon's Bound

- Let's take another look at OTP:

One-Time Pad

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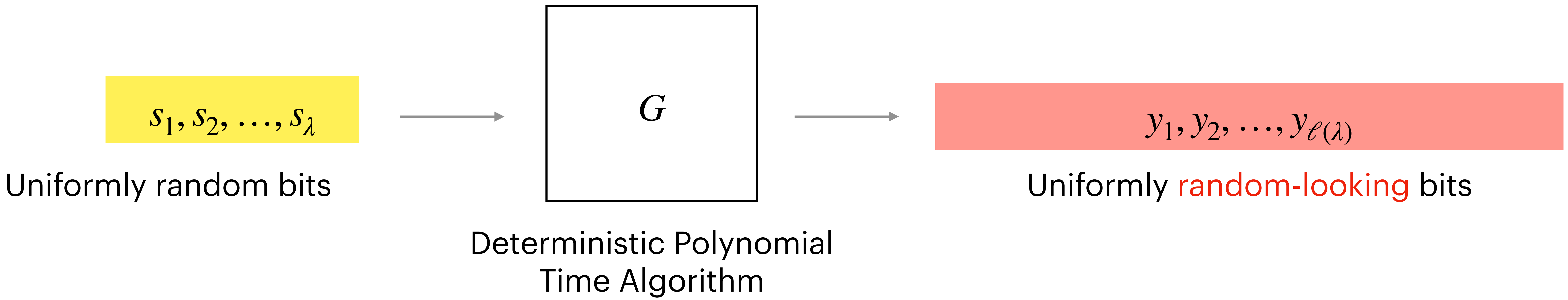
  - What if we can expand a few random bits into many random "looking" bits?

Pseudorandom Generator



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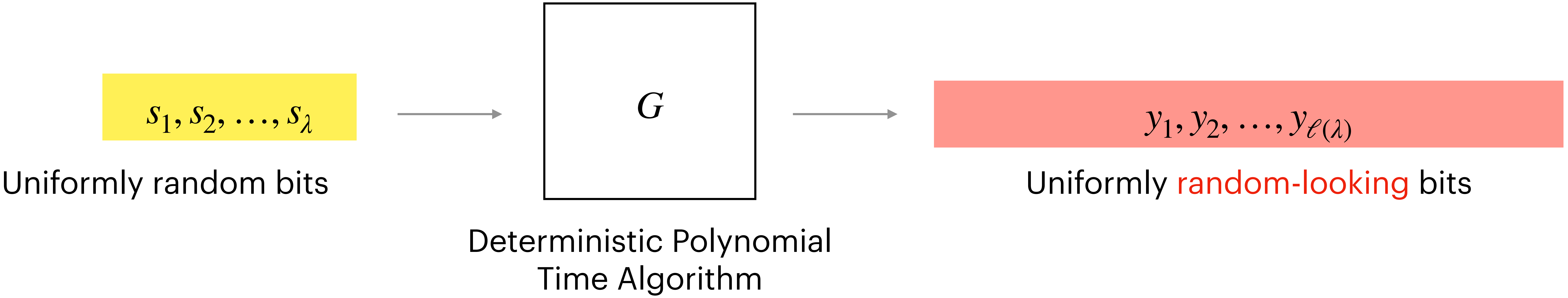

- A **pseudorandom generator** $G : \{0,1\}^\lambda \rightarrow \{0,1\}^{\ell(\lambda)}$ takes a short, uniformly random seed $s \in \{0,1\}^\lambda$ and outputs a longer pseudorandom string $y \in \{0,1\}^{\ell(\lambda)}$.

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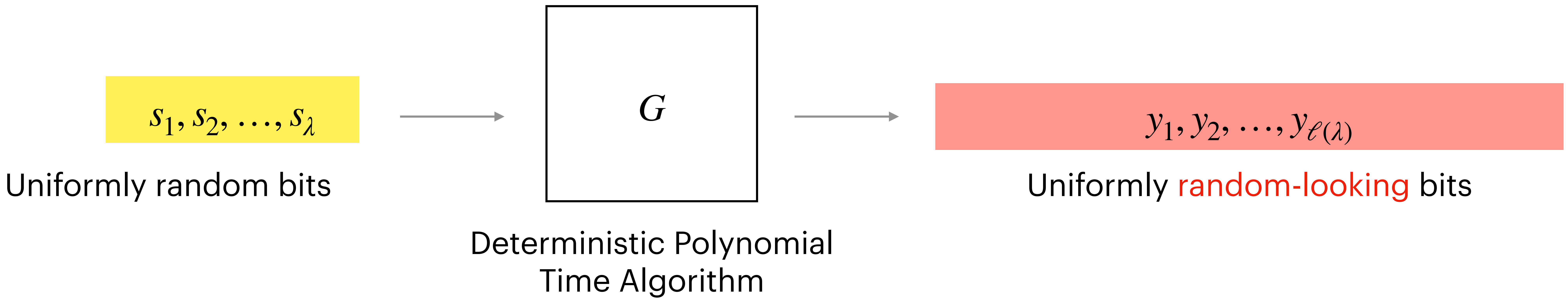

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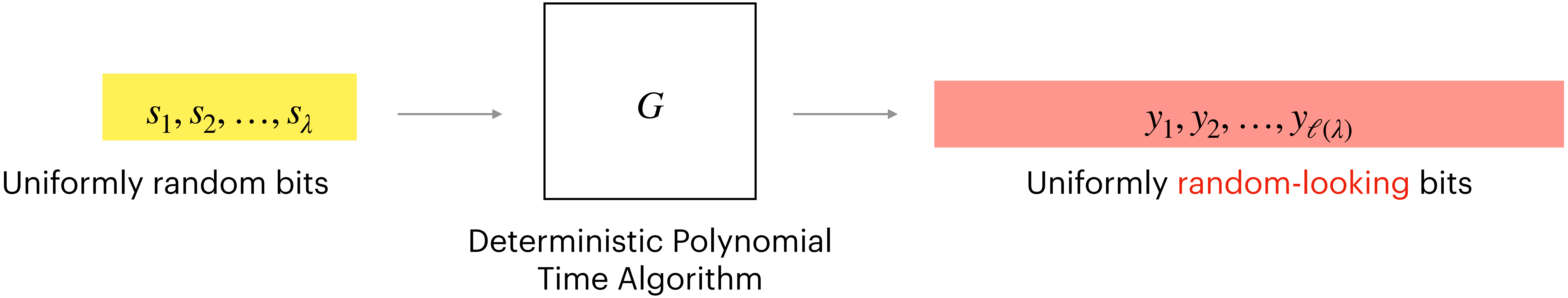

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Pseudorandom Generator

Pseudorandom Generator

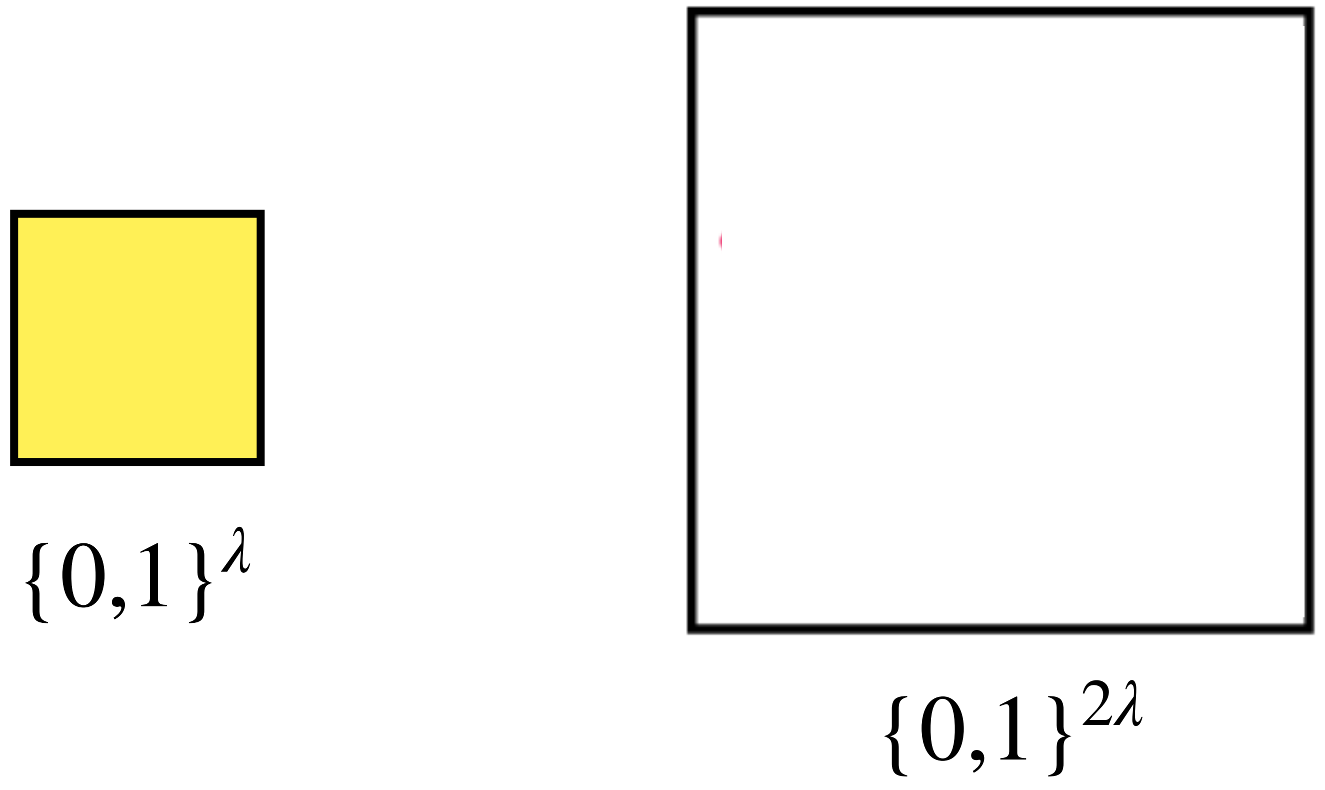
A deterministic algorithm G is called a pseudorandom generator (PRG) if:

- G can be computed in polynomial time,
- On input any $s \in \{0,1\}^\lambda$, G outputs a $\ell(\lambda)$-bit string such that $\ell(\lambda) > \lambda$,
- $\{G(s) : s \stackrel{\$}{\leftarrow} \{0,1\}^\lambda\} \quad \stackrel{c}{\approx} \quad \{r : r \stackrel{\$}{\leftarrow} \{0,1\}^{\ell(\lambda)}\}$

The stretch of G is defined as $\ell(\lambda) - \lambda$.

Why <u>Pseudo</u>random?

Consider a PRG G with λ-bit stretch.



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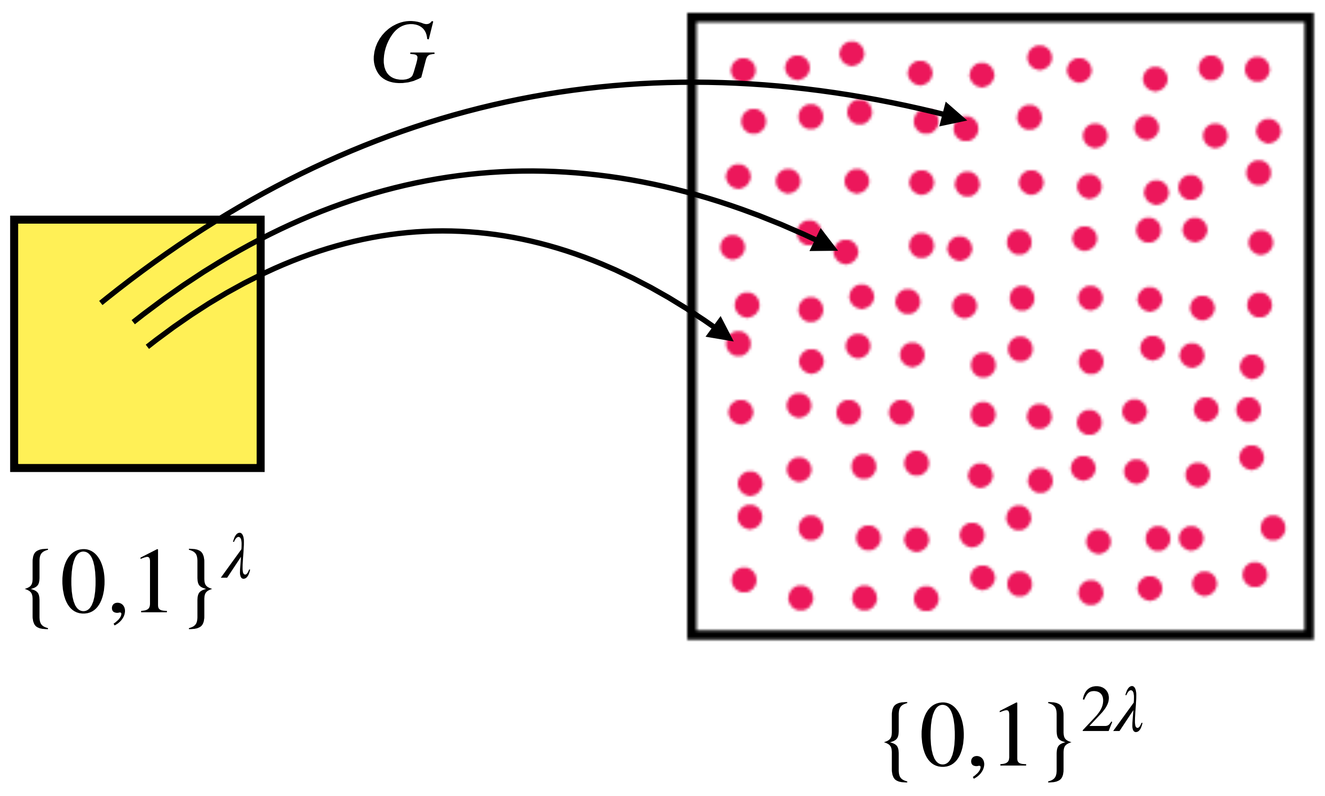

Pseudorandom Distribution

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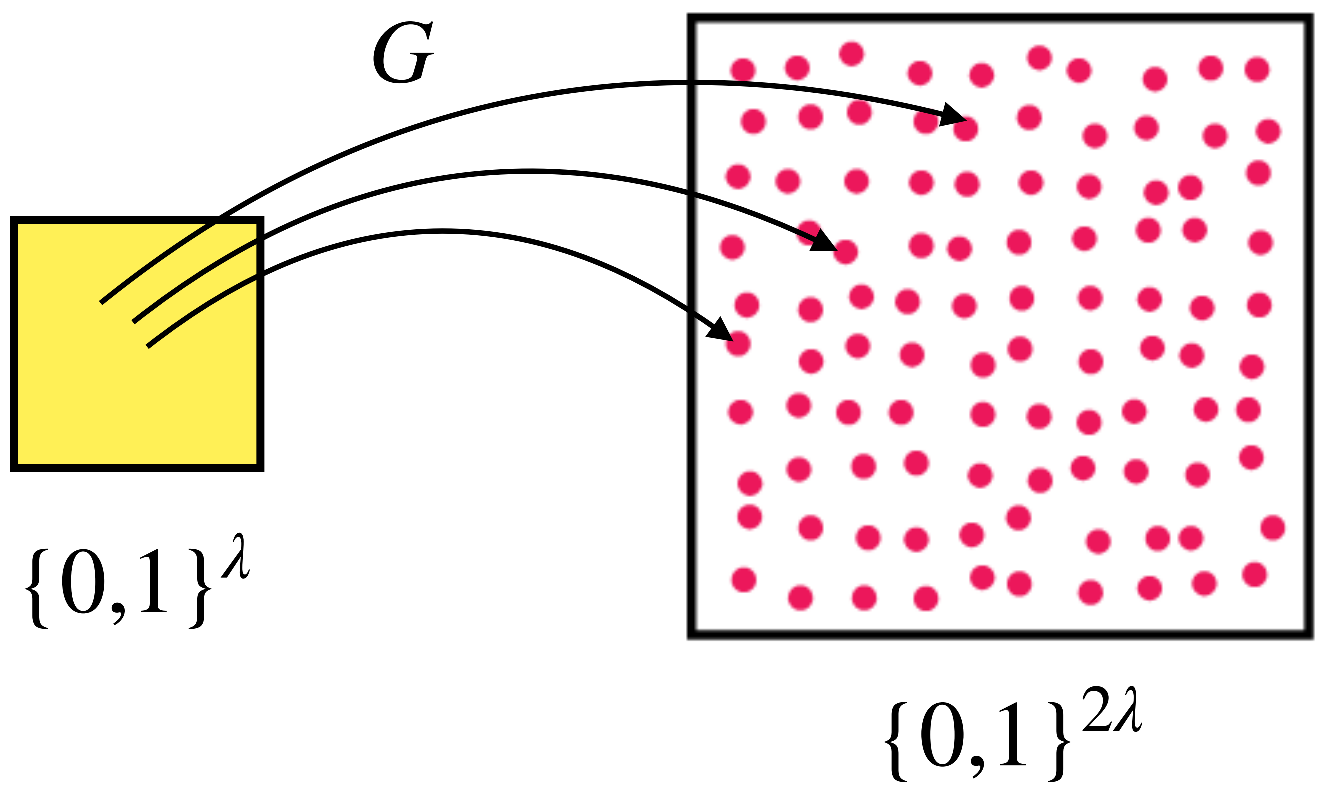

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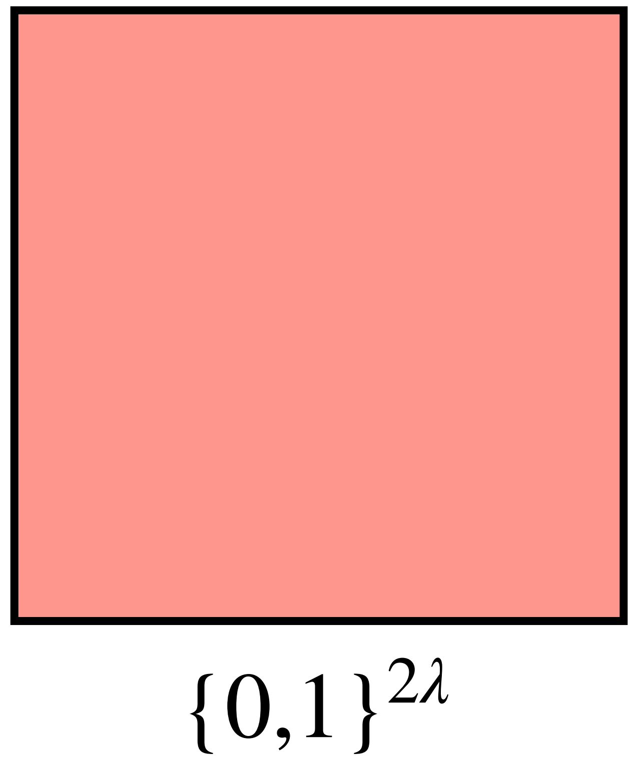

Uniform Distribution

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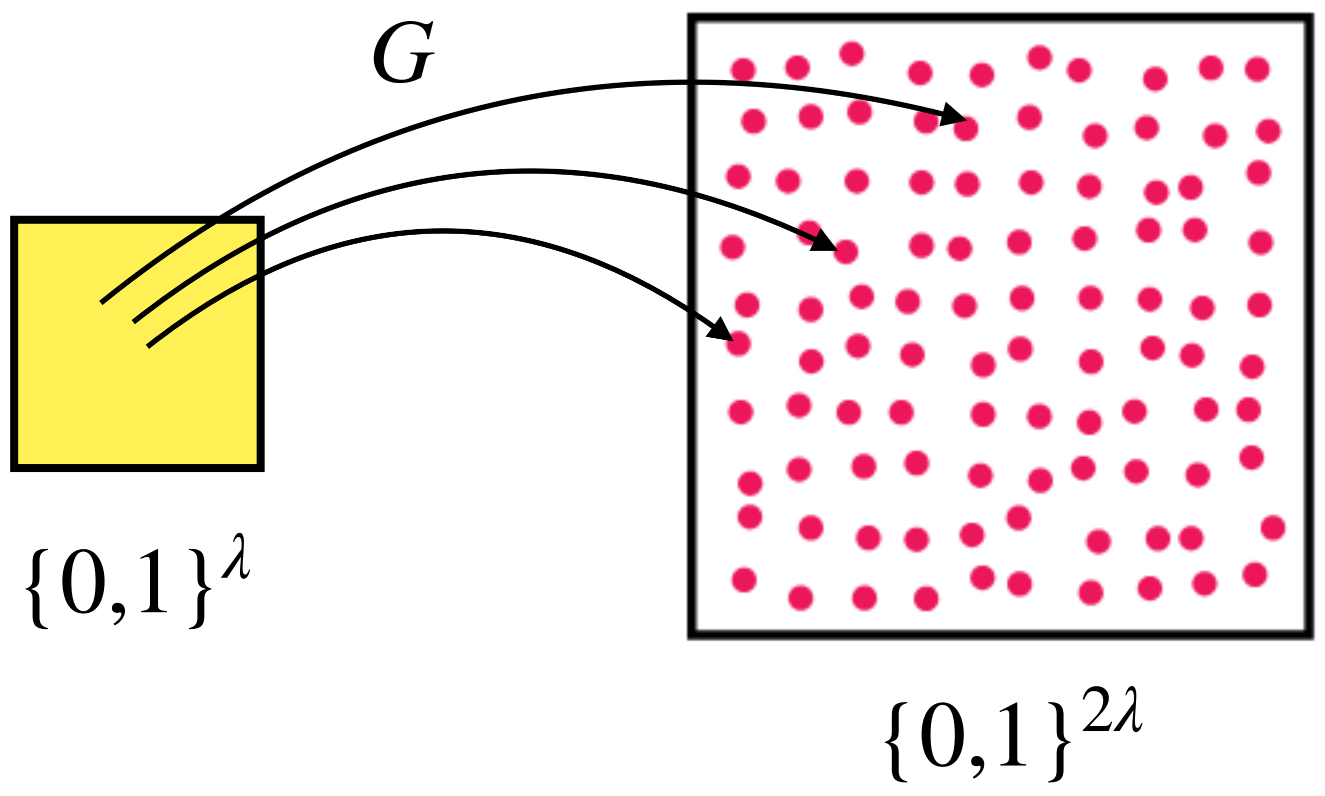

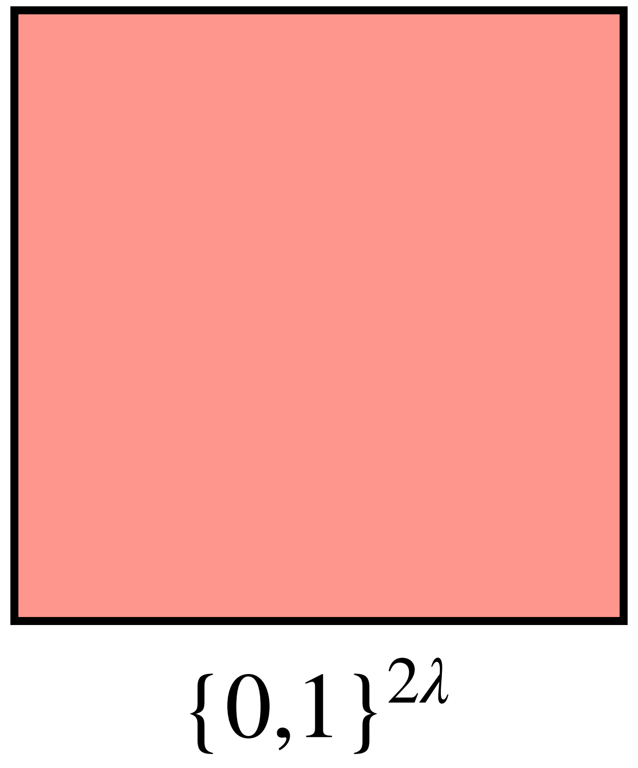

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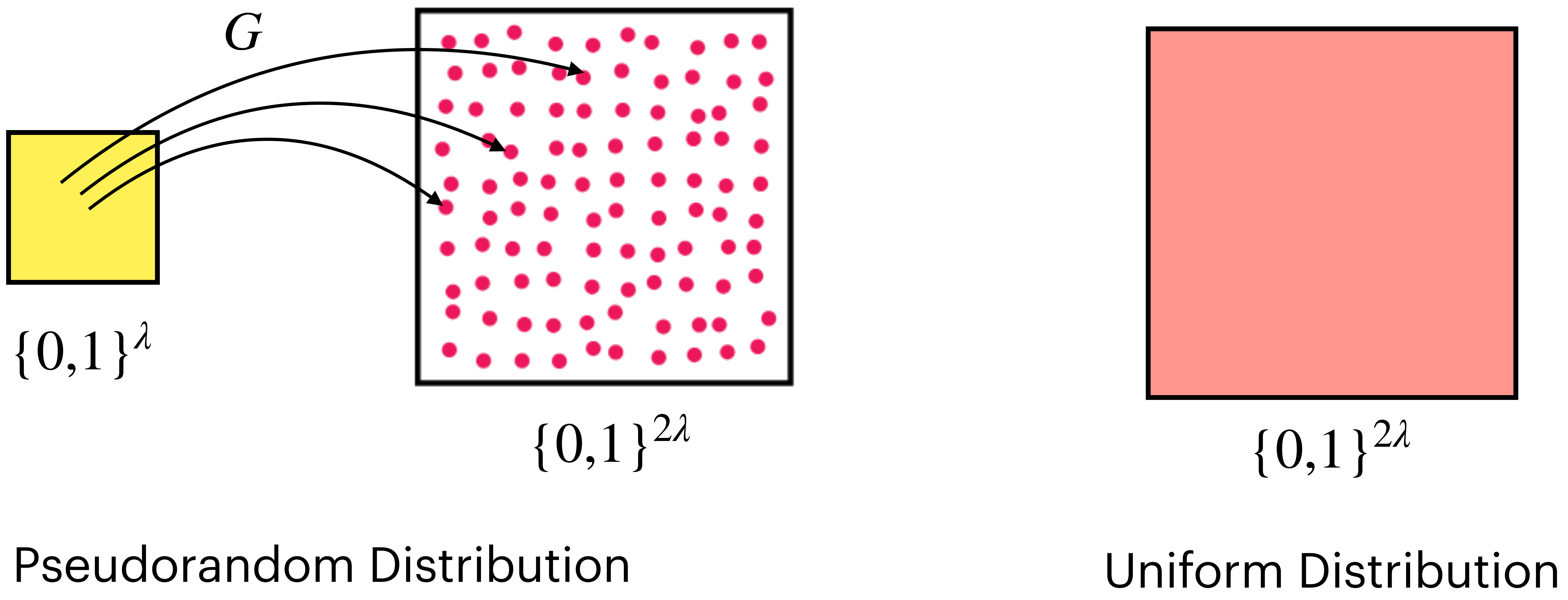

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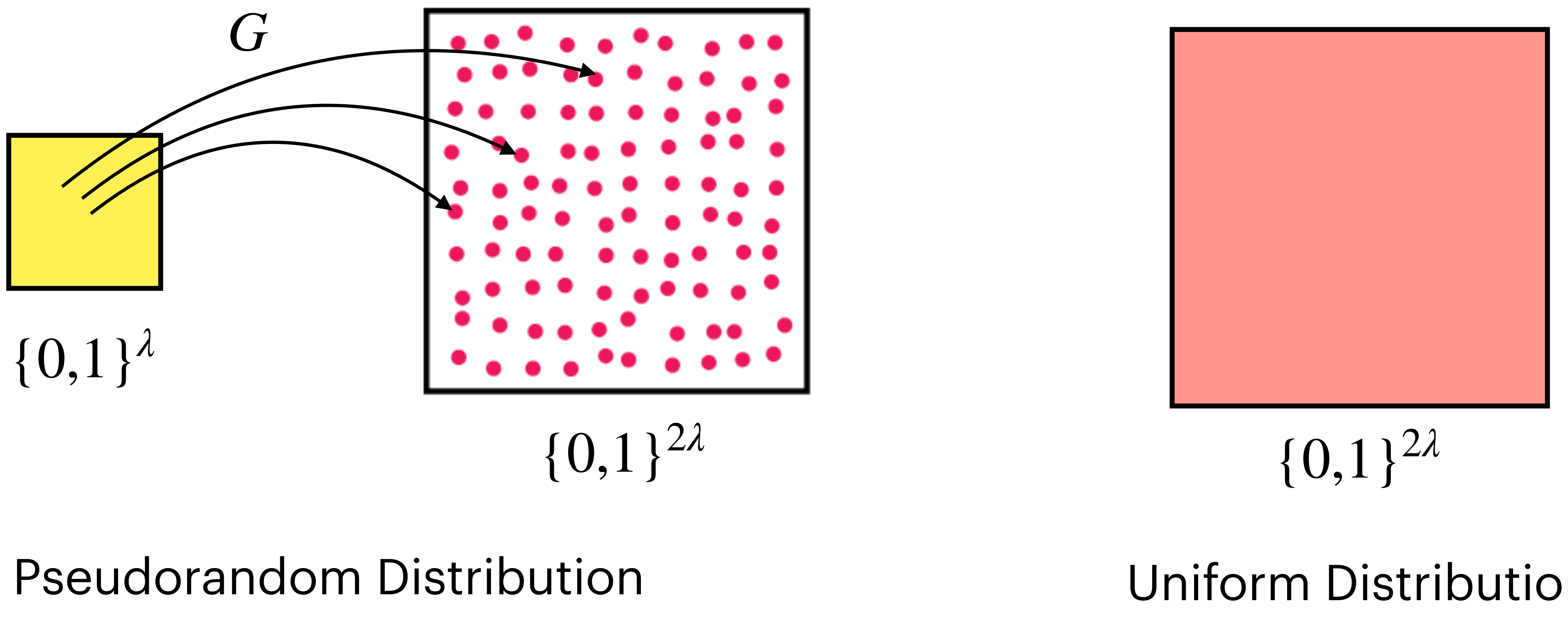

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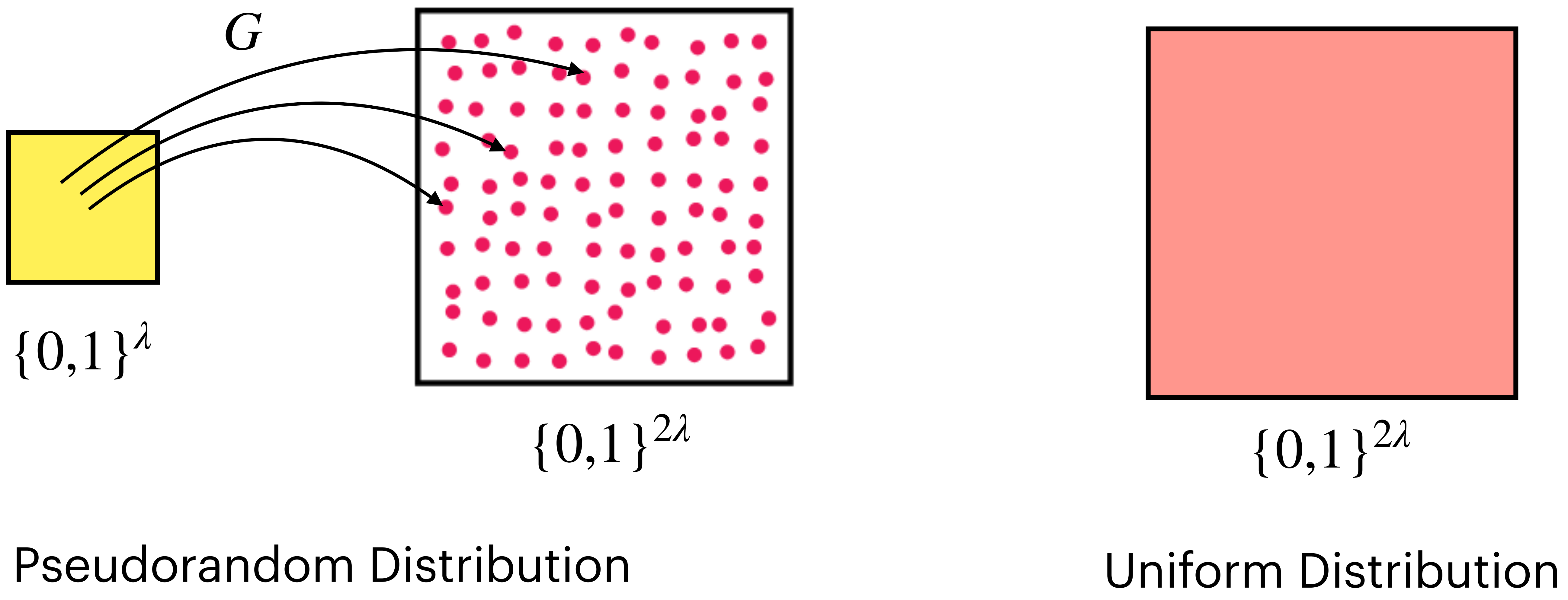

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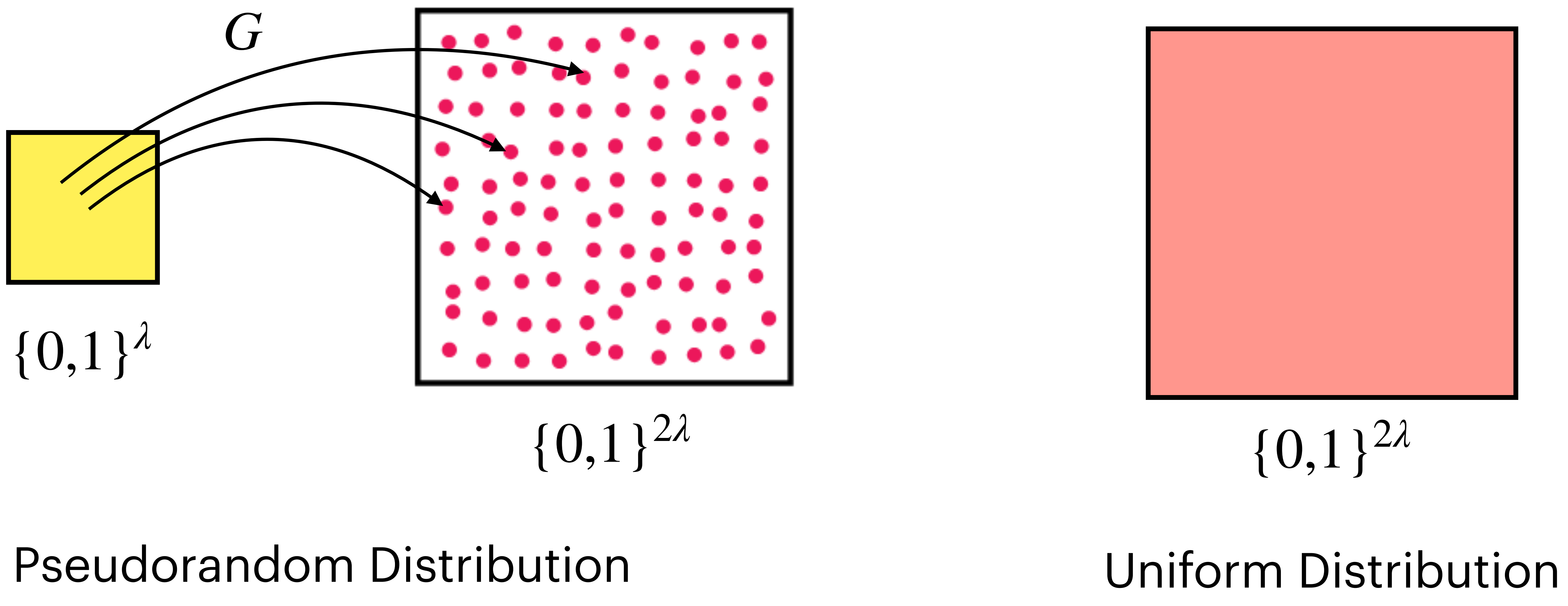

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Any poly-time algorithm that requires a long random string can instead be fed pseudorandomness generated using a short random seed.

Pseudorandom OTP

One-Time Pad

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Security? Is it perfectly secure?

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Similar to $H_0 \overset{c}{\approx} H_1$.

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$H_0 \overset{c}{\approx} H_3$ Hybrid Lemma

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- How do we formally establish that $H_0 \overset{c}{\approx} H_1$?

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Reduction: Use a solution to one problem to solve another related problem.

In crypto, reductions are the standard tool to "transfer" the hardness of one problem to another.

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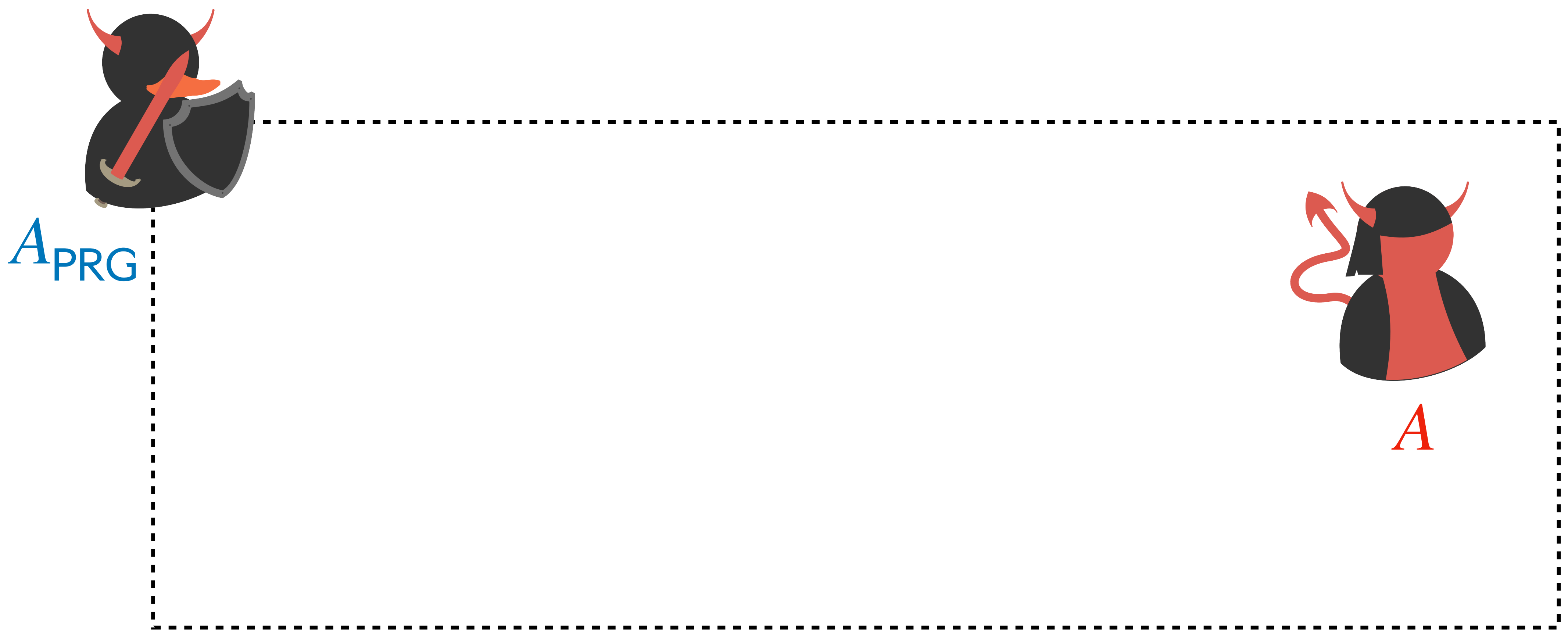

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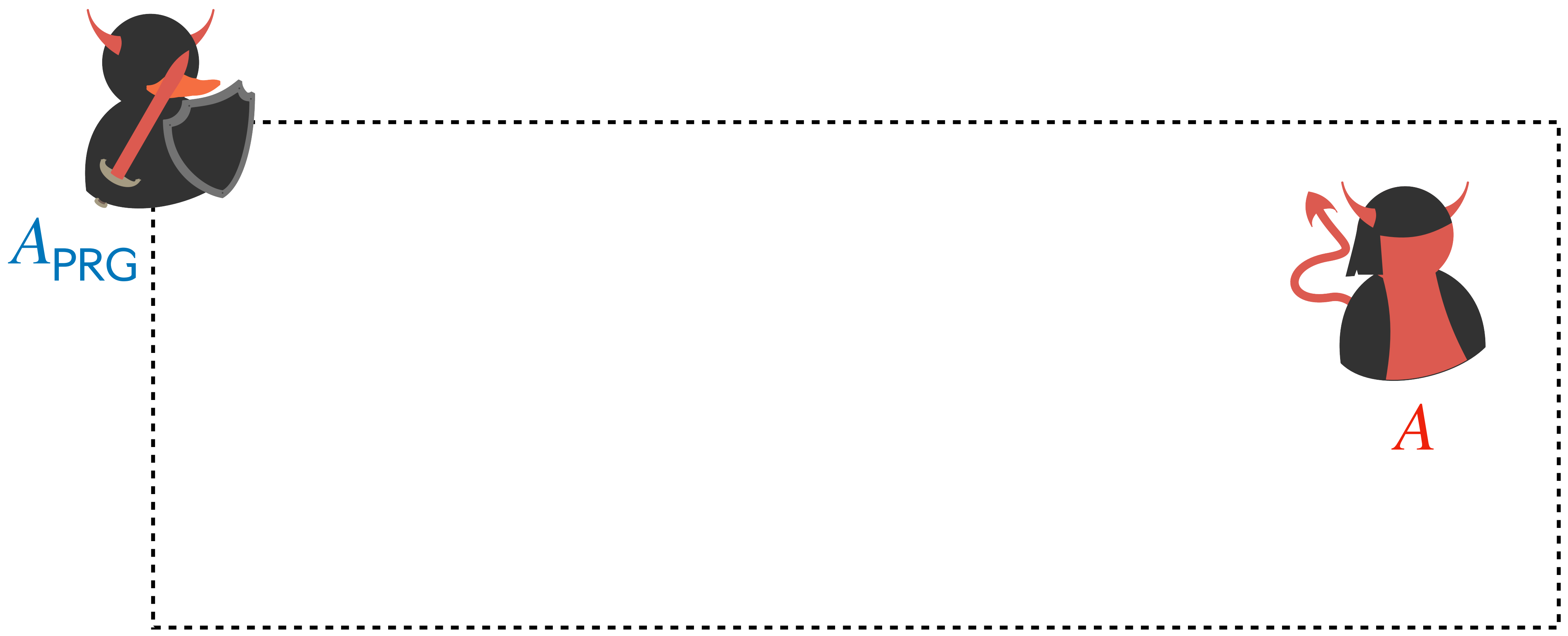

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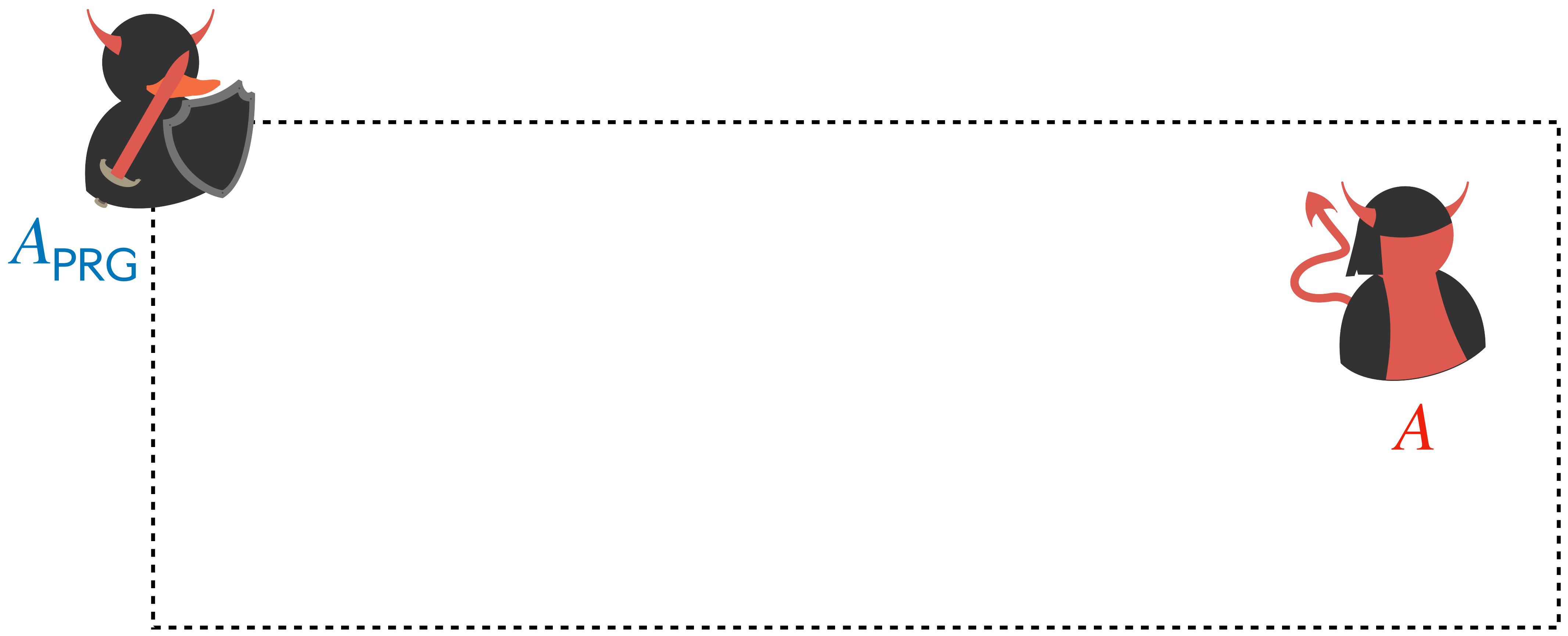

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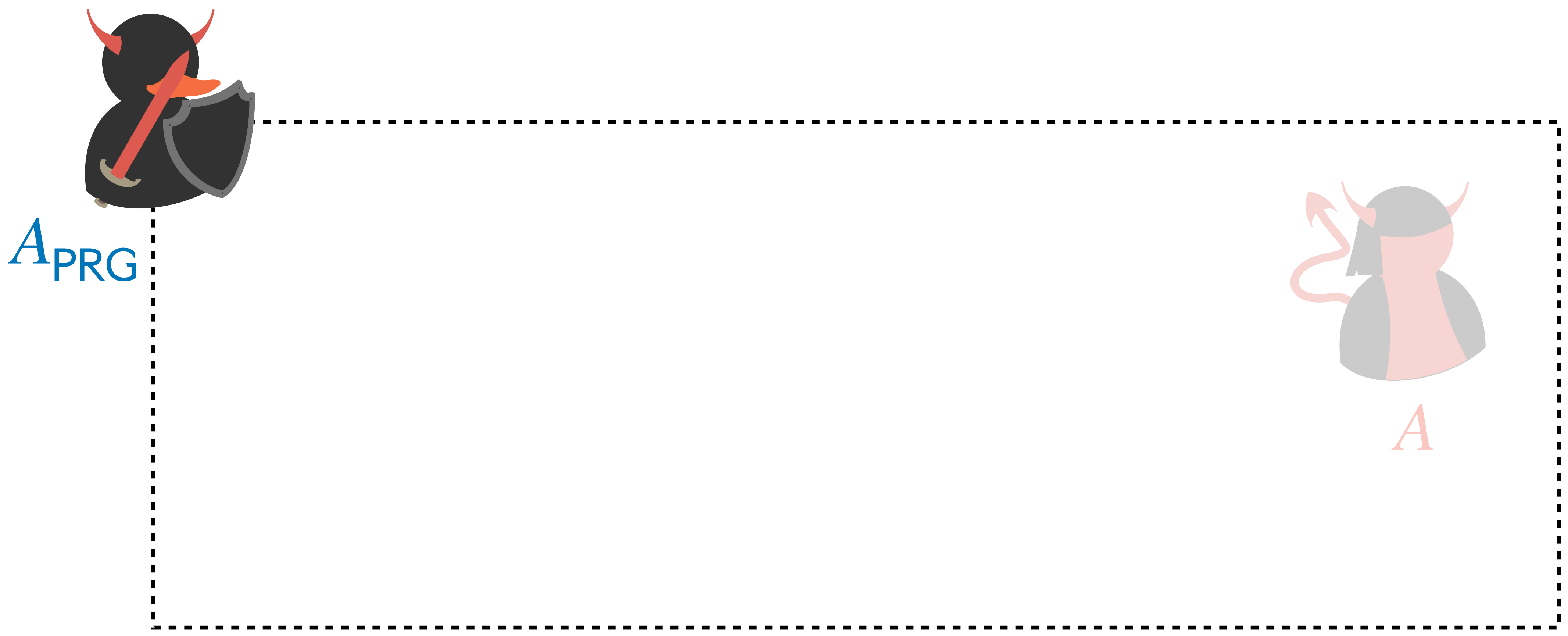

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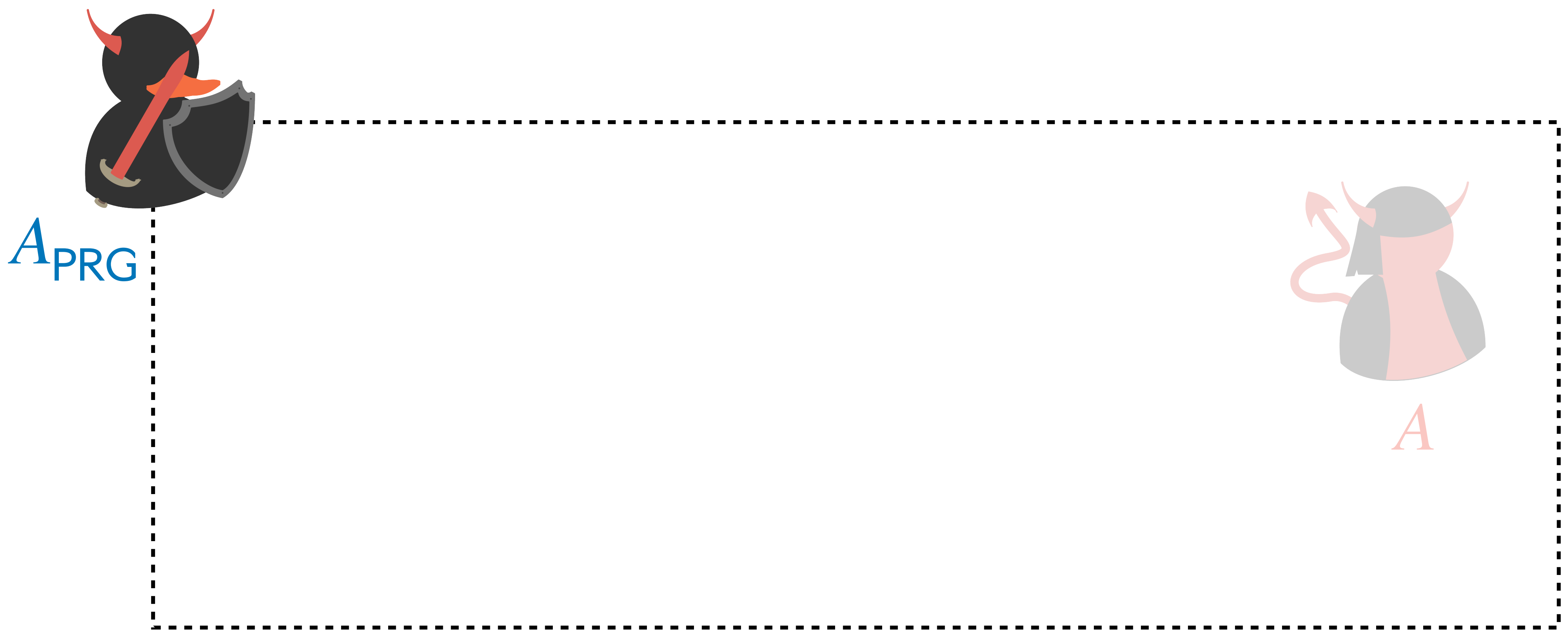

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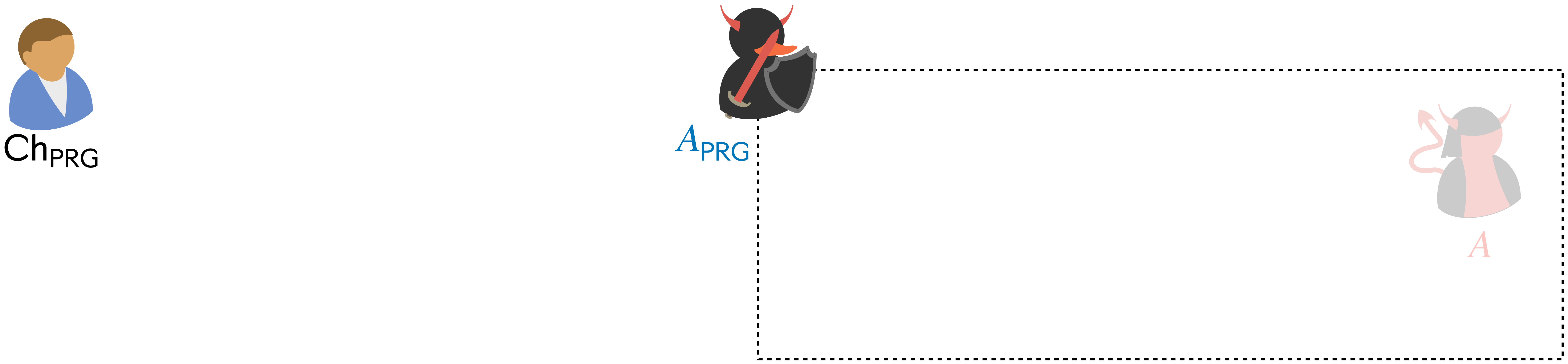

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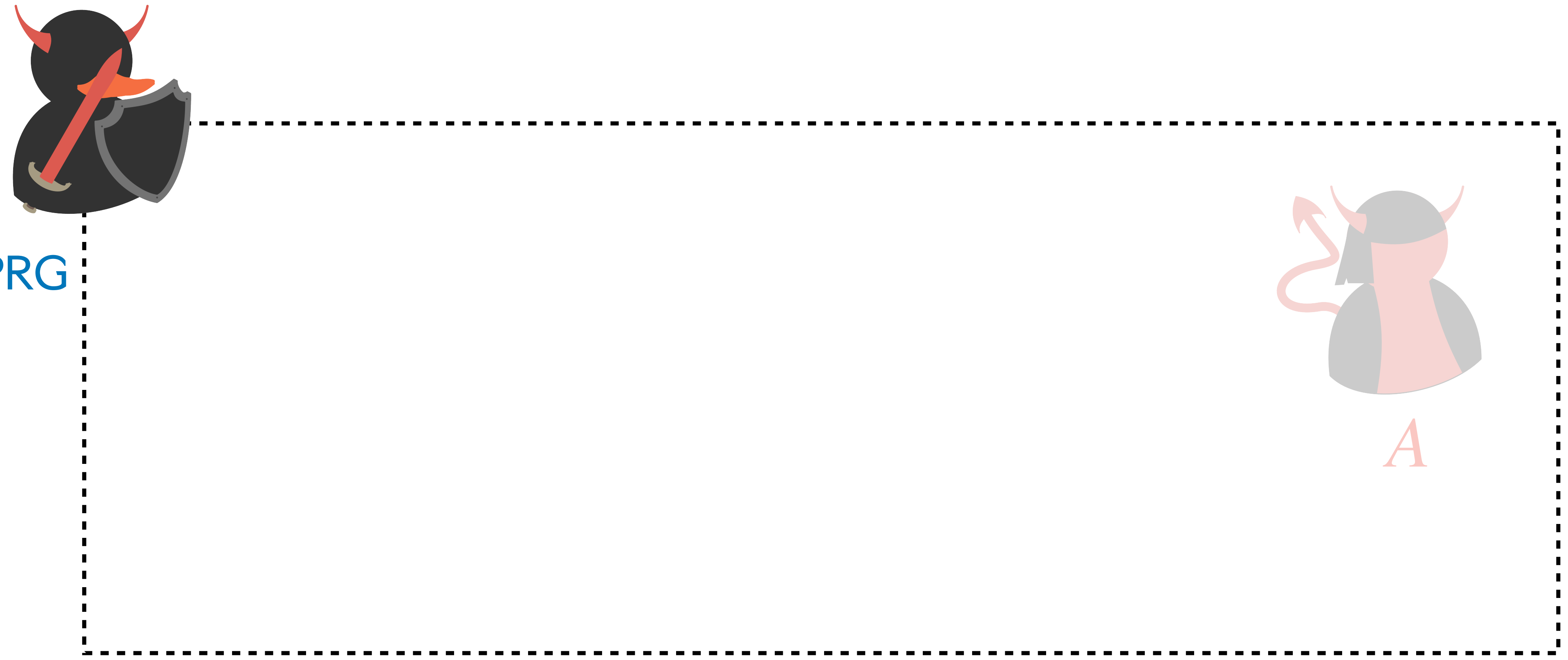

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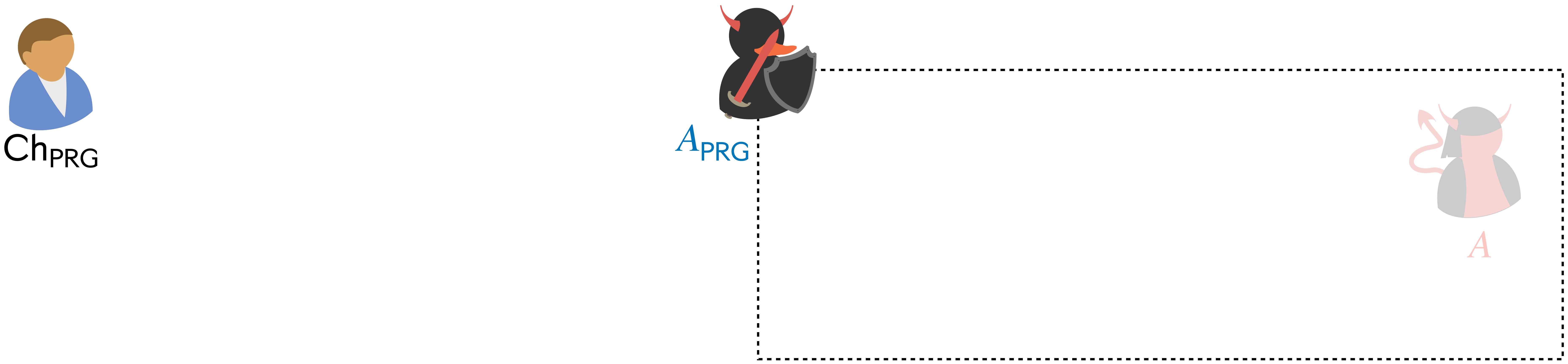

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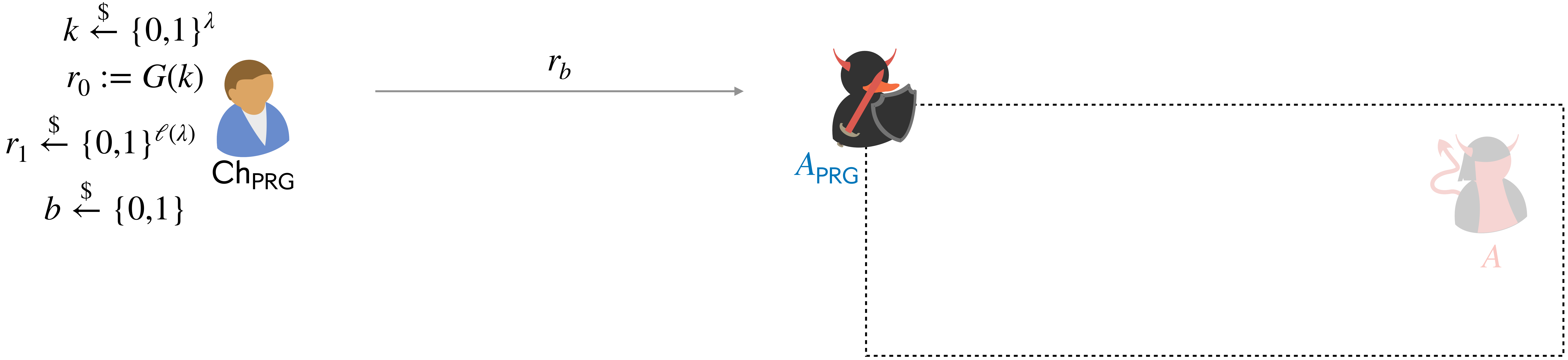

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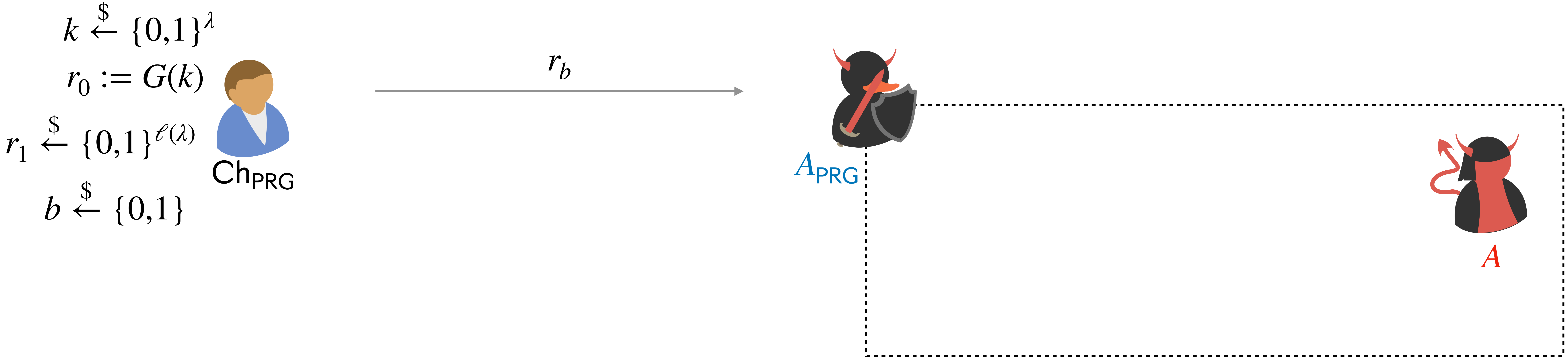

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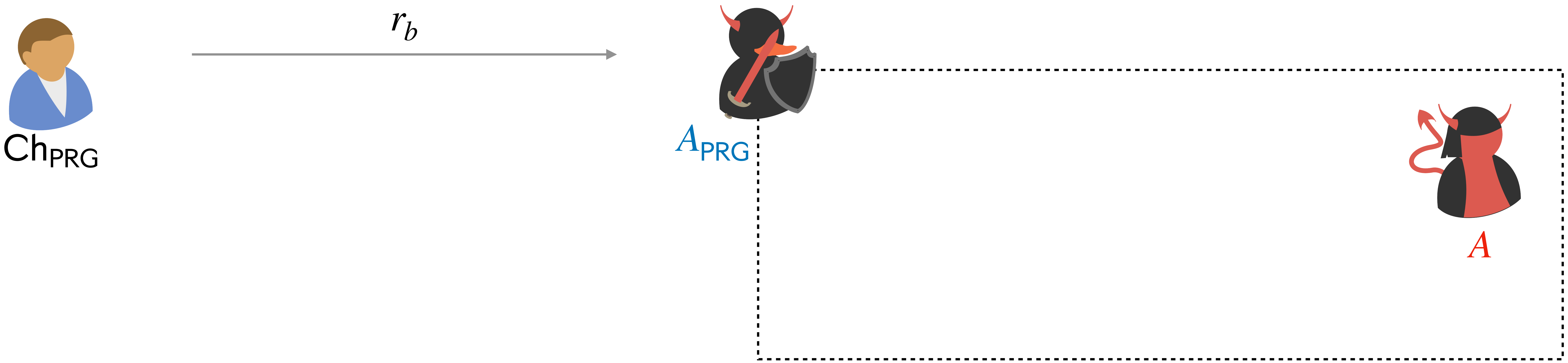

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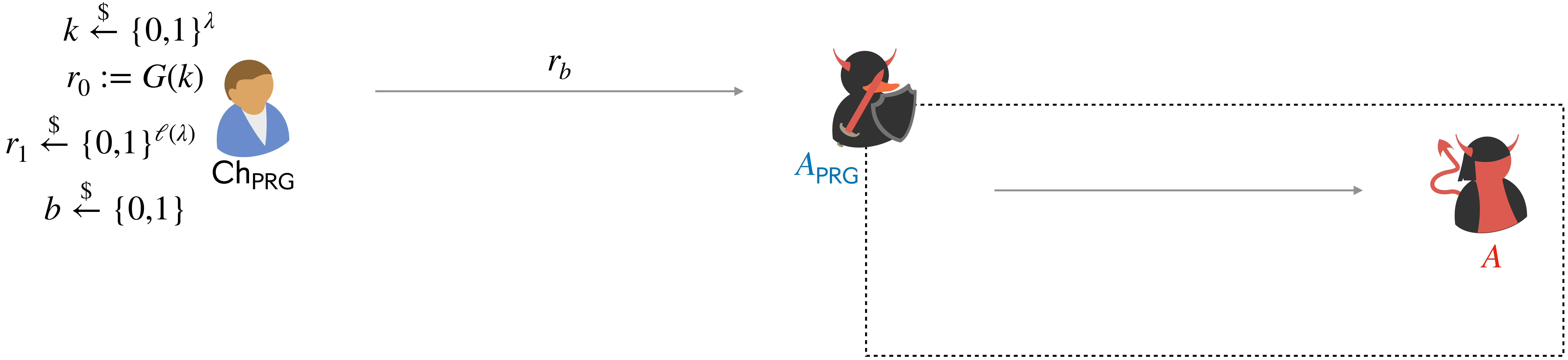

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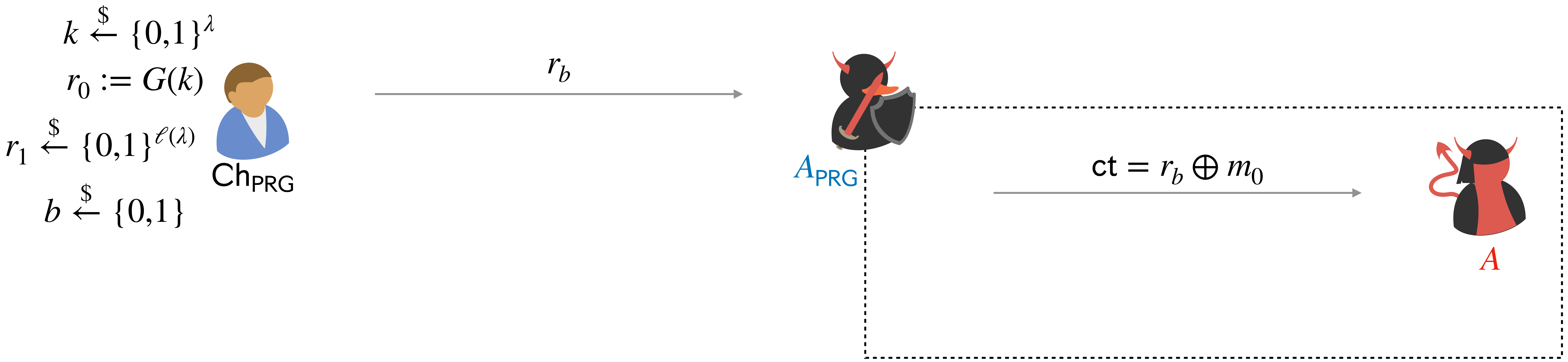

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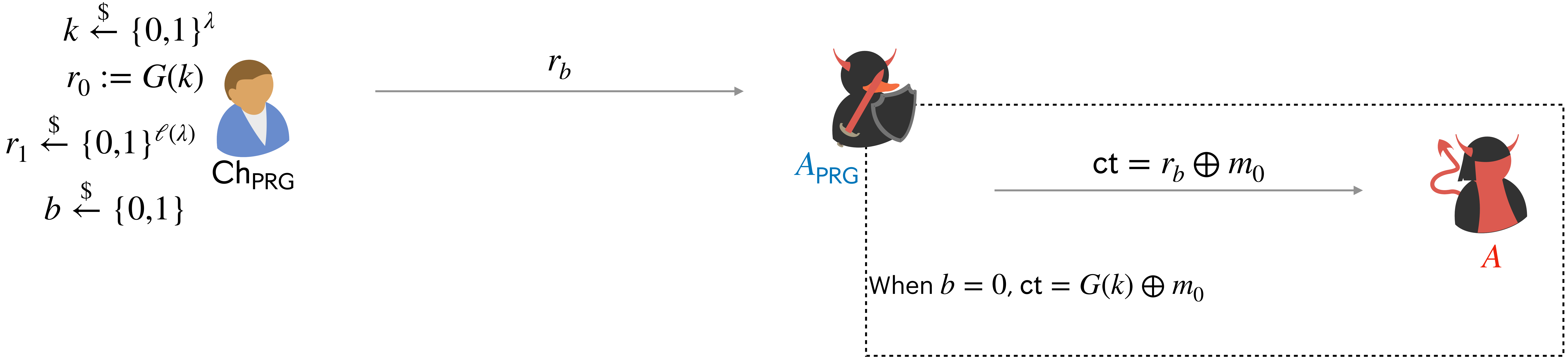

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r_b →

A_{PRG}

$\mathsf{ct} = r_b \oplus m_0$ →

A

When $b = 0$, $\mathsf{ct} = G(k) \oplus m_0 \Longrightarrow A$'s view is equivalent to H_0

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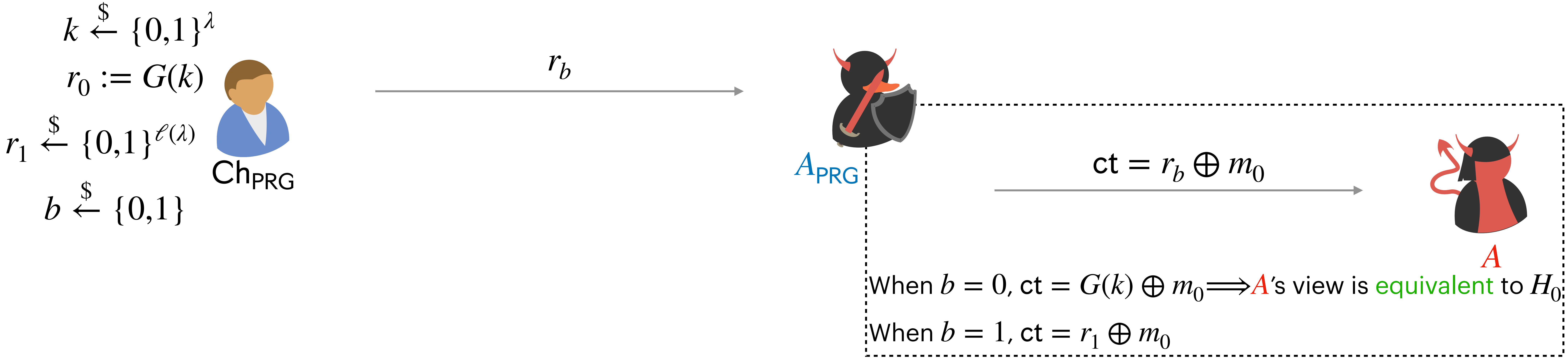

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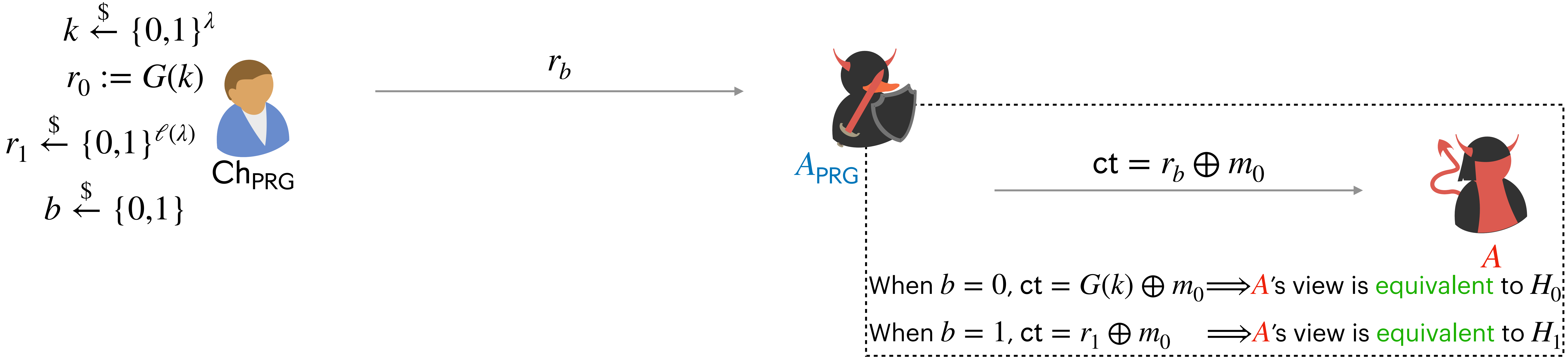

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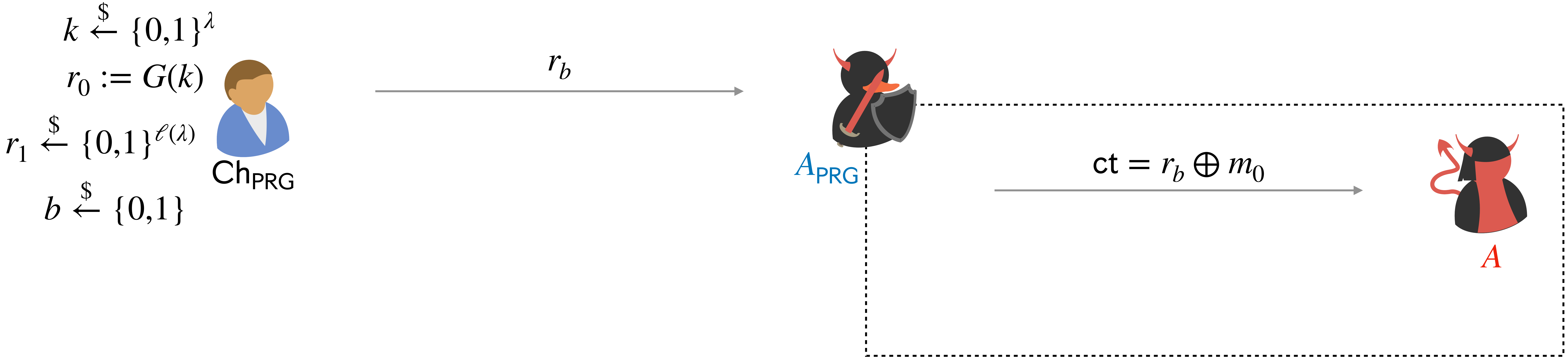

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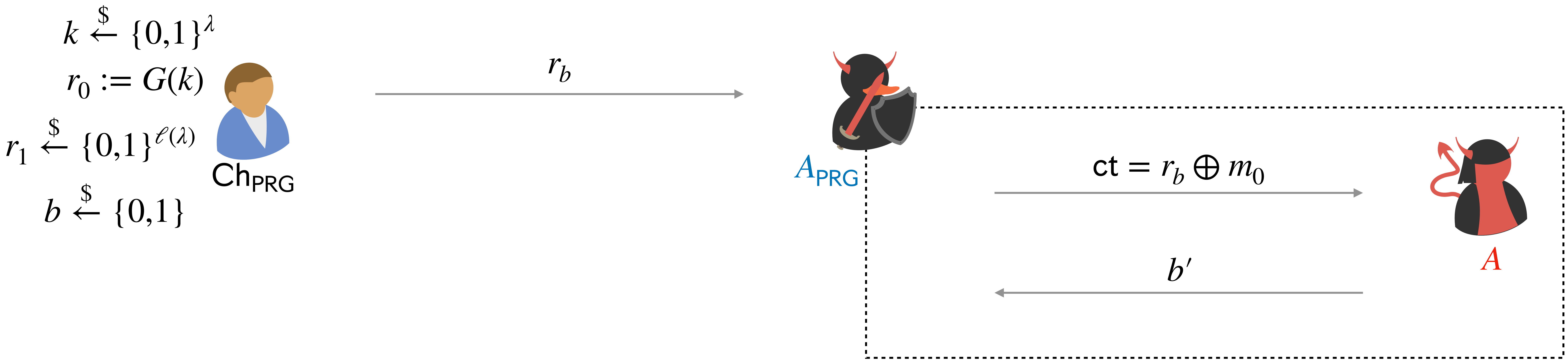

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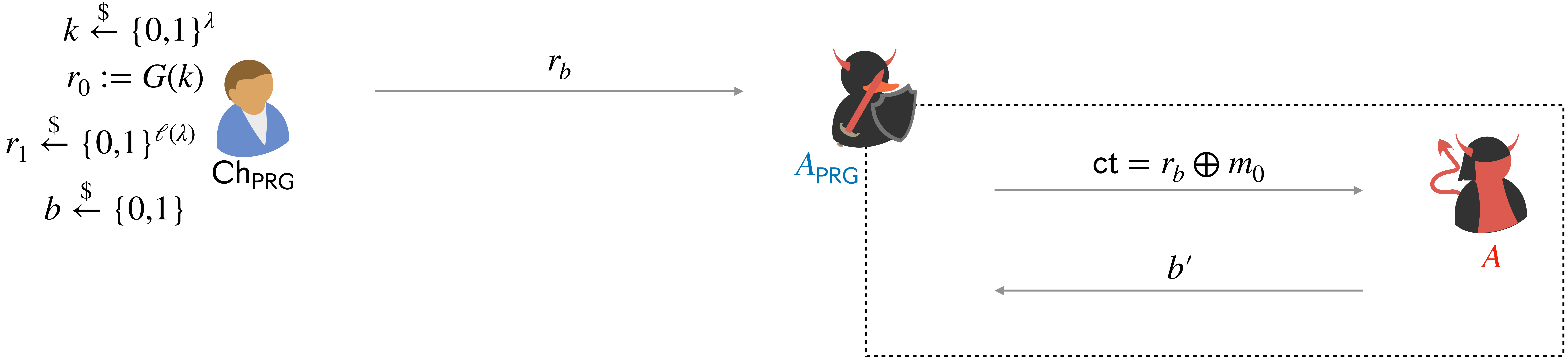

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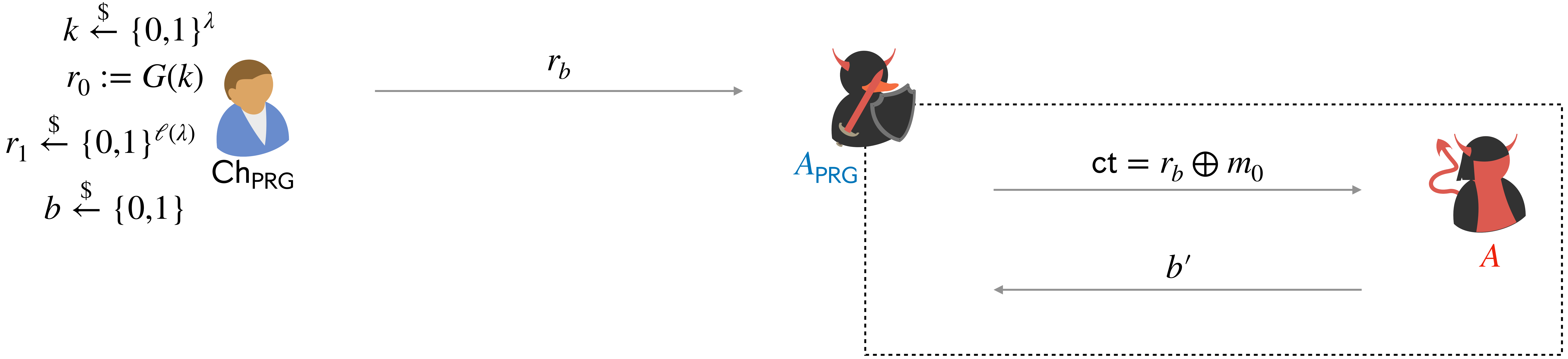

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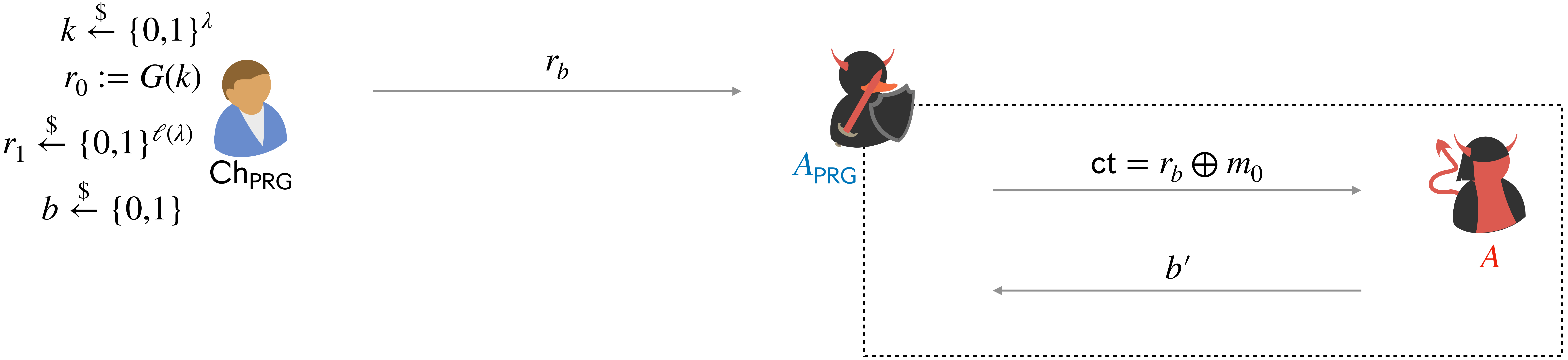

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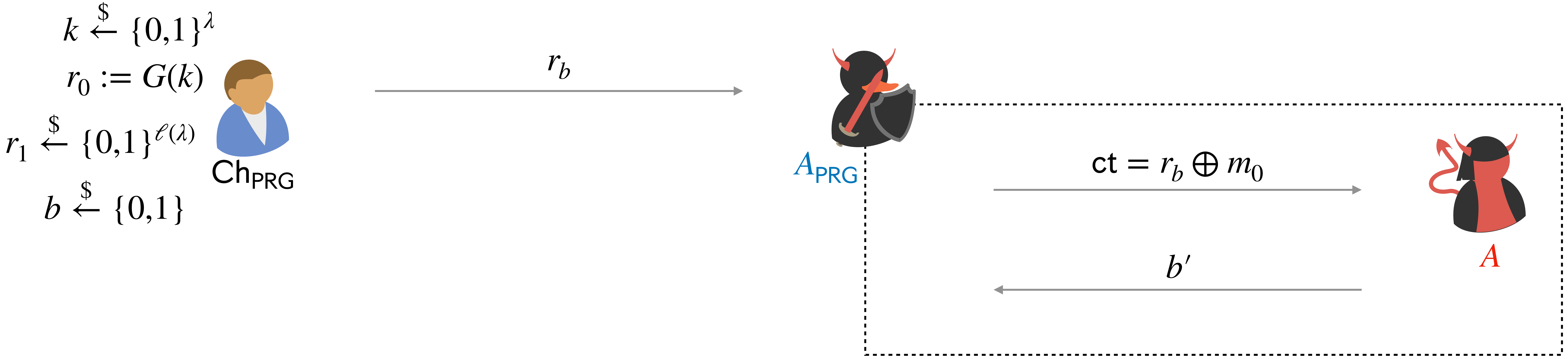

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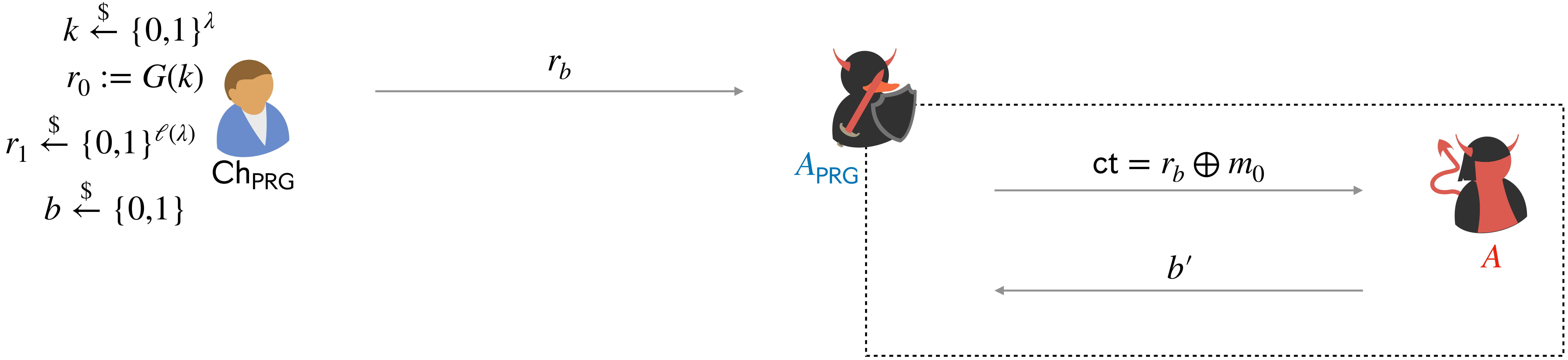

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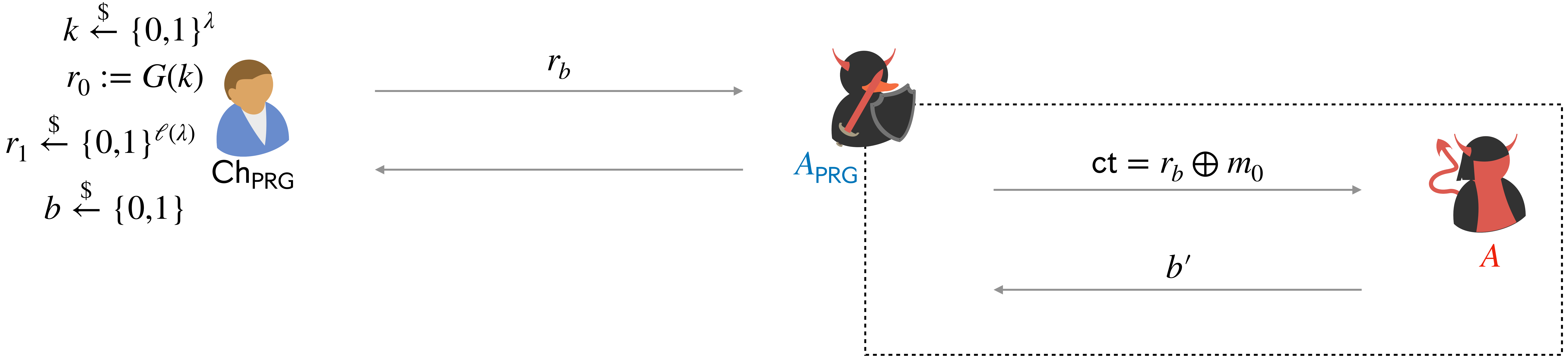

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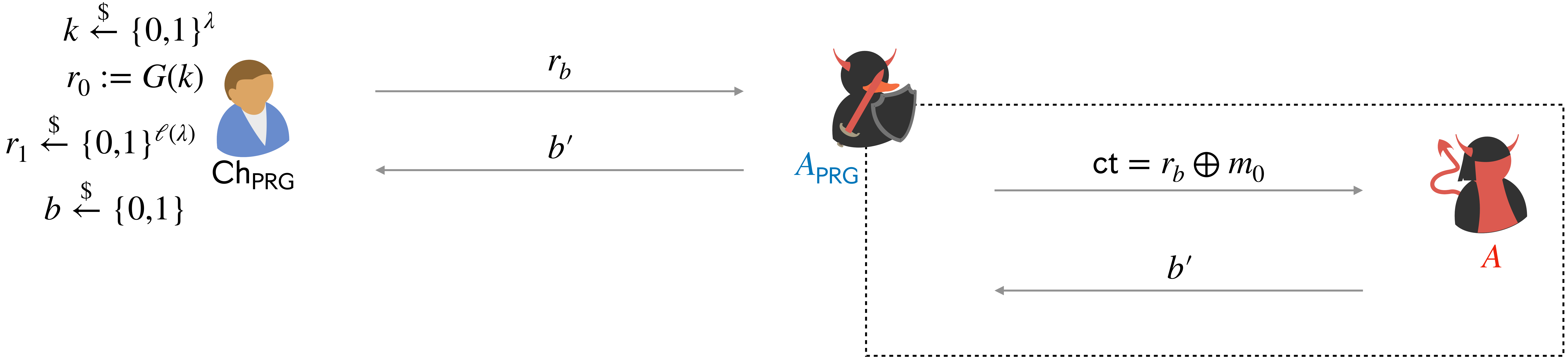

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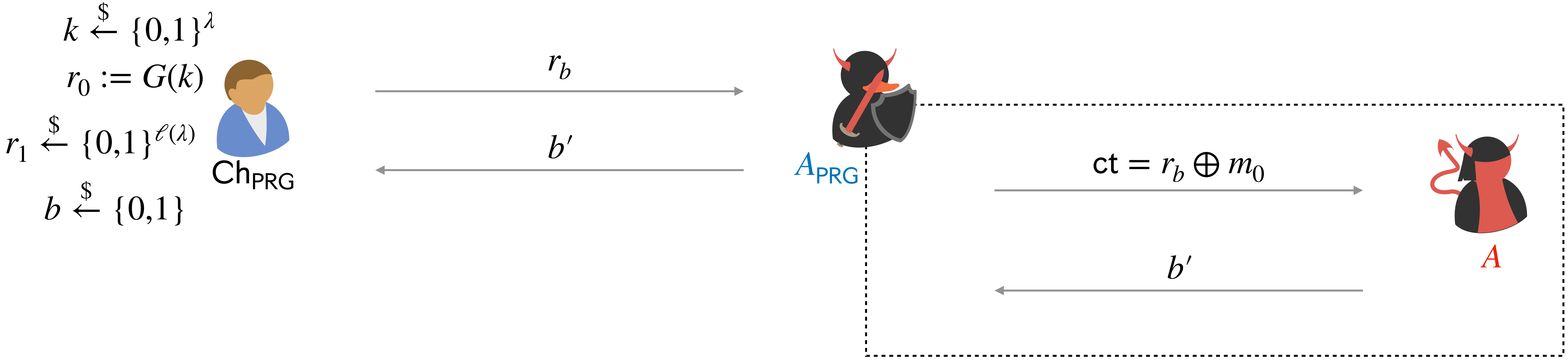

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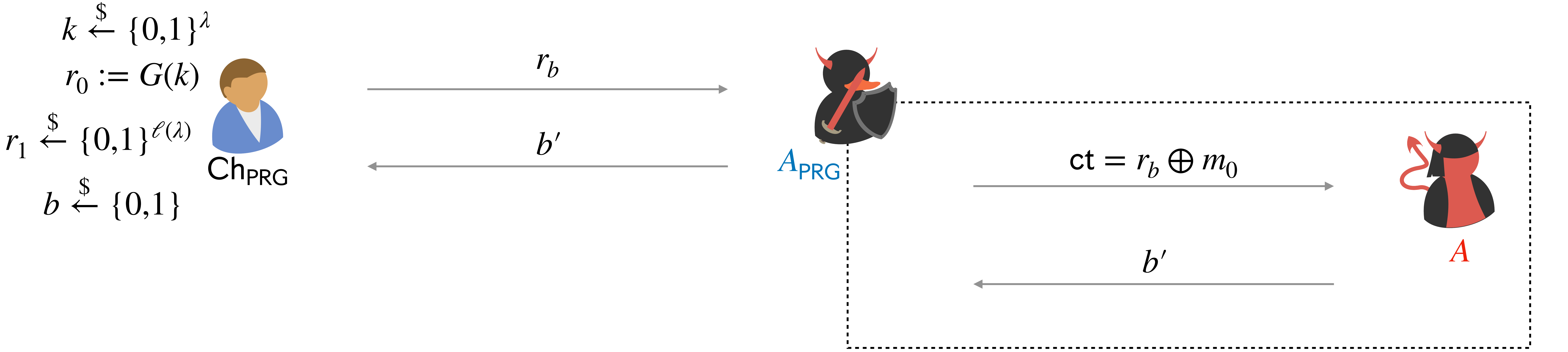

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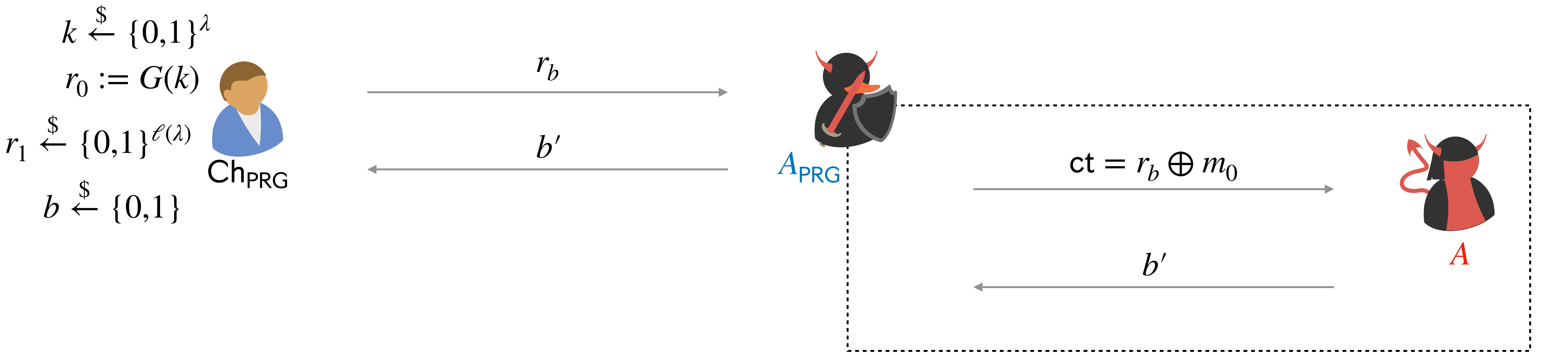

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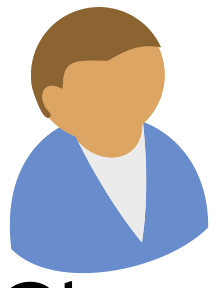

$\mathsf{Ch}_{\mathsf{PRG}}$

r_b →

← b'

A_{PRG}

$\mathsf{ct} = r_b \oplus m_0$ →

← b'

A

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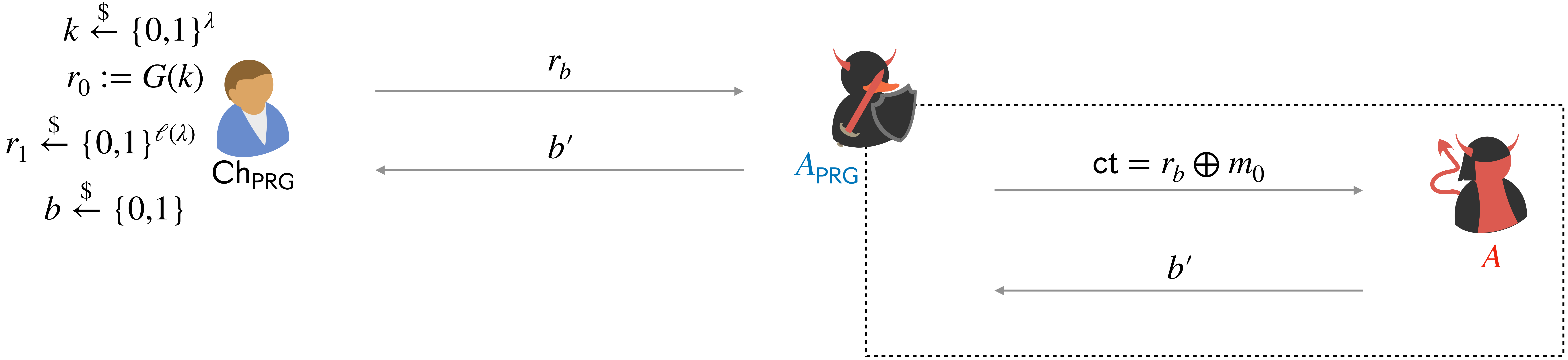

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$\mathsf{Ch}_{\mathsf{PRG}}$ → r_b → A_{PRG}

A_{PRG} → b' → $\mathsf{Ch}_{\mathsf{PRG}}$

A_{PRG} → $\mathsf{ct} = r_b \oplus m_0$ → A

A → b' → A_{PRG}

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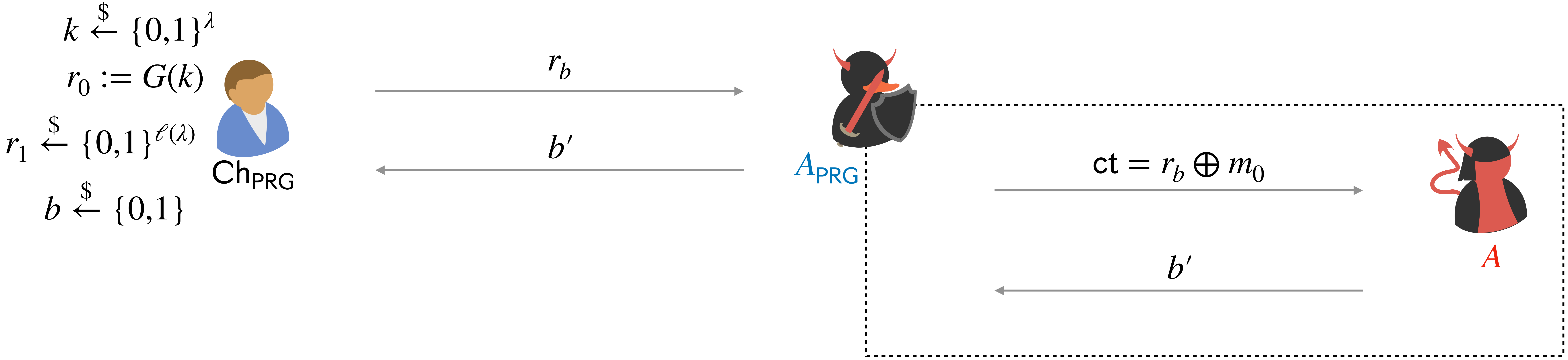

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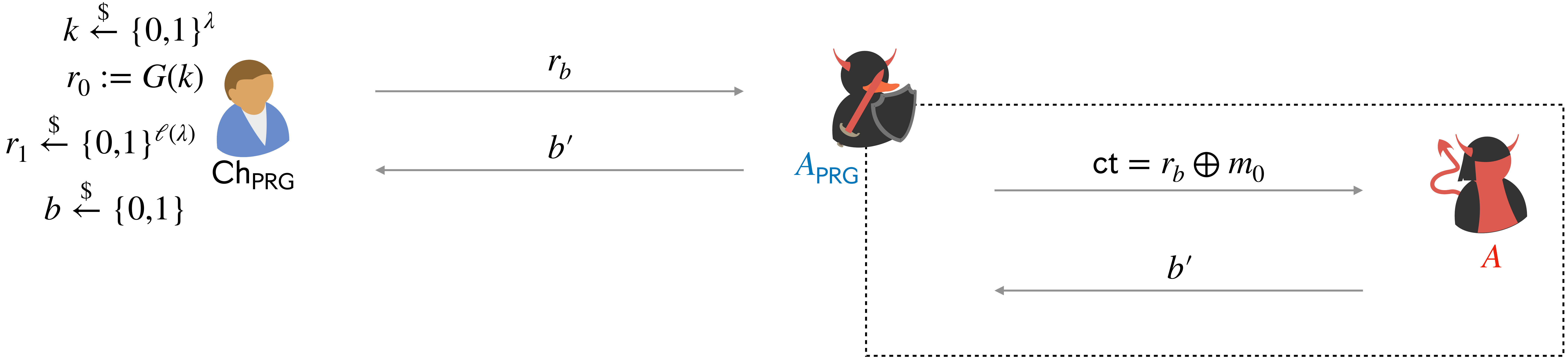

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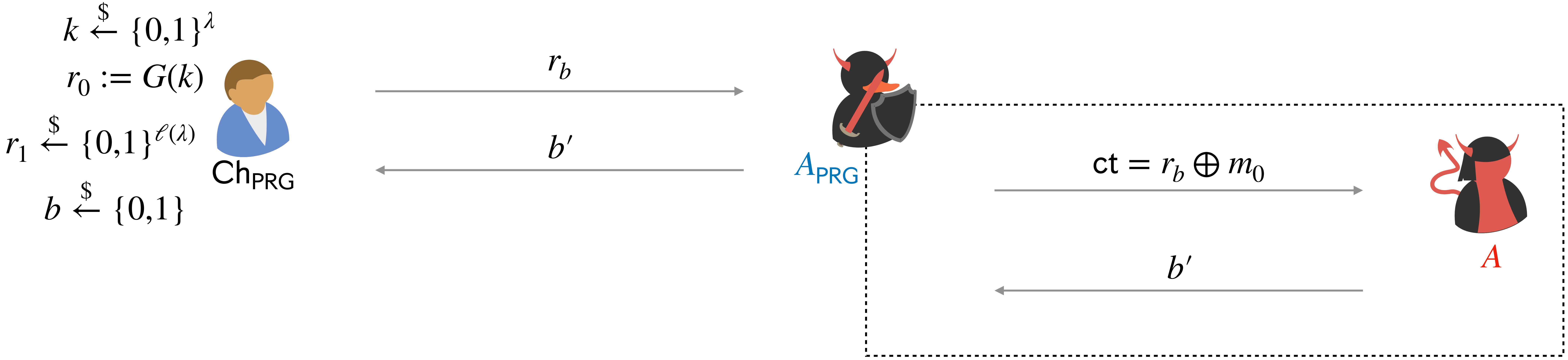

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Therefore, $\epsilon = \epsilon_{\mathsf{PRG}} = \mathsf{negl}(\lambda)$.

Security of Pseudorandom OTP

Theorem: Pseudorandom OTP is one-time computational secure.

Intuition:

$$H_0 = \left\{ G(k) \oplus m_0 \ : \ k \xleftarrow{\$} \{0,1\}^{\lambda} \right\}$$

$$H_0 \overset{c}{\approx} H_1$$

$$H_1 = \left\{ r \oplus m_0 \ : \ r \xleftarrow{\$} \{0,1\}^{\ell(\lambda)} \right\}$$

$$H_1 \equiv H_2$$

$$H_2 = \left\{ r \oplus m_1 \ : \ r \xleftarrow{\$} \{0,1\}^{\ell(\lambda)} \right\}$$

$$H_2 \overset{c}{\approx} H_3$$

$$H_3 = \left\{ G(k) \oplus m_1 \ : \ k \xleftarrow{\$} \{0,1\}^{\lambda} \right\}$$

$$H_0 \overset{c}{\approx} H_3$$

Reductions

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Claim: If $X_0 \overset{c}{\approx} X_1$ then $Y_0 \overset{c}{\approx} Y_1$.

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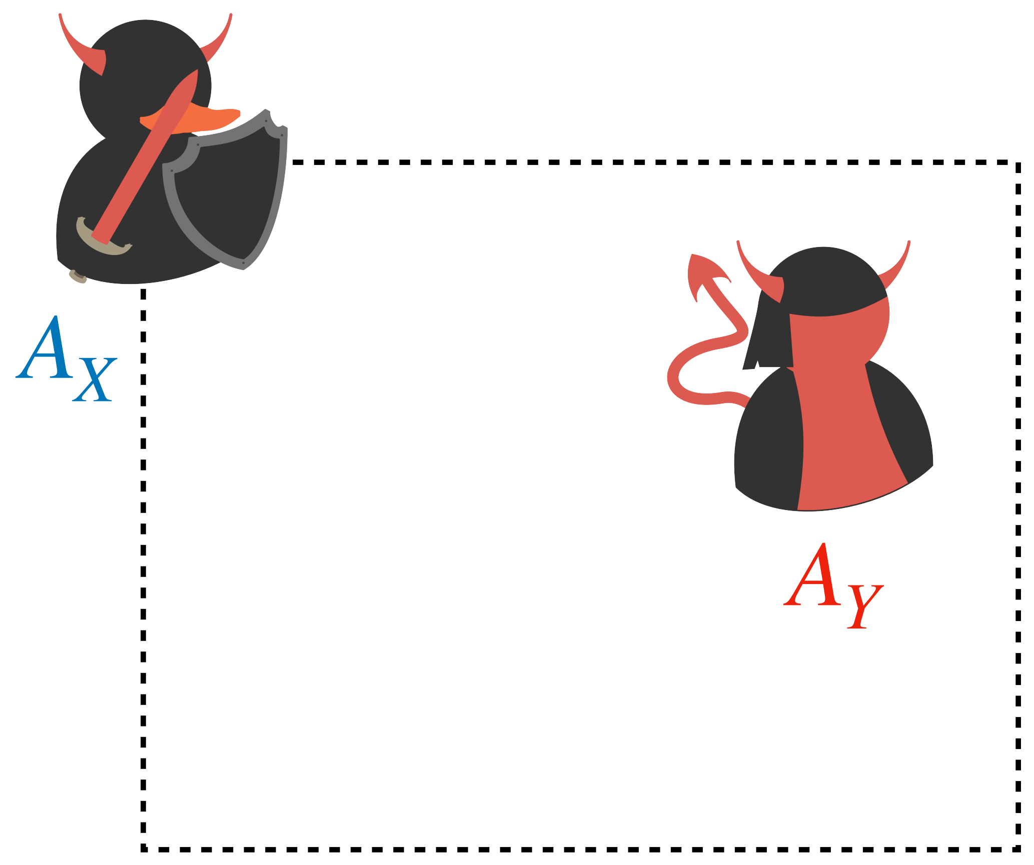
- We construct A_X that distinguishes between X_0 and X_1 with probability ϵ_X.



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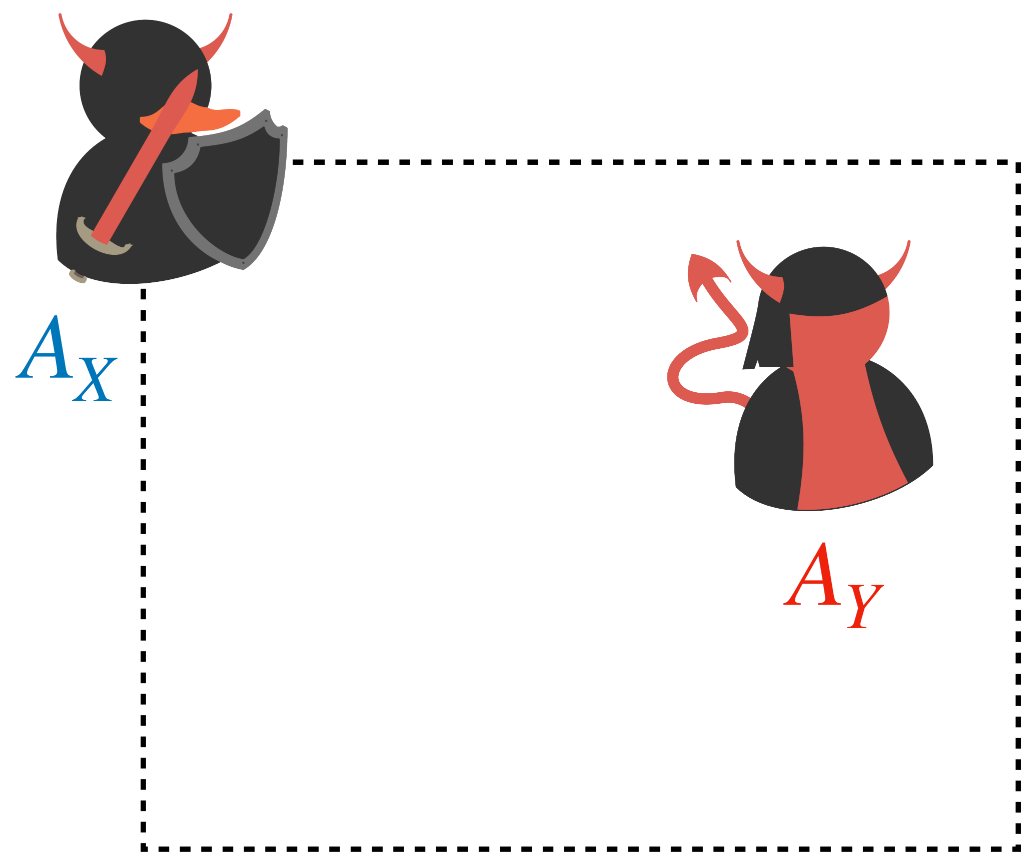
- Given A_Y that distinguishes between Y_0 and Y_1 with probability ϵ_Y.
- We construct A_X that distinguishes between X_0 and X_1 with probability ϵ_X.
  - ϵ_X will be negligibly close to ϵ_Y i.e., $\epsilon_Y = \epsilon_X + \mathsf{negl}(\lambda)$.



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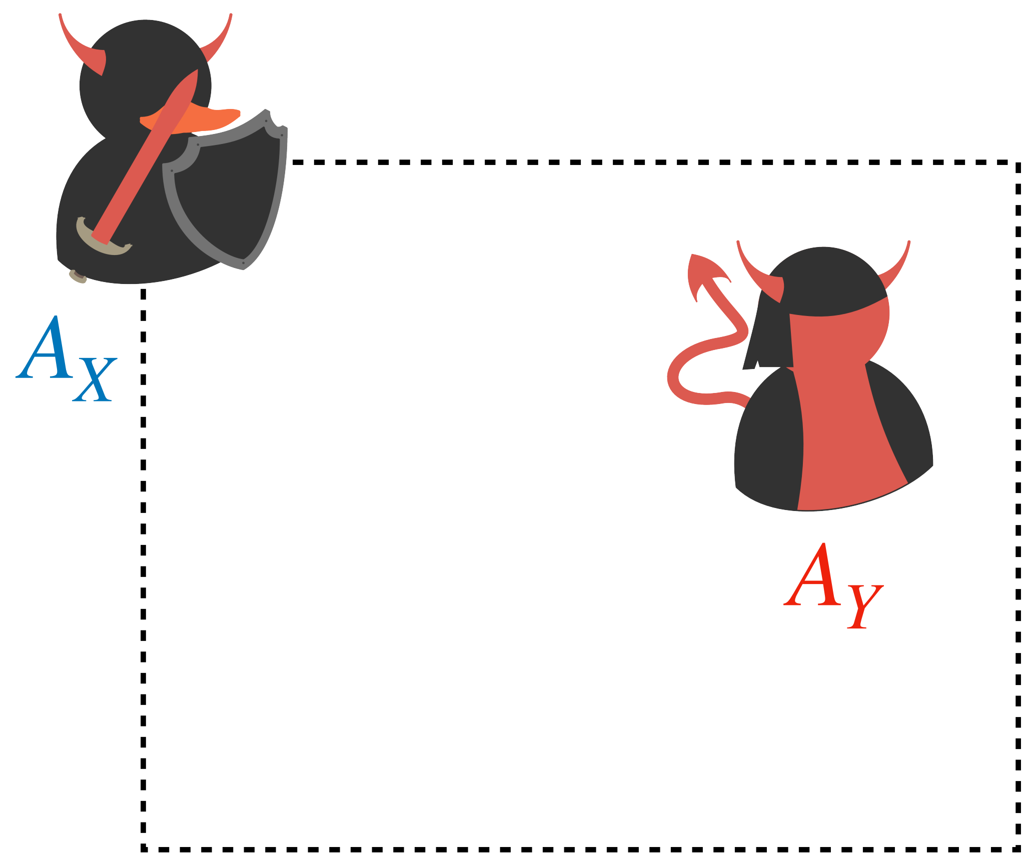
  - ϵ_X will be negligibly close to ϵ_Y i.e., $\epsilon_Y = \epsilon_X + \text{negl}(\lambda)$.
  - Since $X_0 \overset{c}{\approx} X_1$, $\epsilon_X = \text{negl}'(\lambda)$.



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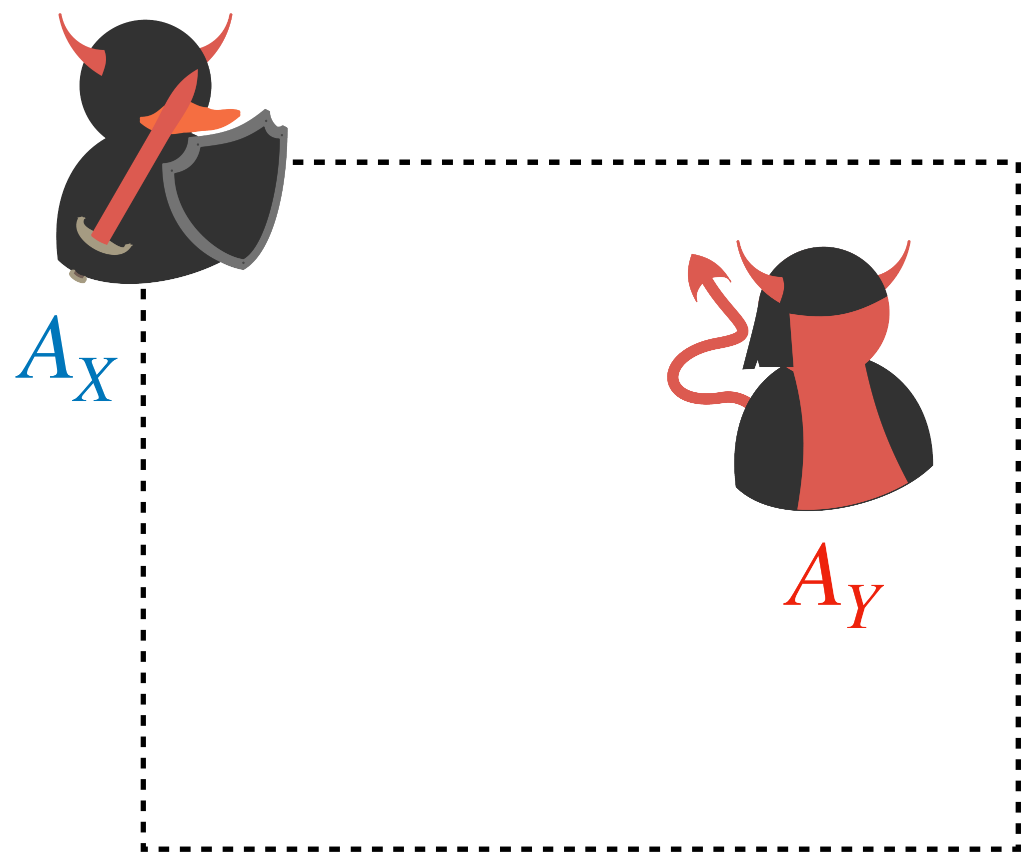
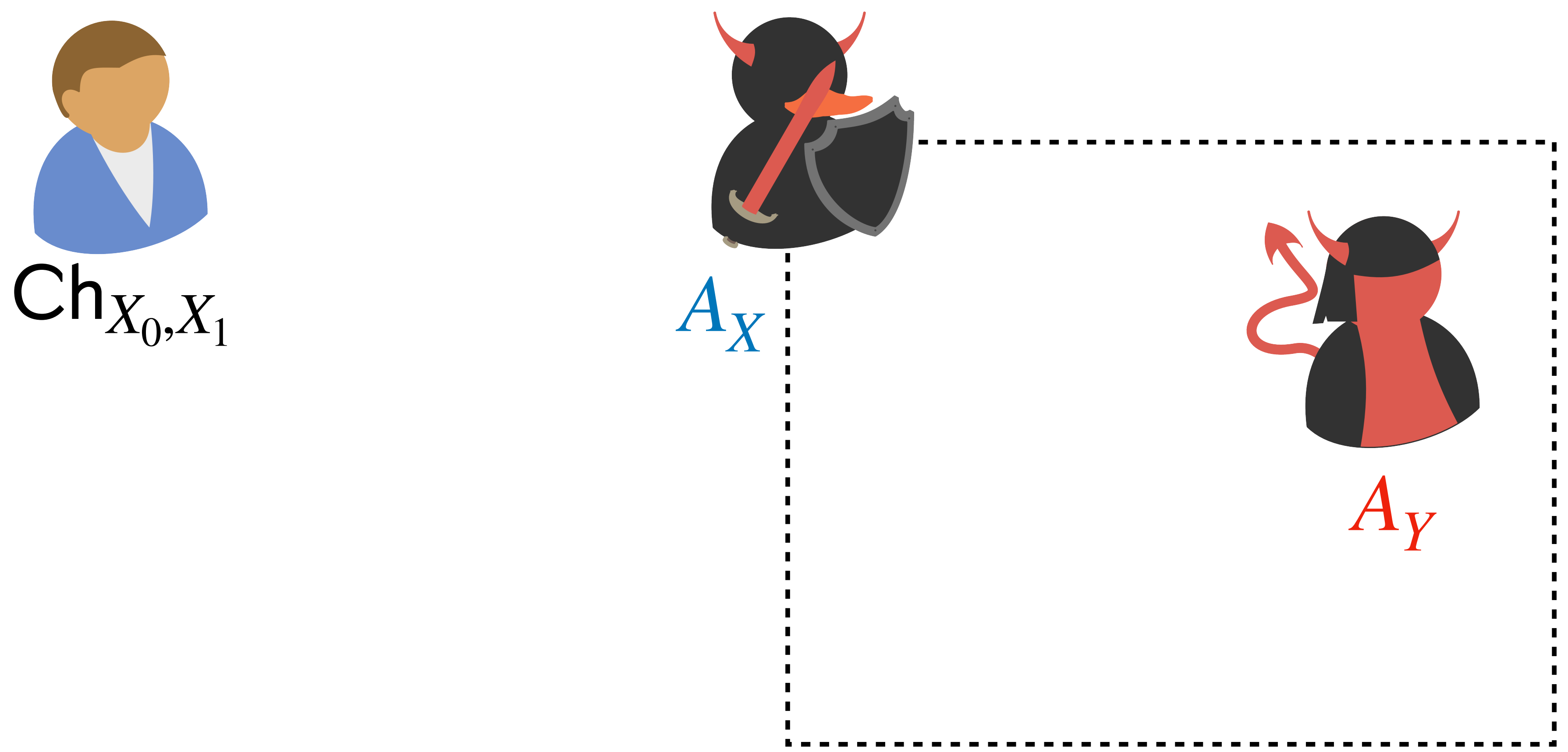
  - ϵ_X will be negligibly close to ϵ_Y i.e., $\epsilon_Y = \epsilon_X + \mathsf{negl}(\lambda)$.
  - Since $X_0 \overset{c}{\approx} X_1$, $\epsilon_X = \mathsf{negl}'(\lambda)$.
  - Therefore, $\epsilon_Y = \mathsf{negl}(\lambda) + \mathsf{negl}'(\lambda) = \mathsf{negl}''(\lambda)$.



Reductions: Key Points

Claim: If $X_0 \overset{c}{\approx} X_1$ then $Y_0 \overset{c}{\approx} Y_1$.



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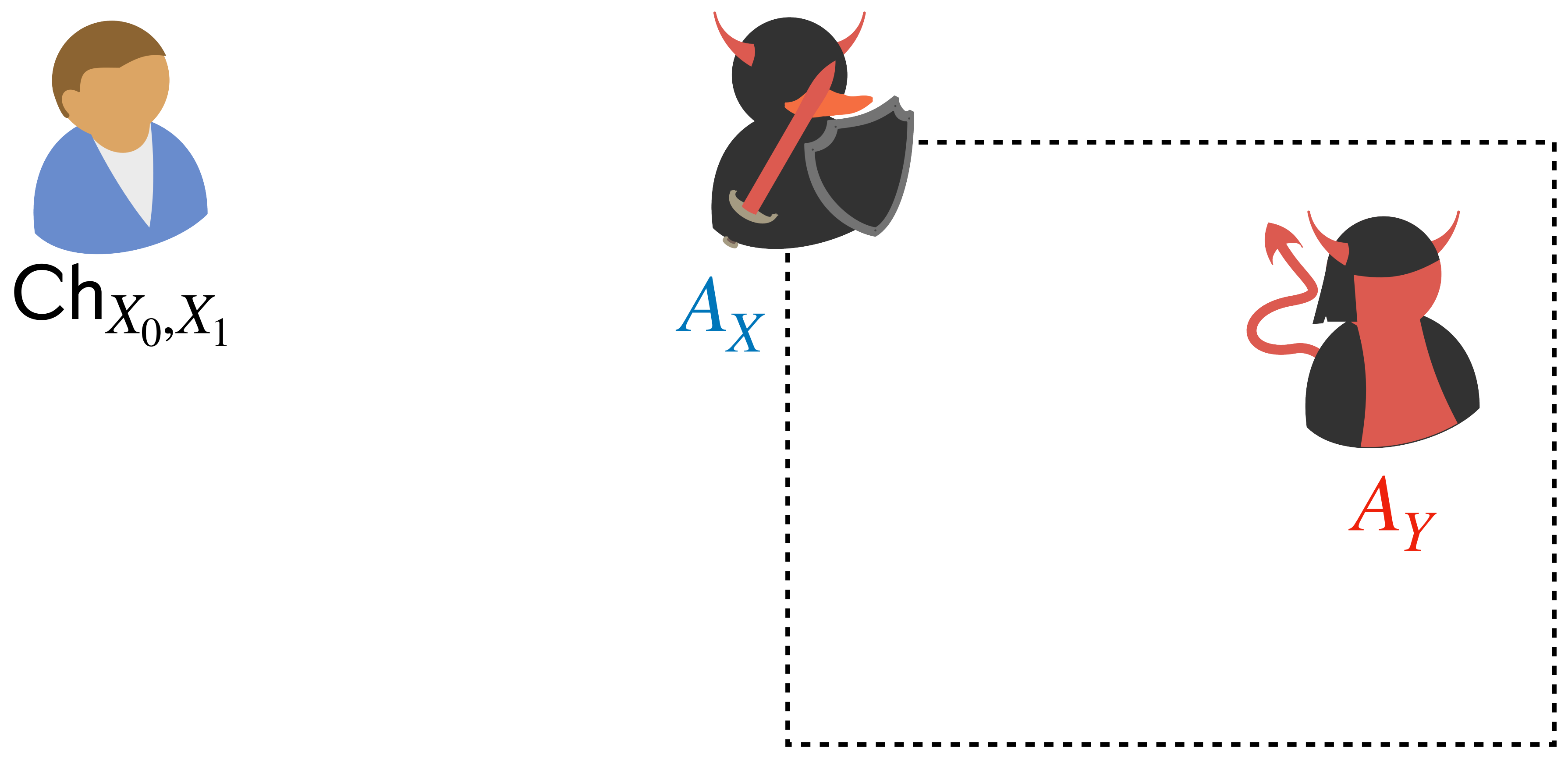
Two things to keep in mind when working through a reduction.



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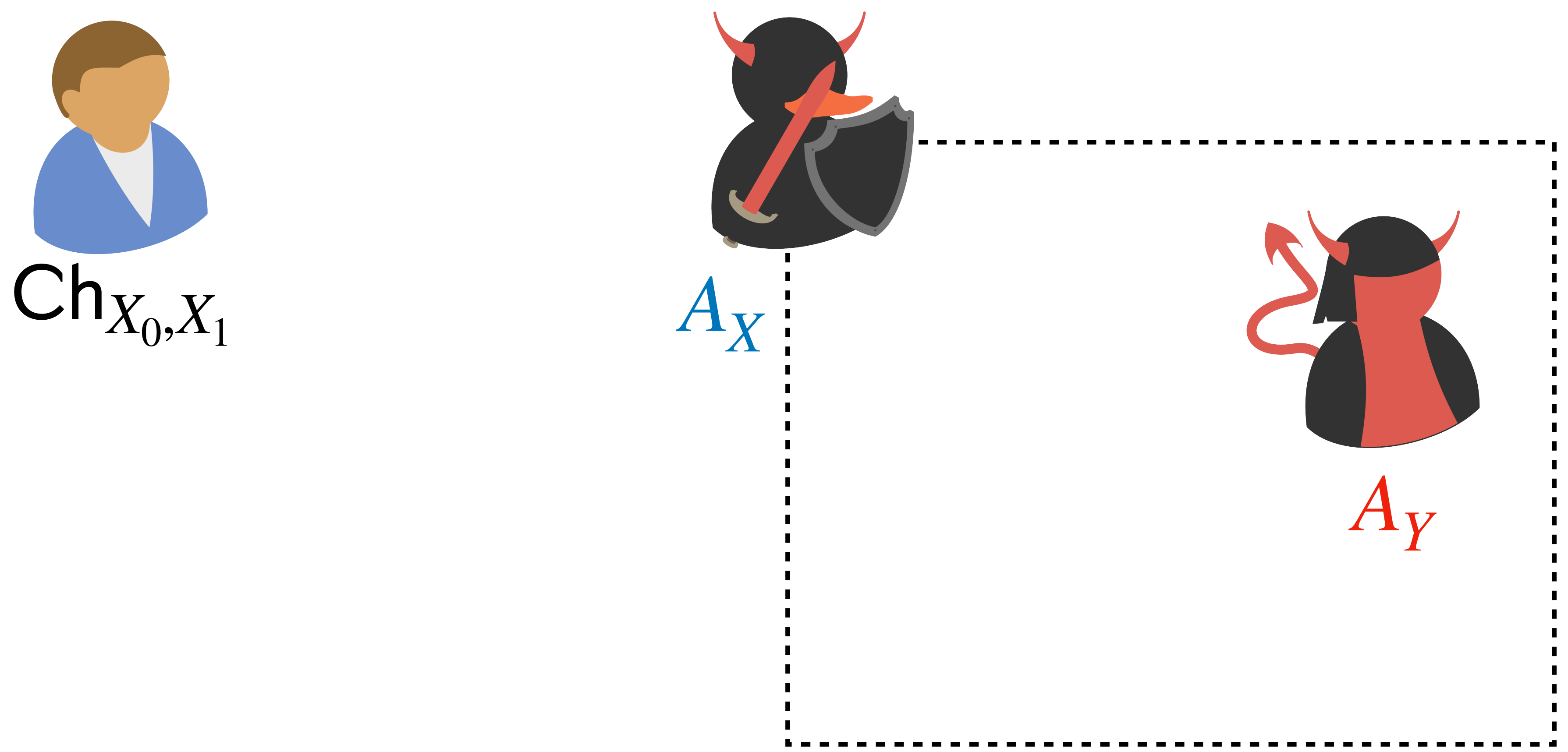
- **Input Mapping:** How can A_X map the input it receives from Ch_{X_0,X_1} to an input for A_Y?



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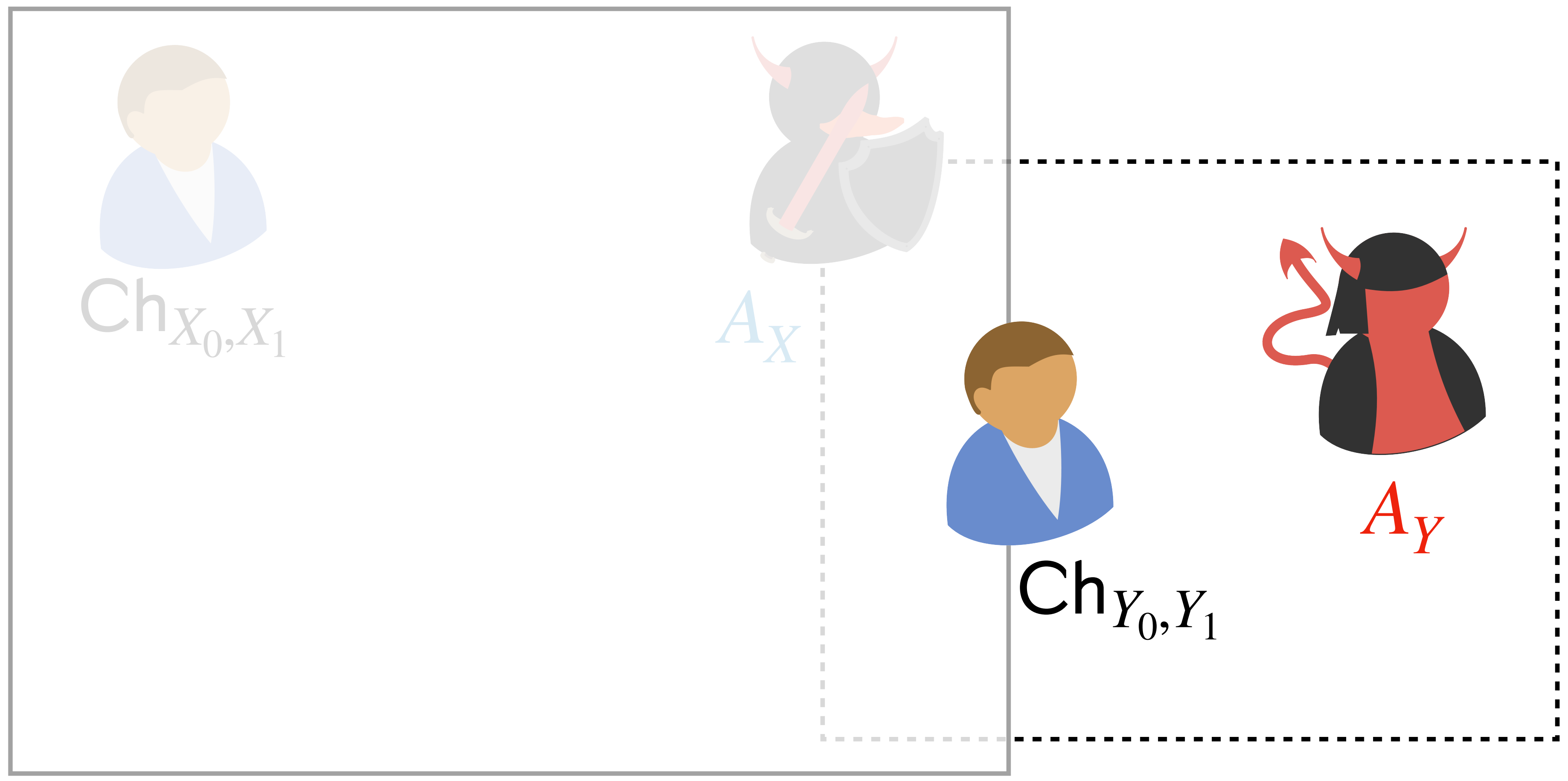

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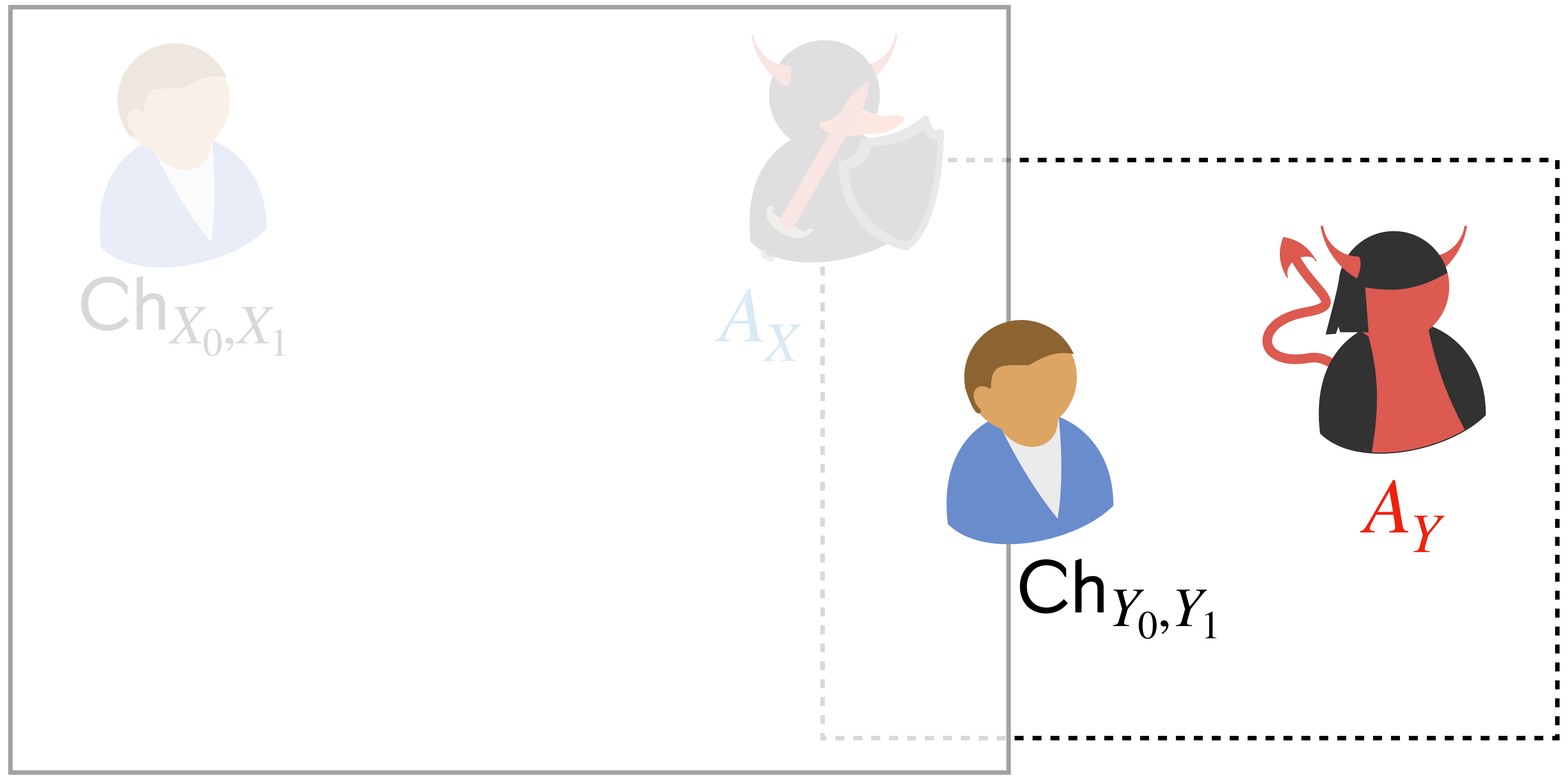

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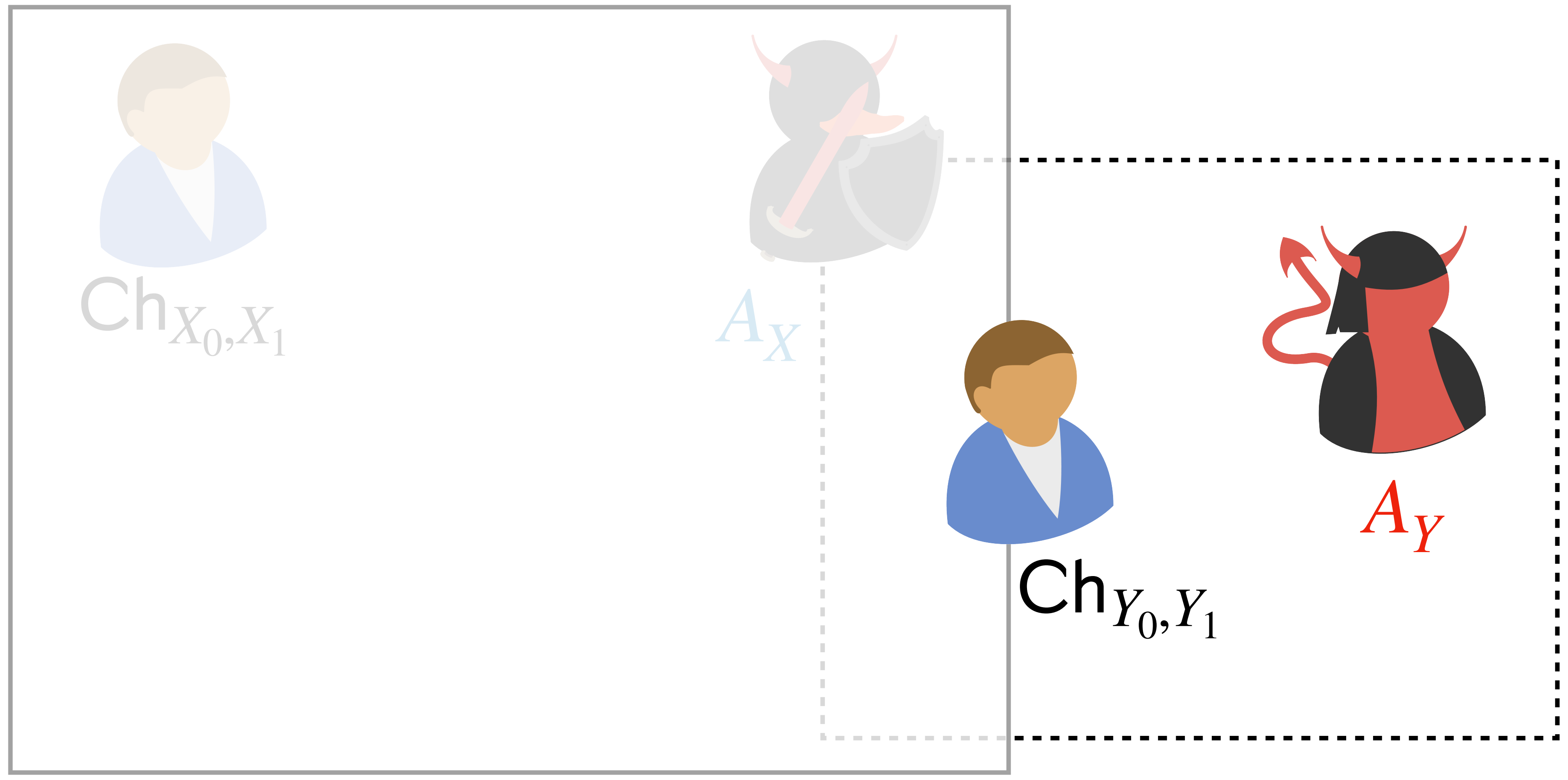

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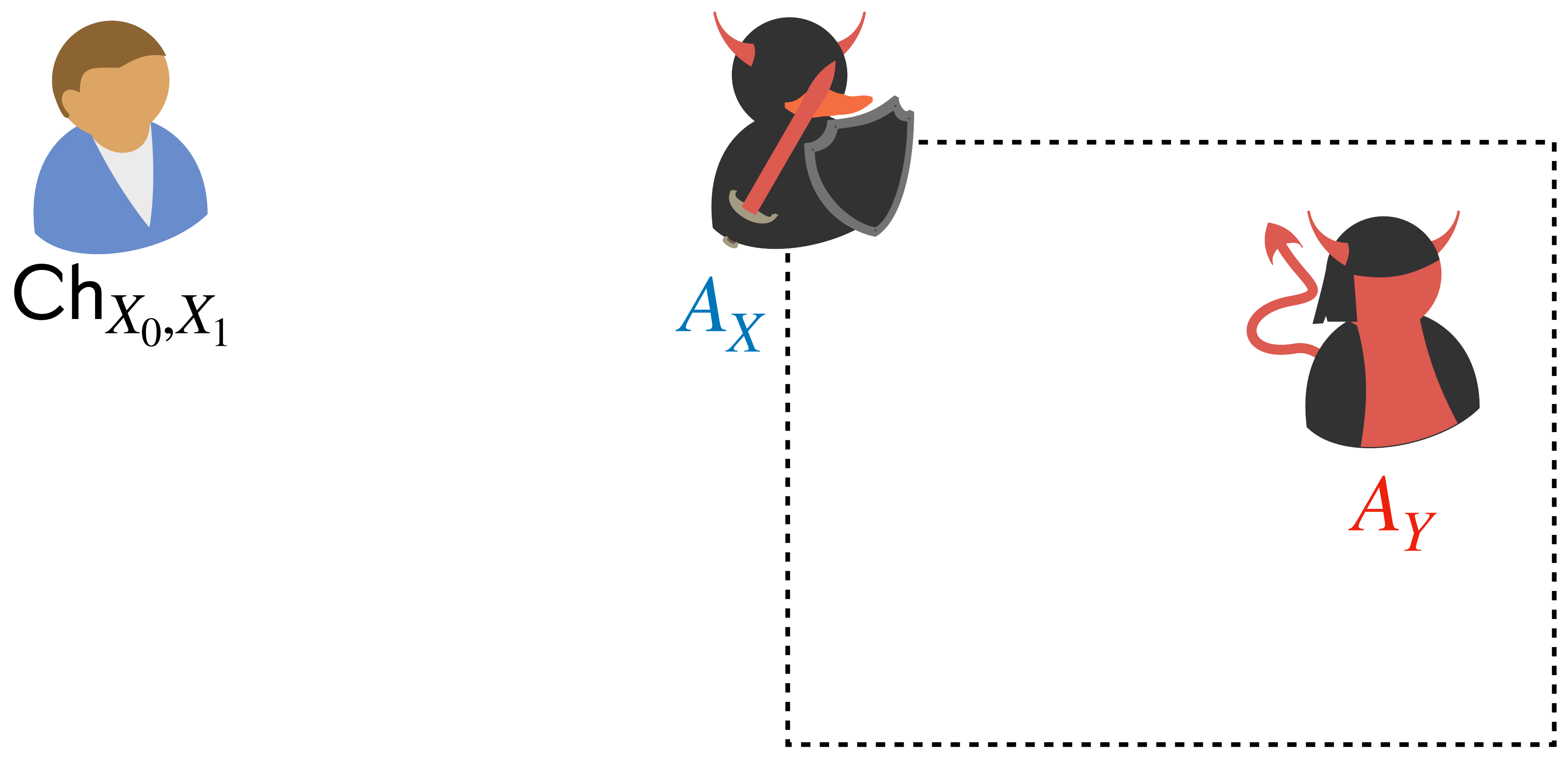

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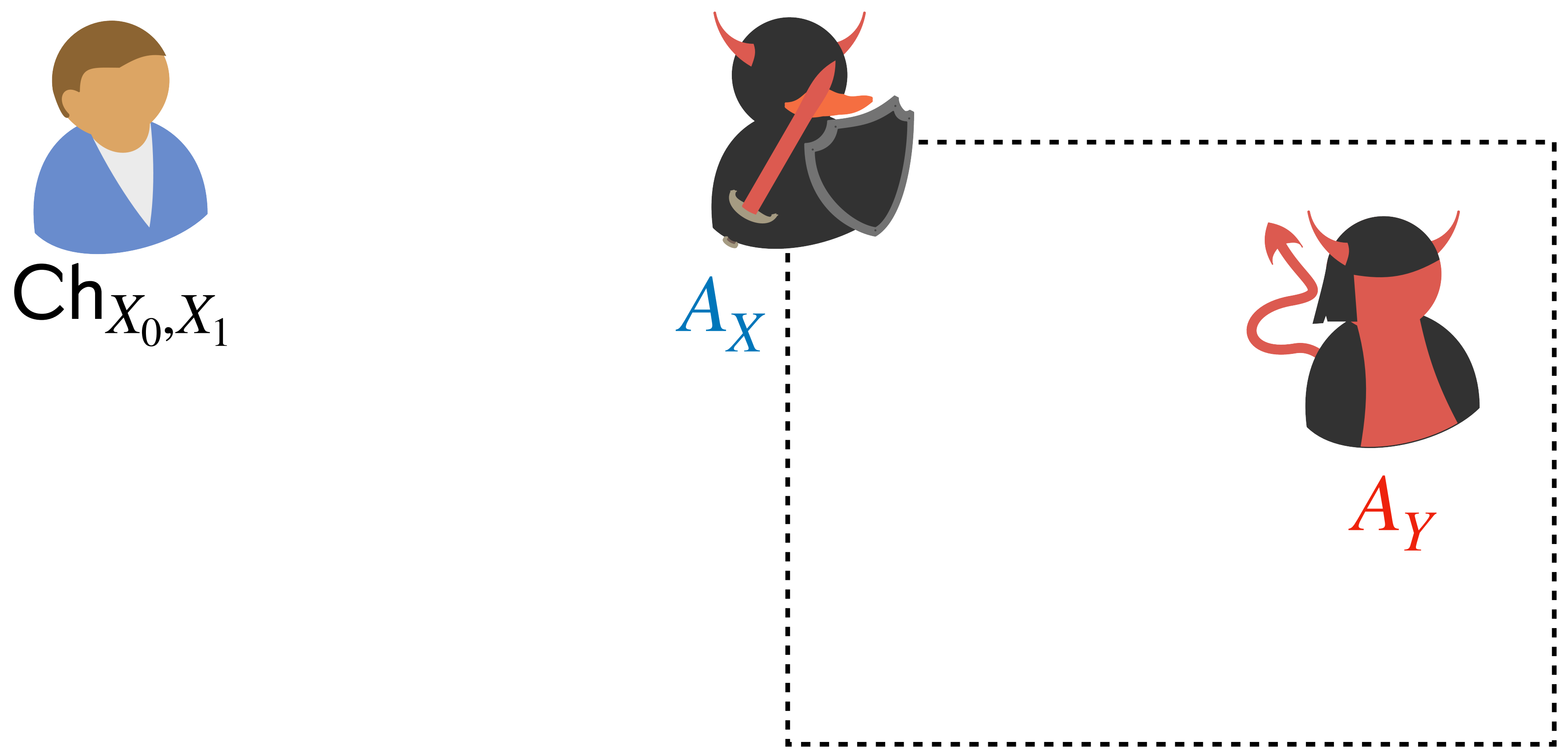

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  - We need to relate ϵ_X with ϵ_Y to then argue that ϵ_Y is negligible.