Logistics

- HW3 due today
- Midterm on Tuesday
  - Questions similar to homework, Boneh-Shoup exercises 4.1 and 3.7
- Definitions sheet on class website

Pseudorandom Functions

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$$\{F_k\}_{k \in \{0,1\}^\lambda} \qquad F_k : X \to Y$$

Pseudorandom Functions

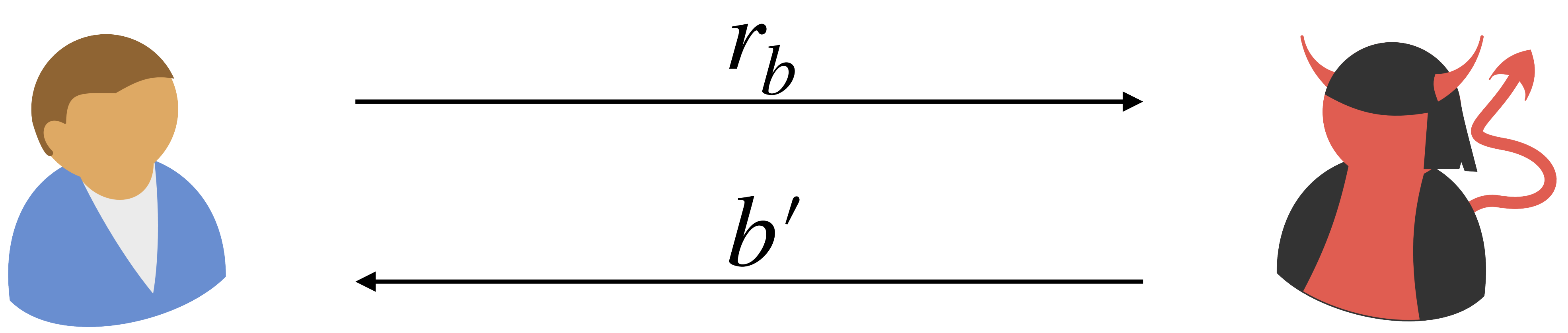
Recap: PRG Game

$$b \xleftarrow{\$} \{0,1\}$$

$$s \xleftarrow{\$} \{0,1\}^{\lambda}$$

$$r_0 \xleftarrow{\$} \{0,1\}^{\ell(\lambda)}$$

$$r_1 := G(s)$$



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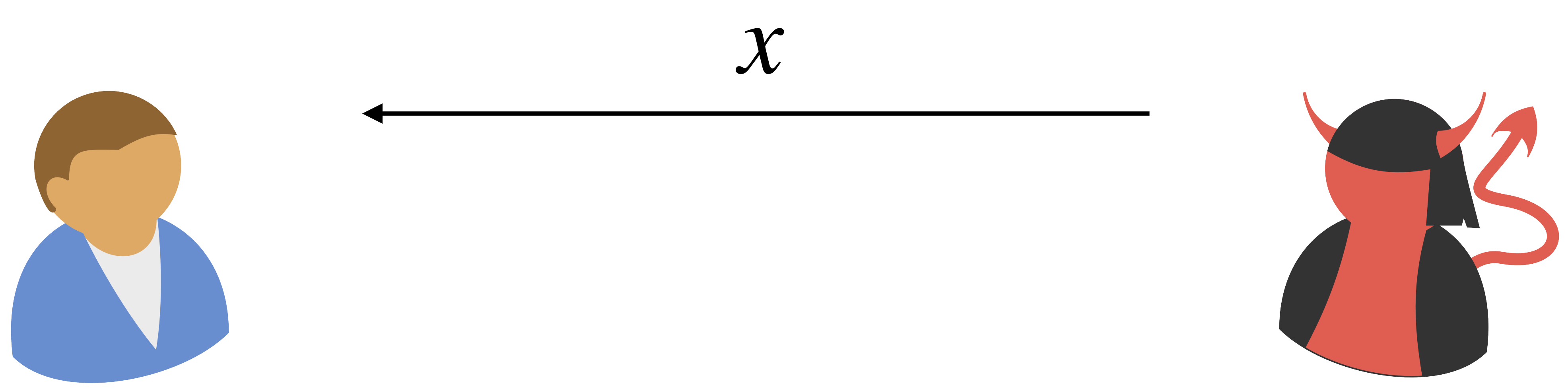

Pseudorandom Functions

Now: PRF Game

$b \xleftarrow{\$} \{0,1\}$

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if $b = 0$

$y = F_k(x)$

x

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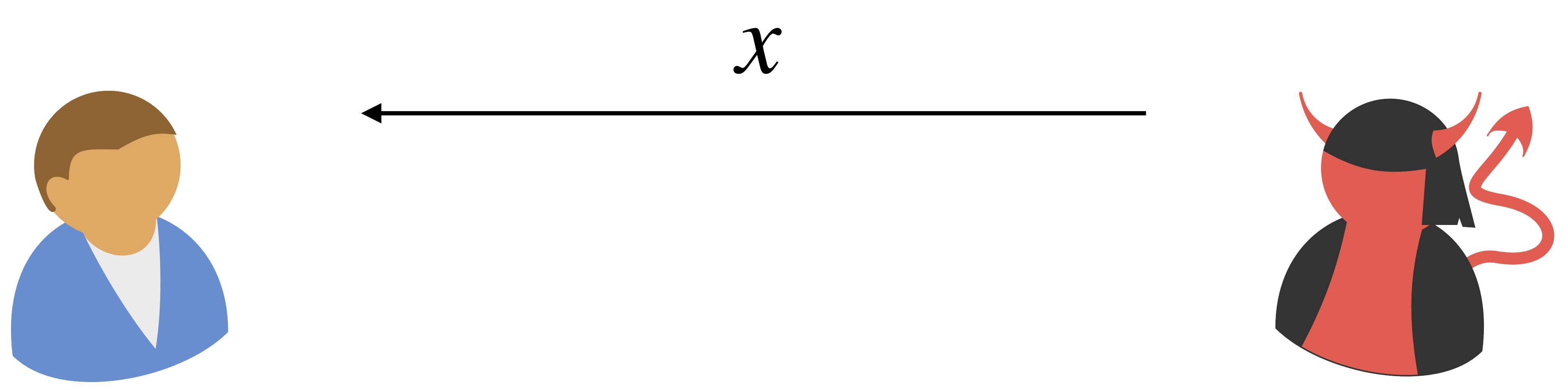

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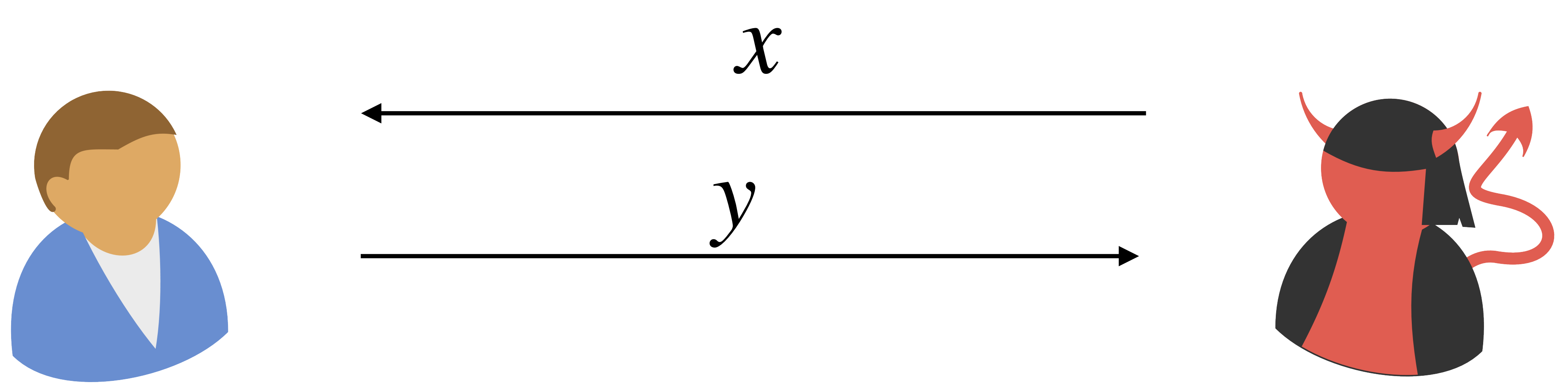

Pseudorandom Functions

Now: PRF Game

Repeat as much as $\mathscr{A}$ wants!

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$\quad$ **if** $x \notin T$

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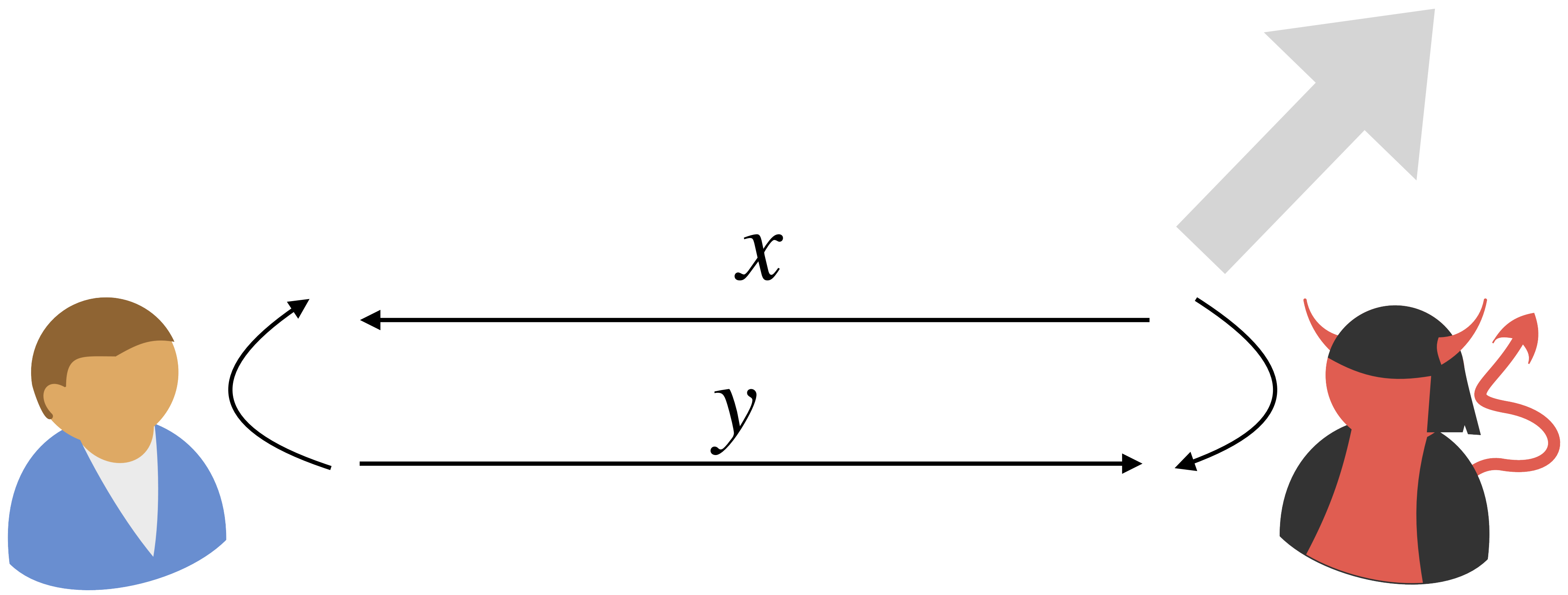

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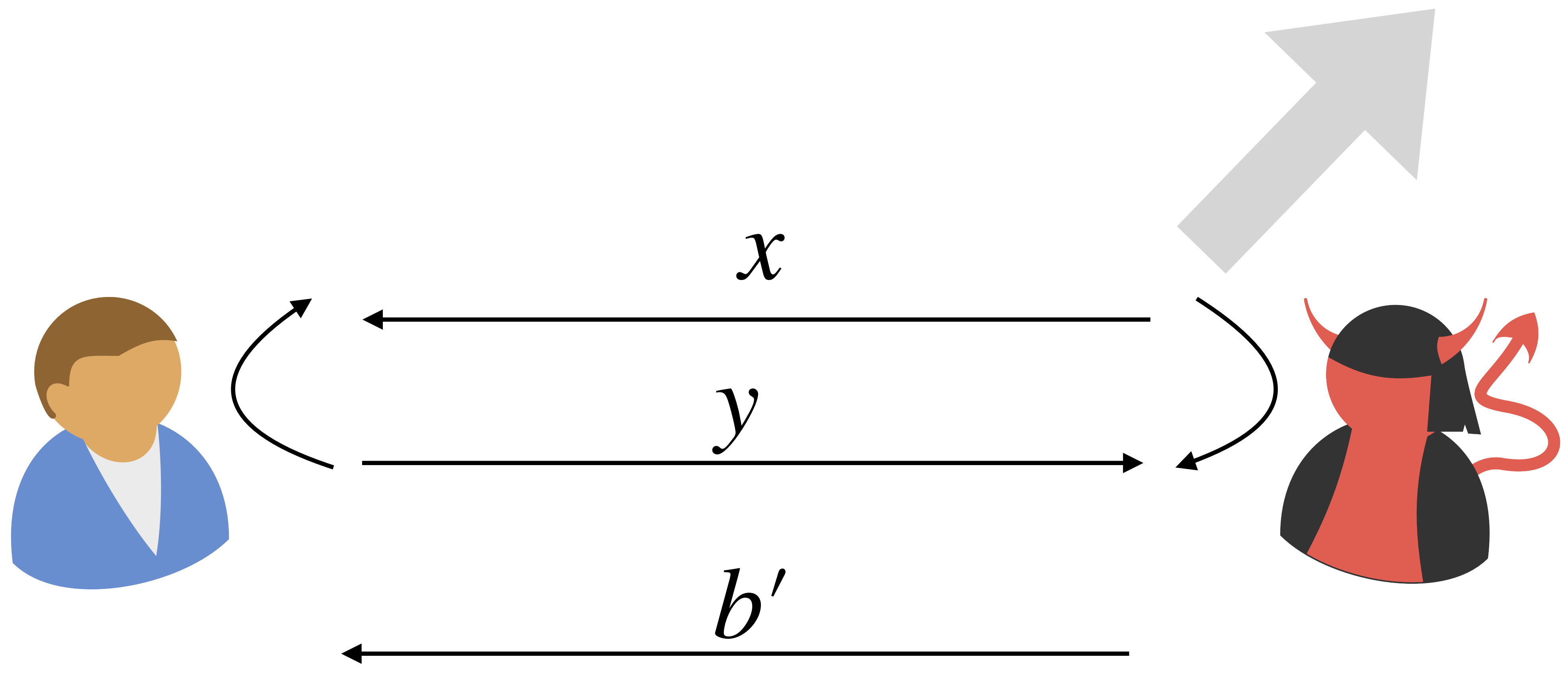

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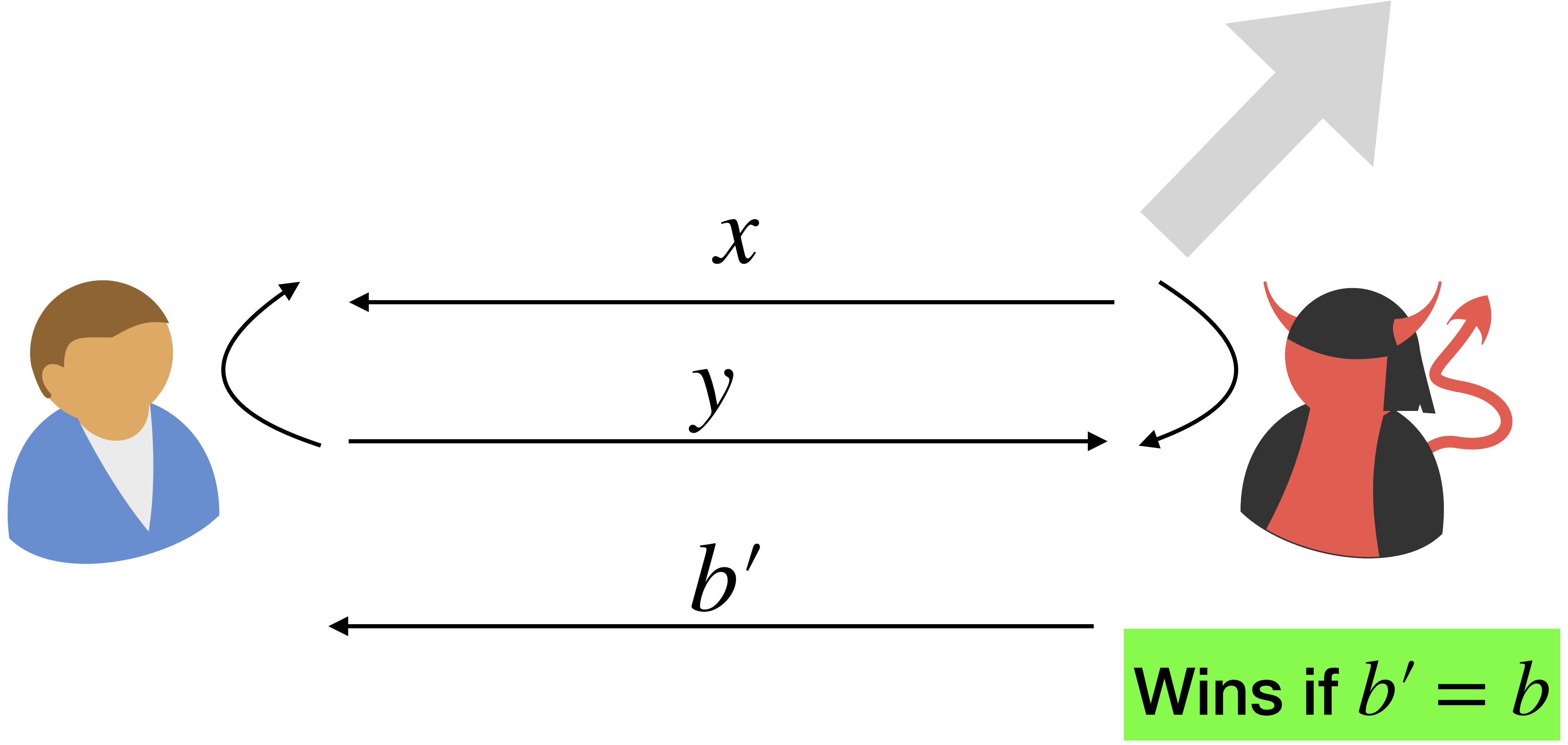

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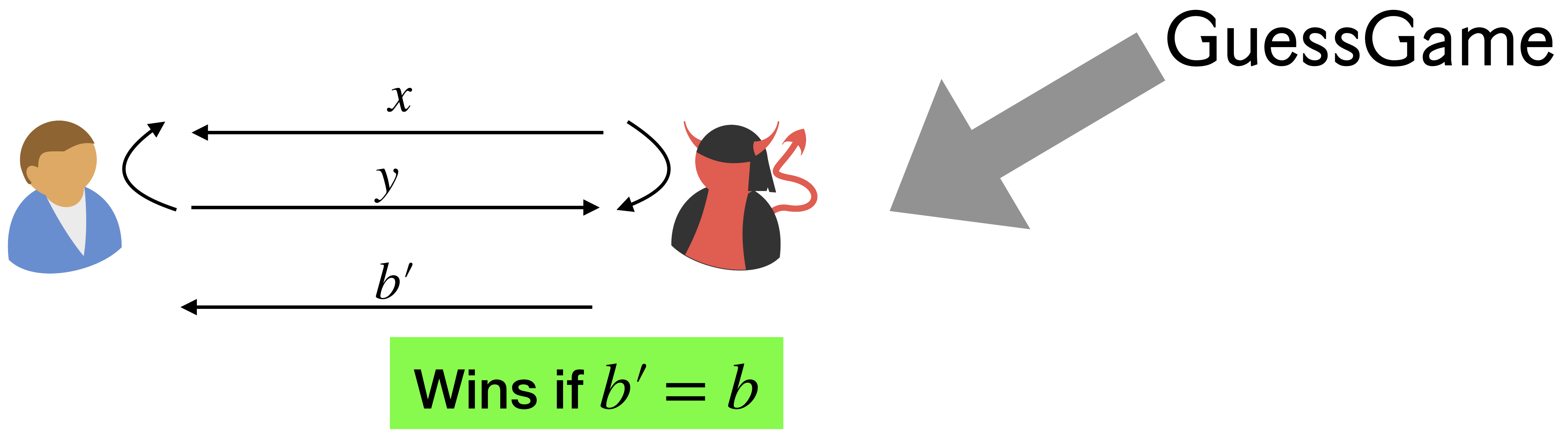

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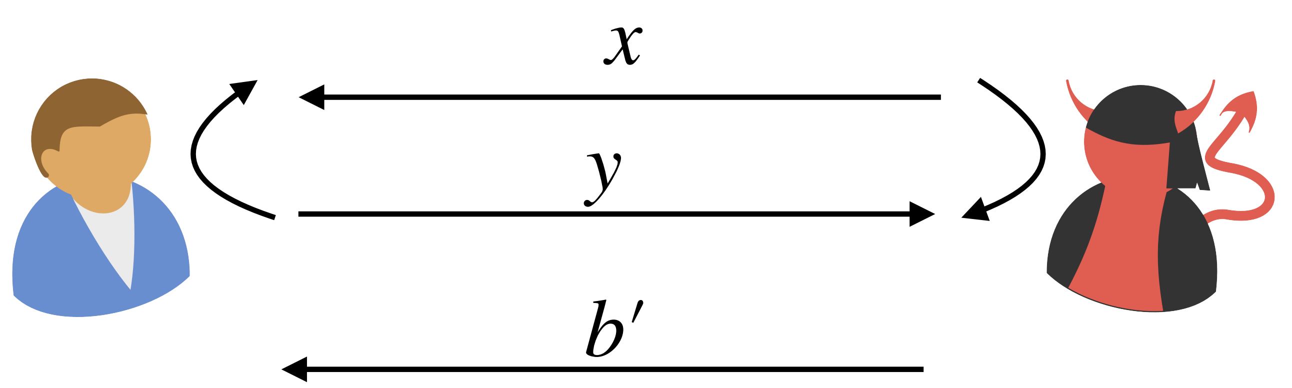

Wins if $b' = b$

This is the first security definition that we have seen that is *adaptive*, the adversary gets to choose its queries based on the responses it gets.

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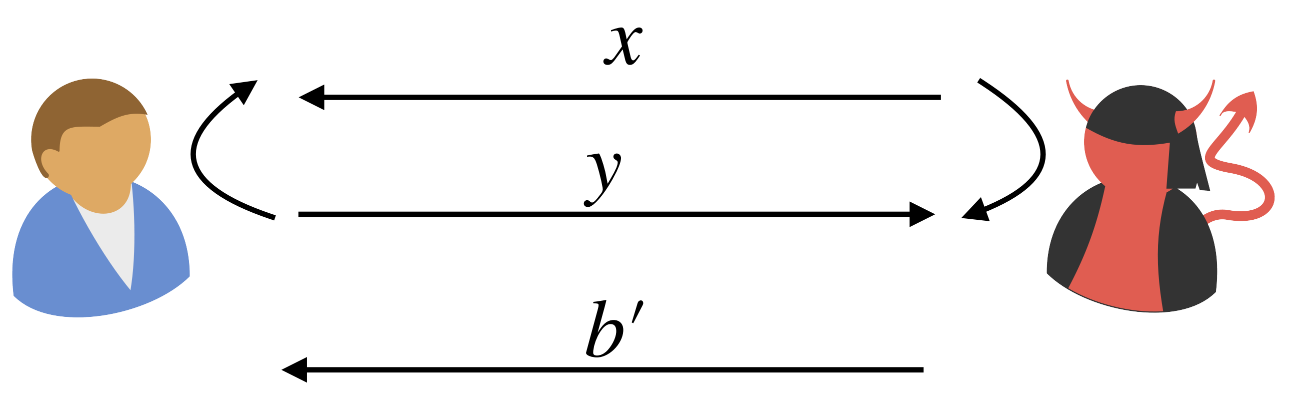

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One of the benefits of thinking in terms of *games* is easily expressing things like this!

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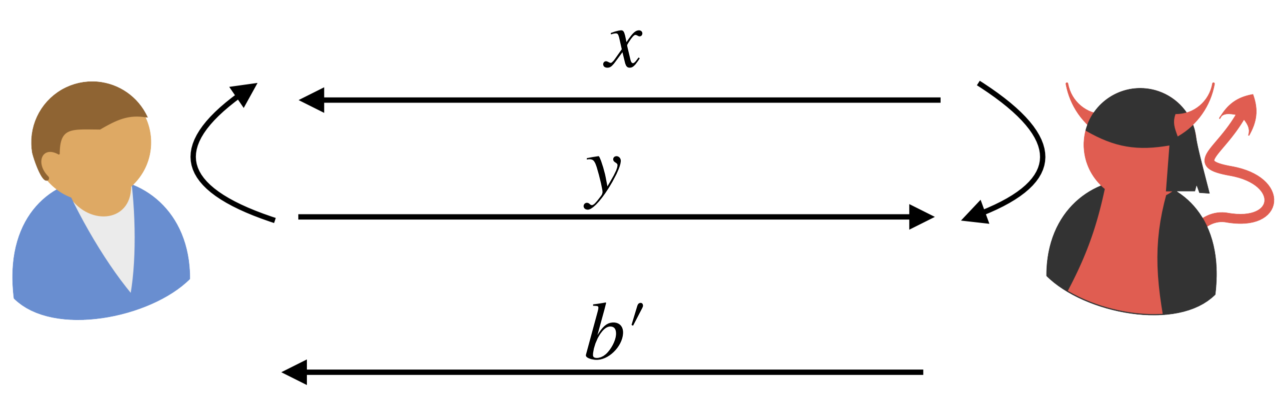

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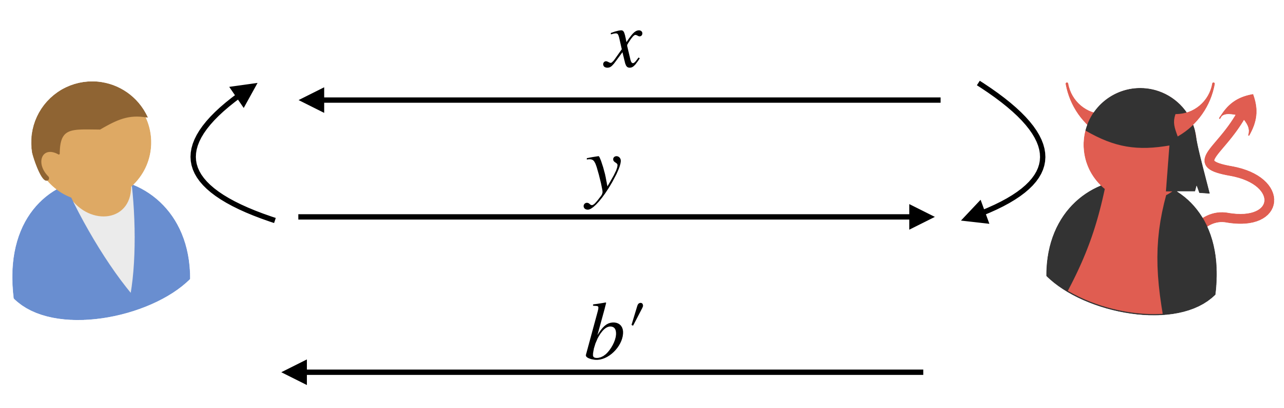

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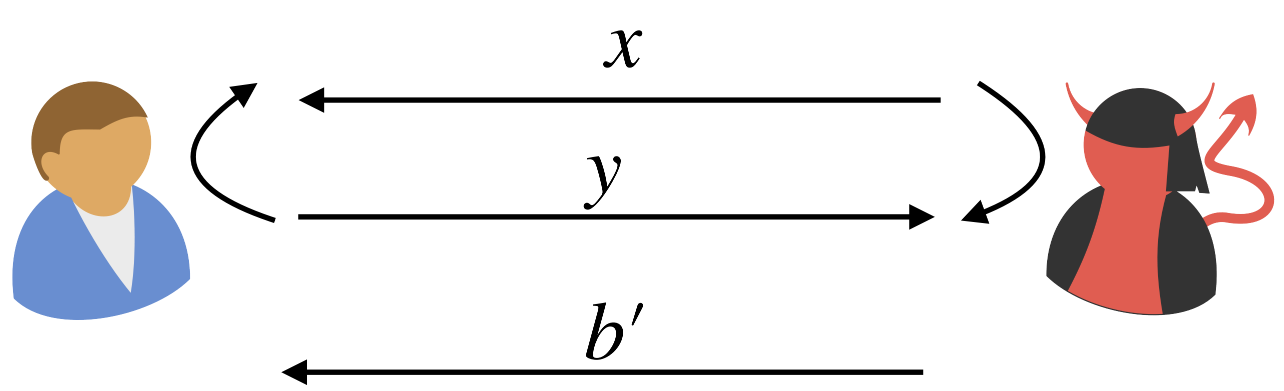

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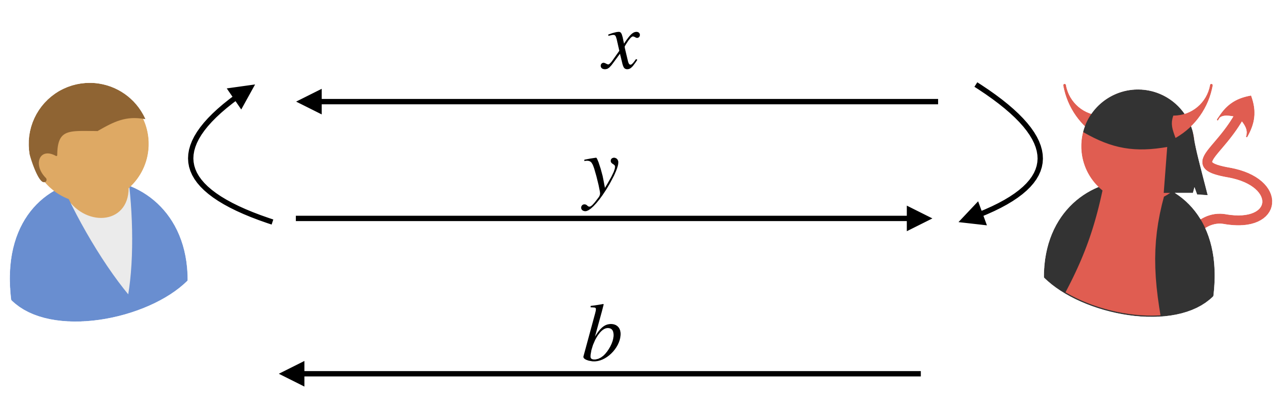
Game_0

$k \xleftarrow{\$} \{0,1\}^\lambda$ $\quad y = F_k(x)$



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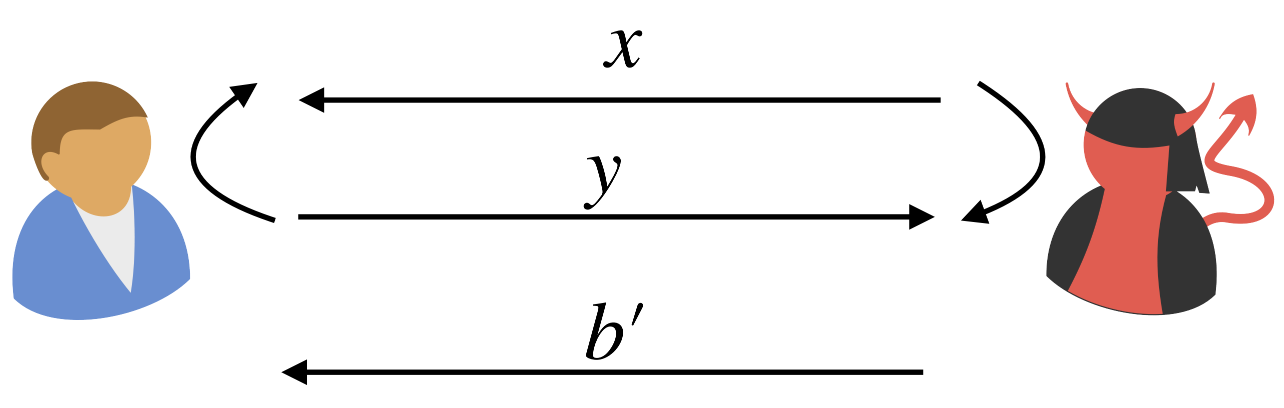

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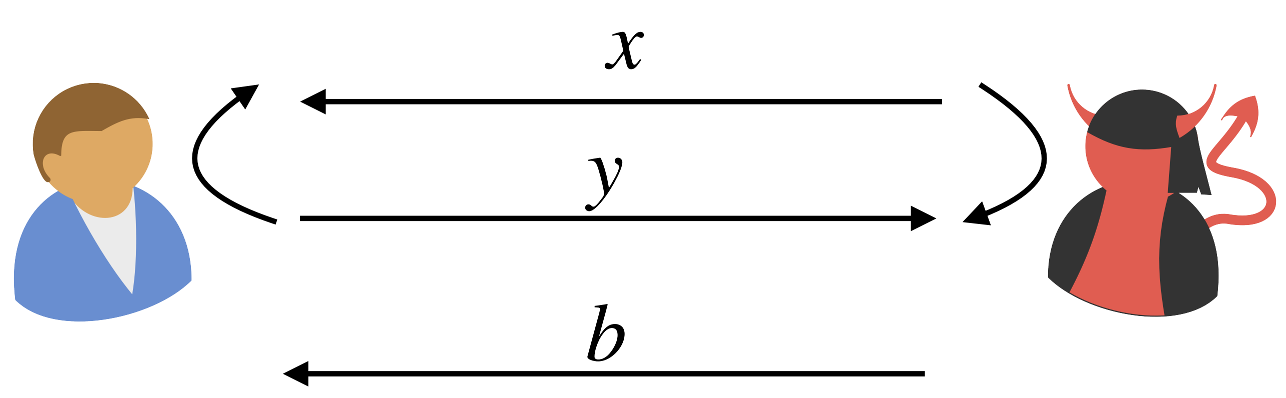

Game_1

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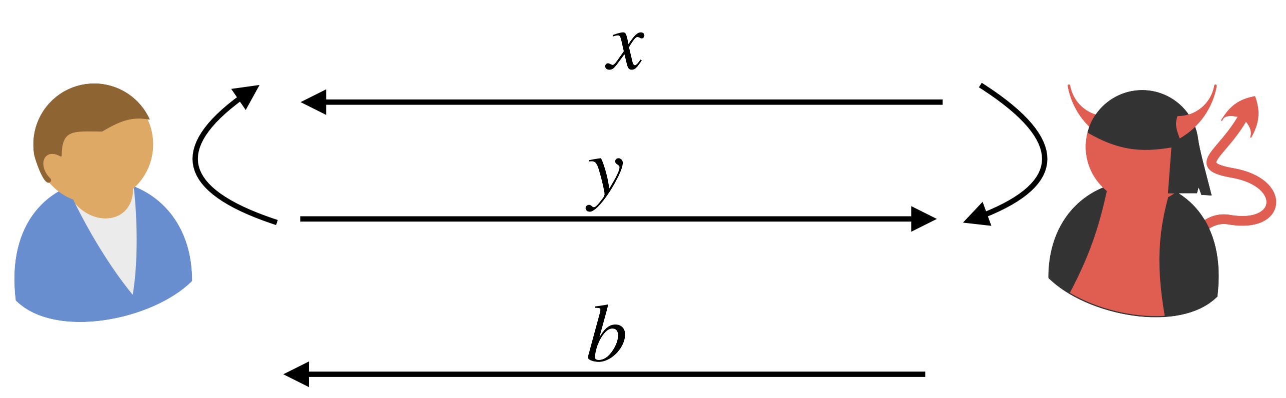

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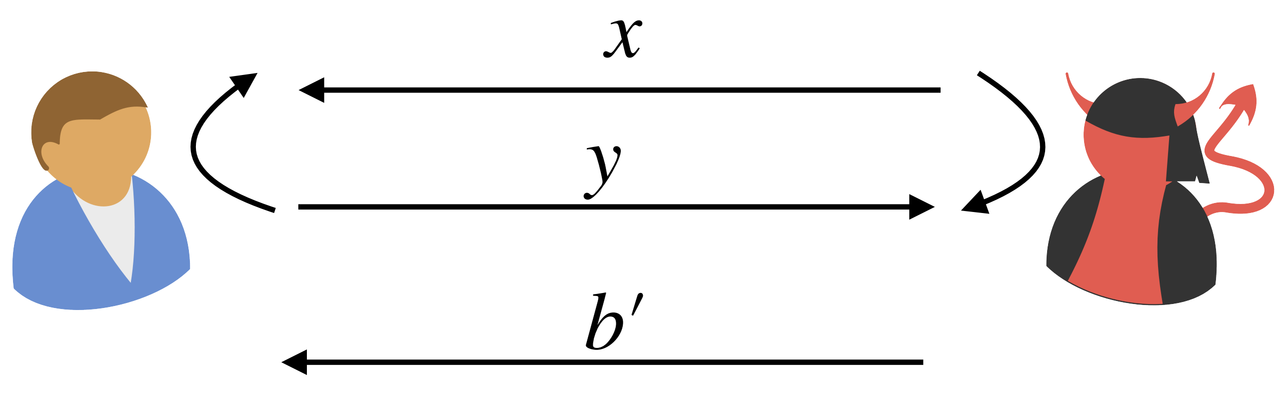

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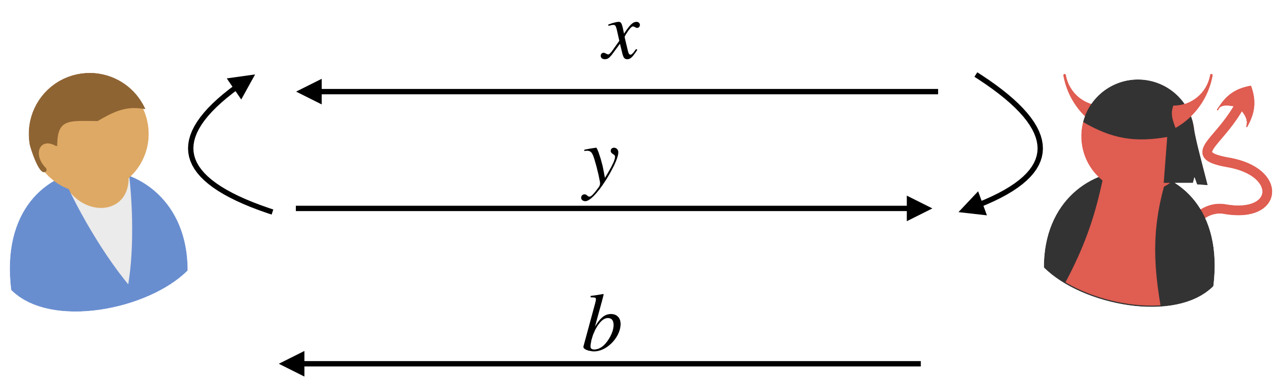

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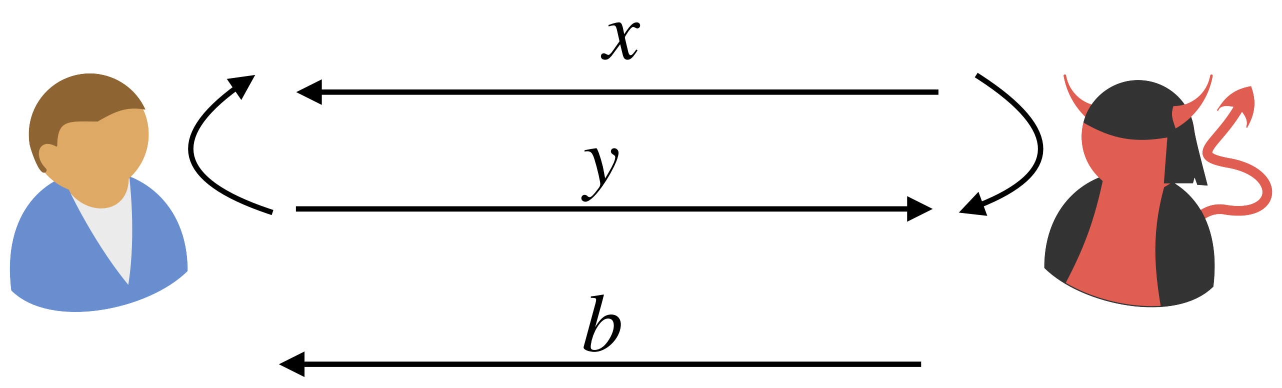

Let W_b be the event that $\mathscr{A}$ outputs 1 in Game_b

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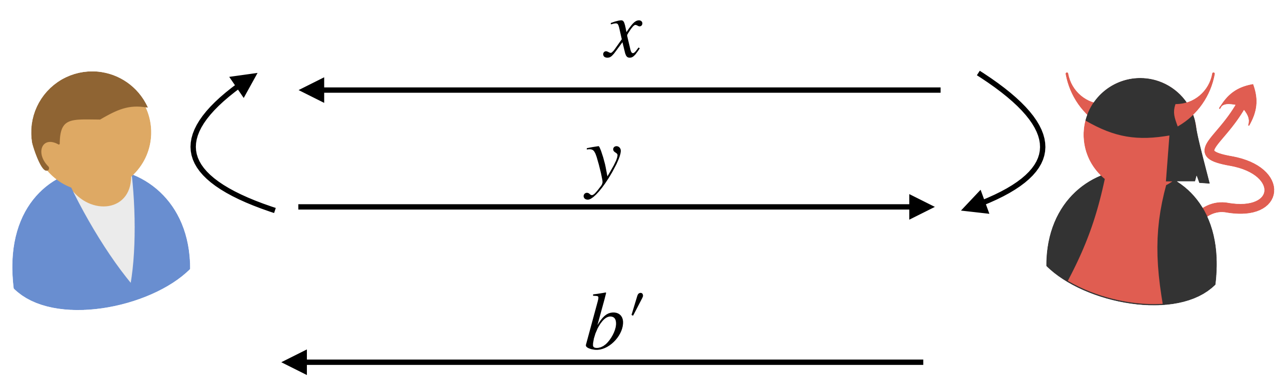

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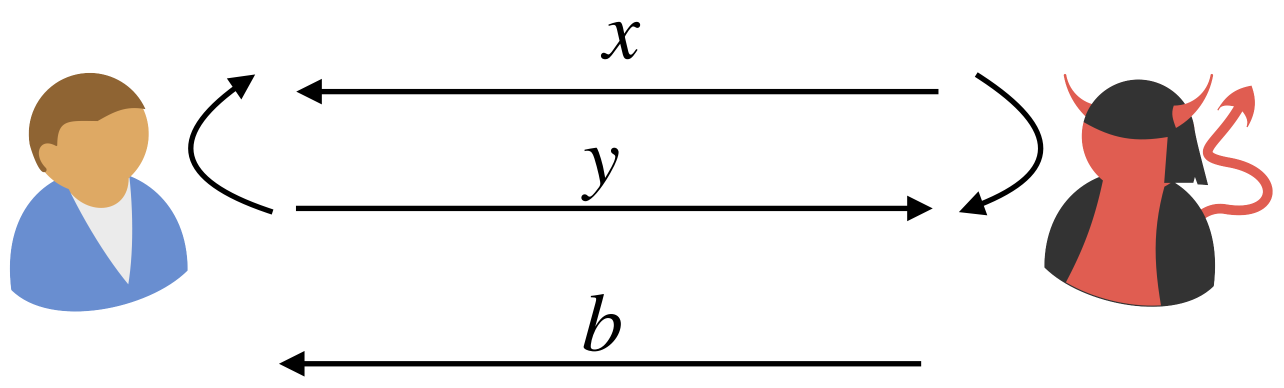

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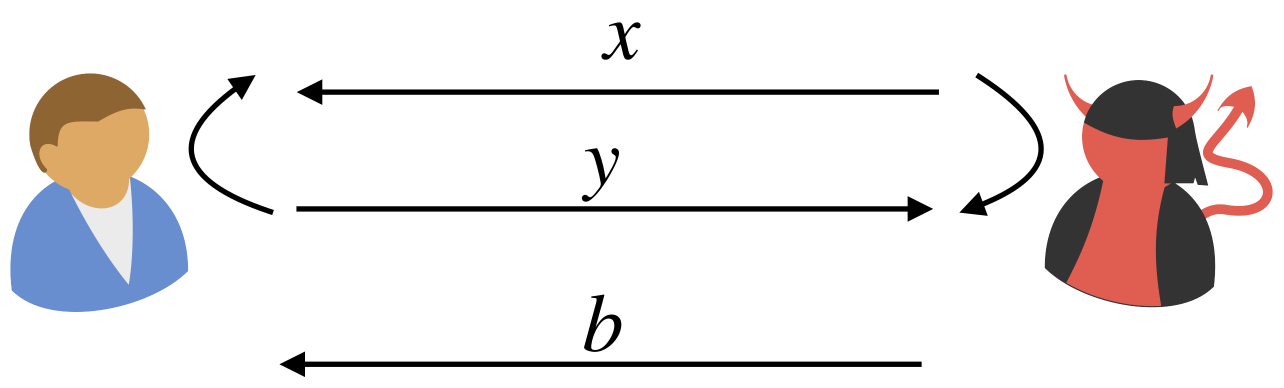

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$$\left| \Pr[W_0] - \Pr[W_1] \right| \leq \text{negl}(\lambda)$$

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$$\left| \Pr[\mathscr{A} \text{ outputs } 1 \text{ in Game}_0] - \Pr[\mathscr{A} \text{ outputs } 1 \text{ in Game}_1] \right| \leq \nu(\lambda)$$

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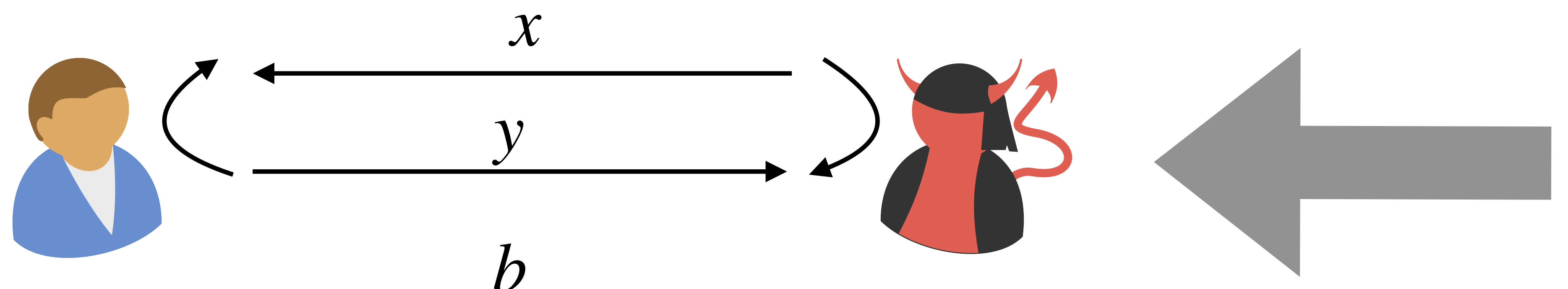

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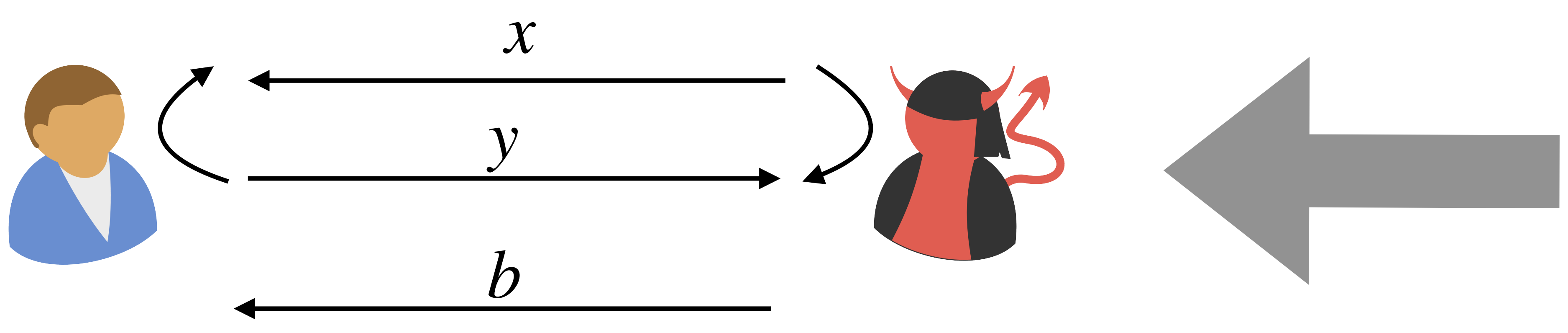

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