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    - Will start Thursday with a review session on whatever you think would be most useful

Proof Techniques

10th February 2026

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  - e.g. "If $G : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{\lambda+1}$ is a PRG, then $G'(s) = G(G(s)[1...\lambda])$ is a PRG"

Proof Techniques

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    - Done by specifying an adversary and analyzing its advantage

A reduction also involves specifying an adversary!

Proof Example: PRG

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If $\exists$ an adversary A_{H_0,H_1} that distinguishes between H_0 and H_1 with probability $\nu(\lambda)$, then $\exists$ an adversary that distinguishes between $G(s)$ and $r \xleftarrow{\$} \{0,1\}^\lambda$ with probability $\nu(\lambda)$

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C_G

$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^\lambda$

$r_0 := G(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{\lambda+1}$

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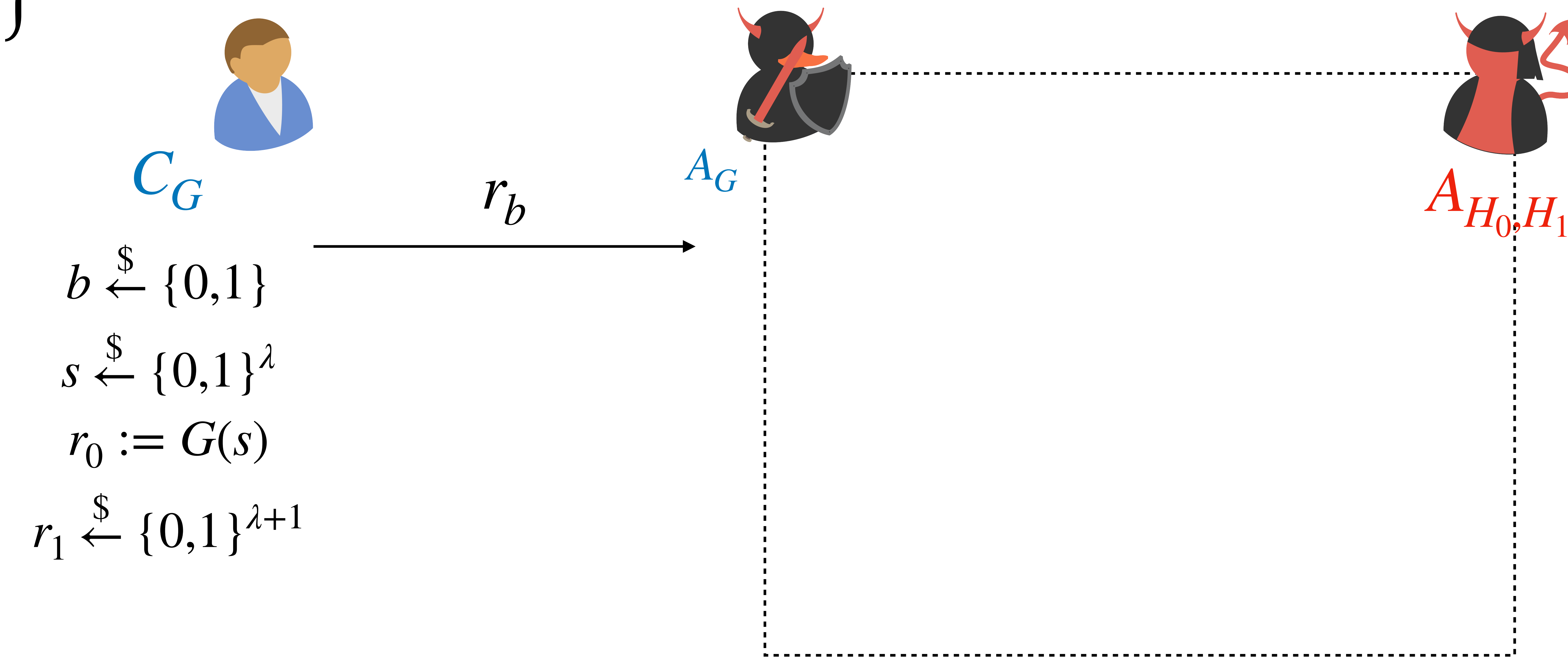

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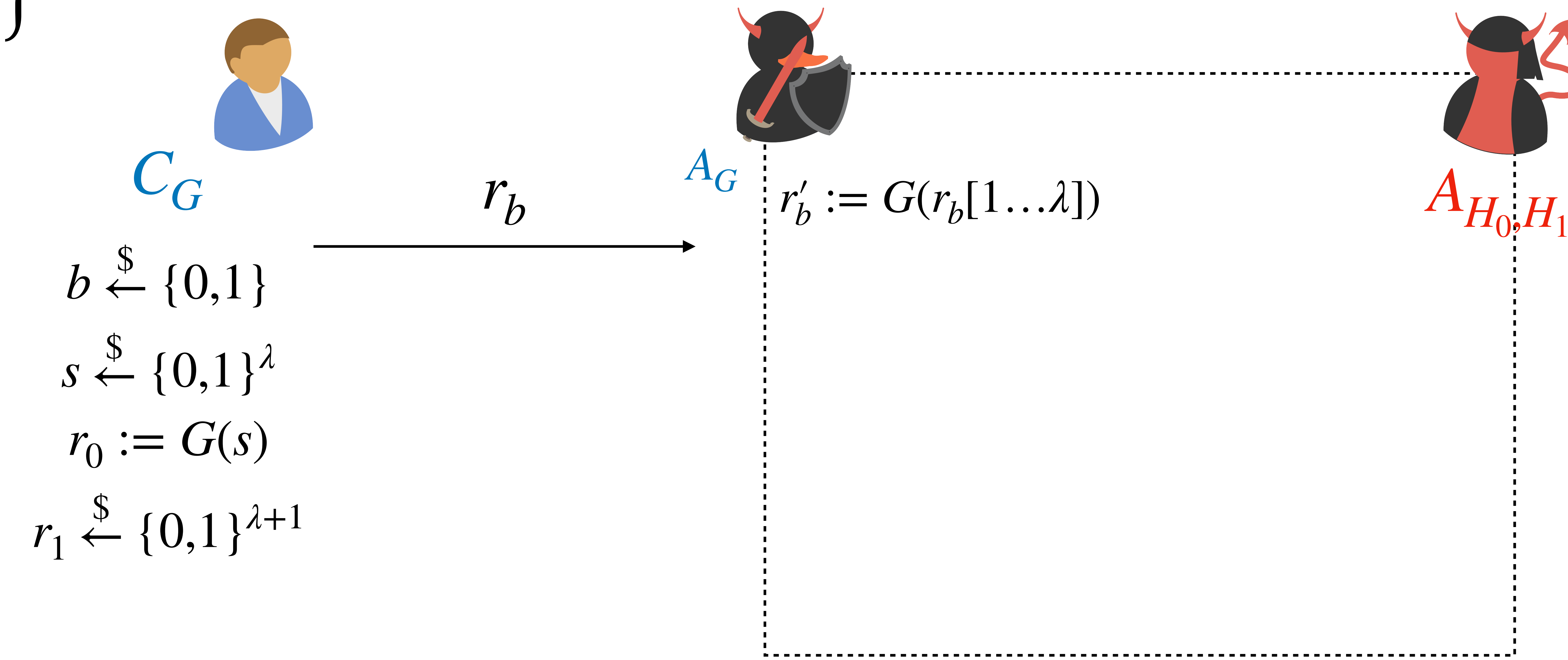

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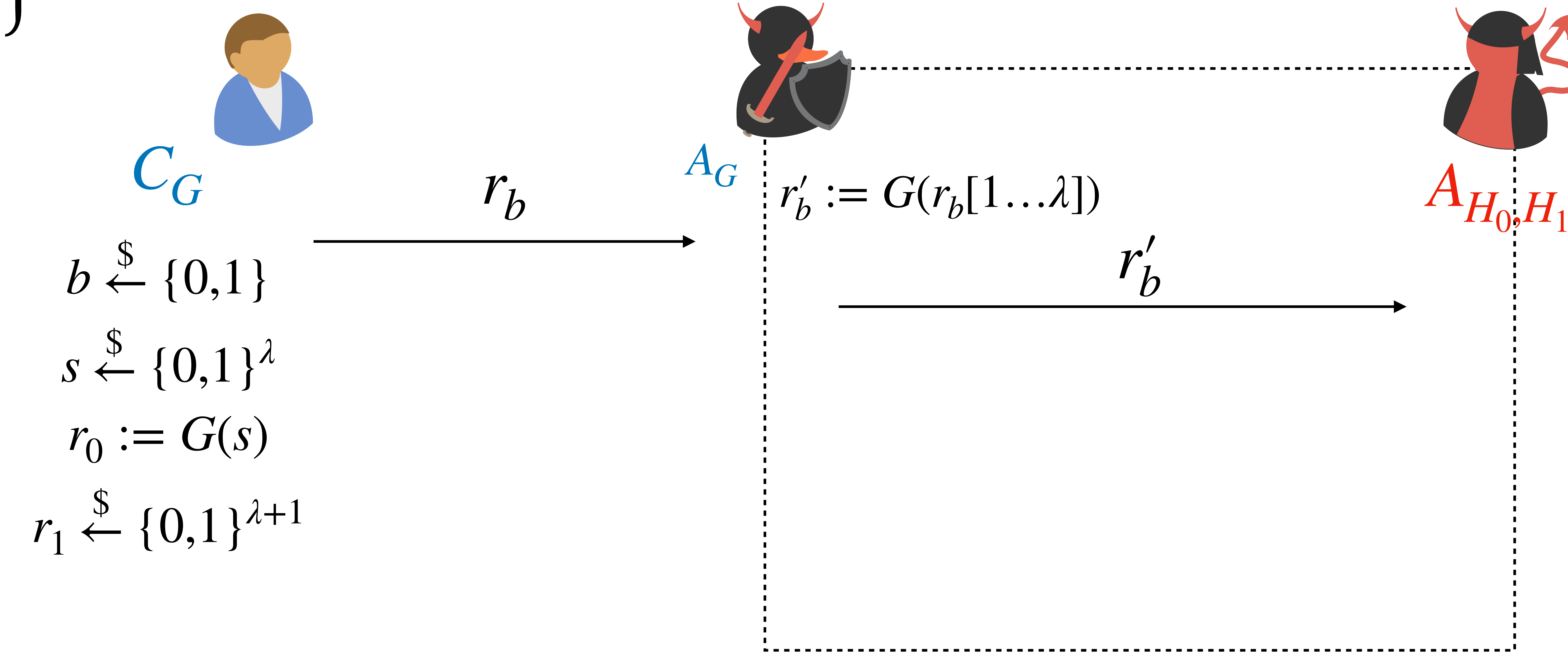

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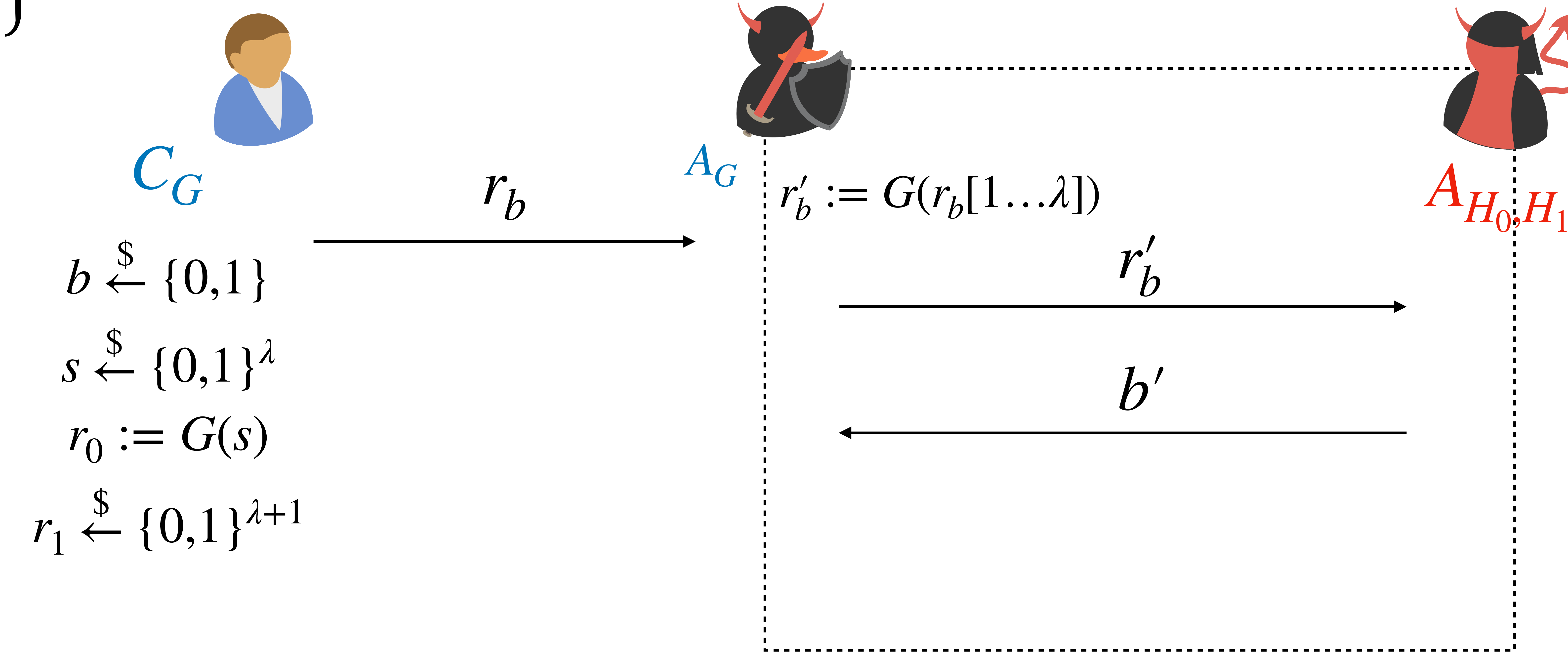

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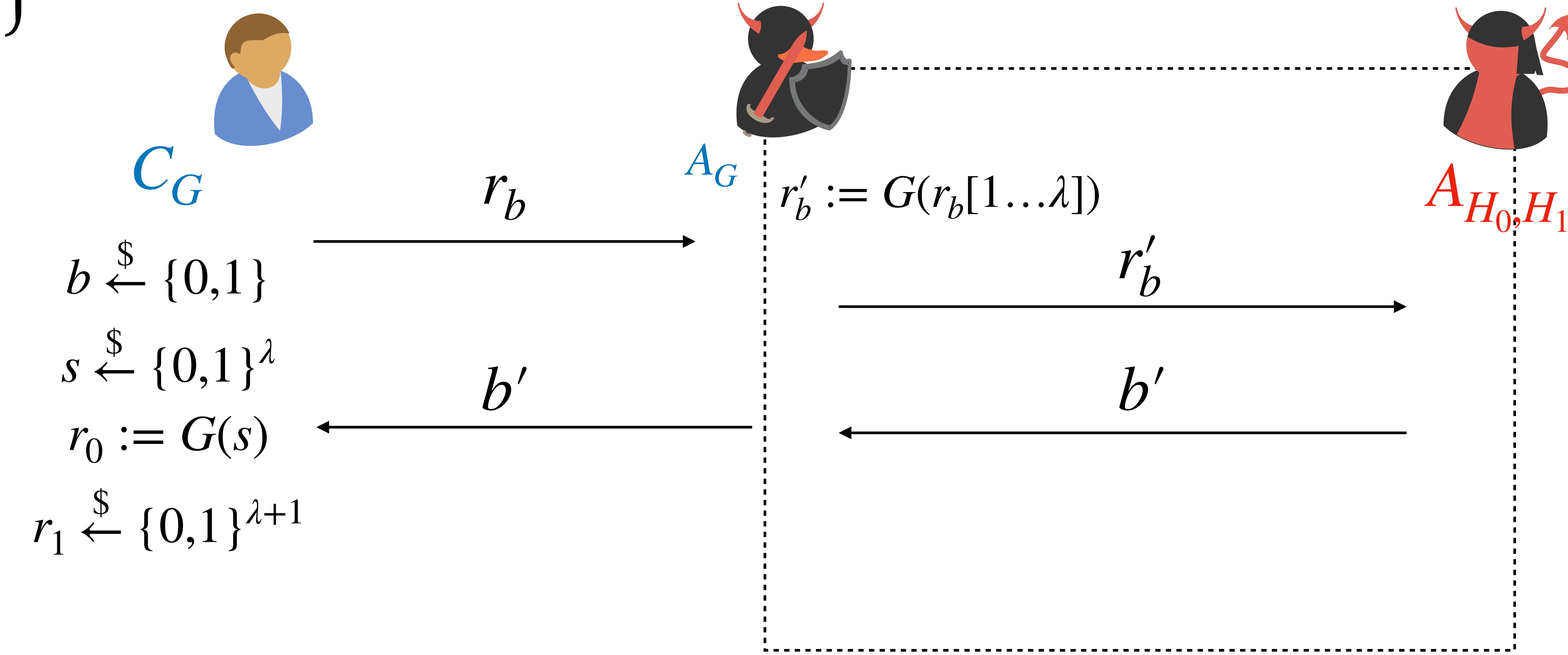

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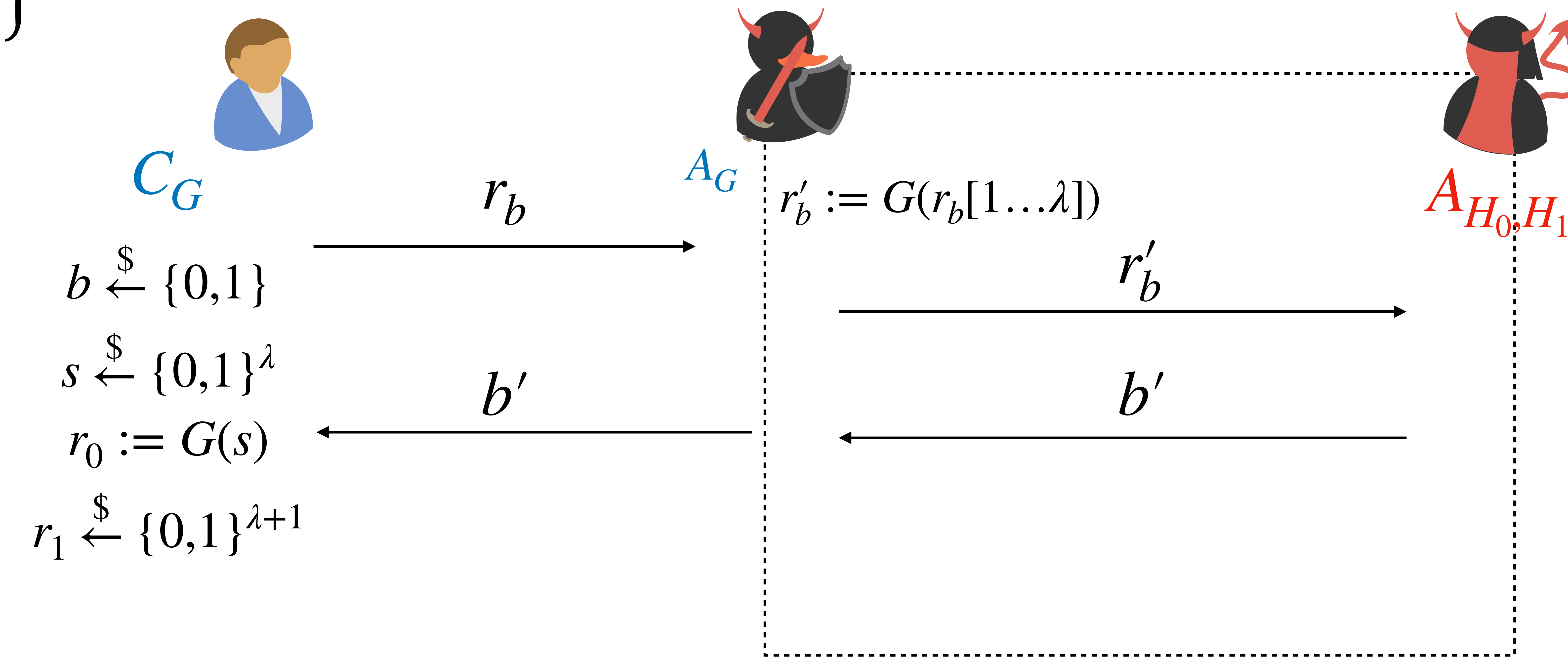

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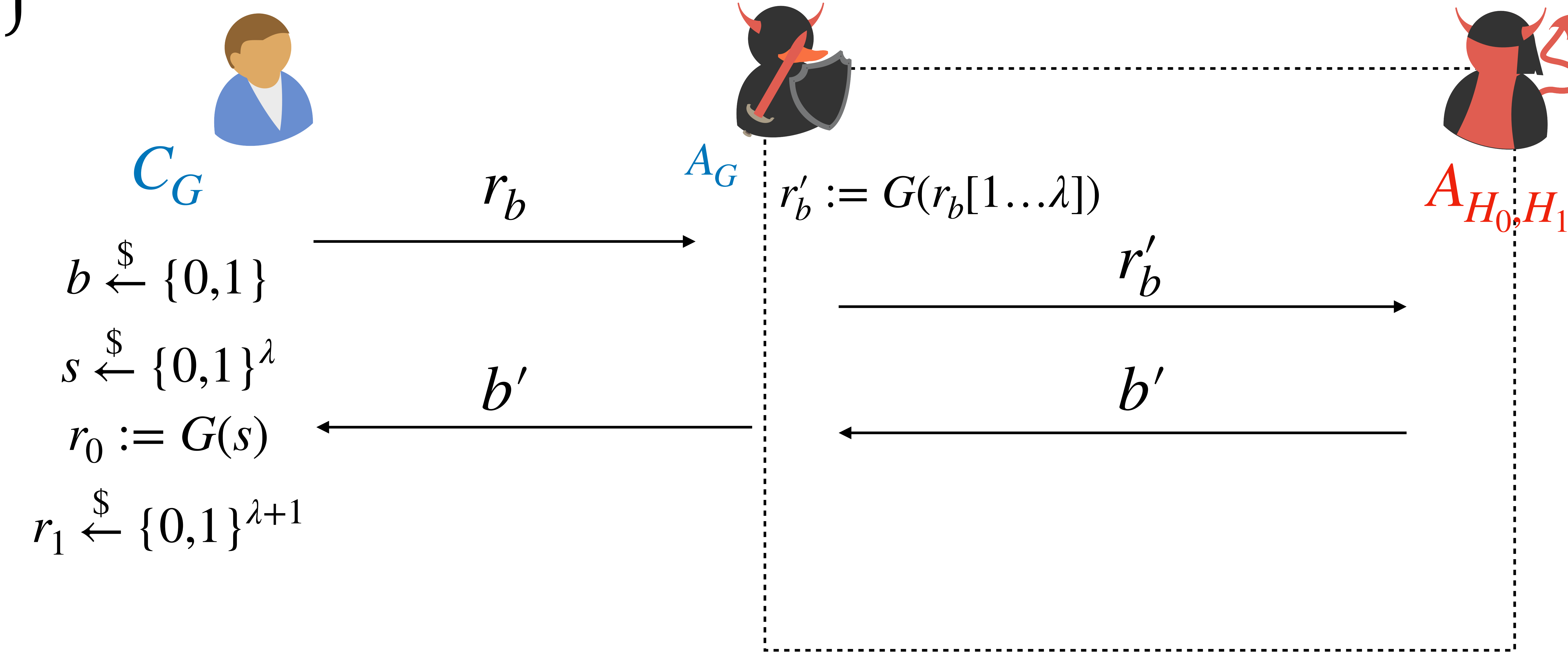
When $b = 0$, A_{H_0,H_1} sees $G(G(s)[1\ldots\lambda])$, the same as in H_0!



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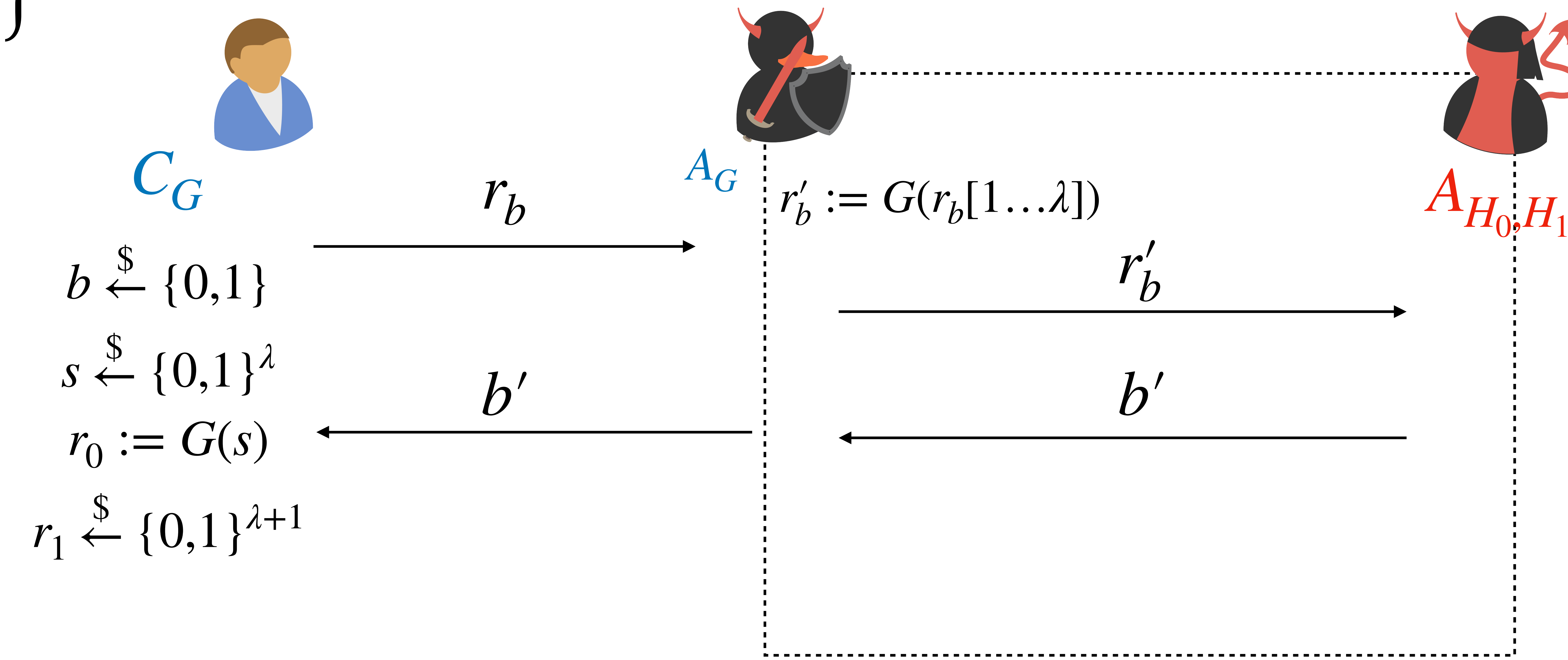

When $b = 0$, A_{H_0,H_1} sees $G(G(s)[1\ldots\lambda])$, the same as in H_0!

When $b = 1$, A_{H_0,H_1} sees $G(r[1\ldots\lambda])$, the same as in H_1!

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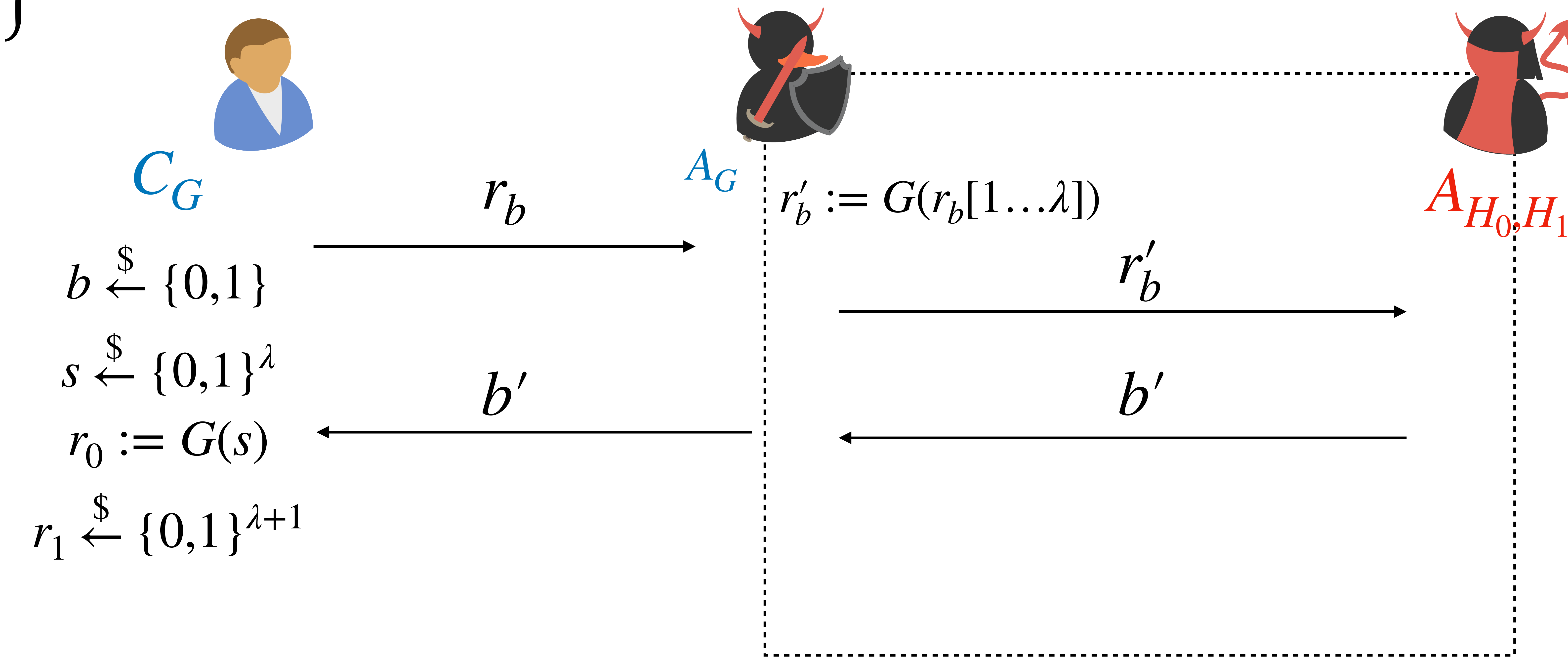
Must show that:

$$\left\{ G'(s) : s \xleftarrow{\$} \{0,1\}^\lambda \right\} \stackrel{c}{\approx} \left\{ r : r \xleftarrow{\$} \{0,1\}^{\lambda+1} \right\}$$

$$H_0 : \left\{ G(G(s)[1\ldots\lambda]) : s \xleftarrow{\$} \{0,1\}^{\lambda+1} \right\}$$

$$H_1 : \left\{ G(r[1\ldots\lambda]) : r \xleftarrow{\$} \{0,1\}^{\lambda+1} \right\}$$



When $b = 0$, A_{H_0,H_1} sees $G(G(s)[1\ldots\lambda])$, the same as in H_0!

When $b = 1$, A_{H_0,H_1} sees $G(r[1\ldots\lambda])$, the same as in H_1!

Therefore, $Pr[A_G \text{ wins}] = \Pr[A_{H_0,H_1} \text{ wins}]$

Proof Example: PRG

$G : \{0,1\}^\lambda \to \{0,1\}^{\lambda+1}$ is a PRG

$\underline{G'(s)}$:
return $G(G(s)[1\ldots\lambda])$

Given:

$$\left\{G(s) : s \xleftarrow{\$} \{0,1\}^\lambda\right\} \stackrel{c}{\approx} \left\{r : r \xleftarrow{\$} \{0,1\}^{\lambda+1}\right\}$$

Must show that:

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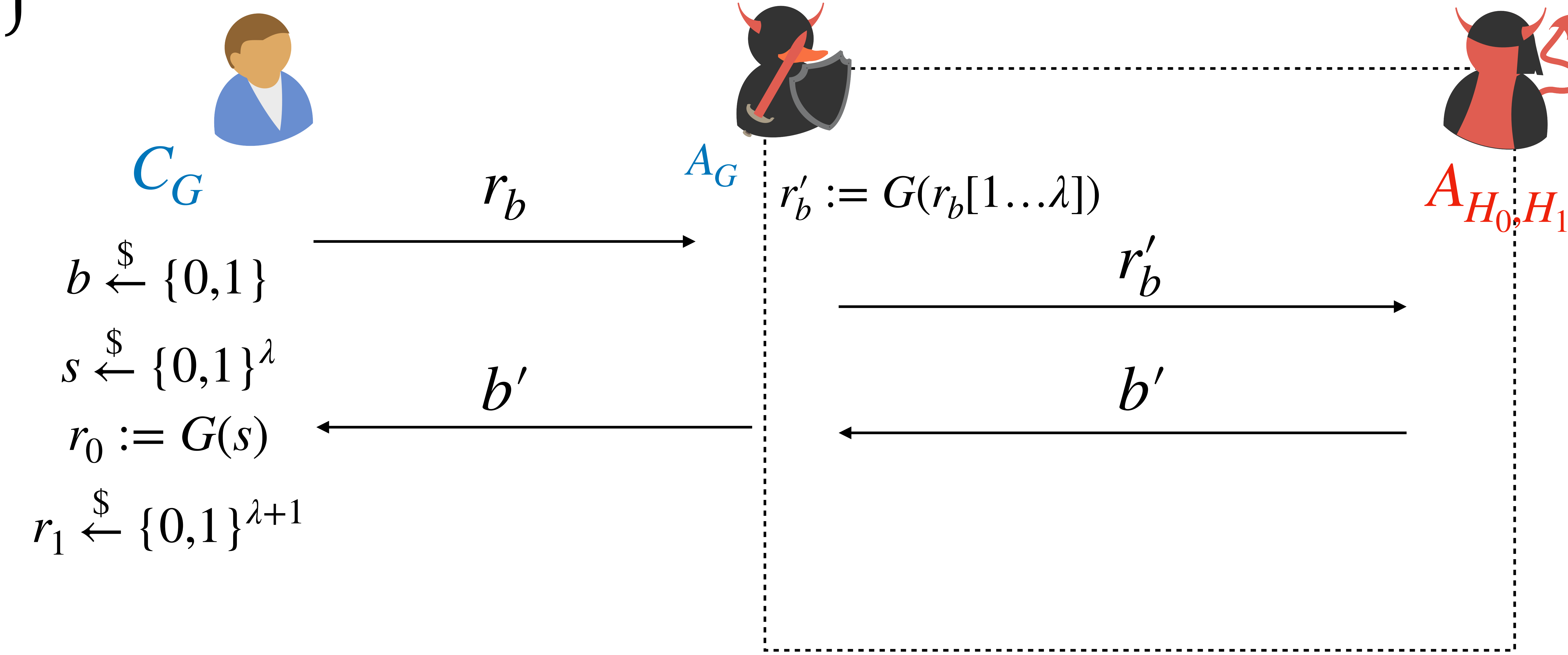

When $b = 0$, A_{H_0,H_1} sees $G(G(s)[1\ldots\lambda])$, the same as in H_0!

When $b = 1$, A_{H_0,H_1} sees $G(r[1\ldots\lambda])$, the same as in H_1!

Therefore, $\text{negl}(\lambda) = \Pr[A_{H_0,H_1}$ wins$]$

Proof Example: PRG

$G : \{0,1\}^\lambda \to \{0,1\}^{\lambda+1}$ is a PRG

Given:

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Must show that:

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$\underline{G'(s)}$:
return $G(G(s)[1\ldots\lambda])$

$$H_0 : \left\{ G(G(s)[1\ldots\lambda]) : s \xleftarrow{\$} \{0,1\}^{\lambda+1} \right\}$$

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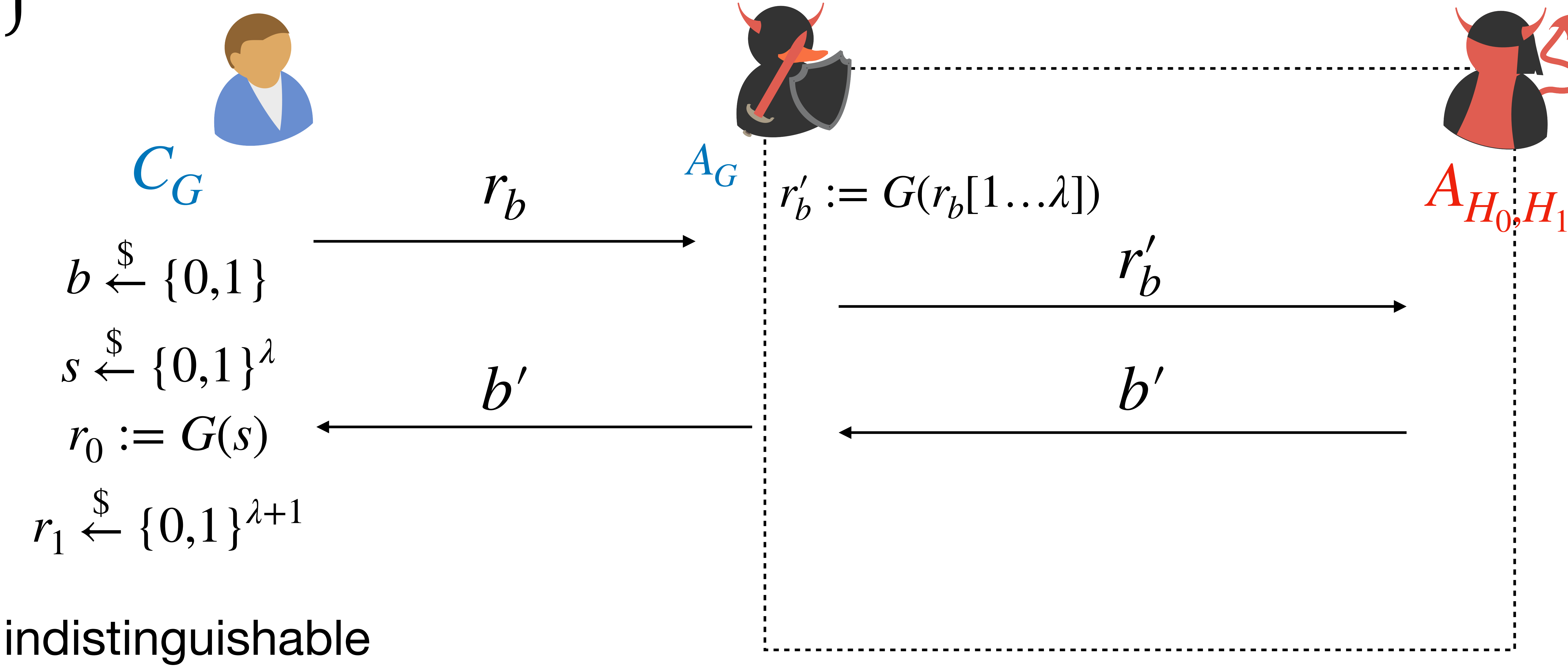

When $b = 0$, A_{H_0,H_1} sees $G(G(s)[1\ldots\lambda])$, the same as in H_0!

When $b = 1$, A_{H_0,H_1} sees $G(r[1\ldots\lambda])$, the same as in H_1!

Therefore, H_0 and H_1 are computationally indistinguishable

Proof Example: PRG

$G : \{0,1\}^\lambda \rightarrow \{0,1\}^{\lambda+1}$ is a PRG

Given:

$$\left\{ G(s) : s \xleftarrow{\$} \{0,1\}^\lambda \right\} \stackrel{c}{\approx} \left\{ r : r \xleftarrow{\$} \{0,1\}^{\lambda+1} \right\}$$

Must show that:

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return $G(G(s)[1\ldots\lambda])$

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$$H_0 : \left\{ G(G(s)[1\ldots\lambda]) : s \xleftarrow{\$} \{0,1\}^{\lambda+1} \right\}$$

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$$H_2 : \left\{ r : r \xleftarrow{\$} \{0,1\}^{\lambda+1} \right\}$$

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Prove indistinguishable via a reduction

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$$H_2 : \left\{ r : r \xleftarrow{\$} \{0,1\}^{\lambda+1} \right\}$$

Prove indistinguishable via a reduction

By the hybrid lemma $H_0 \overset{c}{\approx} H_2$, and so G' is a PRG

Proof Example: Not a PRG

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$G : \{0,1\}^\lambda \rightarrow \{0,1\}^{\lambda+1}$ is a PRG

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return $G(s) || s$

$G' : \{0,1\}^\lambda \to \{0,1\}^{2\lambda+1}$

Proof Example: Not a PRG

$G : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{\lambda+1}$ is a PRG

$\underline{A_{G'}(r)}$:

$\underline{G'(s)}$:

return $G(s) \,||\, s$

$G' : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{2\lambda+1}$

Proof Example: Not a PRG

$G : \{0,1\}^\lambda \to \{0,1\}^{\lambda+1}$ is a PRG

$\underline{G'(s)}$:

return $G(s) || s$

$G' : \{0,1\}^\lambda \to \{0,1\}^{2\lambda+1}$

$\underline{A_{G'}(r)}$:

$x = r[1 \ldots \lambda + 1]$

$y = r[\lambda + 2 \ldots 2\lambda + 1]$

Proof Example: Not a PRG

$G : \{0,1\}^\lambda \to \{0,1\}^{\lambda+1}$ is a PRG

$\underline{A_{G'}(r)}$:

$x = r[1\ldots\lambda+1]$

$y = r[\lambda+2\ldots 2\lambda+1]$

if $G(y) = x$: return 0

else return 1

$\underline{G'(s)}$:

return $G(s)||s$

$G' : \{0,1\}^\lambda \to \{0,1\}^{2\lambda+1}$

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$G : \{0,1\}^\lambda \rightarrow \{0,1\}^{\lambda+1}$ is a PRG

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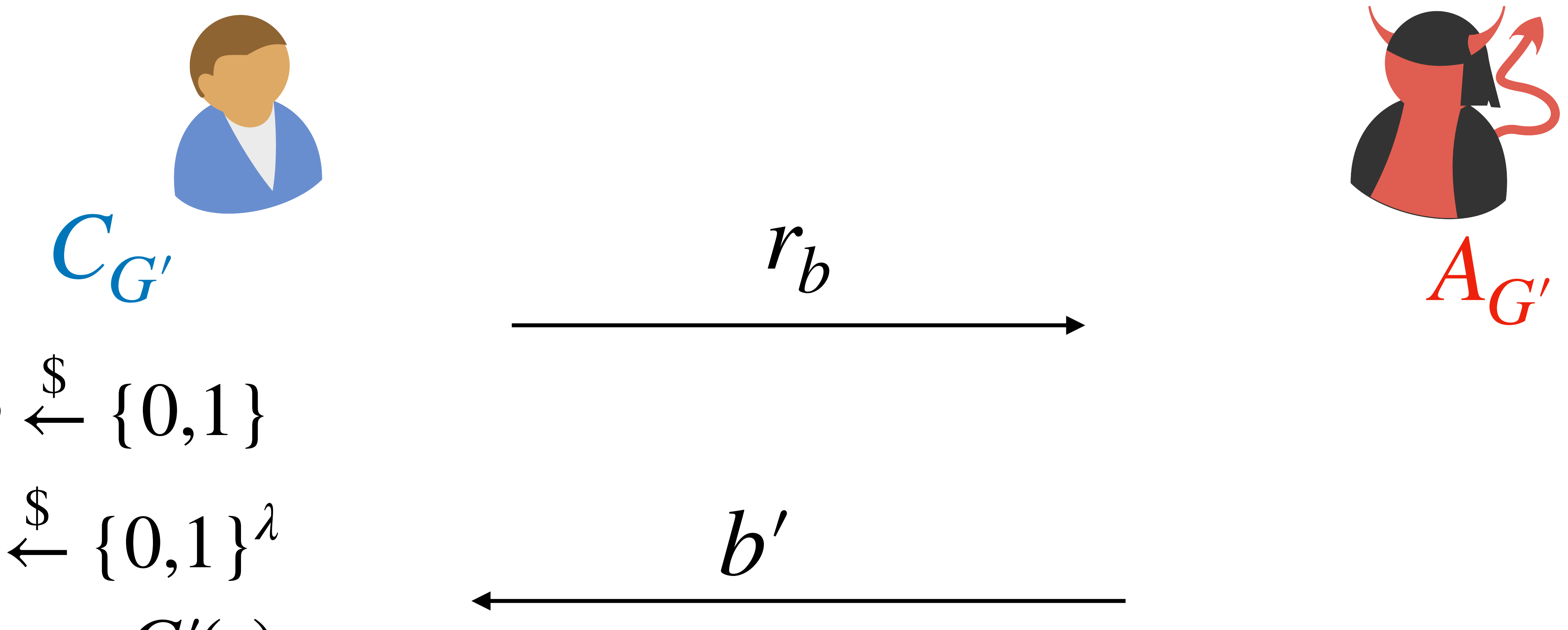

$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^\lambda$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

Proof Example: Not a PRG

$G : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{\lambda+1}$ is a PRG

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$\underline{G'(s)}$:

return $G(s)||s$

$G' : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{2\lambda+1}$

$C_{G'}$

$A_{G'}$

r_b →

← b'

$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^{\lambda}$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

$Pr[b = b'] =$

Proof Example: Not a PRG

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$\underline{G'(s)}$:

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$G' : \{0,1\}^\lambda \rightarrow \{0,1\}^{2\lambda+1}$

$C_{G'}$

$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^\lambda$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

r_b →

← b'

$A_{G'}$

$$Pr[b = b'] = \frac{1}{2}Pr[b = b'|b = 0] + \frac{1}{2}Pr[b = b'|b = 1]$$

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$G : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{\lambda+1}$ is a PRG

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$\underline{G'(s)}$:

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$G' : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{2\lambda+1}$

$$Pr[b = b'] = \frac{1}{2}Pr[b = b' | b = 0] + \frac{1}{2}Pr[b = b' | b = 1]$$

$C_{G'}$

$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^{\lambda}$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

r_b →

← b'

$A_{G'}$

$$Pr[b = b' | b = 0]$$

Proof Example: Not a PRG

$G : \{0,1\}^\lambda \rightarrow \{0,1\}^{\lambda+1}$ is a PRG

$\underline{A_{G'}(r)}$:

$x = r[1\ldots\lambda + 1]$

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if $G(y) = x$: return 0

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$\underline{G'(s)}$:

return $G(s)||s$

$G' : \{0,1\}^\lambda \rightarrow \{0,1\}^{2\lambda+1}$

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$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^\lambda$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

r_b →

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$A_{G'}$

$$Pr[b = b'] = \frac{1}{2}Pr[b = b' | b = 0] + \frac{1}{2}Pr[b = b' | b = 1]$$

$$Pr[b = b' | b = 0]$$

$$Pr[0 = b' | b = 0]$$

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r_b →

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$A_{G'}$

$$Pr[b = b'] = \frac{1}{2}Pr[b = b' | b = 0] + \frac{1}{2}Pr[b = b' | b = 1]$$

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$$Pr[G(s) = G(s)]$$

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return $G(s)||s$

$G':\{0,1\}^{\lambda}\rightarrow\{0,1\}^{2\lambda+1}$

$C_{G'}$

$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^{\lambda}$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

r_b →

← b'

$A_{G'}$

$$Pr[b=b'] = \frac{1}{2}Pr[b=b'|b=0] + \frac{1}{2}Pr[b=b'|b=1]$$

$Pr[b=b'|b=0]$

$Pr[0=b'|b=0]$

$Pr[G(s)=G(s)]$

$=1$

Proof Example: Not a PRG

$G : \{0,1\}^\lambda \to \{0,1\}^{\lambda+1}$ is a PRG

$\underline{A_{G'}(r)}$:

$x = r[1 \ldots \lambda + 1]$

$y = r[\lambda + 2 \ldots 2\lambda + 1]$

if $G(y) = x$: return 0

else return 1

$\underline{G'(s)}$:
return $G(s) \| s$

$G' : \{0,1\}^\lambda \to \{0,1\}^{2\lambda+1}$

$C_{G'}$

$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^\lambda$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

r_b →

← b'

$A_{G'}$

$$Pr[b = b'] = \frac{1}{2}Pr[b = b' \mid b = 0] + \frac{1}{2}Pr[b = b' \mid b = 1]$$

$Pr[b = b' \mid b = 0]$

$Pr[0 = b' \mid b = 0]$

$Pr[G(s) = G(s)]$

$= 1$

$Pr[b = b' \mid b = 1]$

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r_b →

← b'

$A_{G'}$

$$Pr[b = b'] = \frac{1}{2}Pr[b = b' | b = 0] + \frac{1}{2}Pr[b = b' | b = 1]$$

$Pr[b = b' | b = 0]$

$Pr[0 = b' | b = 0]$

$Pr[G(s) = G(s)]$

$= 1$

$Pr[b = b' | b = 1]$

$Pr[1 = b' | b = 0]$

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$s \xleftarrow{\$} \{0,1\}^{\lambda}$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

$\xrightarrow{\quad r_b \quad}$

$\xleftarrow{\quad b' \quad}$

$A_{G'}$

$$Pr[b = b'] = \frac{1}{2}Pr[b = b' | b = 0] + \frac{1}{2}Pr[b = b' | b = 1]$$

$Pr[b = b' | b = 0]$

$Pr[0 = b' | b = 0]$

$Pr[G(s) = G(s)]$

$= 1$

$Pr[b = b' | b = 1]$

$Pr[1 = b' | b = 0]$

$Pr[G(y) \neq x | x \xleftarrow{\$} \{0,1\}^{\lambda+1}]$

Proof Example: Not a PRG

$G : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{\lambda+1}$ is a PRG

$\underline{G'(s)}$:

return $G(s) || s$

$G' : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{2\lambda+1}$

$\underline{A_{G'}(r)}$:

$x = r[1 \ldots \lambda + 1]$

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if $G(y) = x$: return 0

else return 1

$C_{G'}$

$b \overset{\$}{\leftarrow} \{0,1\}$

$s \overset{\$}{\leftarrow} \{0,1\}^{\lambda}$

$r_0 := G'(s)$

$r_1 \overset{\$}{\leftarrow} \{0,1\}^{2\lambda+1}$

r_b →

← b'

$A_{G'}$

$$Pr[b = b'] = \frac{1}{2} Pr[b = b' | b = 0] + \frac{1}{2} Pr[b = b' | b = 1]$$

$Pr[b = b' | b = 0]$

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$Pr[G(s) = G(s)]$

$= 1$

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$= 1 - \frac{1}{2^{\lambda+1}}$

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$G' : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{2\lambda+1}$

$$Pr[b = b'] = \frac{1}{2} + \frac{1}{2} - \frac{1}{2^{\lambda+2}} = 1 - \frac{1}{2^{\lambda+2}}$$

$C_{G'}$

$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^{\lambda}$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

r_b →

← b'

$A_{G'}$

$$Pr[b = b' \,|\, b = 0]$$
$$Pr[0 = b' \,|\, b = 0]$$
$$Pr[G(s) = G(s)]$$
$$= 1$$

$$Pr[b = b' \,|\, b = 1]$$
$$Pr[1 = b' \,|\, b = 0]$$
$$Pr[G(y) \neq x \,|\, x \xleftarrow{\$} \{0,1\}^{\lambda+1}]$$
$$= 1 - \frac{1}{2^{\lambda+1}}$$

Proof Example: Not a PRG

$G : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{\lambda+1}$ is a PRG

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$G' : \{0,1\}^{\lambda} \rightarrow \{0,1\}^{2\lambda+1}$

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$b \xleftarrow{\$} \{0,1\}$

$s \xleftarrow{\$} \{0,1\}^{\lambda}$

$r_0 := G'(s)$

$r_1 \xleftarrow{\$} \{0,1\}^{2\lambda+1}$

r_b →

← b'

$A_{G'}$

$$Pr[b = b'] = \frac{1}{2} + \frac{1}{2} - \frac{1}{2^{\lambda+2}} = 1 - \frac{1}{2^{\lambda+2}}$$

$Pr[b = b'$

$Pr[0 = b'$

$Pr[G(s) =$

$= 1$

NOT negligibly close to $\frac{1}{2}$, and so G' is *not* a PRG

$= 1 - \frac{\quad}{2^{\lambda+1}}$

Proof Techniques

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Proof Techniques

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  - This may be harder than the example we just gave! It might require "specific" schemes.
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  - This gives you a lot of freedom! You can choose G to be *whatever you want*, as long as it is a PRG!

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Proof Techniques

- Proving that a construction satisfies a definition
- Proving that a construction *does not* satisfy a definition
- Proving that a definition implies another definition
    - Done via a reduction (see uniform ciphertext security -> perfect security)

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- Proving that a construction satisfies a definition
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Proof Techniques

When approaching a proof, first ask: "Which type is it going to be?"

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Pseudorandomness II

601.442/642 Modern Cryptography

5th February 2026

Multi-Message Security

One-Time Computational Security

An encryption scheme with message length $\ell := \ell(\lambda)$ is one-time computationally secure if $\forall m_0, m_1 \in \{0,1\}^\ell$

$$D_0 = \left\{ \mathsf{ct} : \begin{matrix} k \leftarrow \mathsf{KeyGen}(1^\lambda) \\ c \leftarrow \mathsf{Enc}(k, m_0) \end{matrix} \right\} \overset{c}{\approx} D_1 = \left\{ \mathsf{ct} : \begin{matrix} k \leftarrow \mathsf{KeyGen}(1^\lambda) \\ c \leftarrow \mathsf{Enc}(k, m_1) \end{matrix} \right\}$$

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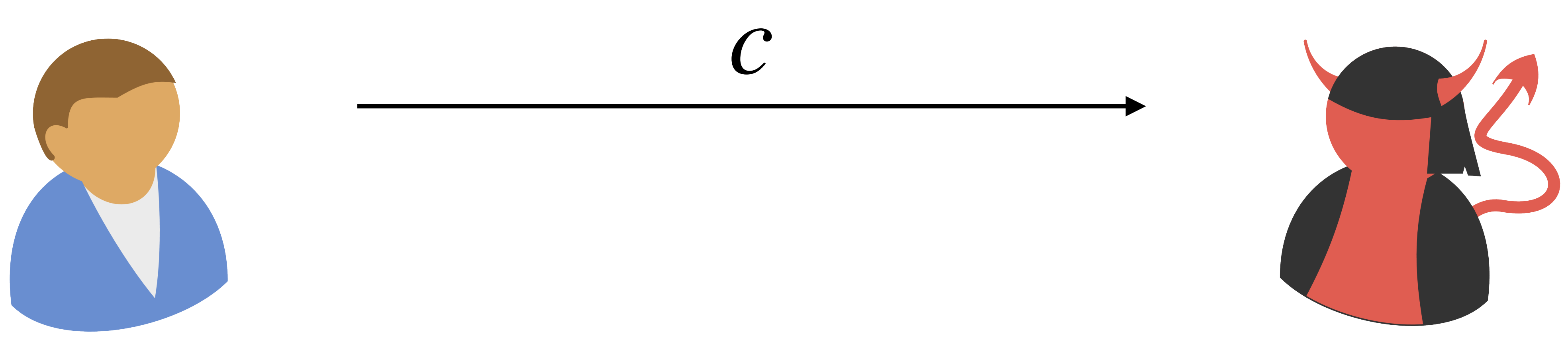

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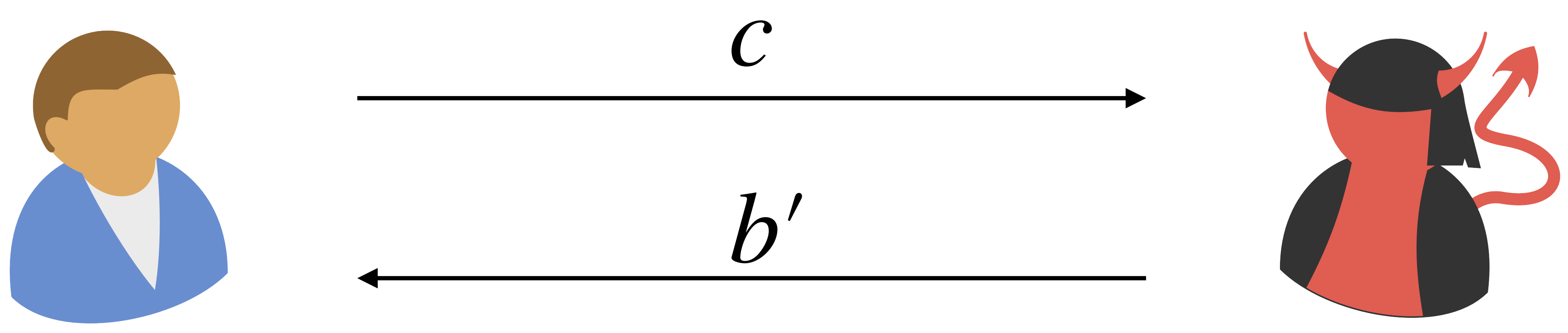

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c →

← b'

Wins if $b' = b$

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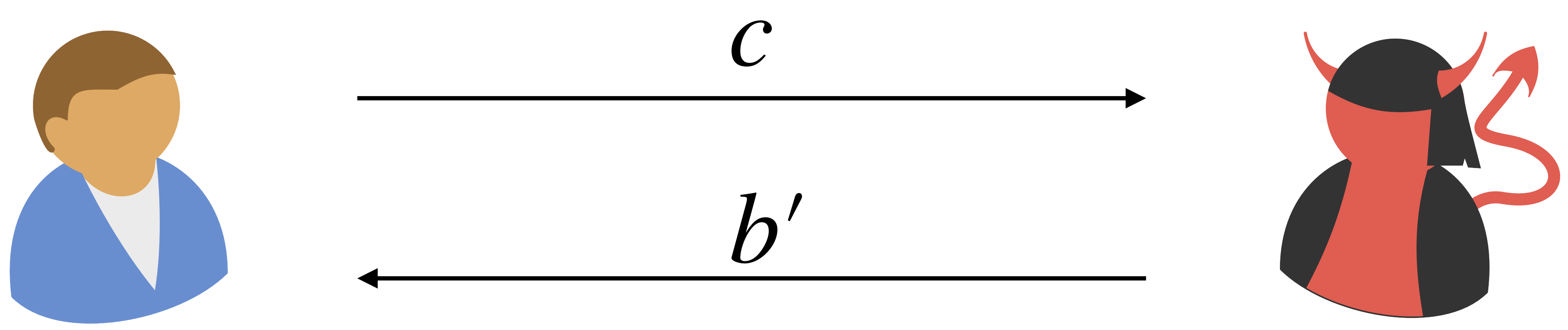

Wins if $b' = b$

$$\forall \mathscr{A}, \forall m_0, m_1,$$
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$q(\lambda)$ ***pairs*** of messages

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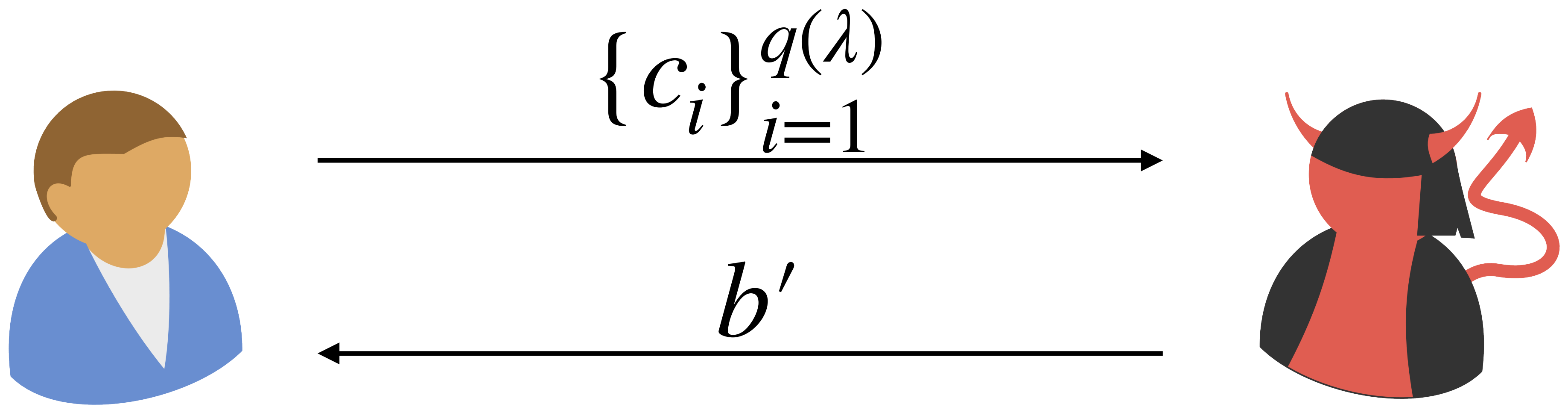

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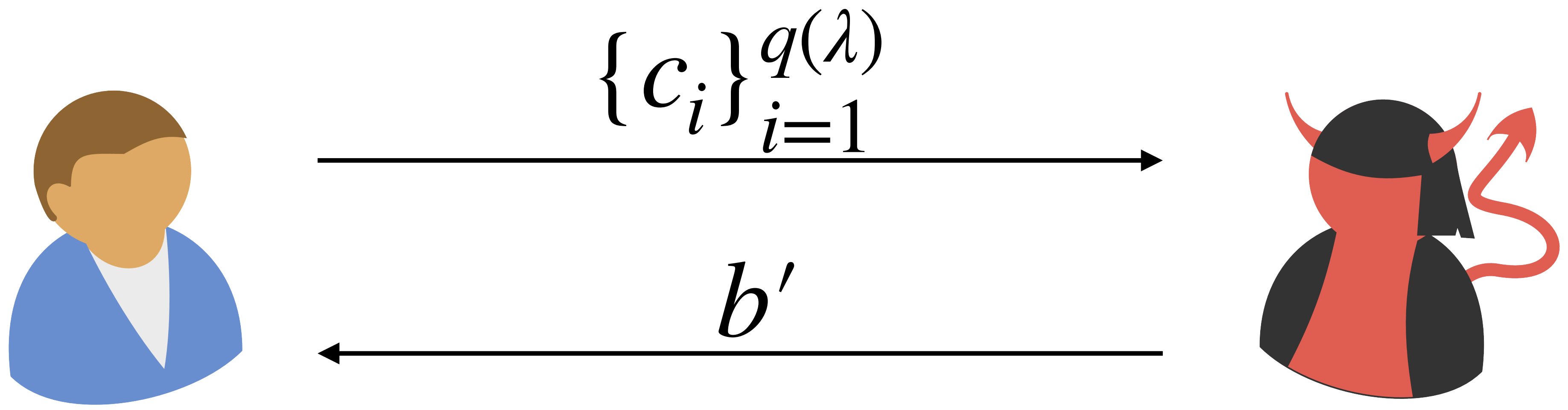

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Does Pseudorandom OTP satisfy this definition?

Pseudorandom OTP

$$\mathsf{KeyGen}(1^\lambda) : k \overset{\$}{\leftarrow} \{0,1\}$$

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Idea: Can we design a multi-message secure encryption scheme that is **stateful**?

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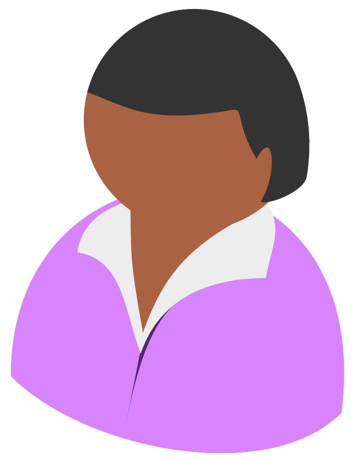

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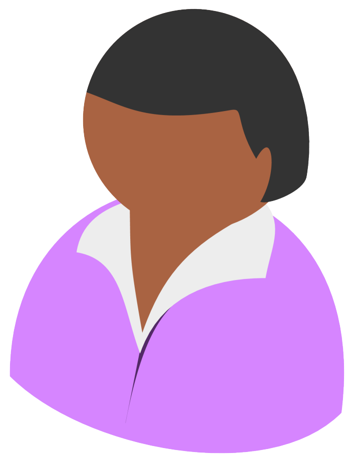
k



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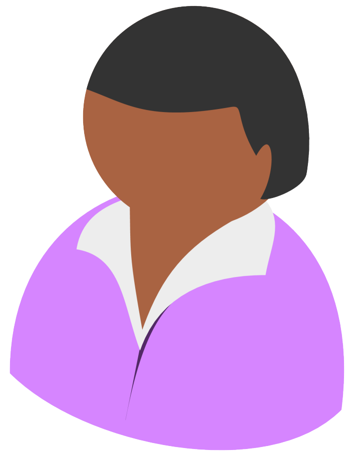
k



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k

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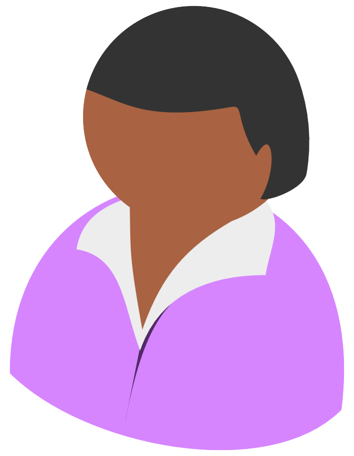
$\xrightarrow{c_0 = m_0 \oplus G(k)[0]}$

k



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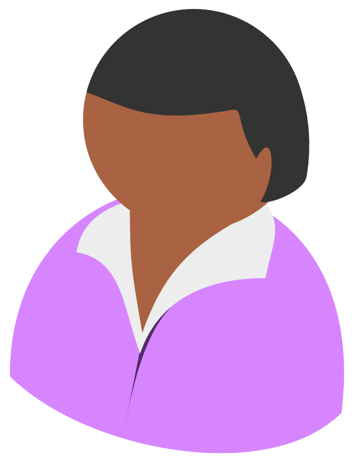

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k

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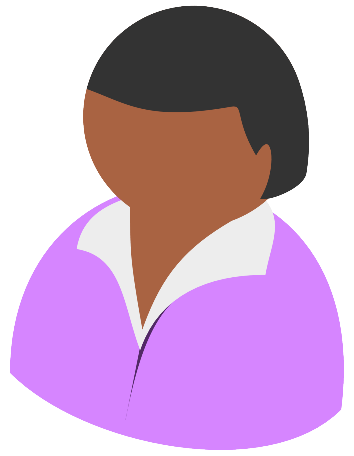

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Stateful Multi-Message Secure Encryption

We just saw a PRG that can output a *polynomial* number of pseudorandom bits.

What if we use one chunk at a time?

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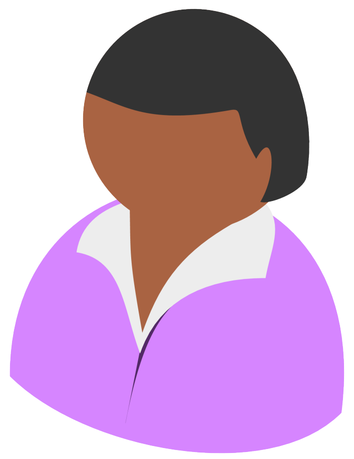

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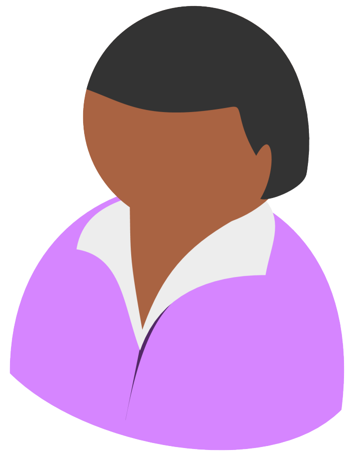

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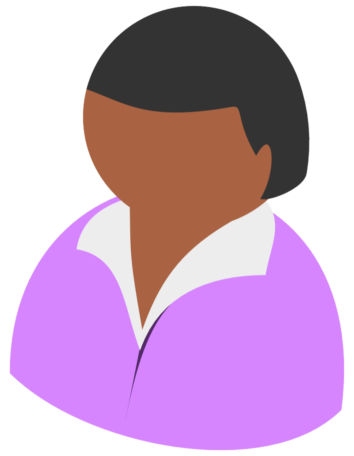

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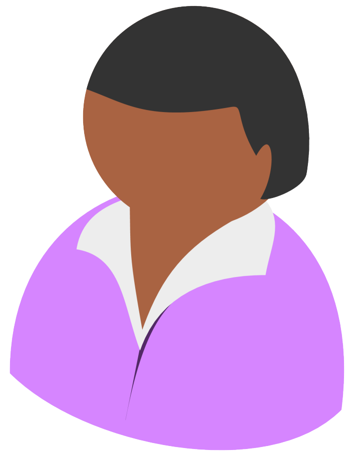

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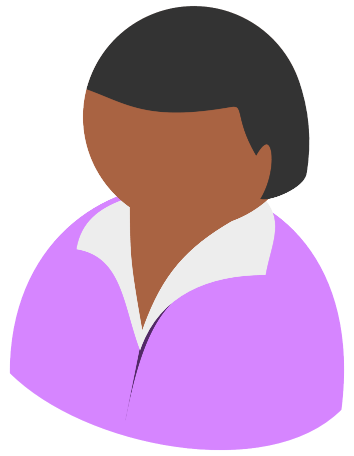

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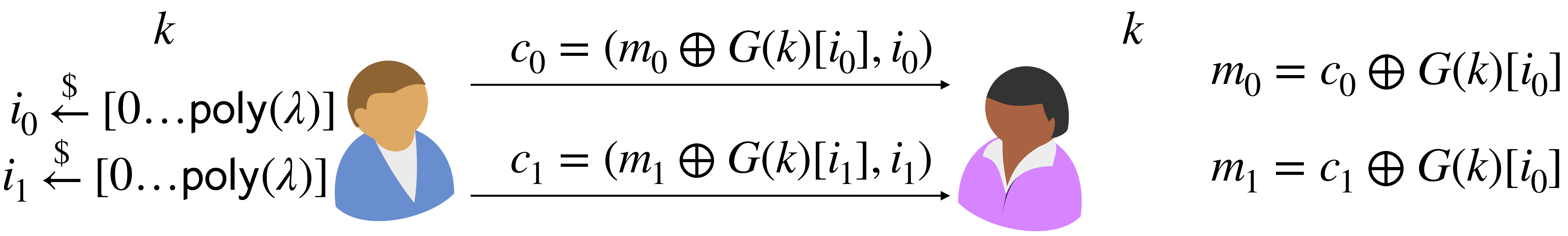

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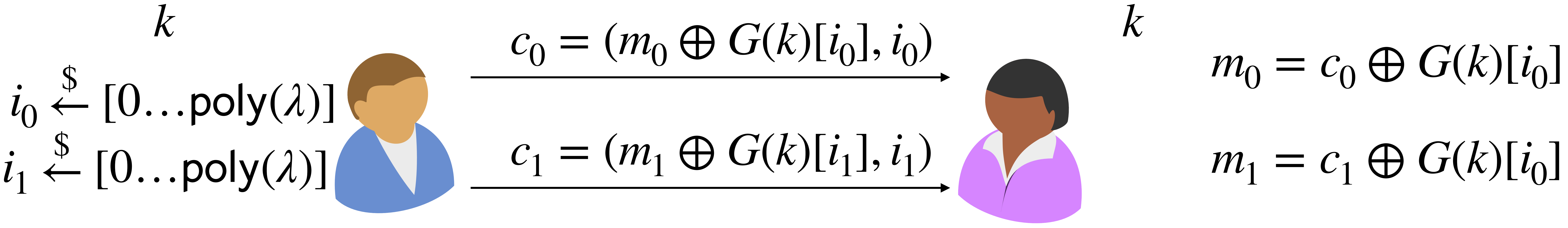

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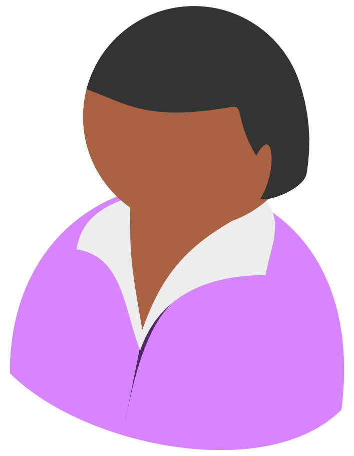

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Idea: What if we could index into an *exponential* amount of randomness?

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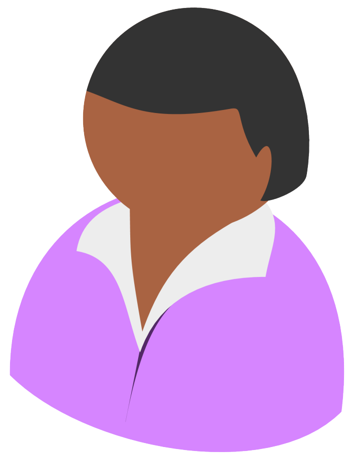

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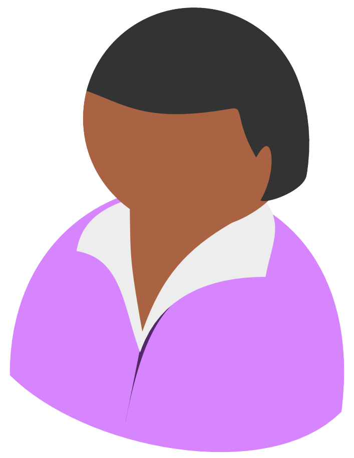

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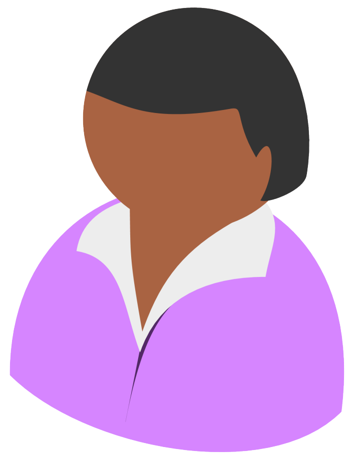

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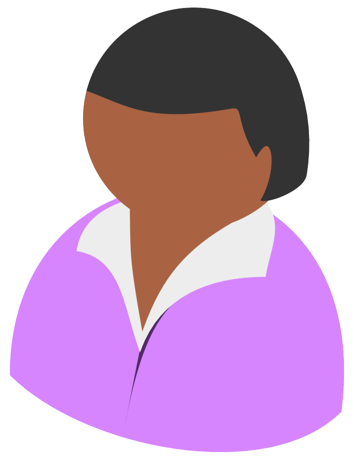

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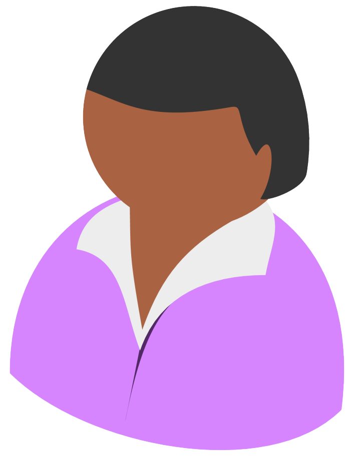

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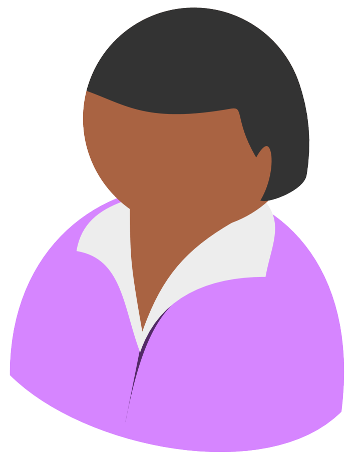

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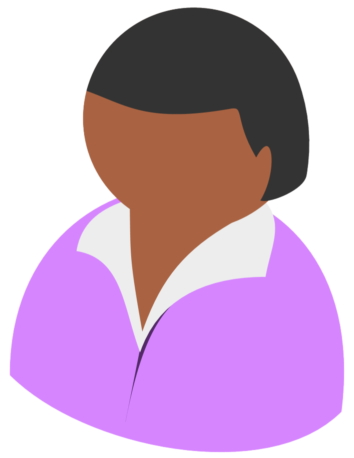

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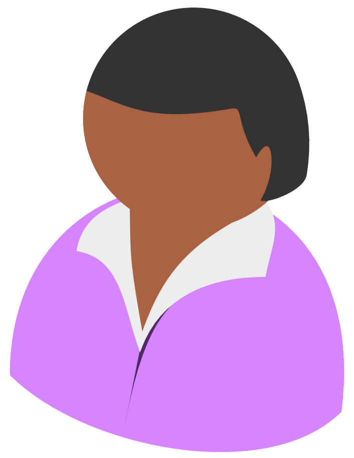

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If all you have is oracle access to F (i.e. you can get outputs, but don't have the function's *description*), these two are *identical*. They have the same output distribution

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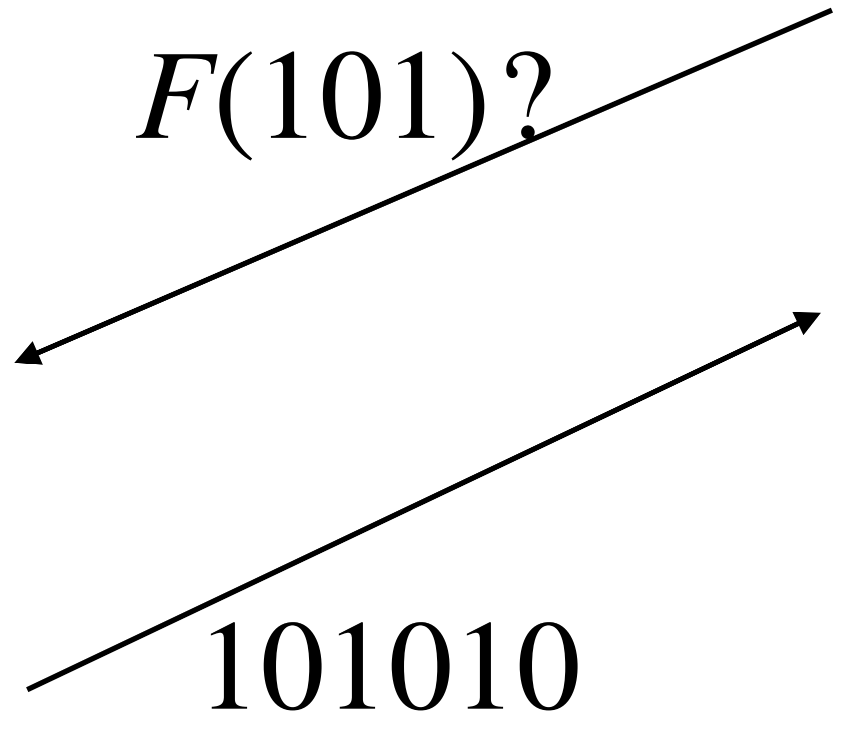

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Computational Indistinguishability!

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