Encryption II

601.442/642 Modern Cryptography

24th February 2026

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- HW4 due Thursday
- I am traveling next week, will not be holding office hours. Send me an email to schedule a specific time!

Recap: Public Key Encryption (PKE)

Public Key Encryption Scheme Syntax

A *public key encryption scheme* consists of three (possibly probabilistic) algorithms:

- KeyGen$(1^\lambda) \to (sk, pk)$ outputs a secret key $sk \in \mathcal{K}_s$ and a public key $pk \in \mathcal{K}_p$
- Enc$(pk, m) \to$ ct takes a public key pk and a message $m \in \mathcal{M}$ and outputs a ciphertext ct $\in \mathcal{C}$
- Dec$(sk,$ ct$) \to m$ takes a secret key sk and a ciphertext ct and outputs a message m

Recap: PKE Security

IND-CPA Security

A public key encryption scheme (KeyGen, Enc, Dec) satisfies *indistinguishability under chosen-plaintext attack* (IND-CPA) if for all NUPPT $\mathcal{A}$, there exists a negligible function $\nu(\cdot)$ such that $\forall \lambda \in \mathbb{N}$

$$\Pr[\mathcal{A} \text{ wins GuessGame}] \leq \frac{1}{2} + \nu(\lambda)$$

$b \xleftarrow{\$} \{0,1\}$

$(pk, sk) \leftarrow \text{KeyGen}(1^\lambda)$

$\text{ct}_b \leftarrow \text{Enc}(pk, m_b)$

pk →

← (m_0, m_1)

ct_b →

← b'

Wins if $b' = b$

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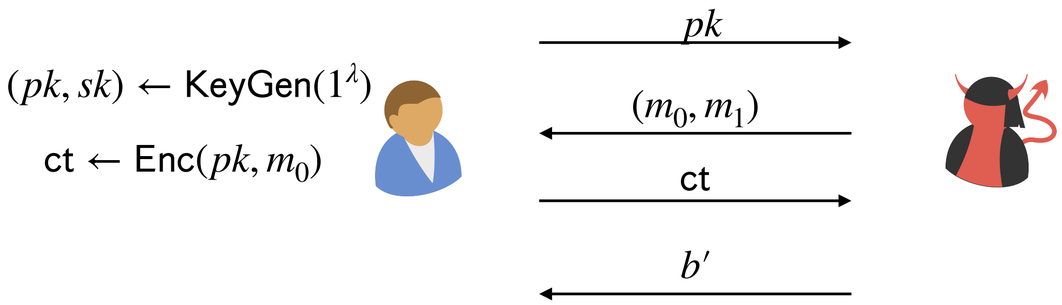
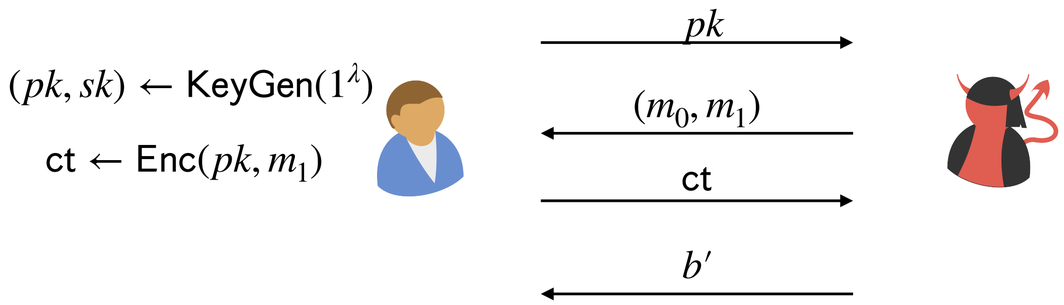
$$\left|\Pr[\mathscr{A} \text{ outputs } 1 \text{ in Game}_0] - \Pr[\mathscr{A} \text{ outputs } 1 \text{ in Game}_1]\right| \leq \nu(\lambda)$$

Game$_0$



Game$_1$



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- Our assumption: given g^a for some randomly sampled a, it should be hard to find a

Recap: Discrete Logarithm Assumption

Discrete Logarithm Assumption

Let $(G, *)$ be a cyclic group of order p (where p is a safe prime) with generator g, then for every NUPPT adversary $\mathcal{A}$, there exists a negligible function ν such that

$$\Pr\left[\mathcal{A} \text{ wins DLGame}\right] \leq \nu(\lambda)$$

$a \xleftarrow{\$} \{0, \ldots, p-1\}$

(G, p, g, g^a)

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Wins if $a' = a$

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Note: This is a *search problem*. $\mathcal{A}$ needs to actually *find* the discrete logarithm

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- Remember: $g^x * g^y = g^{x+y}$, so no trivial attack!

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- How do we model indistinguishability? A bit-guessing game!
- Challenger samples a bit b, samples (x, y, r), then either sends the adversary g^{xy} when $b = 0$, or g^r when $b = 1$. Adversary has to guess b.

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DDH ⇒ CDH ⇒ DL

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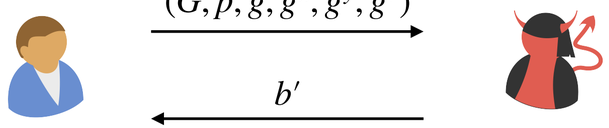

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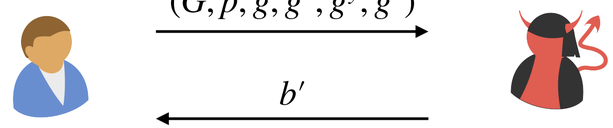

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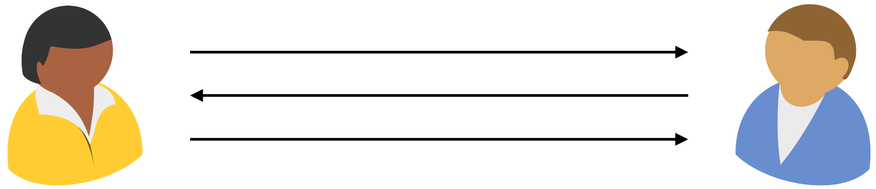

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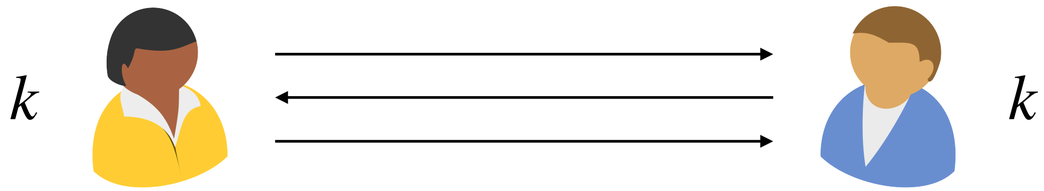

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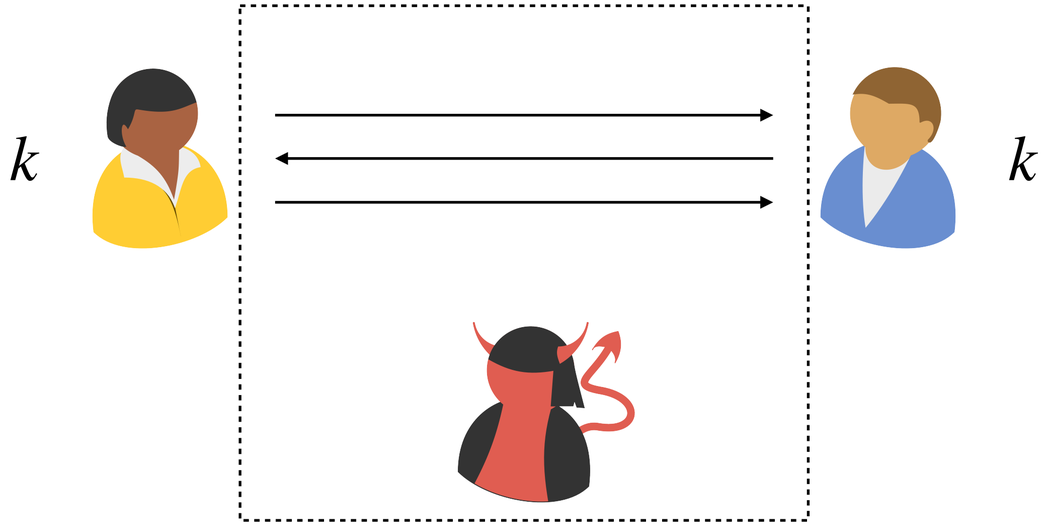

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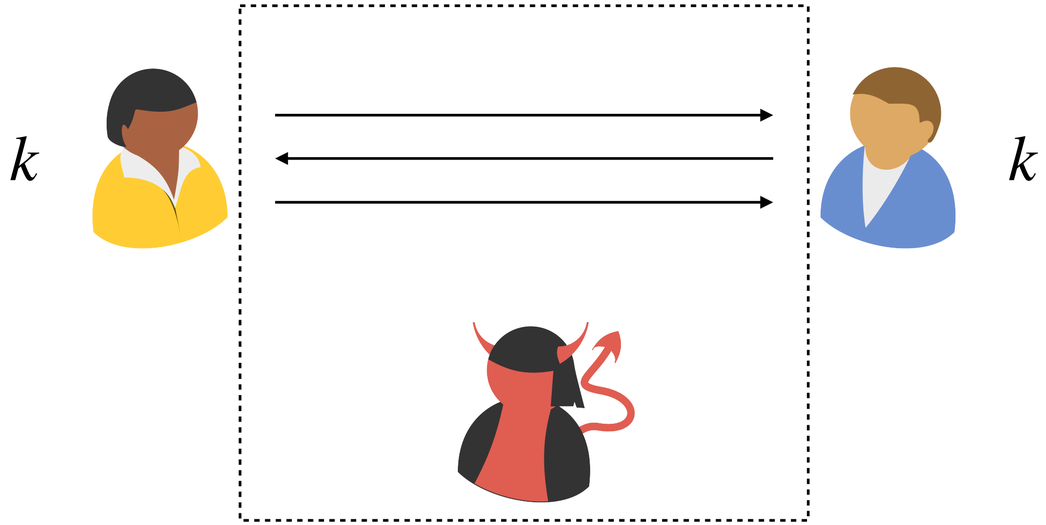

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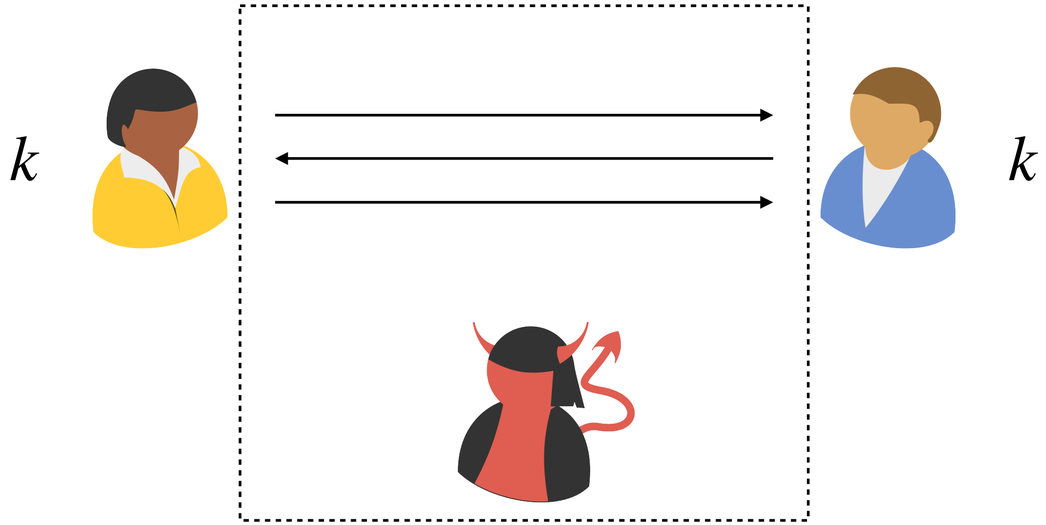

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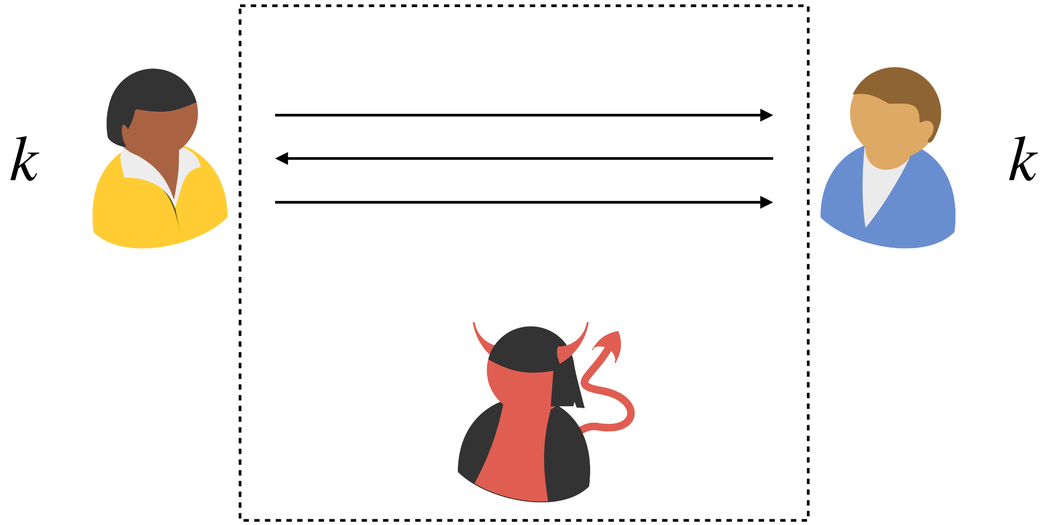

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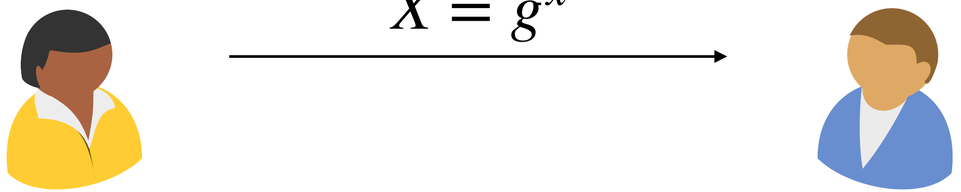

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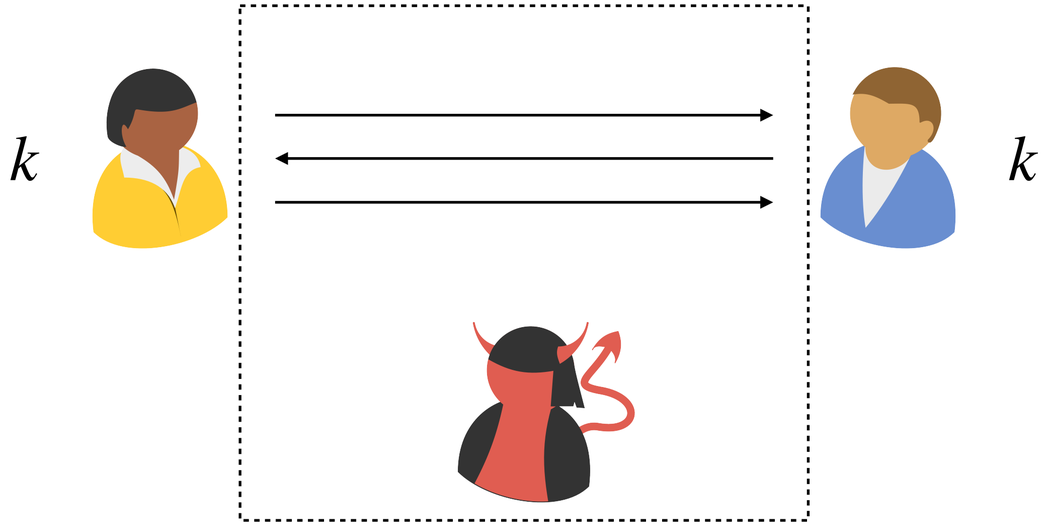

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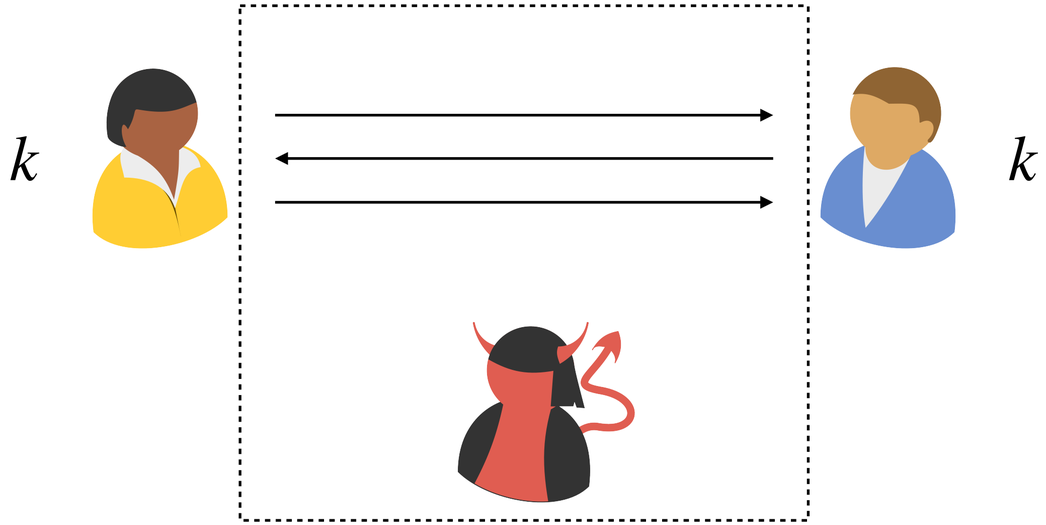

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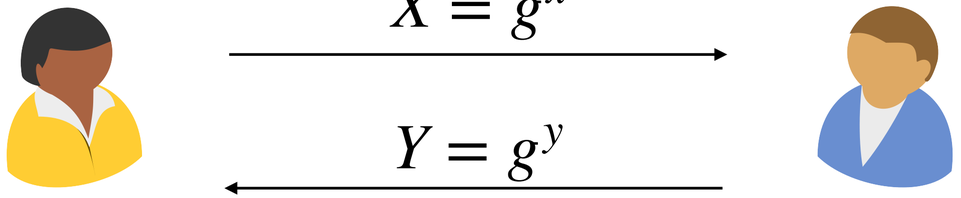

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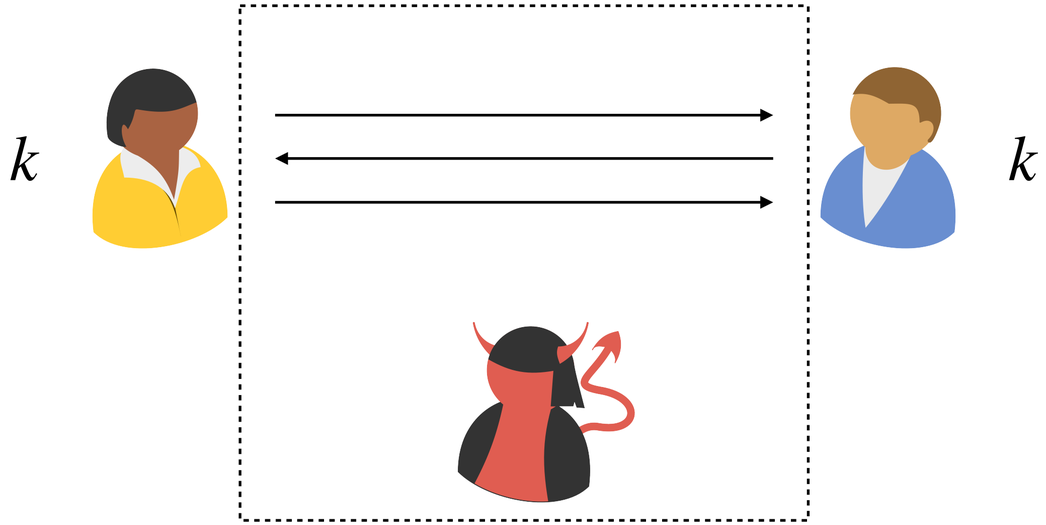

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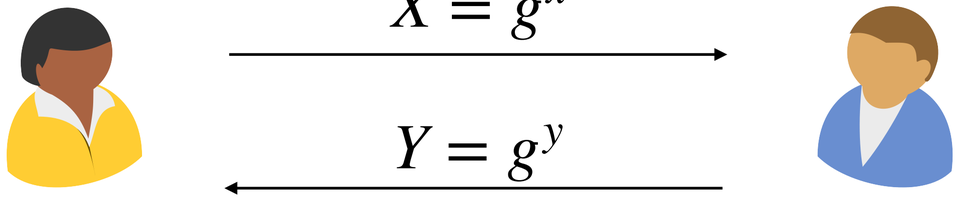

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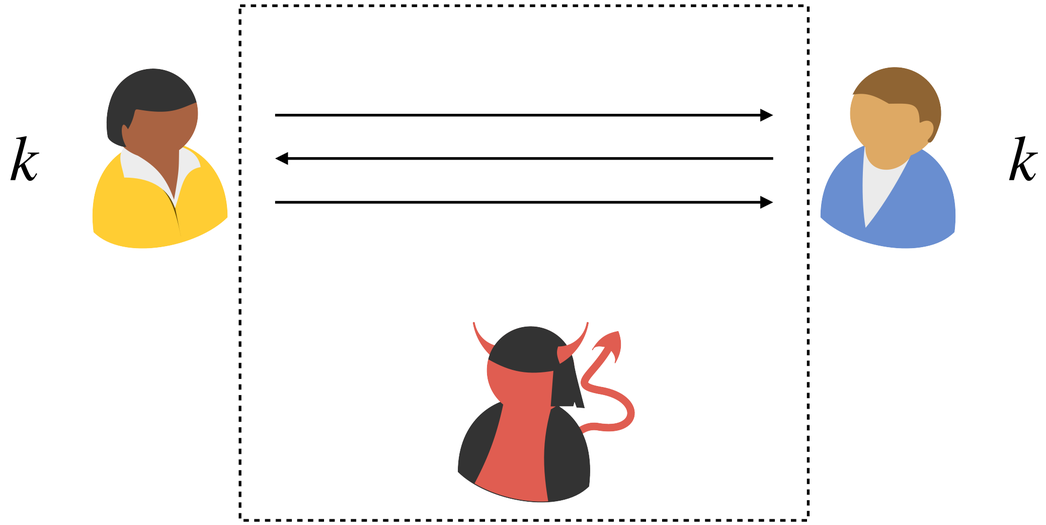

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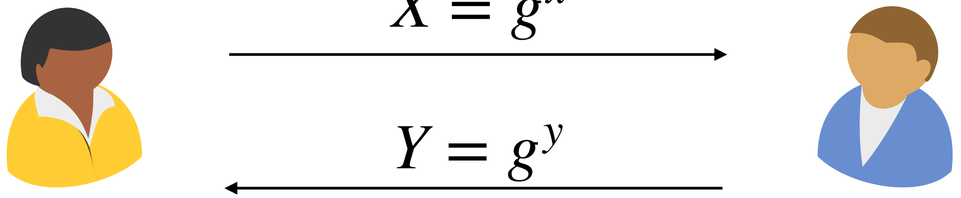

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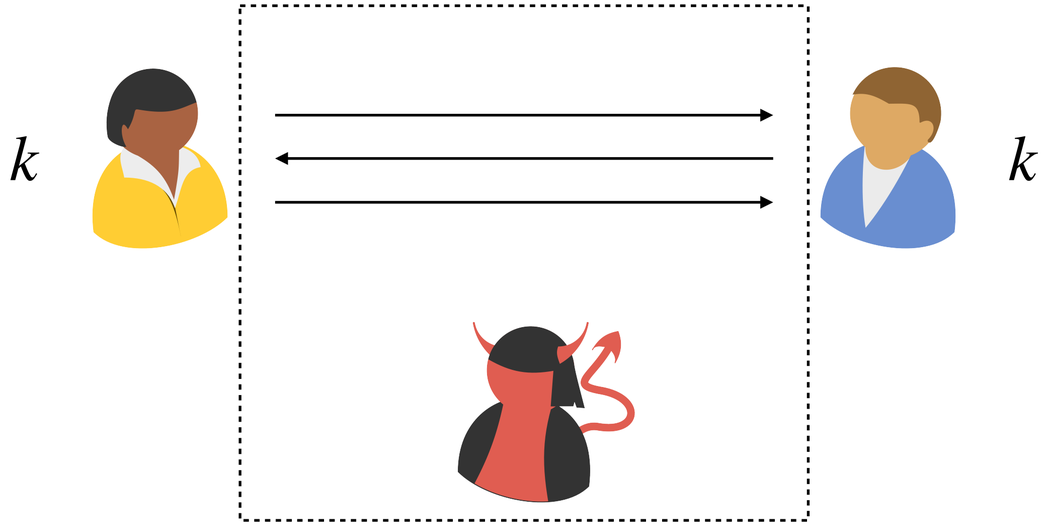

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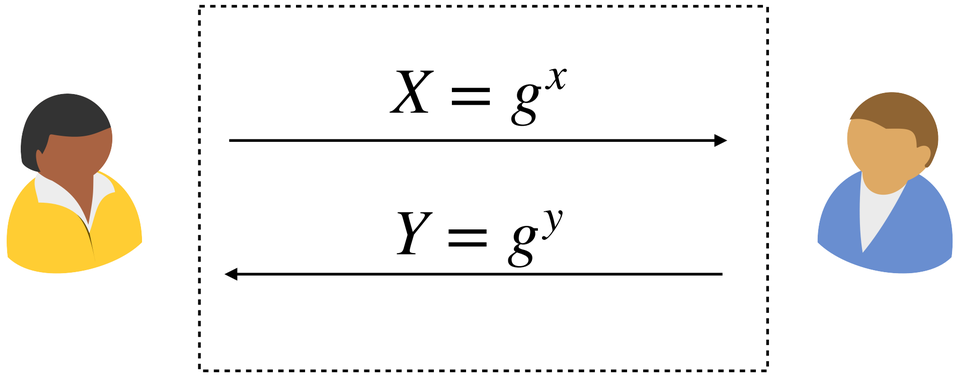

$y \xleftarrow{\$} \{0,\ldots,p-1\}$

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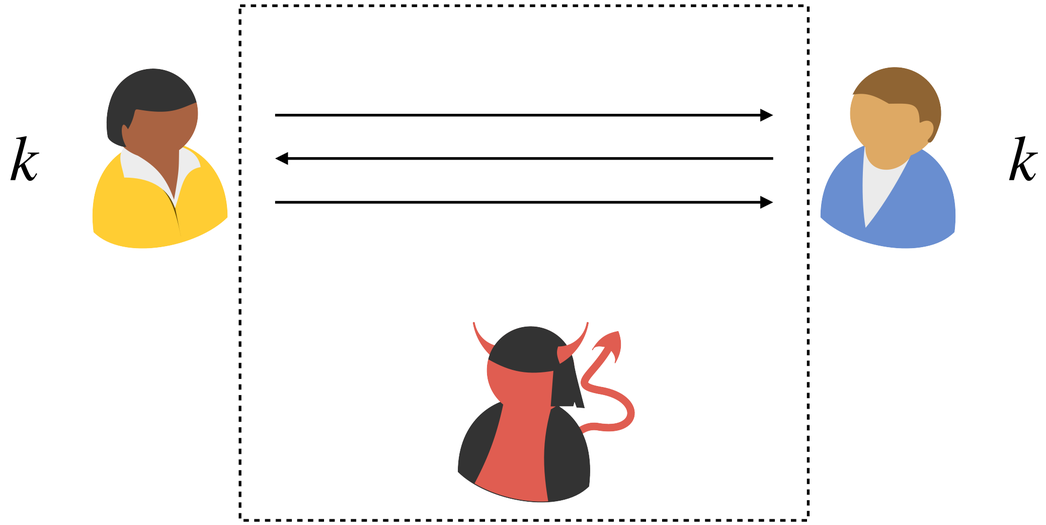
$\mathcal{A}$ sees g^x and g^y. The key is g^{xy}. By DDH k is indistinguishable from random!

Key Exchange

This protocol is known as "Diffie-Hellman Key Exchange"



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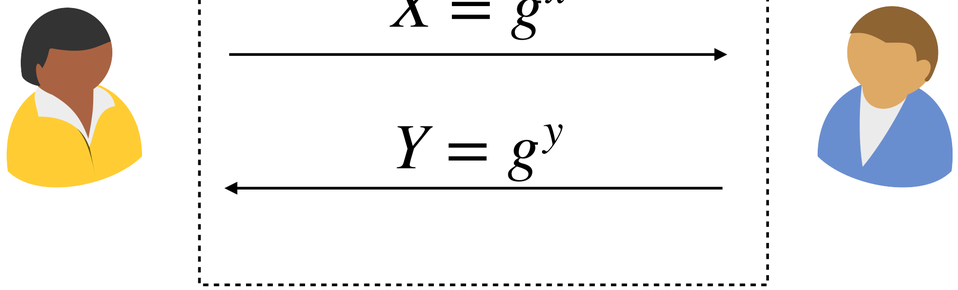

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ElGamal Encryption

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$k = Y^x$

$X = g^x$

$Y = g^y$

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ElGamal Encryption

If Bob *published* his value g^y, Alice could do this protocol *non-interactively*

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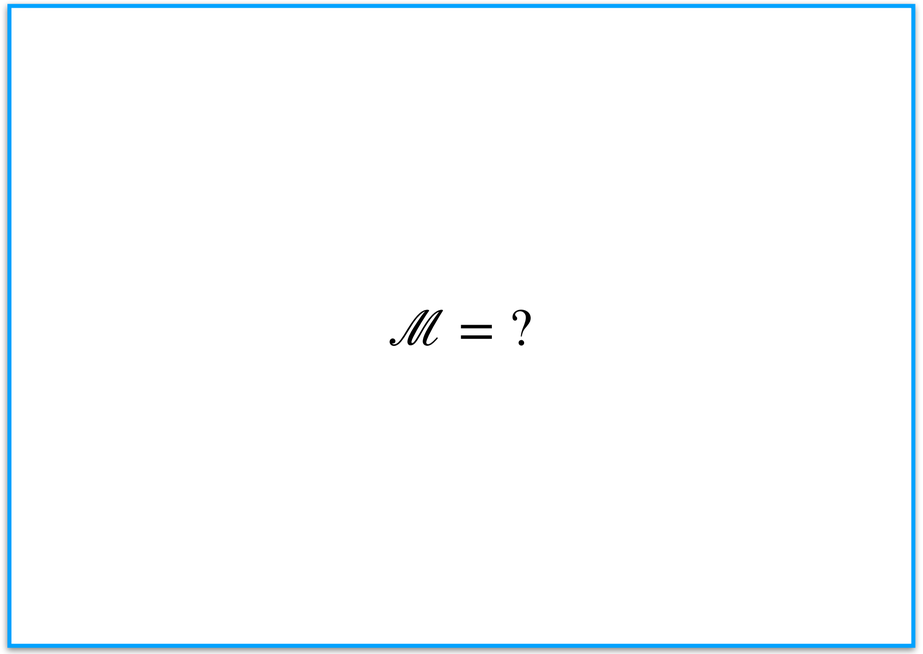

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$$\mathscr{M} = G$$
$$\mathscr{K}_s = ?$$

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$$\mathcal{M} = G$$
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$$\mathcal{M} = G$$
$$\mathcal{K}_s = \{0,\ldots,p-1\}$$
$$\mathcal{C} = ?$$

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$$\mathscr{M} = G$$
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Correctness:

$$m := c_1 * (c_2^{sk})^{-1}$$
$$m := c_1 * ((g^x)^{sk})^{-1}$$
$$m := c_1 * ((g^x)^y)^{-1}$$
$$m := c_1 * g^{-xy}$$
$$m := m * pk^x * g^{-xy}$$
$$m := m * g^{xy} * g^{-xy}$$
$$m := m$$

ElGamal: Proof of Security

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ElGamal: Proof of Security

H_0

$y \xleftarrow{\$} \{0,\ldots,p-1\}$

$sk := y,\ pk := g^y$

pk →

(m_0, m_1) ←

$x \xleftarrow{\$} \{0,\ldots,p-1\}$

$\mathsf{ct} := (m_0 * pk^x, g^x)$

ct →

b' ←

$\mathsf{KeyGen}(1^\lambda)$: $y \xleftarrow{\$} \{0,\ldots,p-1\}$

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pk →

(m_0, m_1) ←

ct →

b' ←

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H_1

$y \xleftarrow{\$} \{0,\ldots,p-1\}$

$sk := y, pk := g^y$

pk →

(m_0, m_1) ←

ct →

b' ←

$r \xleftarrow{\$} \{0,\ldots,p-1\}$

$x \xleftarrow{\$} \{0,\ldots,p-1\}$

$\text{ct} := (m_0 * g^r, g^x)$

ElGamal: Proof of Security

H_0

$y \xleftarrow{\$} \{0,\ldots,p-1\}$

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$x \xleftarrow{\$} \{0,\ldots,p-1\}$

$\mathsf{ct} := (m_0 * pk^x, g^x)$

pk →

(m_0, m_1) ←

ct →

b' ←

Prove $H_0 \overset{c}{\approx} H_1$ via a reduction to DDH

H_1

$y \xleftarrow{\$} \{0,\ldots,p-1\}$

$sk := y,\ pk := g^y$

$r \xleftarrow{\$} \{0,\ldots,p-1\}$

$x \xleftarrow{\$} \{0,\ldots,p-1\}$

$\mathsf{ct} := (m_0 * g^r, g^x)$

pk →

(m_0, m_1) ←

ct →

b' ←

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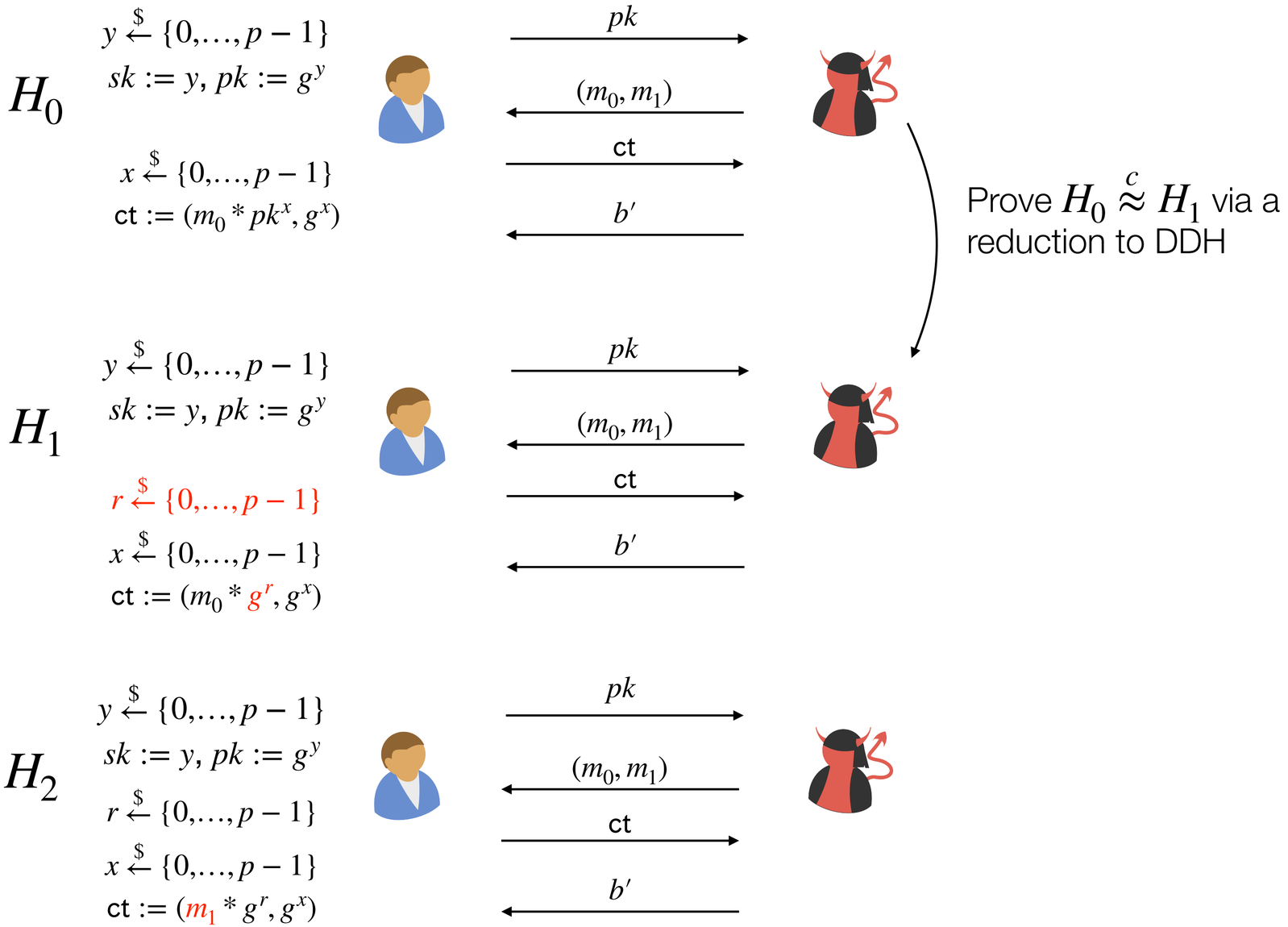

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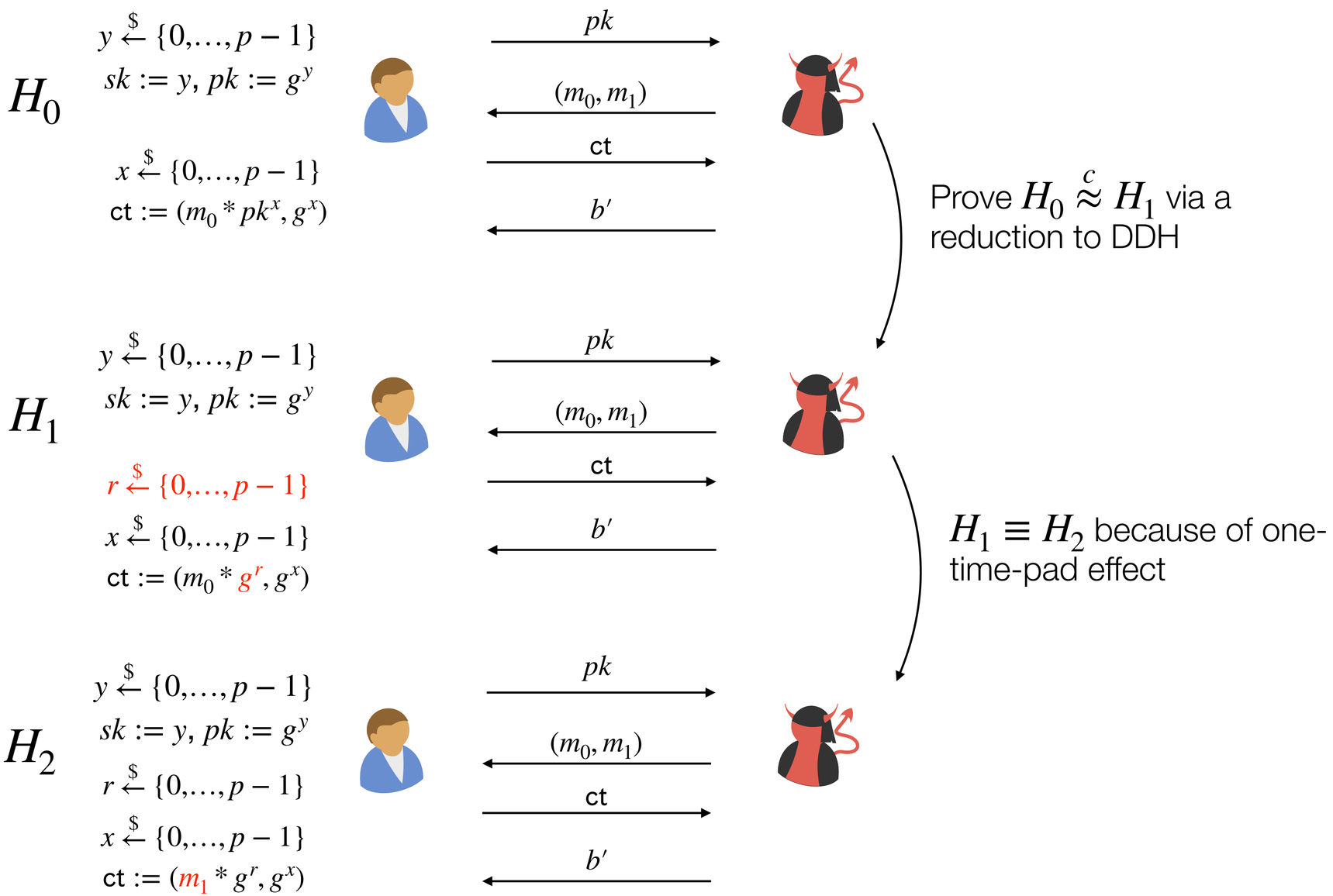

ElGamal: Proof of Security

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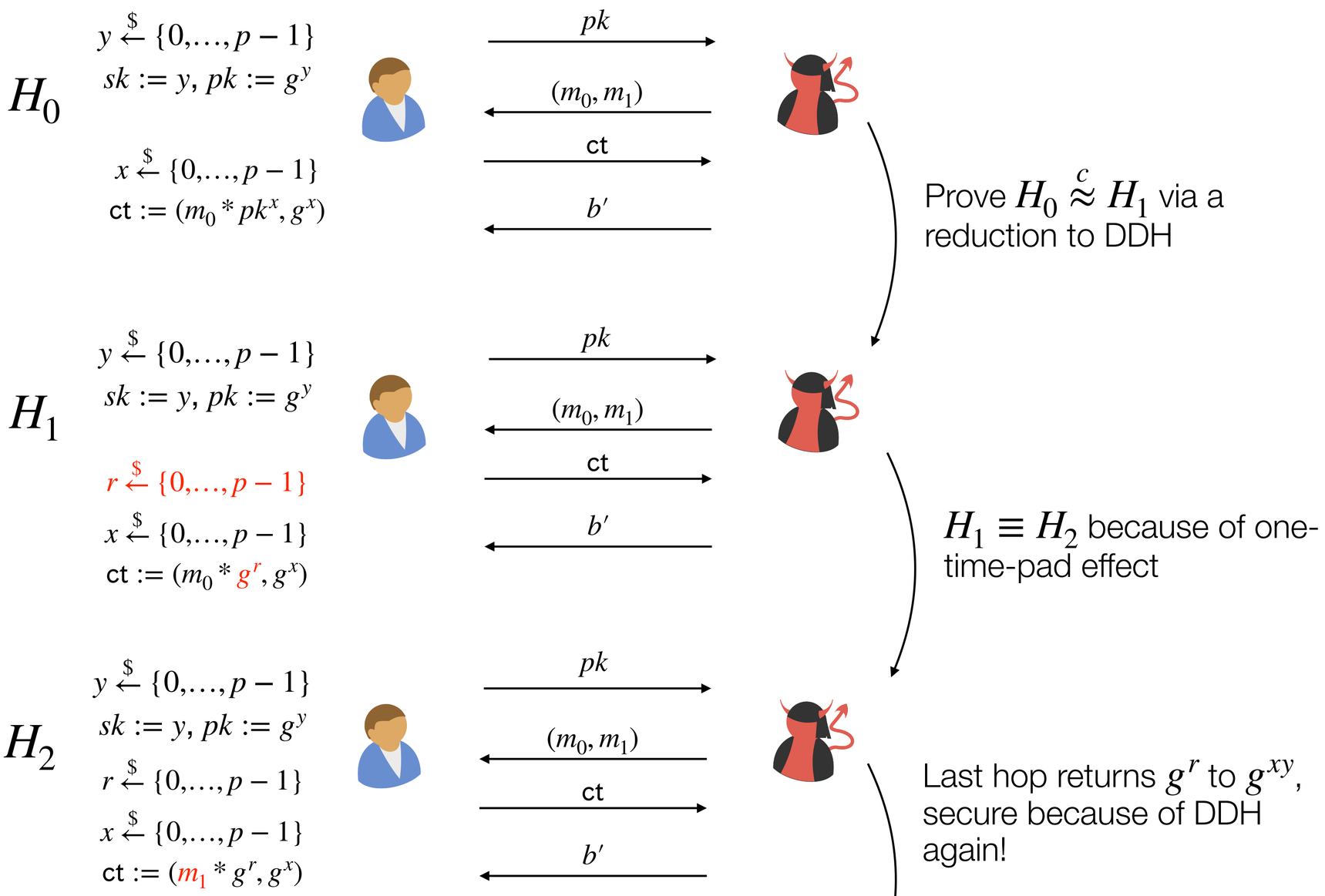

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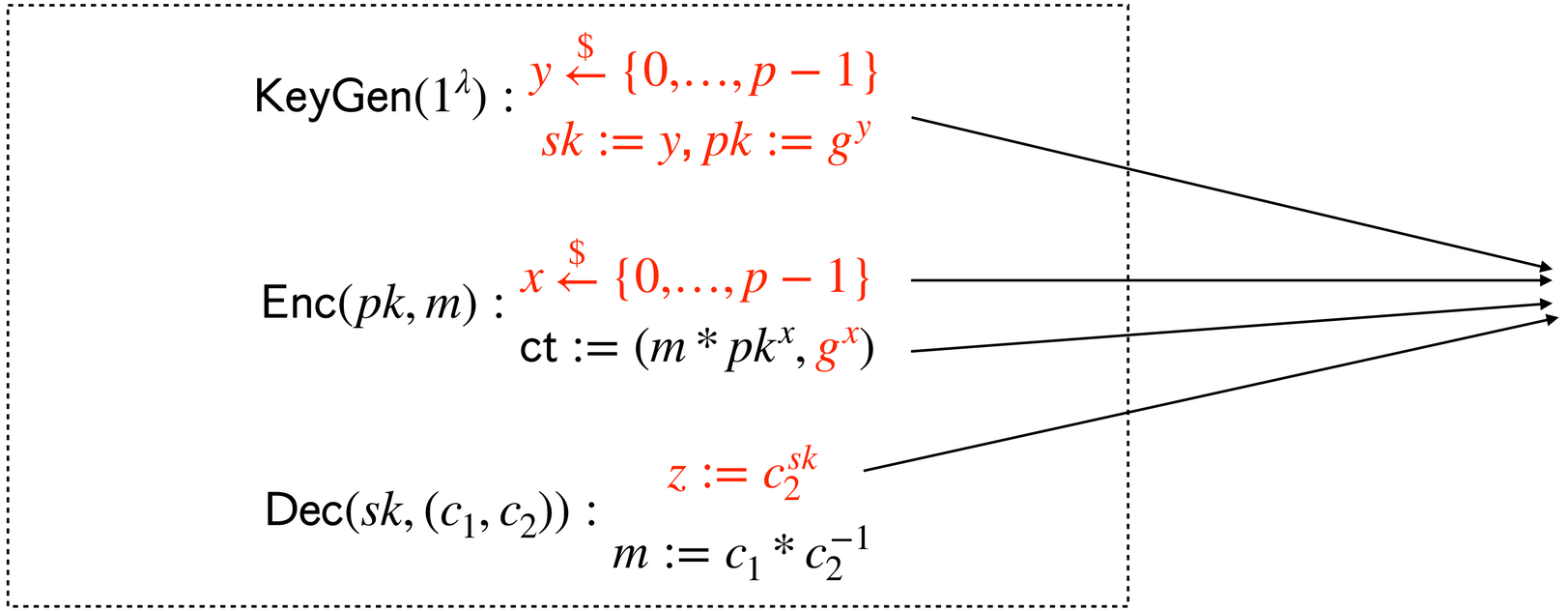

Ciphertext is one part Diffie-Hellman key exchange, and one part OTP with the shared key!

Non-Interactive Key Exchange (NIKE)

Non-Interactive Key Exchange Scheme Syntax

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Correctness: $$\Pr\left[\mathsf{Derive}(sk_0, pk_1) = \mathsf{Derive}(sk_1, pk_0) \;:\; \begin{matrix}(sk_0, pk_0) \leftarrow \mathsf{KeyGen}(\lambda) \\ (sk_1, pk_1) \leftarrow \mathsf{KeyGen}(\lambda)\end{matrix}\right] = 1$$

NIKE Security

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A non-interactive key exchange scheme (KeyGen, Derive) is *secure* if for all NUPPT $\mathscr{A}$, there exists a negligible function $\nu(\cdot)$ such that $\forall \lambda \in \mathbb{N}$

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$$\left| \Pr[\mathscr{A} \text{ outputs } 1 \text{ in Game}_0] - \Pr[\mathscr{A} \text{ outputs } 1 \text{ in Game}_1] \right| \leq \nu(\lambda)$$

Game_0

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Prove $H_1 \overset{c}{\approx} H_2$ via a reduction to IND-CPA security of encryption scheme

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Last hop returns k to $\mathsf{Derive}(sk', pk)$, secure because of NIKE security again!

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  - "Baby-step giant step:" an algorithm for computing discrete log that runs in $O(\sqrt{p})$
    - To get "128 bits of security" (i.e. an assurance that an adversary attacking the scheme successfully must run in time 2^{128}) we need to use groups of order $\geq 2^{256}$!

Usage in practice: Discrete Log Assumptions

Elliptic curve picture from: https://www.globalsign.com/en/blog/elliptic-curve-cryptography

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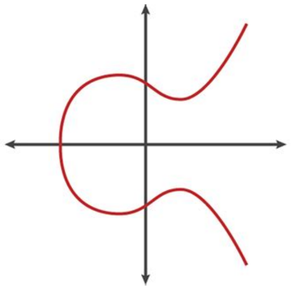

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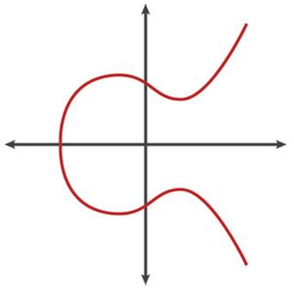

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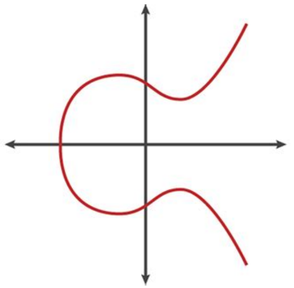
  - Algorithm known as “Elliptic Curve Diffie-Hellman” (ECDH)



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