Client-Server Homomorphic Secret Sharing in the CRS Model

NTT CIS Seminar

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Lawrence Roy

Sacha Servan-Schreiber

Homomorphic Secret Sharing (HSS)

[Boyle-Gilboa-Ishai'16]

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$(x_A, x_B) \leftarrow \mathsf{Share}(x)$

Homomorphic Secret Sharing (HSS)

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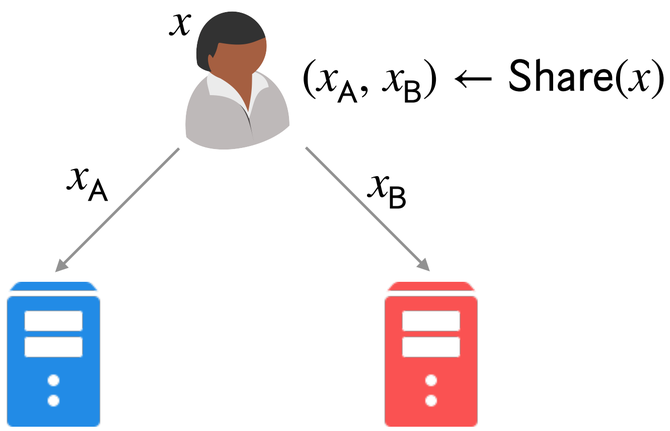

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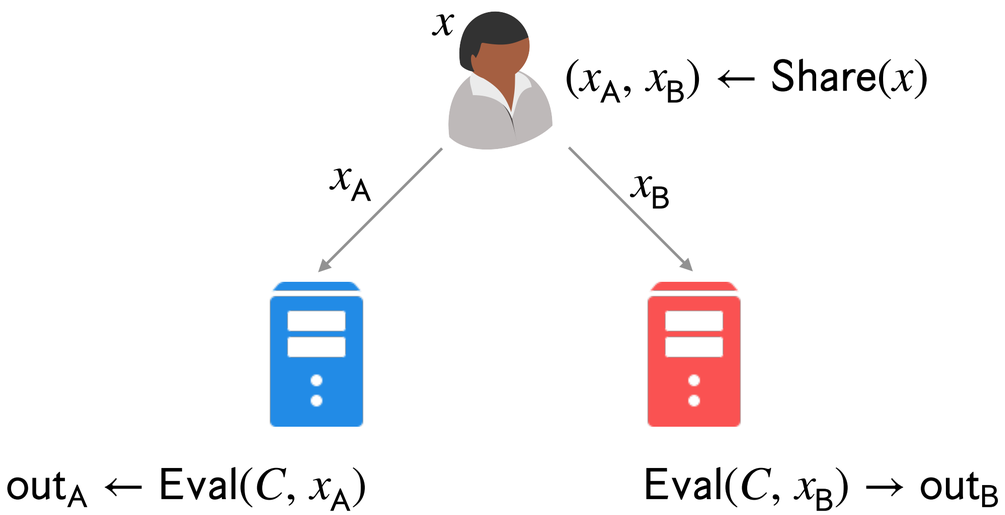
[Boyle-Gilboa-Ishai'16]



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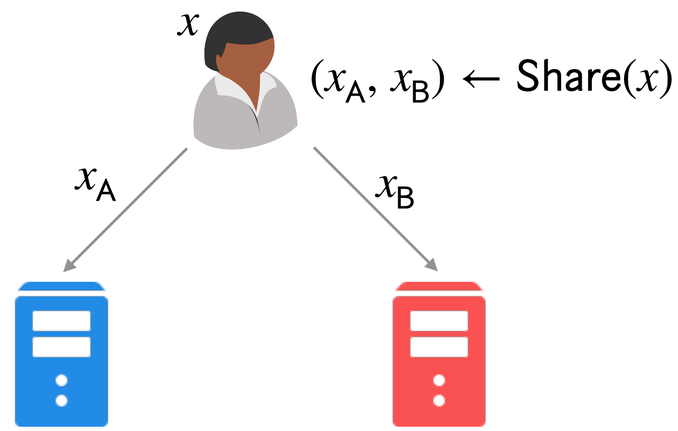
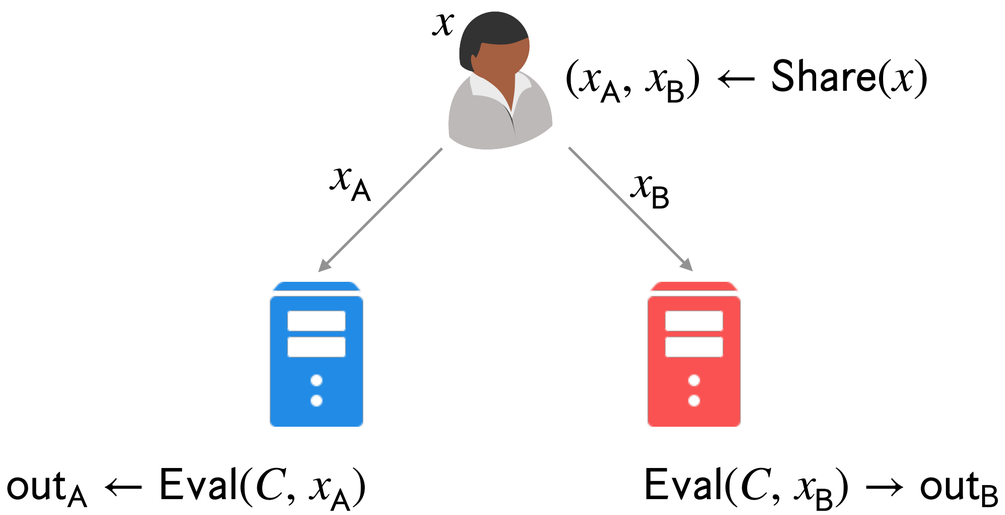


$\text{out}_A \leftarrow \text{Eval}(C, x_A)$ $\quad\quad$ $\text{Eval}(C, x_B) \rightarrow \text{out}_B$

Correctness: $\quad \text{out}_A + \text{out}_B = C(x)$

Homomorphic Secret Sharing (HSS)

[Boyle-Gilboa-Ishai'16]



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Security: x_A ensures privacy of x

x_B ensures privacy of x

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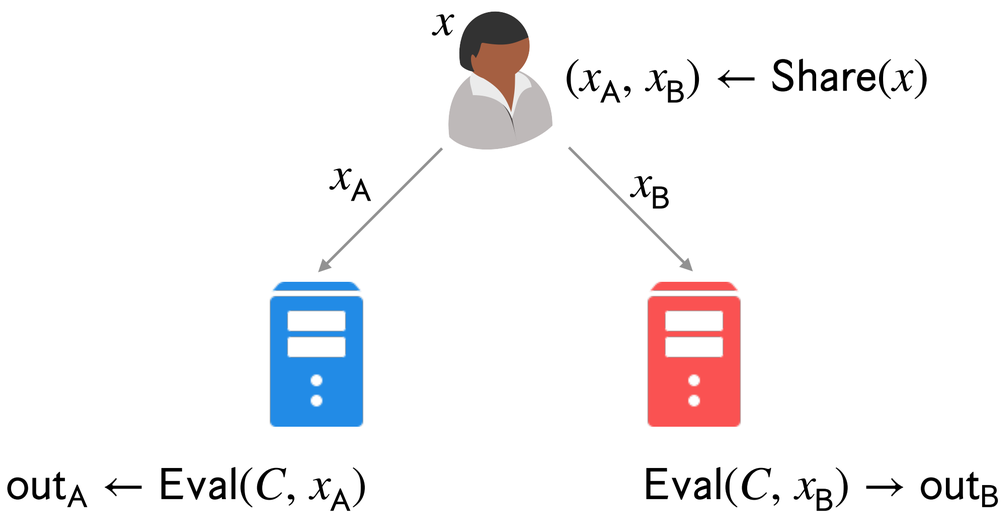

Correctness: $\text{out}_A + \text{out}_B = C(x)$

Security: x_A ensures privacy of x
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Succinctness: Size of x_A and x_B are independent of C

Homomorphic Secret Sharing (HSS)

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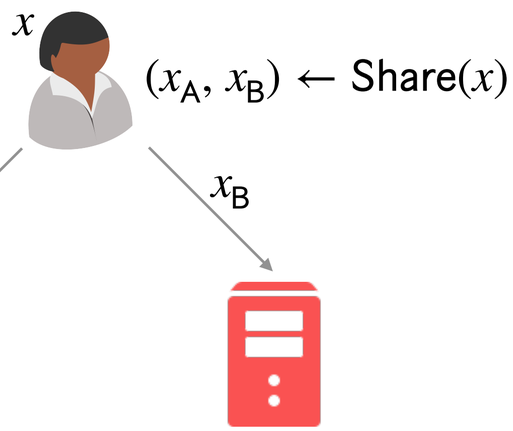
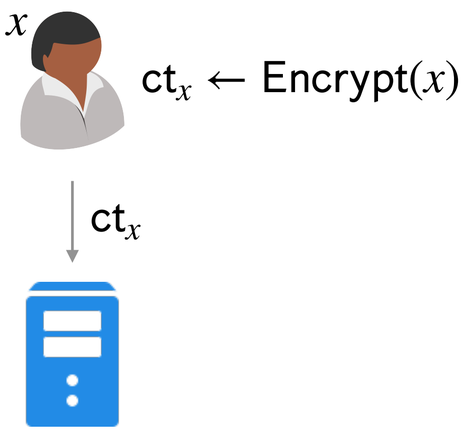
HSS



$\text{out}_A \leftarrow \text{Eval}(C, x_A)$ $\quad$ $\text{Eval}(C, x_B) \rightarrow \text{out}_B$

FHE



$\text{ct}_{C(x)} \leftarrow \text{Eval}(C, \text{ct}_x)$

Correctness: $\text{out}_A + \text{out}_B = C(x)$

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Succinctness: Size of x_A and x_B are independent of C

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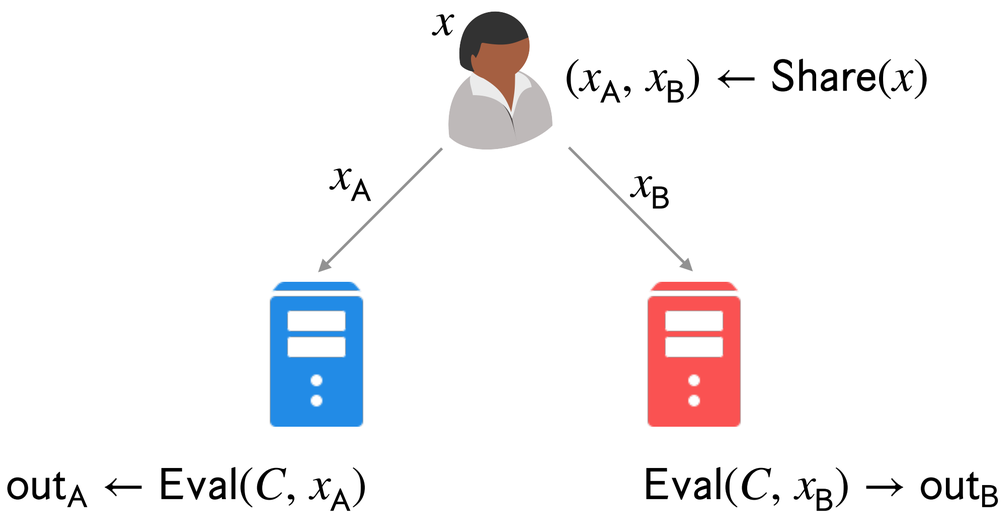

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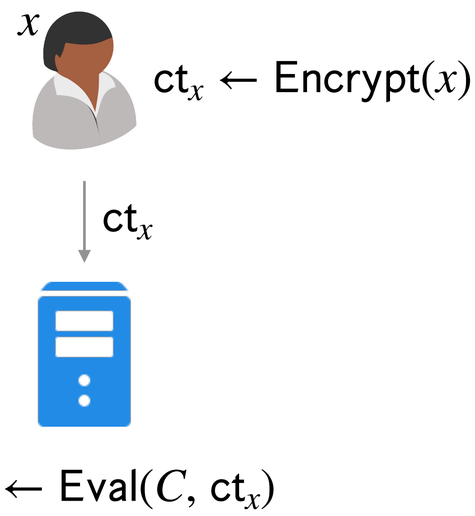

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Succinctness: Size of x_A and x_B are independent of C

HSS is known from assumptions not known to imply FHE

Client-Server HSS

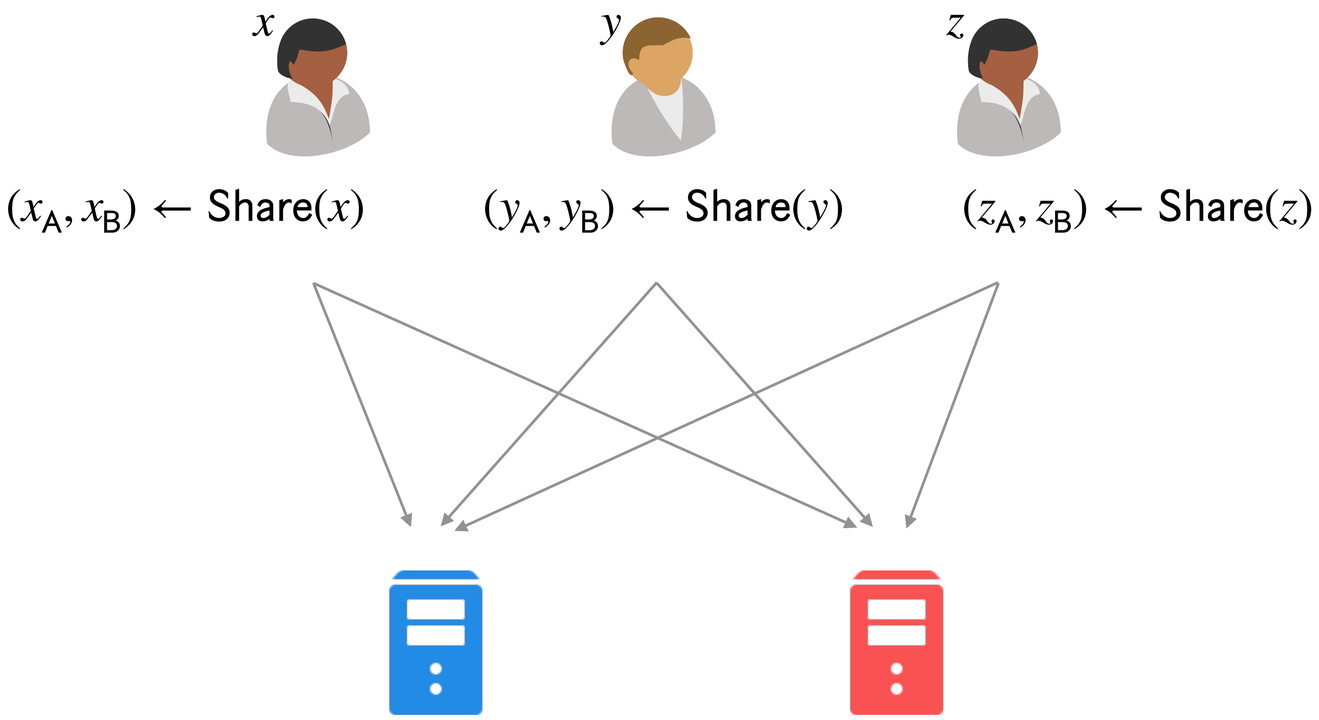
Client-Server HSS

$(x_A, x_B) \leftarrow \text{Share}(x)$

$(y_A, y_B) \leftarrow \text{Share}(y)$

$(z_A, z_B) \leftarrow \text{Share}(z)$

Client-Server HSS



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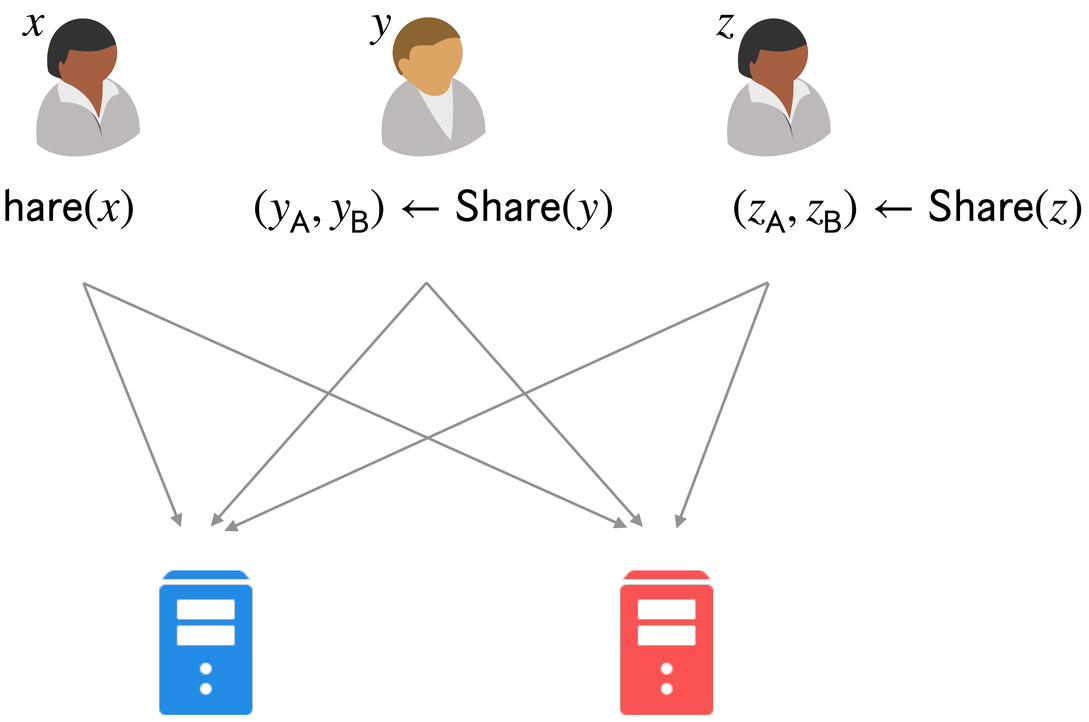
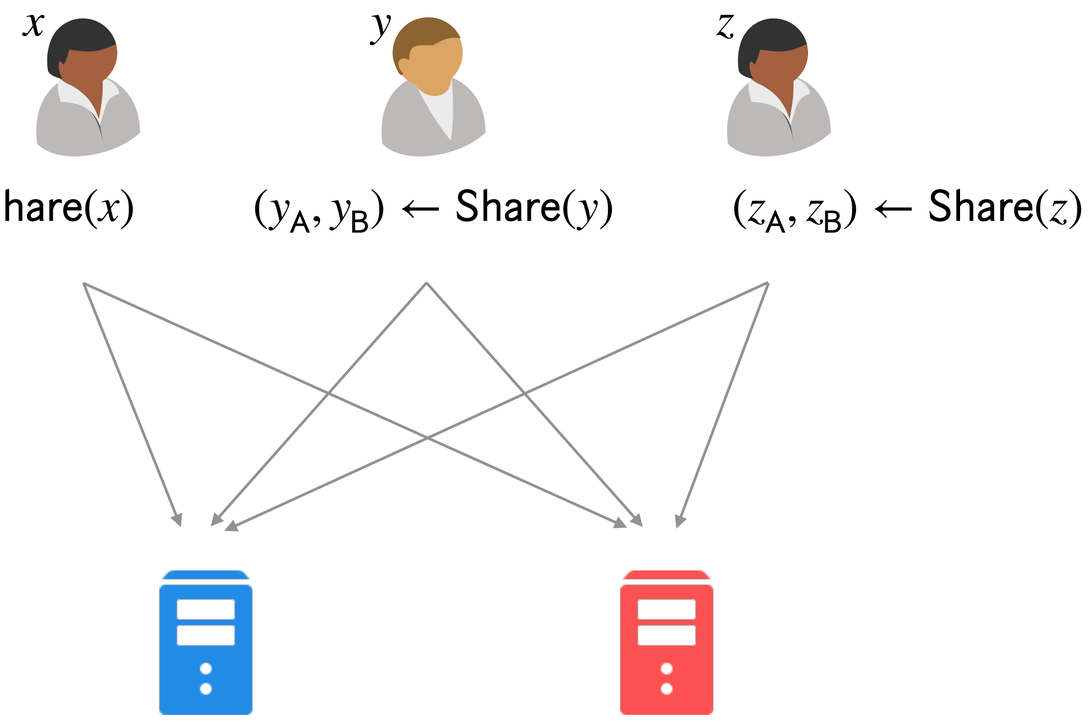


$$\text{out}_A + \text{out}_B = C(x, y, z)$$

Client-Server HSS



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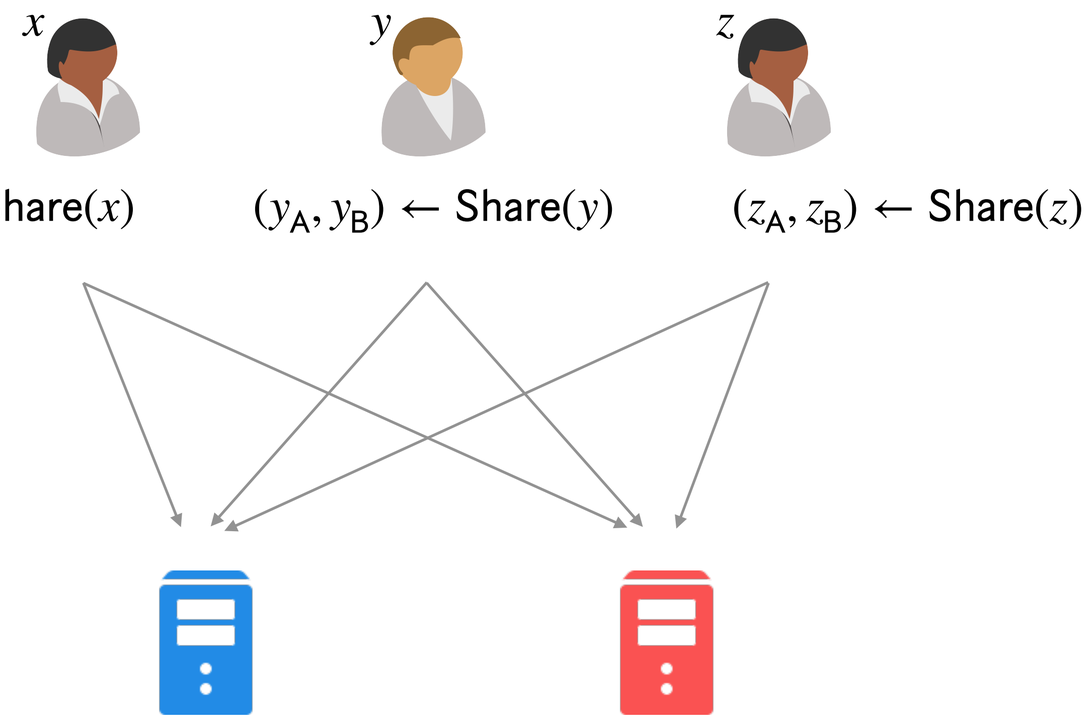
Applications

Two-round succinct MPC

Private Information Retrieval

Pseudorandom Correlation Generators

Client-Server HSS



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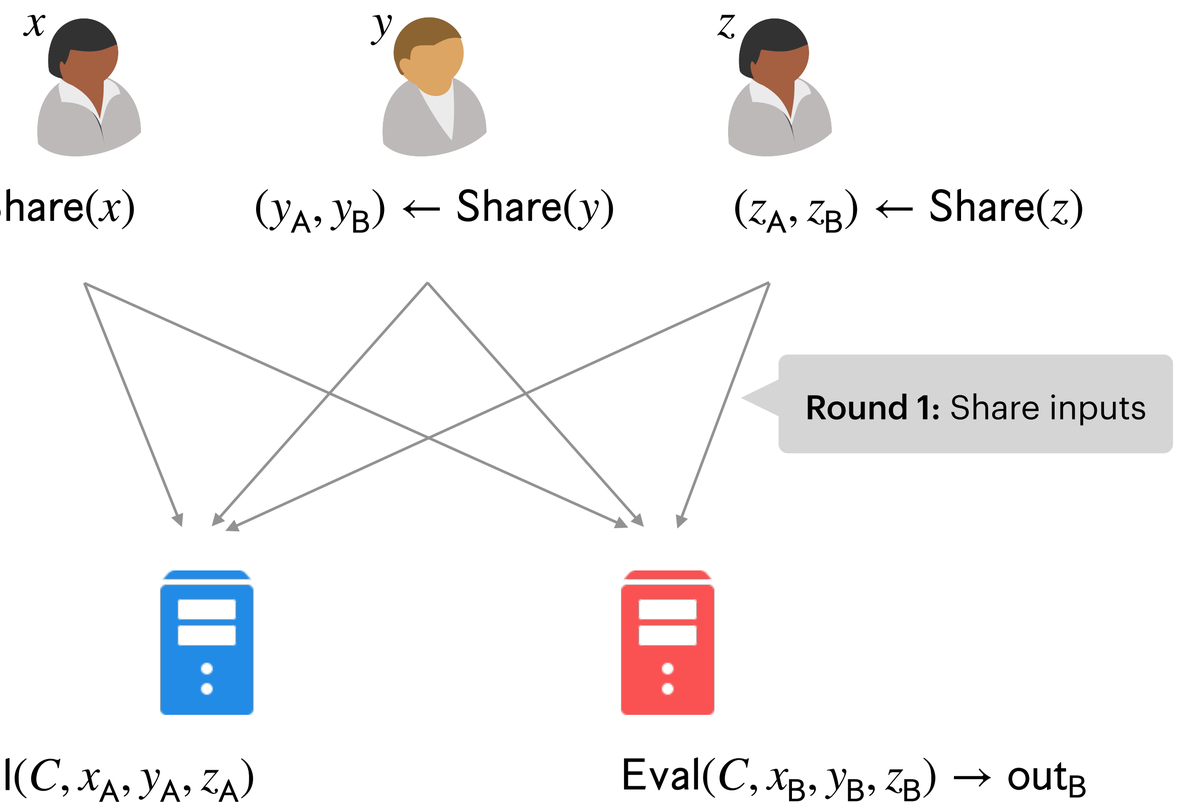

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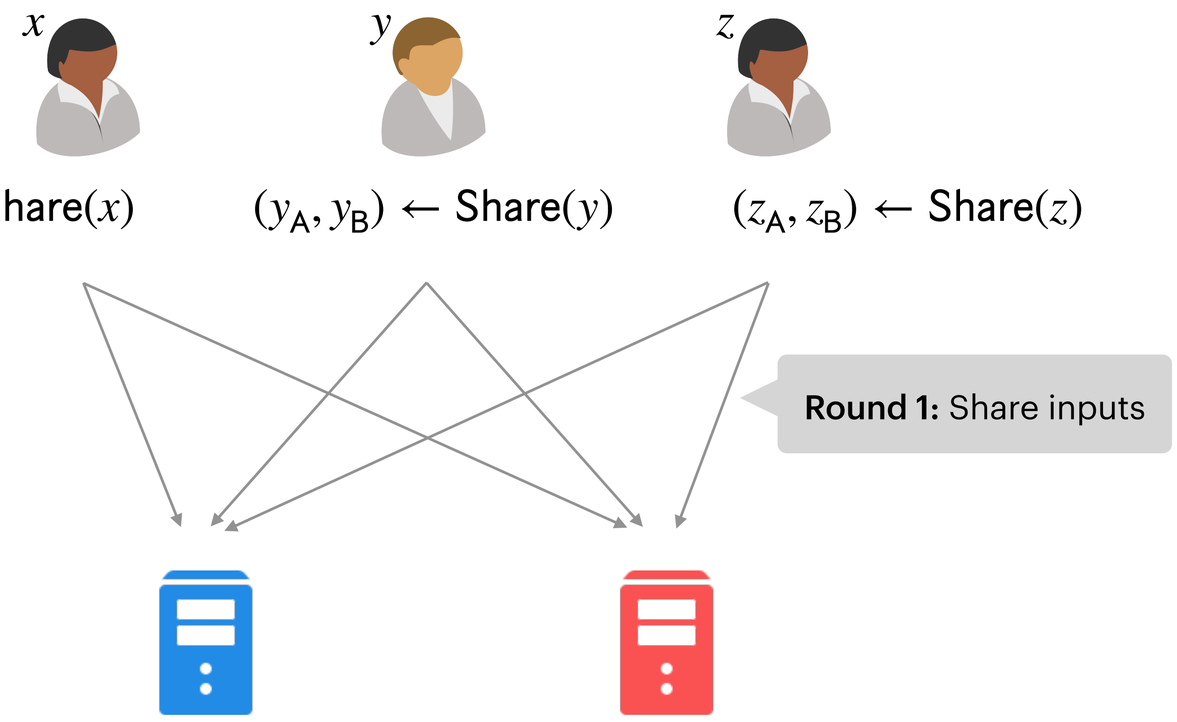

$\text{out}_A + \text{out}_B = C(x, y, z)$ **Round 2:** Reconstruct output

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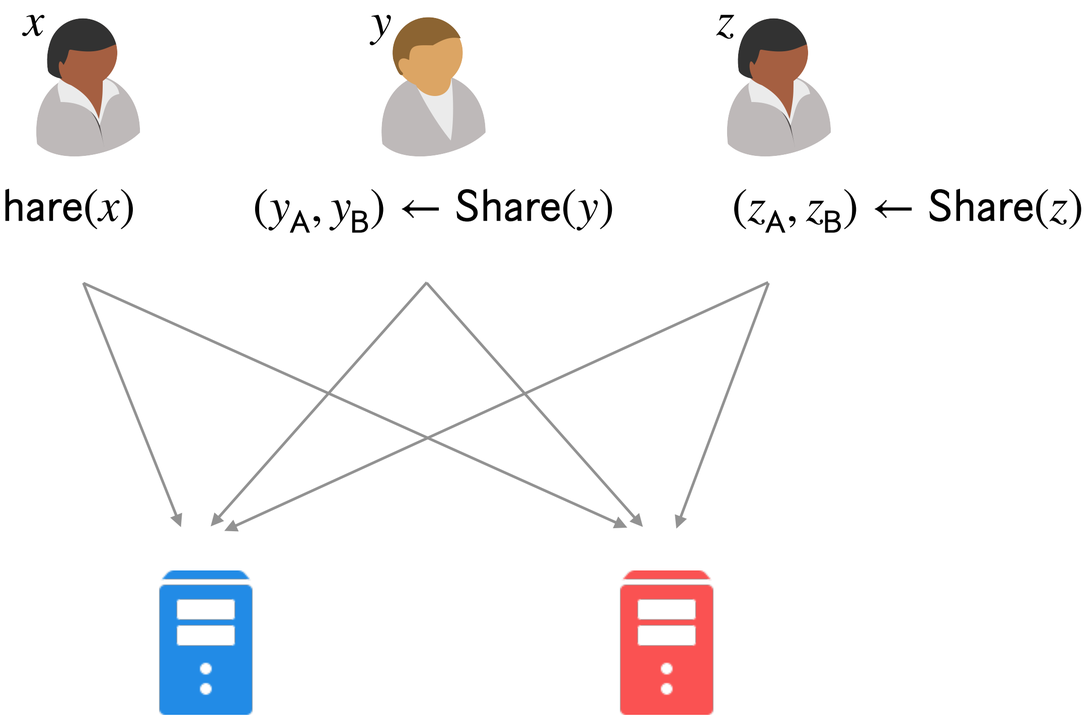

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Existing client-server HSS
require correlated setup

[Boyle-Gilboa-Ishai'16] [Boyle-Kohl-Scholl'19]
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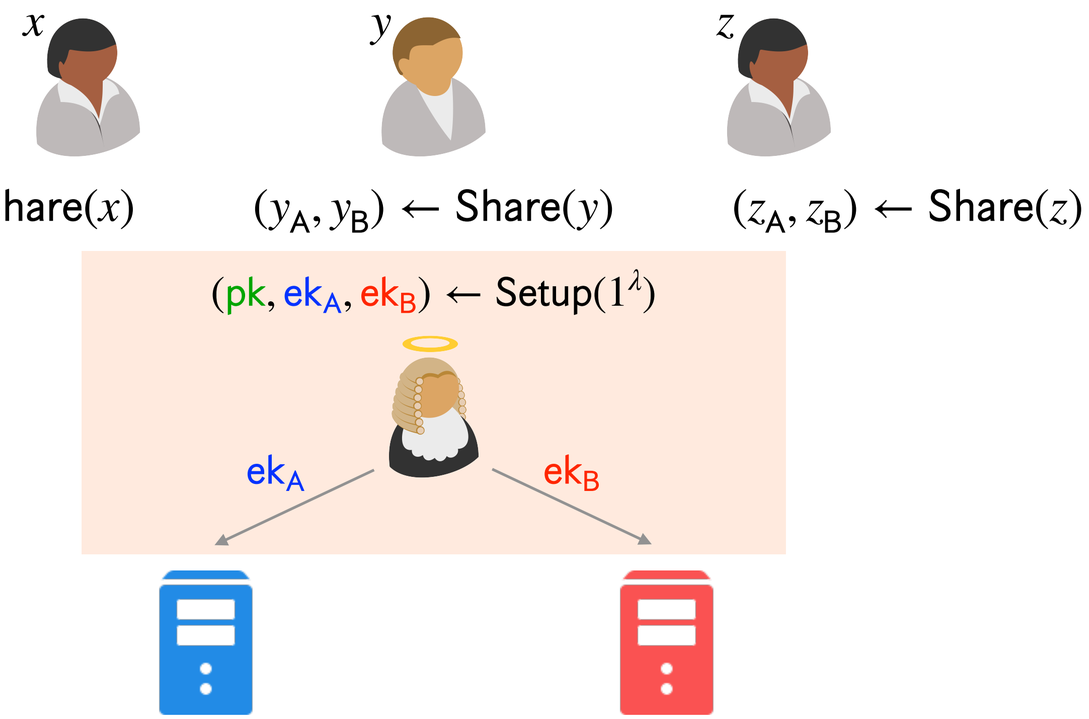

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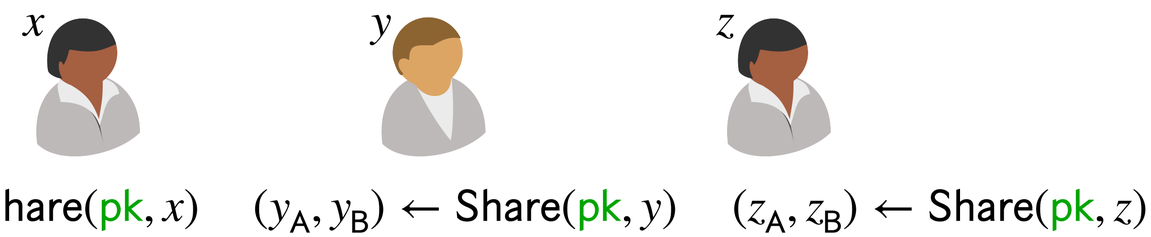
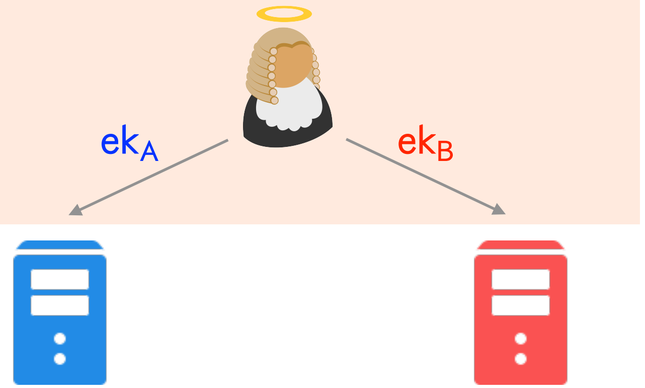


$(x_A, x_B) \leftarrow \text{Share}(\text{pk}, x)$ $(y_A, y_B) \leftarrow \text{Share}(\text{pk}, y)$ $(z_A, z_B) \leftarrow \text{Share}(\text{pk}, z)$

$(\text{pk}, \text{ek}_A, \text{ek}_B) \leftarrow \text{Setup}(1^\lambda)$



$\text{out}_A \leftarrow \text{Eval}(C, x_A, y_A, z_A)$ $\text{Eval}(C, x_B, y_B, z_B) \rightarrow \text{out}_B$

$\text{out}_A + \text{out}_B = C(x, y, z)$

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ek_A ek_B

$\mathsf{out}_A \leftarrow \mathsf{Eval}(\mathsf{ek}_A, C, x_A, y_A, z_A)$ $\mathsf{Eval}(\mathsf{ek}_B, C, x_B, y_B, z_B) \rightarrow \mathsf{out}_B$

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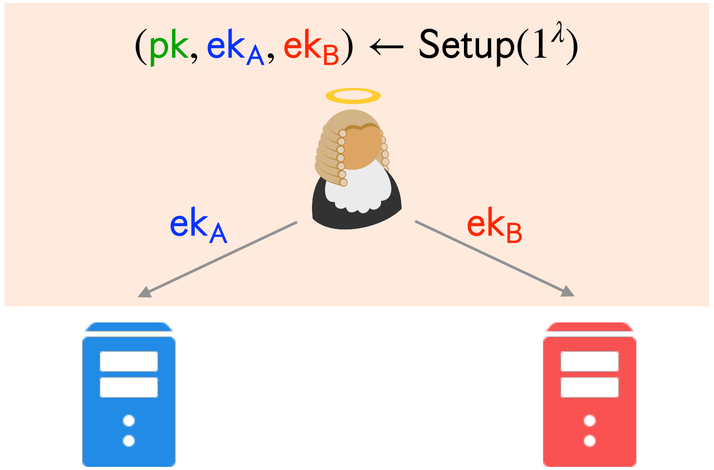

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Multi-key FHE: Multi-input evaluation in the CRS model

[López-Alt–Tromer–Vaikuntanathan'12] [Wichs-Mukherjee'16]

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Common Reference String

$\text{out}_A \leftarrow \text{Eval}(\text{crs}, C, x_A, y_A, z_A)$

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Multi-key FHE: Multi-input evaluation in the CRS model

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Goal: Client-server HSS in the CRS model from assumptions not known to imply FHE

Our Results

Multi-client two-server HSS in the CRS model for evaluating RMS Programs

Our Results

Unbounded polynomial number of clients

Multi-client two-server HSS in the CRS model for evaluating RMS Programs

Our Results

Contains NC^1

Multi-client two-server HSS in the CRS model for evaluating RMS Programs

Our Results

Multi-client two-server HSS in the CRS model for evaluating RMS Programs

DDH

DCR

Class groups

Our Results

Multi-client two-server HSS in the CRS model for evaluating RMS Programs

DDH

DCR

Class groups

Previously known only from LWE or $i\mathcal{O}$ + DDH [Dodis-Halevi-Rothblum-Wichs'16]

Our Results

Multi-client two-server HSS in the CRS model for evaluating RMS Programs

	Client-Server HSS from Prior Works (Require Correlated Setup)
DDH	[Boyle-Gilboa-Ishai'16]
DCR	[Orlandi-Scholl-Yakoubov'21] [Roy-Singh'21]
Class groups	[Abram-Damgård-Orlandi-Scholl'22]

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Our Results

Multi-client two-server HSS in the CRS model for evaluating RMS Programs

Inverse polynomial correctness error

	Client-Server HSS from Prior Works (Require Correlated Setup)
DDH	[Boyle-Gilboa-Ishai'16]
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Our Results

Multi-client two-server HSS in the CRS model for evaluating RMS Programs

		Client-Server HSS from Prior Works (Require Correlated Setup)
Transparent setup	DDH	[Boyle-Gilboa-Ishai'16]
	DCR	[Orlandi-Scholl-Yakoubov'21] [Roy-Singh'21]
Transparent setup	Class groups	[Abram-Damgård-Orlandi-Scholl'22]

Previously known only from LWE or $i\mathcal{O}$ + DDH [Dodis-Halevi-Rothblum-Wichs'16]

Outline

Barriers to Removing Correlated Setup

Our Approach

Extensions

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Our Approach

Extensions

Client-Server HSS with **Correlated Setup**

[Boyle-Gilboa-Ishai'16]

Client-Server HSS with **Correlated Setup**

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$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

Client-Server HSS with **Correlated Setup**

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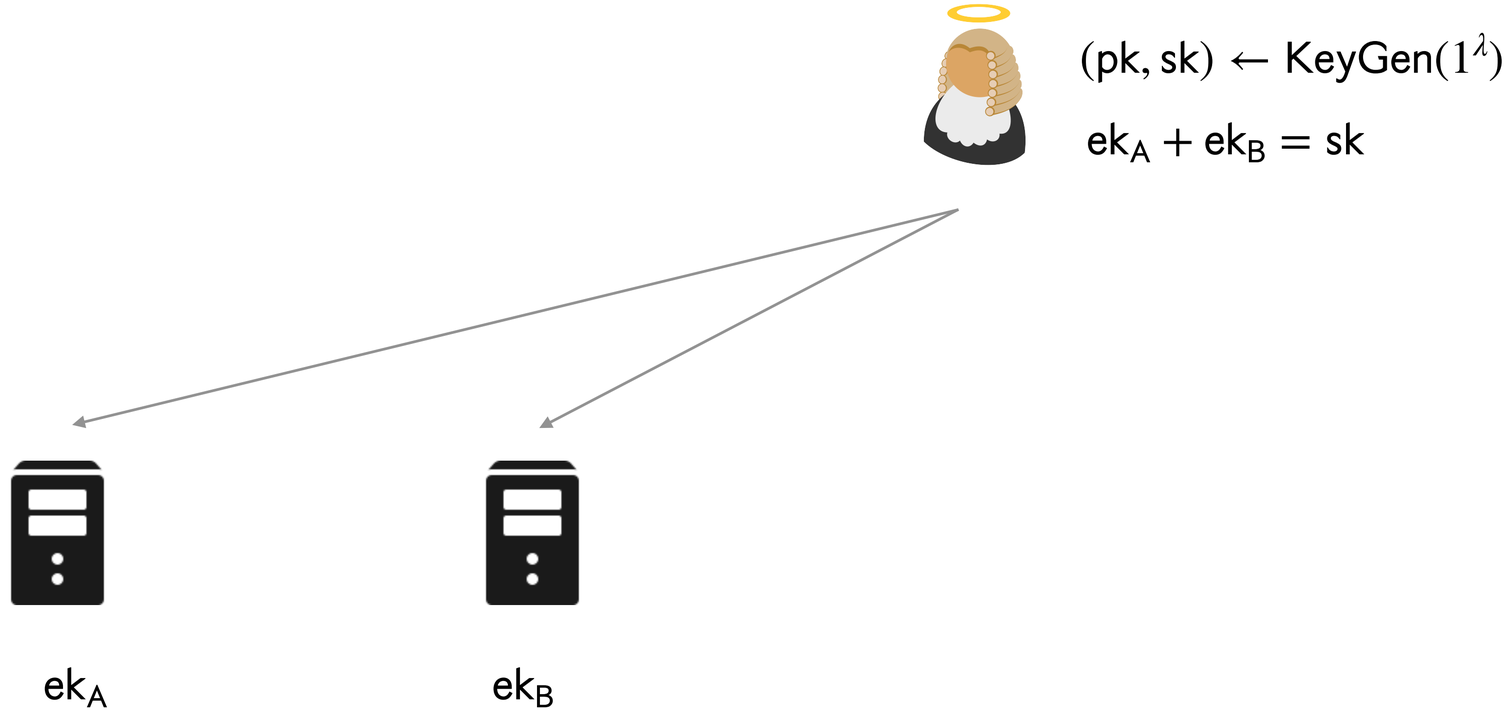
$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{ek}_A + \mathsf{ek}_B = \mathsf{sk}$

Client-Server HSS with **Correlated Setup**

[Boyle-Gilboa-Ishai'16]



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ek_A

ek_B

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x

y

$\mathsf{ct}_x \leftarrow \mathsf{Encrypt}(\mathsf{pk}, x)$

$\mathsf{Encrypt}(\mathsf{pk}, y) \rightarrow \mathsf{ct}_y$

ek_A

ek_B

Client-Server HSS with **Correlated Setup**

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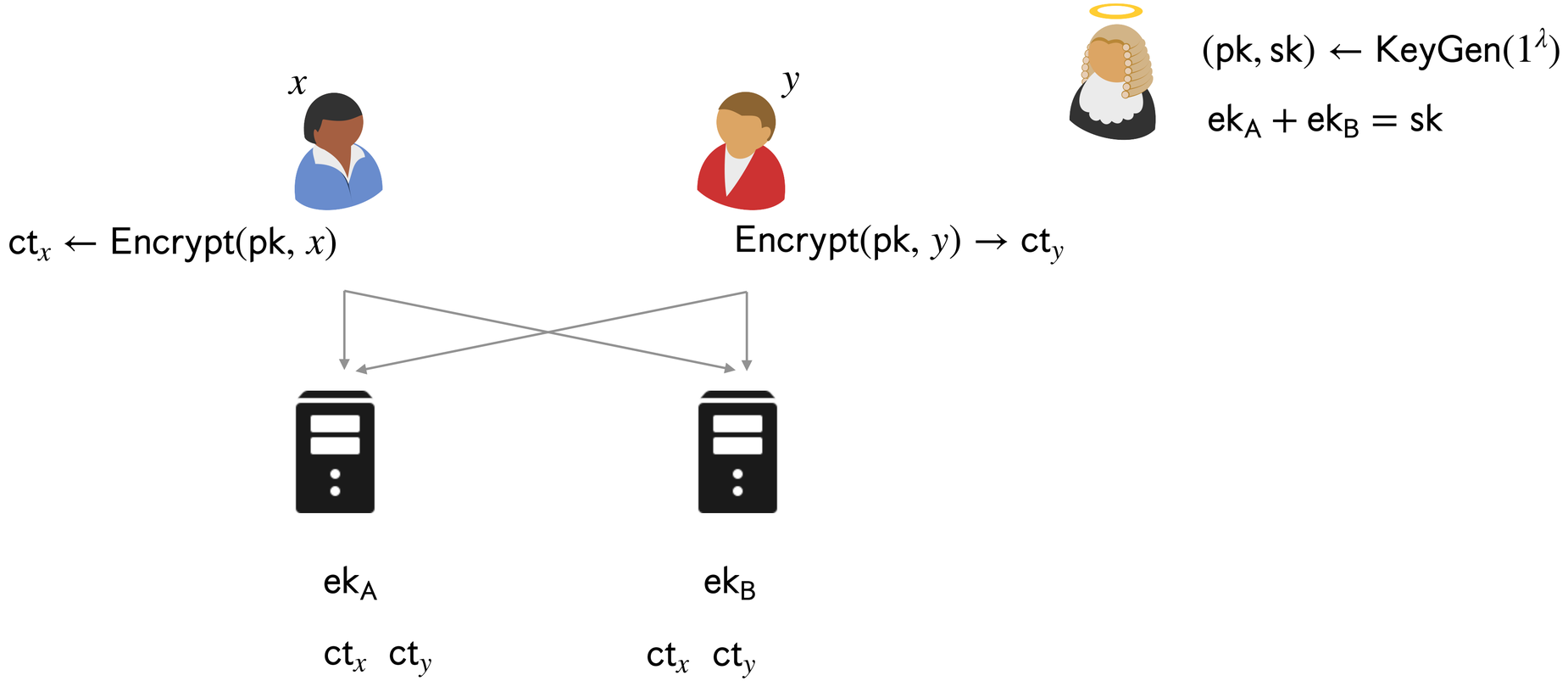

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$\mathsf{Encrypt}(\mathsf{pk}, y) \rightarrow \mathsf{ct}_y$

$[C(x,y)]_A \leftarrow \boxed{\mathsf{Eval}_C} \leftarrow \mathsf{ek}_A,\ \mathsf{ct}_x\ \mathsf{ct}_y$

Any RMS program

$\mathsf{ek}_B \rightarrow,\ \mathsf{ct}_x\ \mathsf{ct}_y \rightarrow \boxed{\mathsf{Eval}_C} \rightarrow [C(x,y)]_B$

Client-Server HSS with **Correlated Setup**

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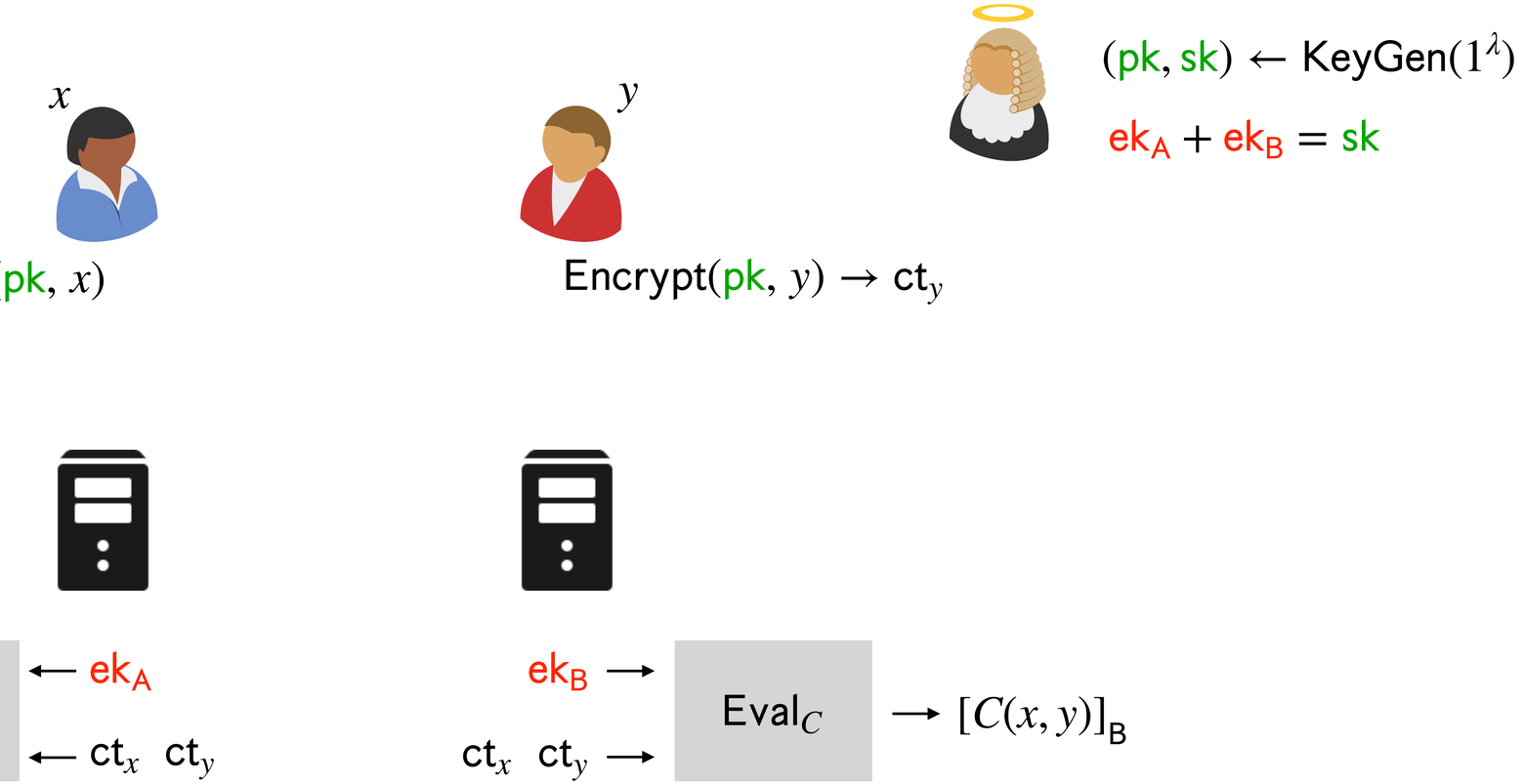
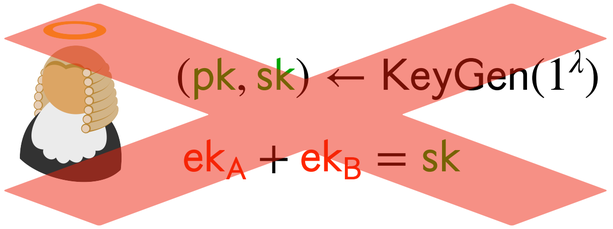


Barrier to Removing Correlated Setup: All inputs must be encrypted under a common key

Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]



$\mathsf{ct}_x \leftarrow \mathsf{Encrypt}(\mathsf{pk}, x)$

$\mathsf{Encrypt}(\mathsf{pk}, y) \rightarrow \mathsf{ct}_y$

$[C(x,y)]_A \leftarrow \boxed{\mathsf{Eval}_C} \leftarrow \mathsf{ek}_A,\ \mathsf{ct}_x\ \mathsf{ct}_y$

$\mathsf{ek}_B \rightarrow,\ \mathsf{ct}_x\ \mathsf{ct}_y \rightarrow \boxed{\mathsf{Eval}_C} \rightarrow [C(x,y)]_B$

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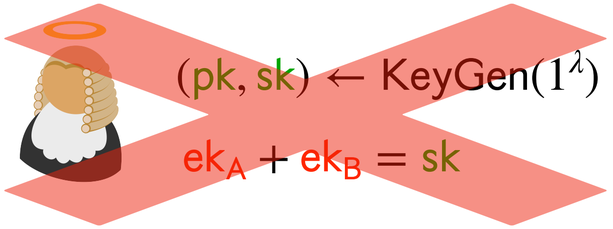


$\mathsf{ct}_x \leftarrow \mathsf{Encrypt}(\mathsf{pk}, x)$

$\mathsf{Encrypt}(\mathsf{pk}, y) \rightarrow \mathsf{ct}_y$

Approach

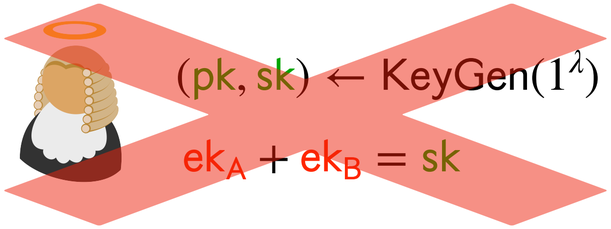
Modify input encoding to use the same evaluation algorithm

$[C(x,y)]_A \leftarrow \mathsf{Eval}_C \leftarrow \mathsf{ek}_A,\ \mathsf{ct}_x,\ \mathsf{ct}_y$

$\mathsf{ek}_B,\ \mathsf{ct}_x,\ \mathsf{ct}_y \rightarrow \mathsf{Eval}_C \rightarrow [C(x,y)]_B$

Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]

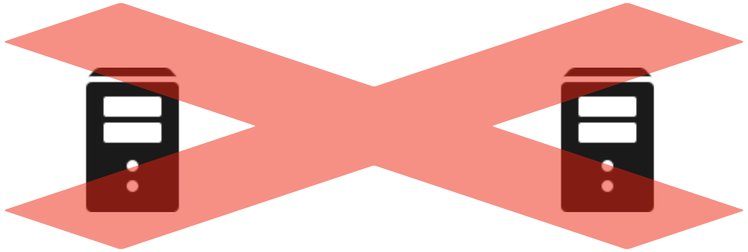


$\mathsf{ct}_x \leftarrow \mathsf{Encrypt}(\mathsf{pk}, x)$

$\mathsf{Encrypt}(\mathsf{pk}, y) \rightarrow \mathsf{ct}_y$

Approach

Modify input encoding to use the same evaluation algorithm



$[C(x,y)]_A \leftarrow \mathsf{Eval}_C \leftarrow \mathsf{ek}_A$

$\leftarrow \mathsf{ct}_x \quad \mathsf{ct}_y$

$\mathsf{ek}_B \rightarrow \mathsf{Eval}_C \rightarrow [C(x,y)]_B$

$\mathsf{ct}_x \quad \mathsf{ct}_y \rightarrow$

Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]

Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]

Common Reference String

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]

Common Reference String

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

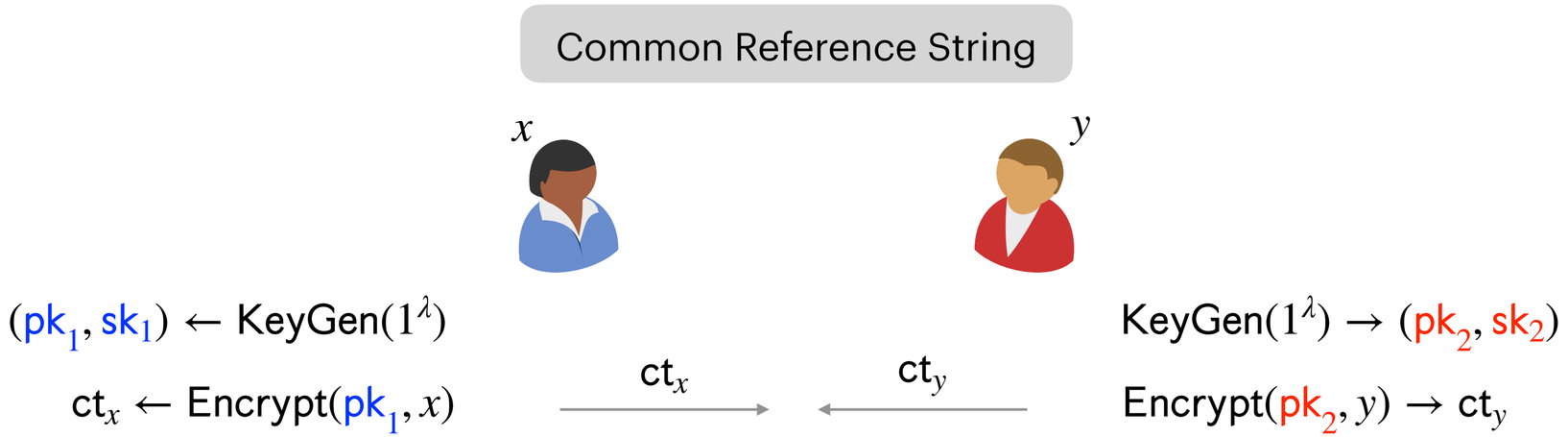
$\mathsf{ct}_x \leftarrow \mathsf{Encrypt}(\mathsf{pk}_1, x)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

$\mathsf{Encrypt}(\mathsf{pk}_2, y) \rightarrow \mathsf{ct}_y$

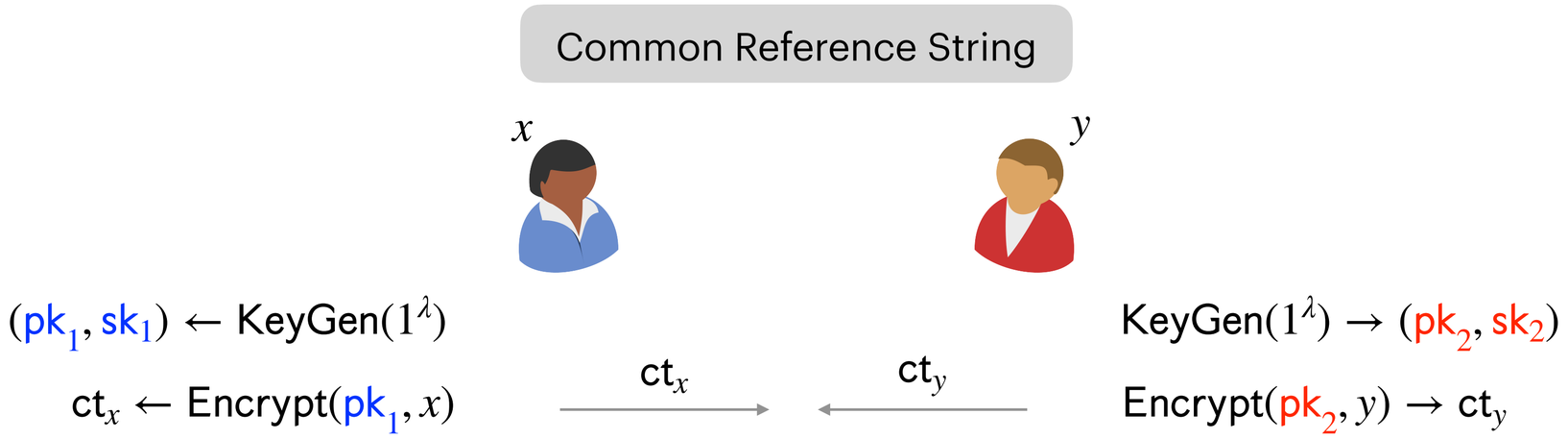
Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]



Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]

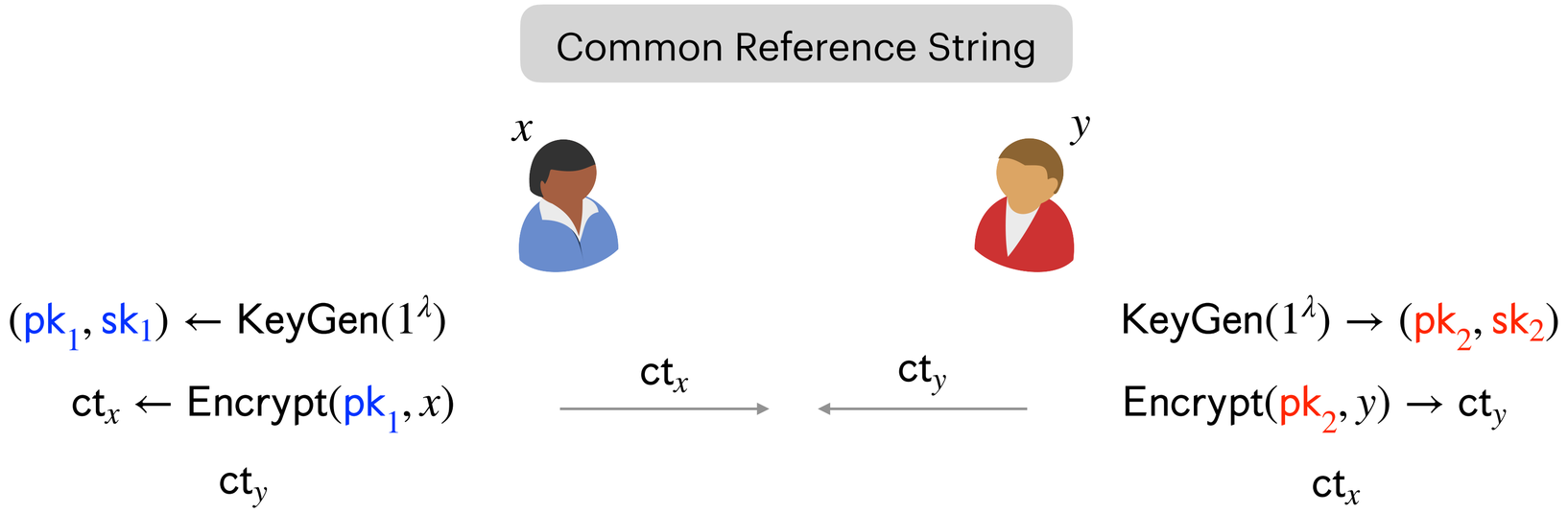


$\mathsf{Encrypt}(\mathsf{pk}_1, x)$
$\mathsf{Encrypt}(\mathsf{pk}_2, y)$
$\rightarrow$ Eval_C

Inputs encrypted under different keys

Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]



Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]

Common Reference String

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{ct}_x \leftarrow \mathsf{Encrypt}(\mathsf{pk}_1, x)$

ct_y

ct_x →

← ct_y

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

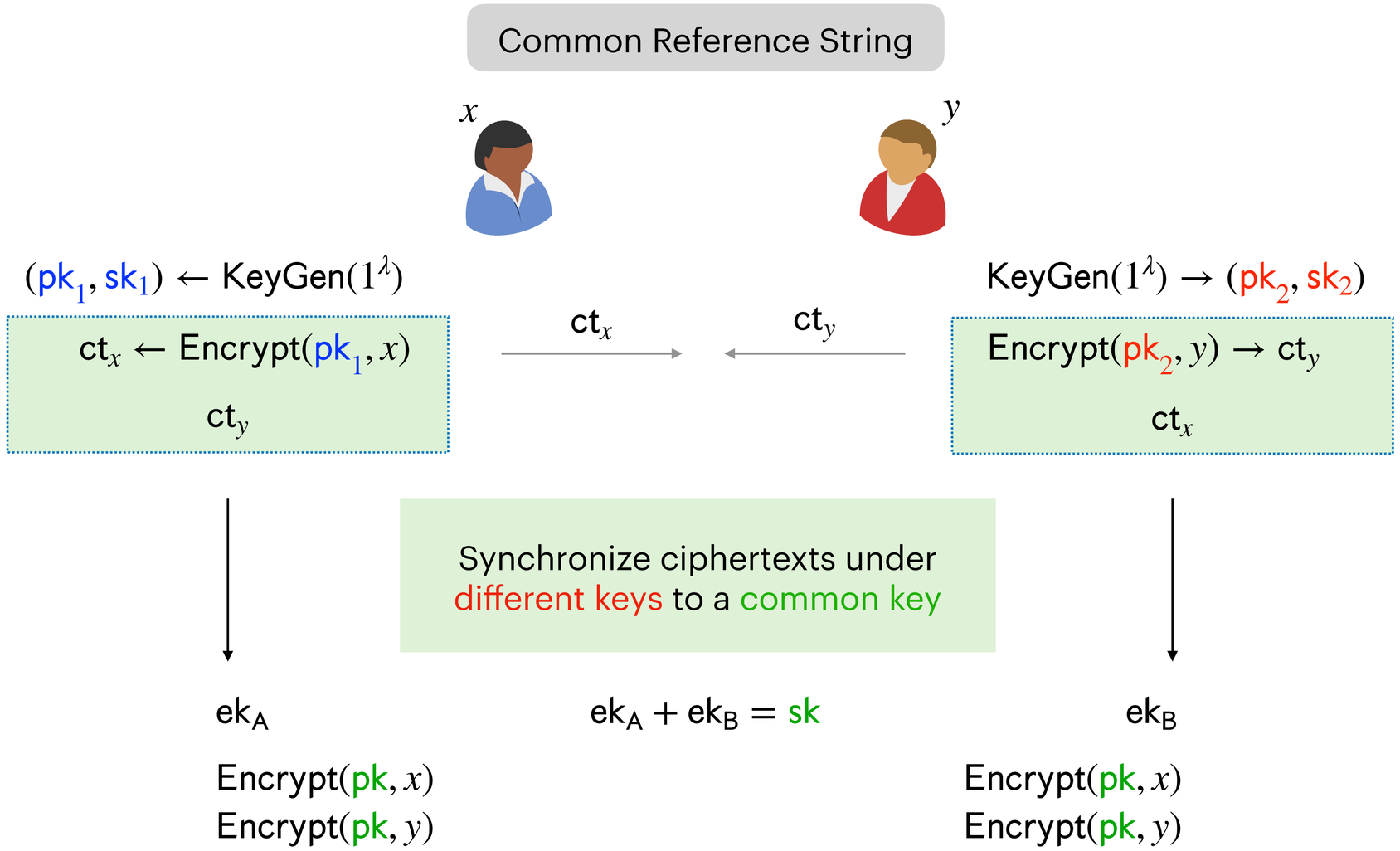
$\mathsf{Encrypt}(\mathsf{pk}_2, y) \rightarrow \mathsf{ct}_y$

ct_x

Synchronize ciphertexts under different keys to a common key

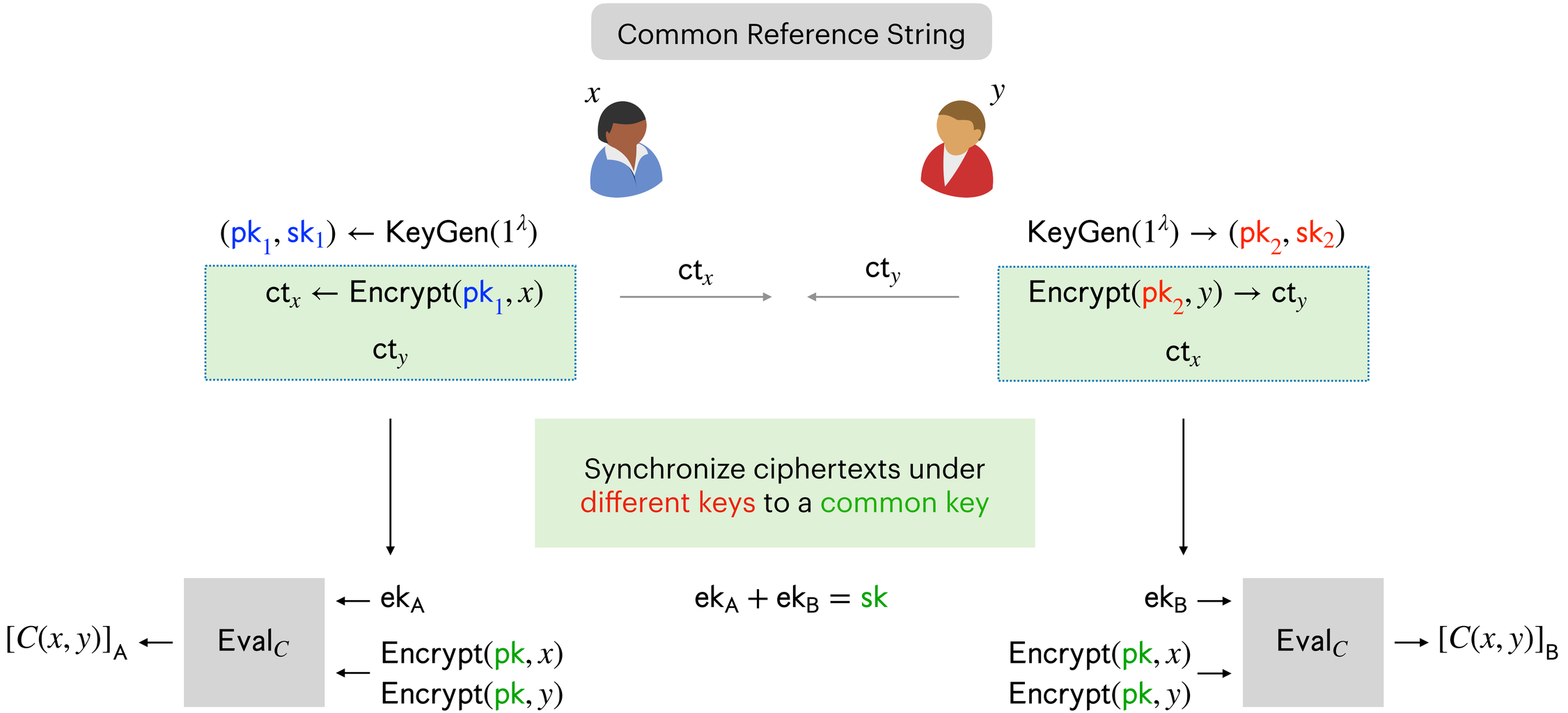
Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]



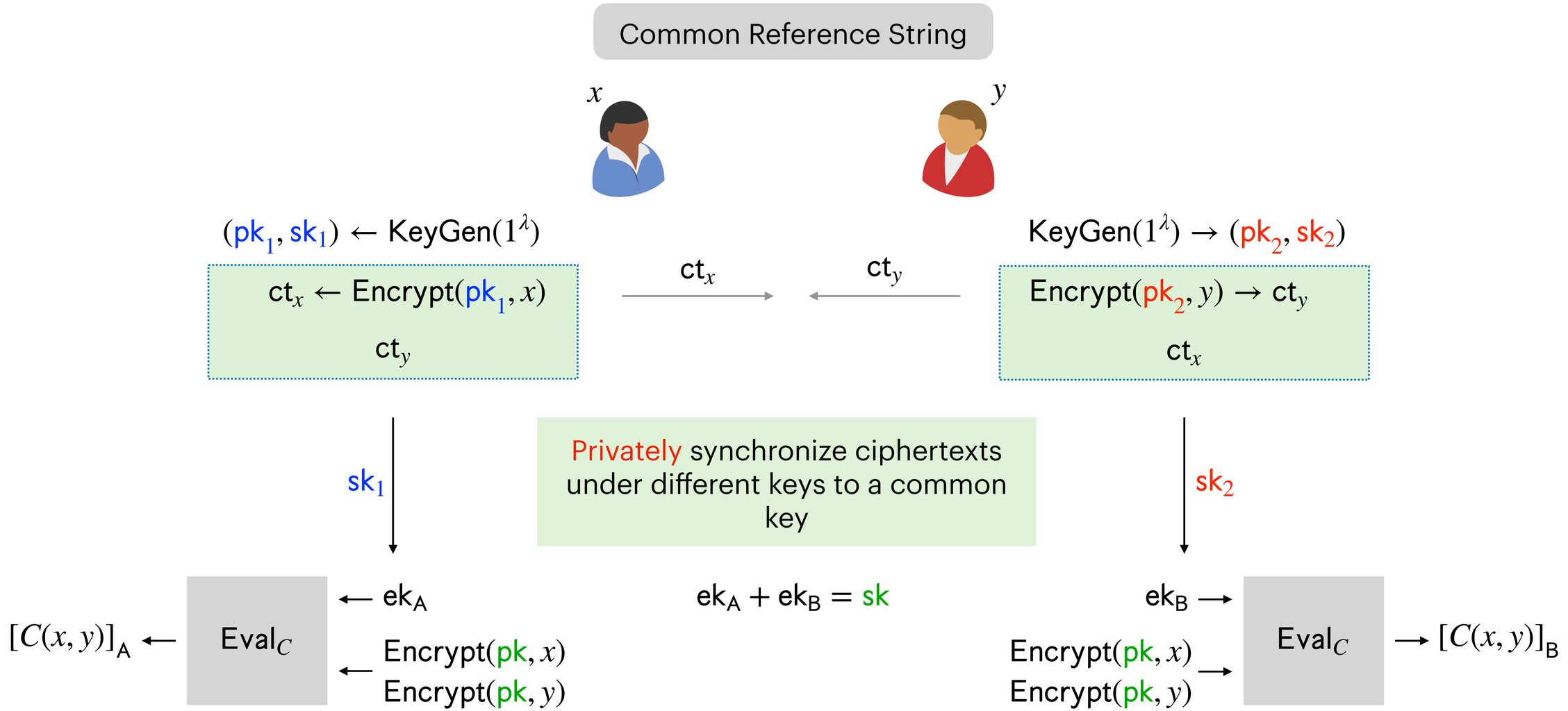
Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]



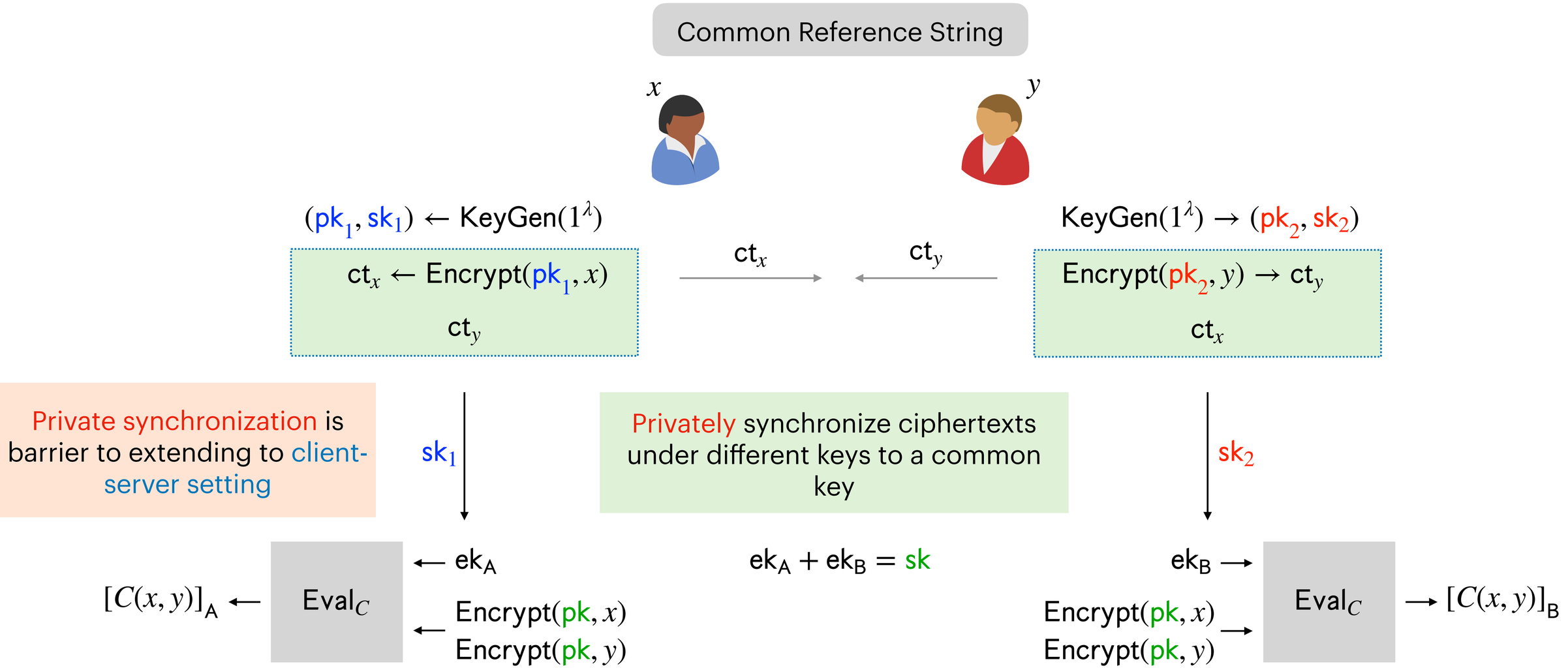
Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]

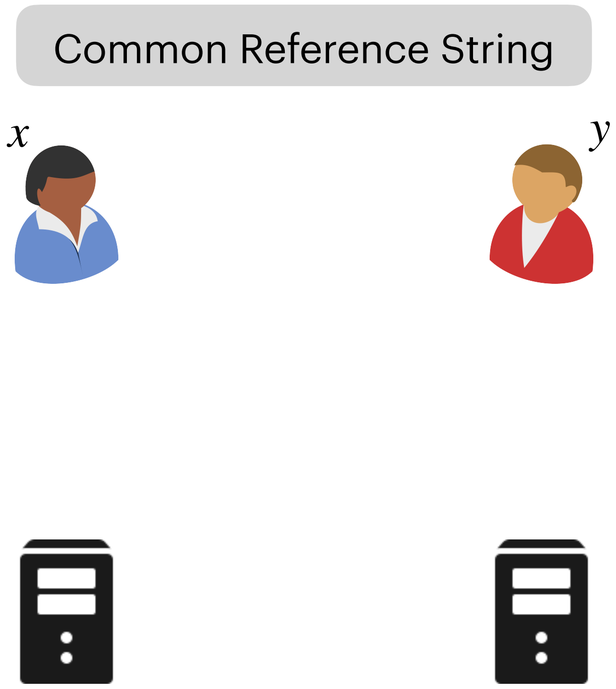


Two-Key HSS

[Couteau–Devadas–H–Jain–Servan-Schreiber'25]



Barriers to **Delegating** Two-key HSS



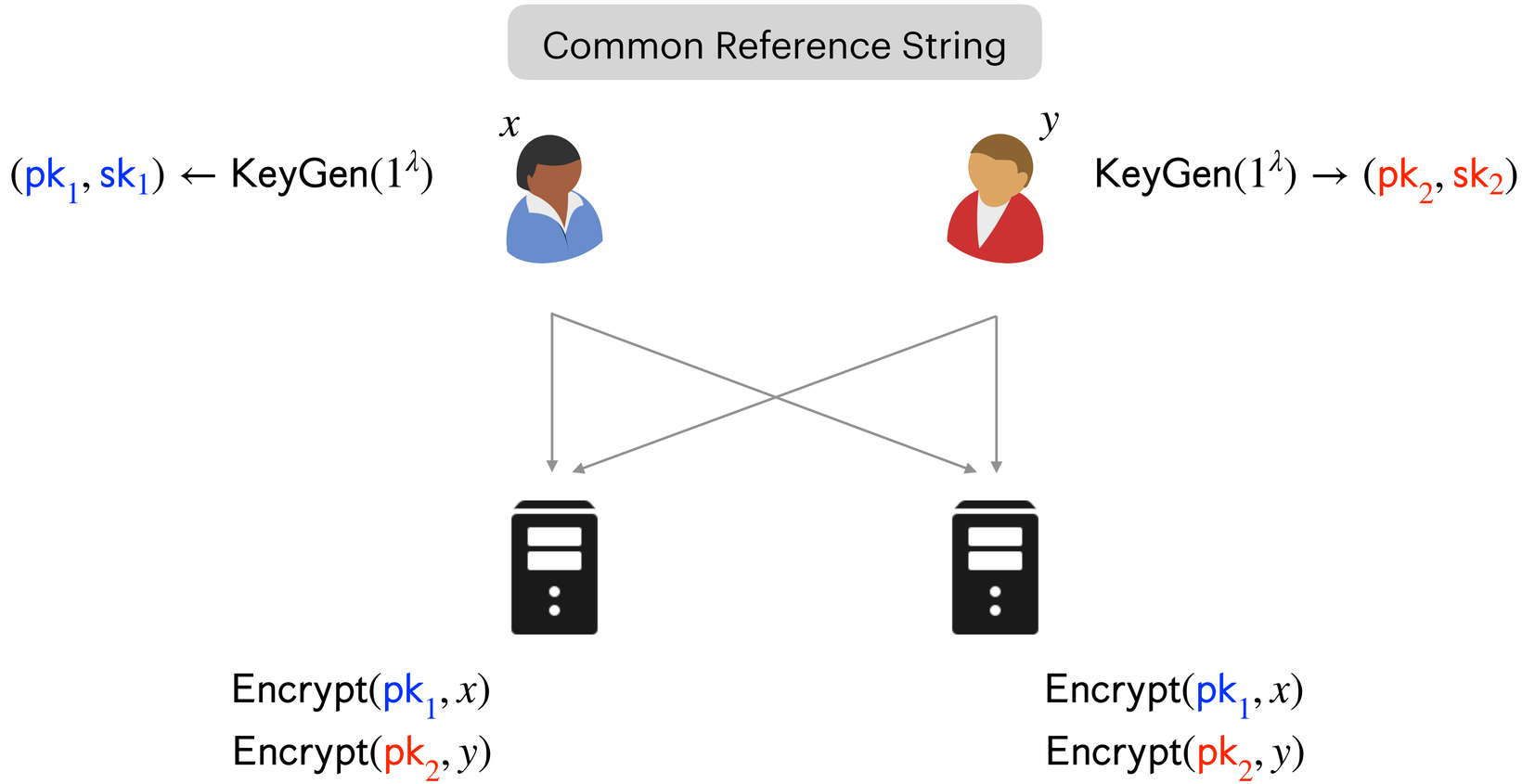
Barriers to **Delegating** Two-key HSS

Common Reference String

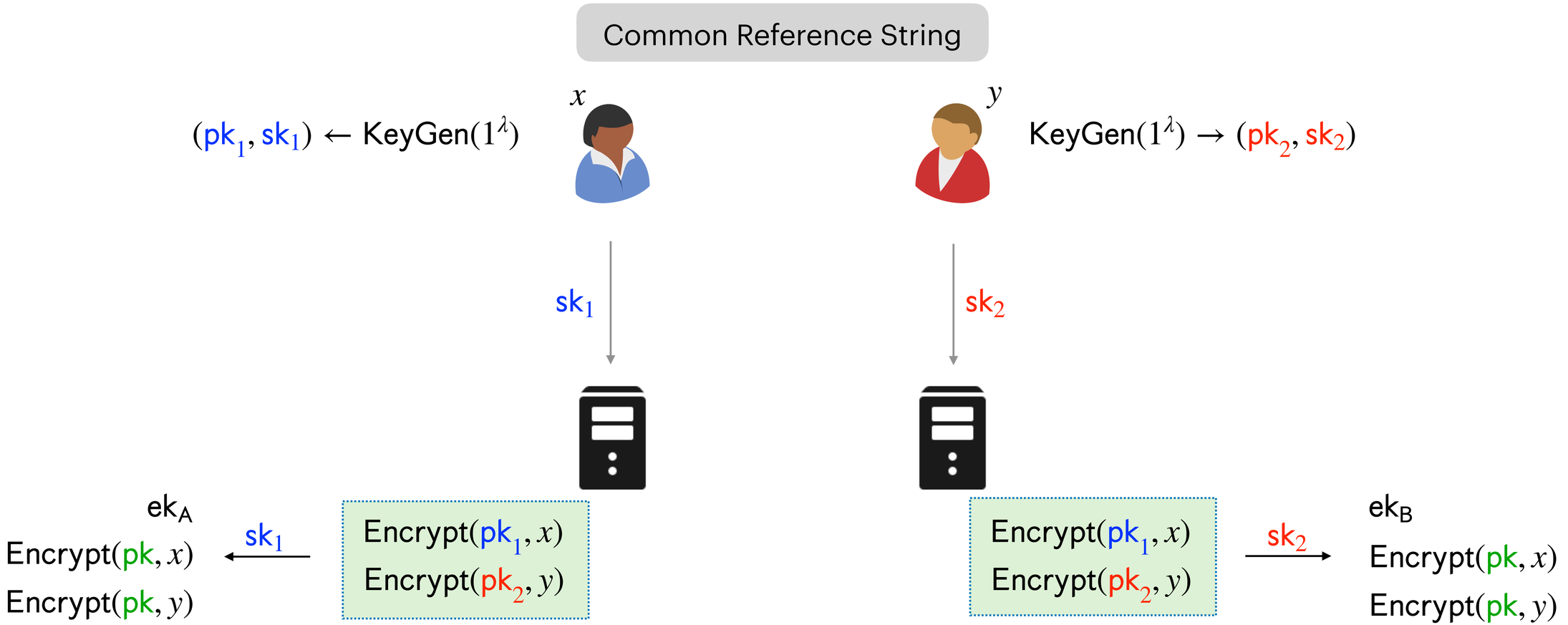
$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

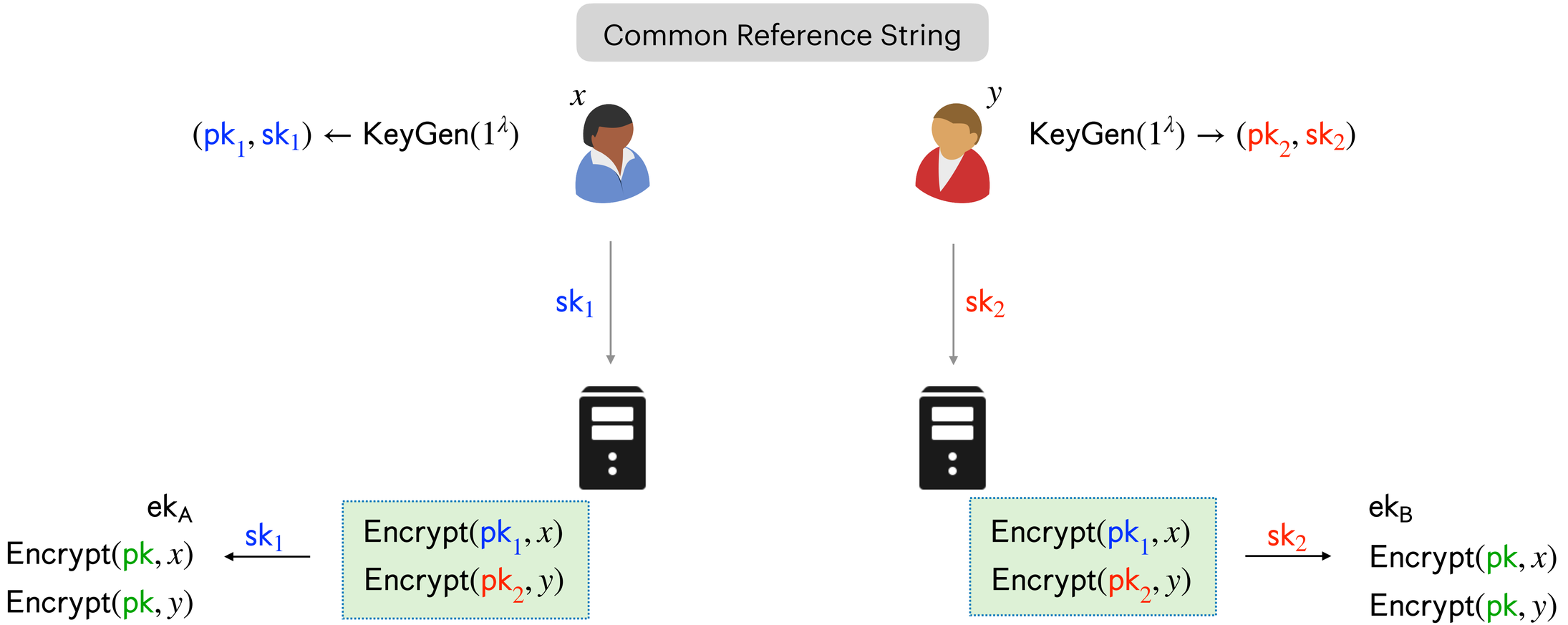
Barriers to **Delegating** Two-key HSS



Barriers to **Delegating** Two-key HSS



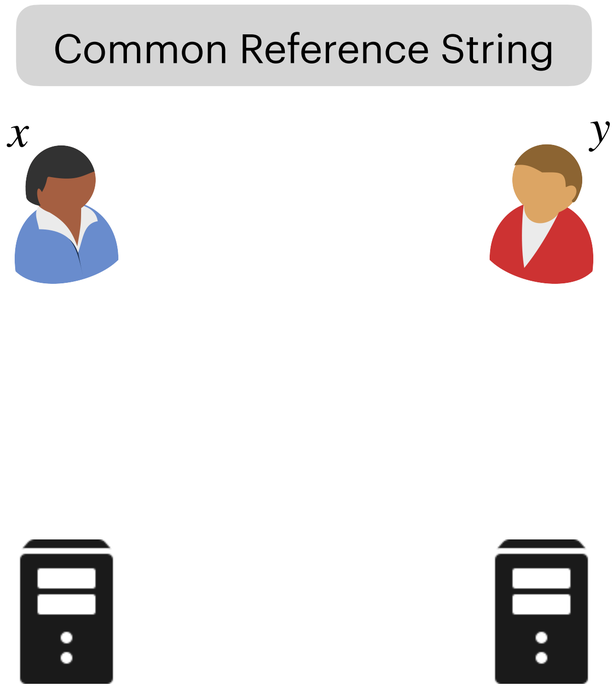
Barriers to **Delegating** Two-key HSS



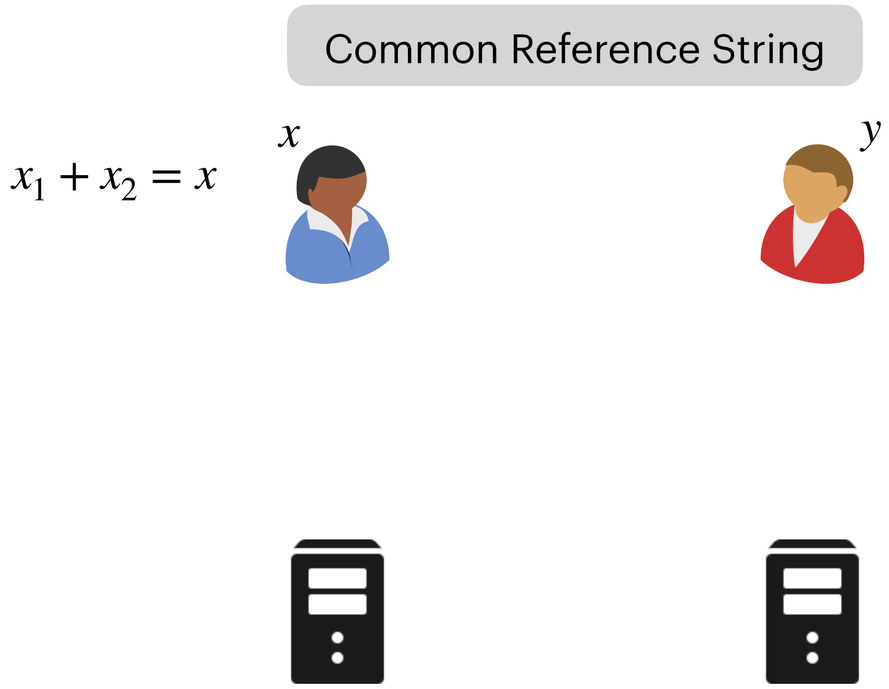
Revealing sk_1 compromises privacy of x

Revealing sk_2 compromises privacy of y

Barriers to **Delegating** Two-key HSS



Barriers to **Delegating** Two-key HSS



Barriers to **Delegating** Two-key HSS

Common Reference String

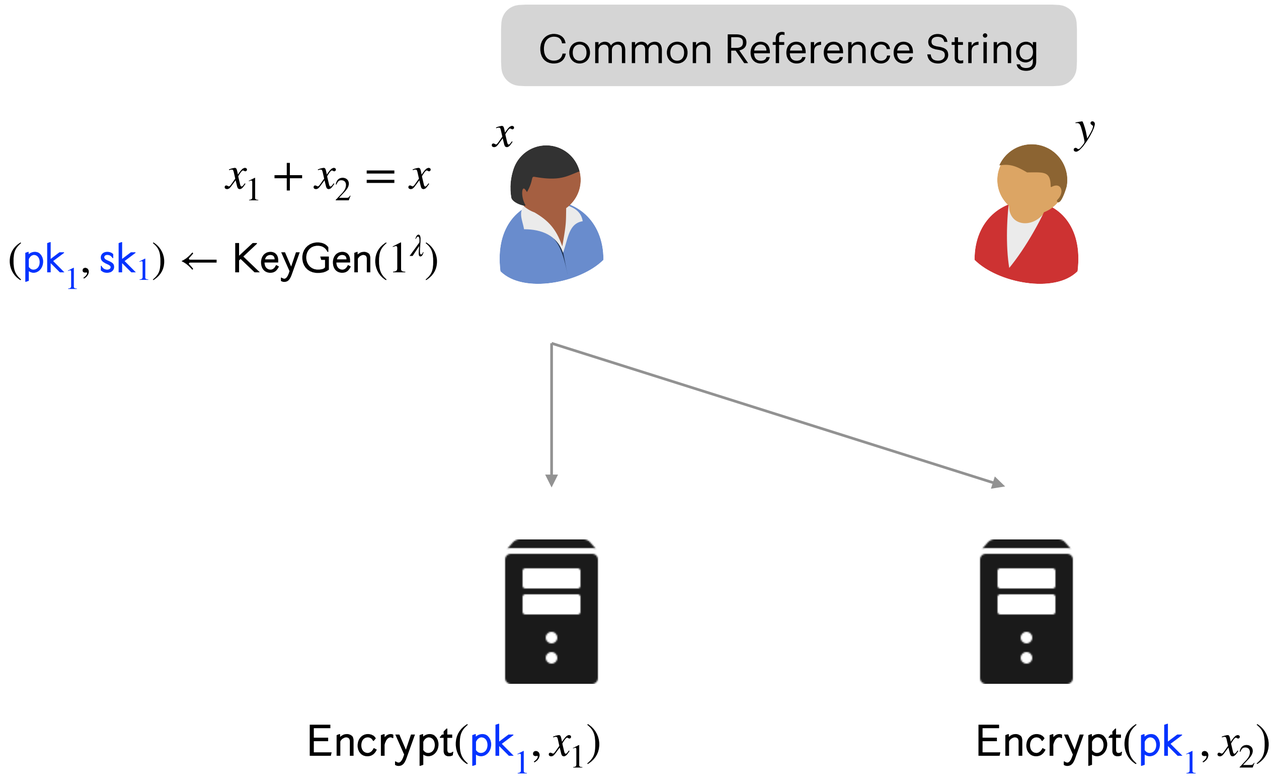
x

y

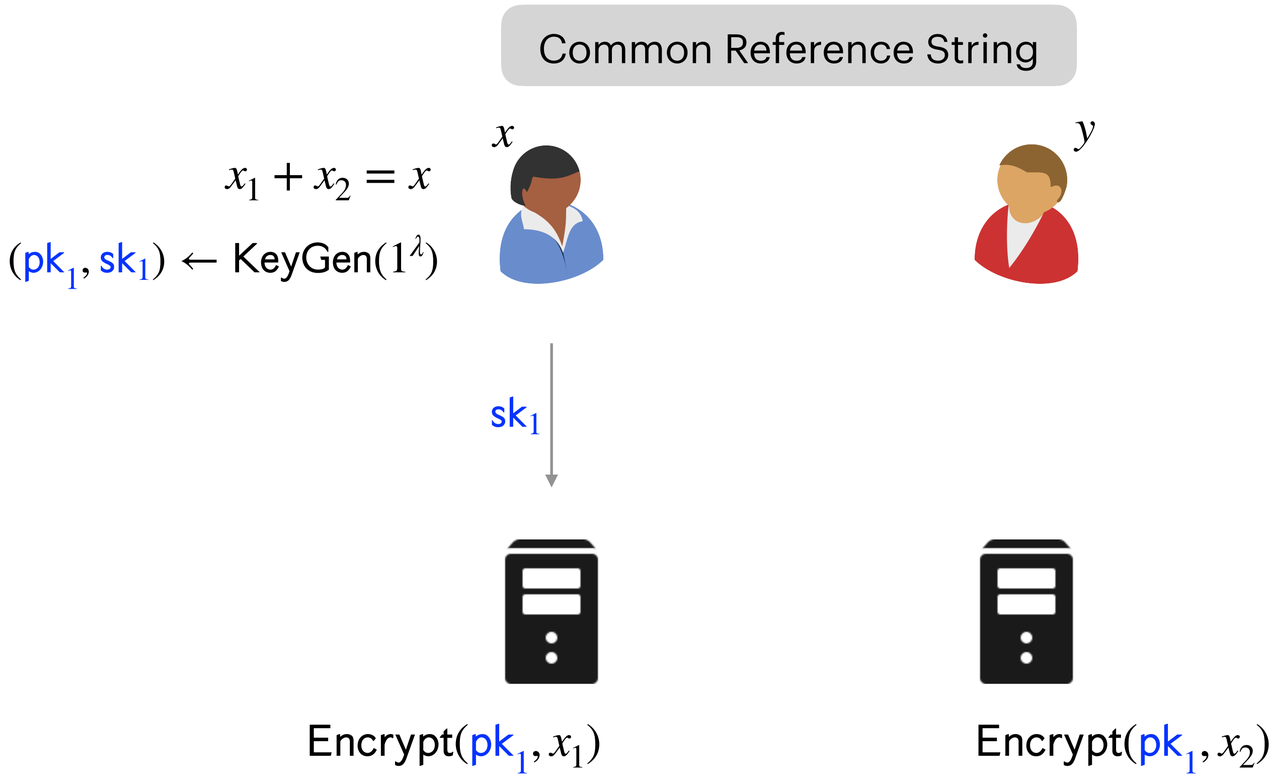
$x_1 + x_2 = x$

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

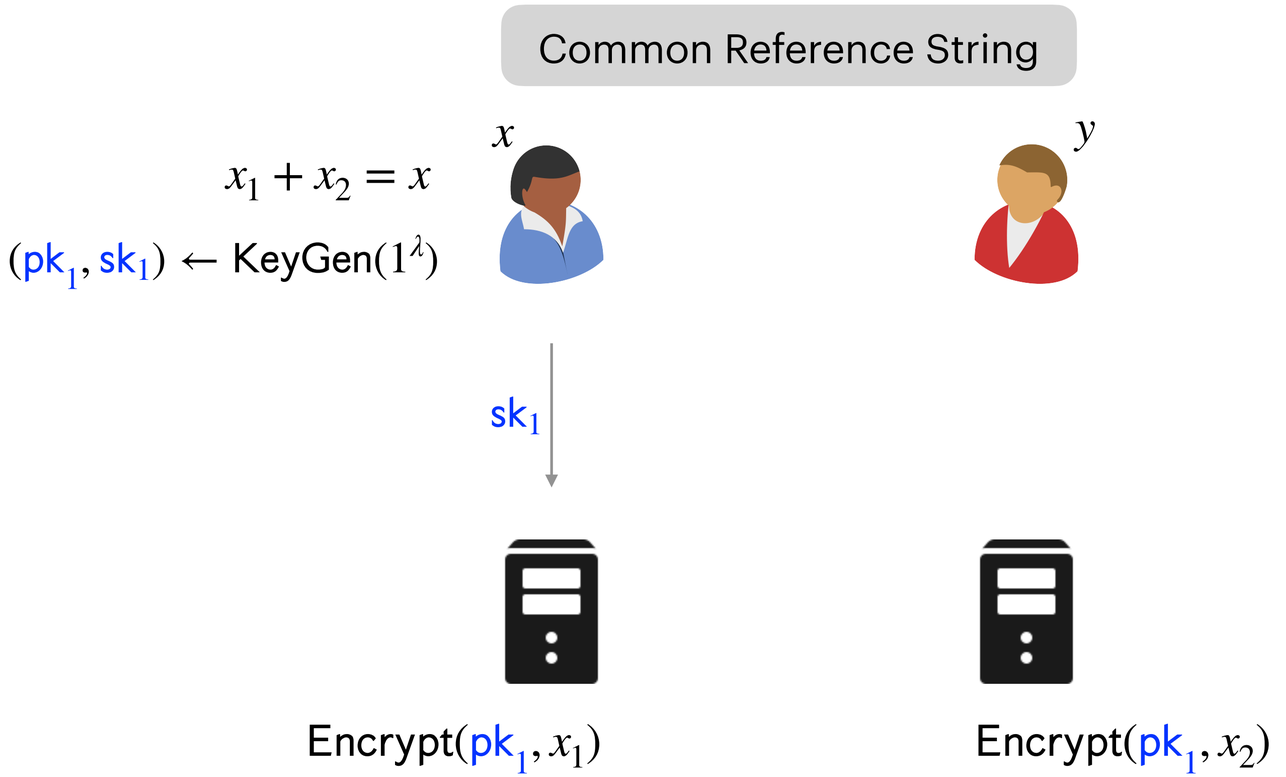
Barriers to **Delegating** Two-key HSS



Barriers to **Delegating** Two-key HSS

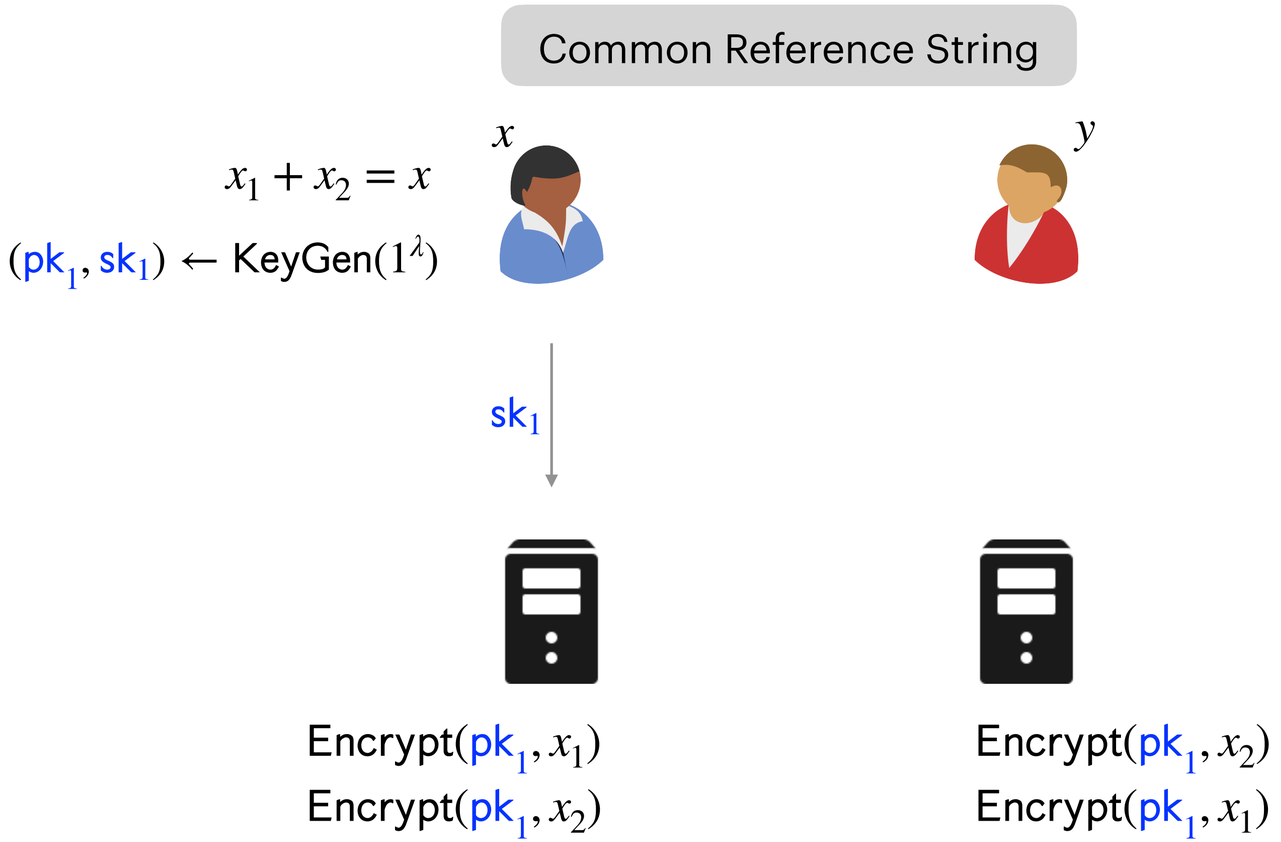


Barriers to **Delegating** Two-key HSS



Evaluation requires encryptions of all input

Barriers to **Delegating** Two-key HSS



Barriers to **Delegating** Two-key HSS

Common Reference String

x

y

$x_1 + x_2 = x$

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

Barriers to **Delegating** Two-key HSS

Common Reference String

x

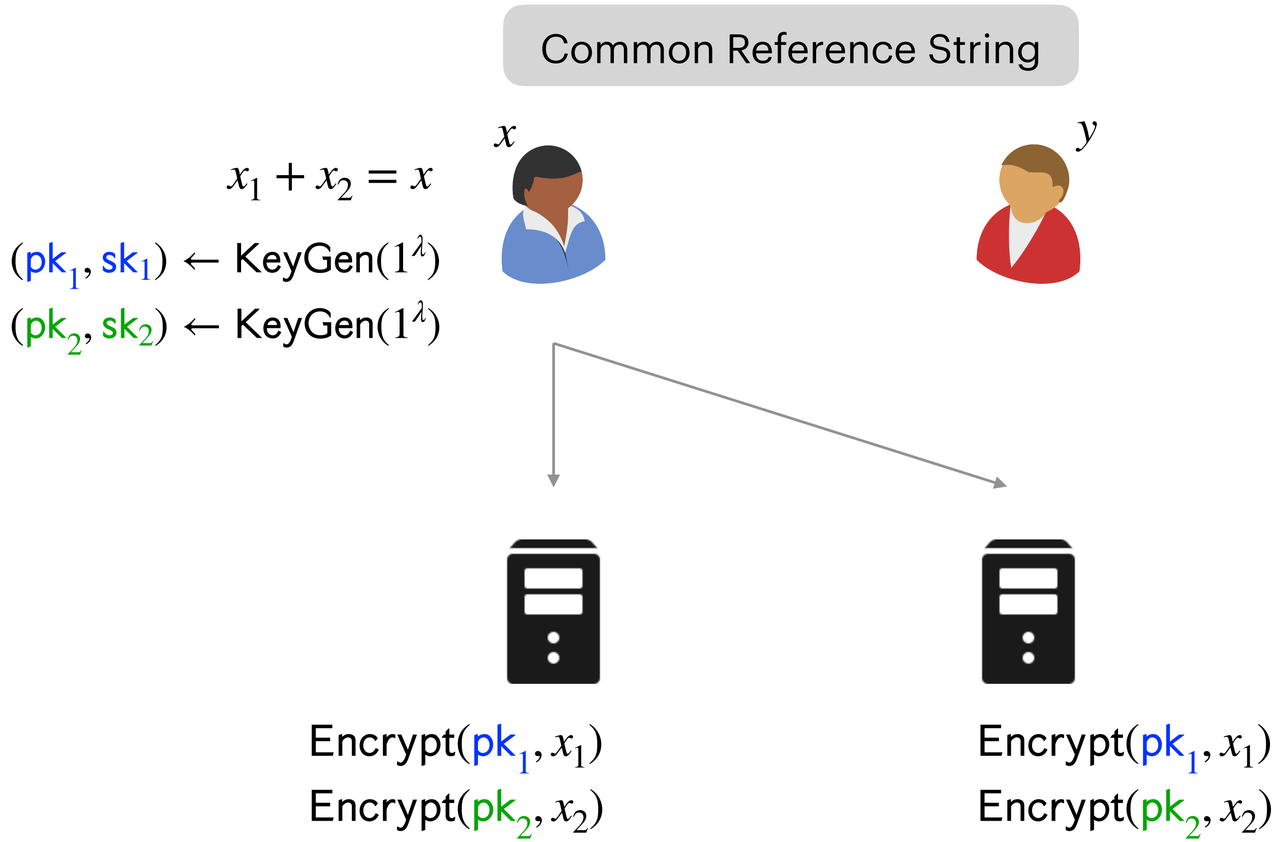
y

$x_1 + x_2 = x$

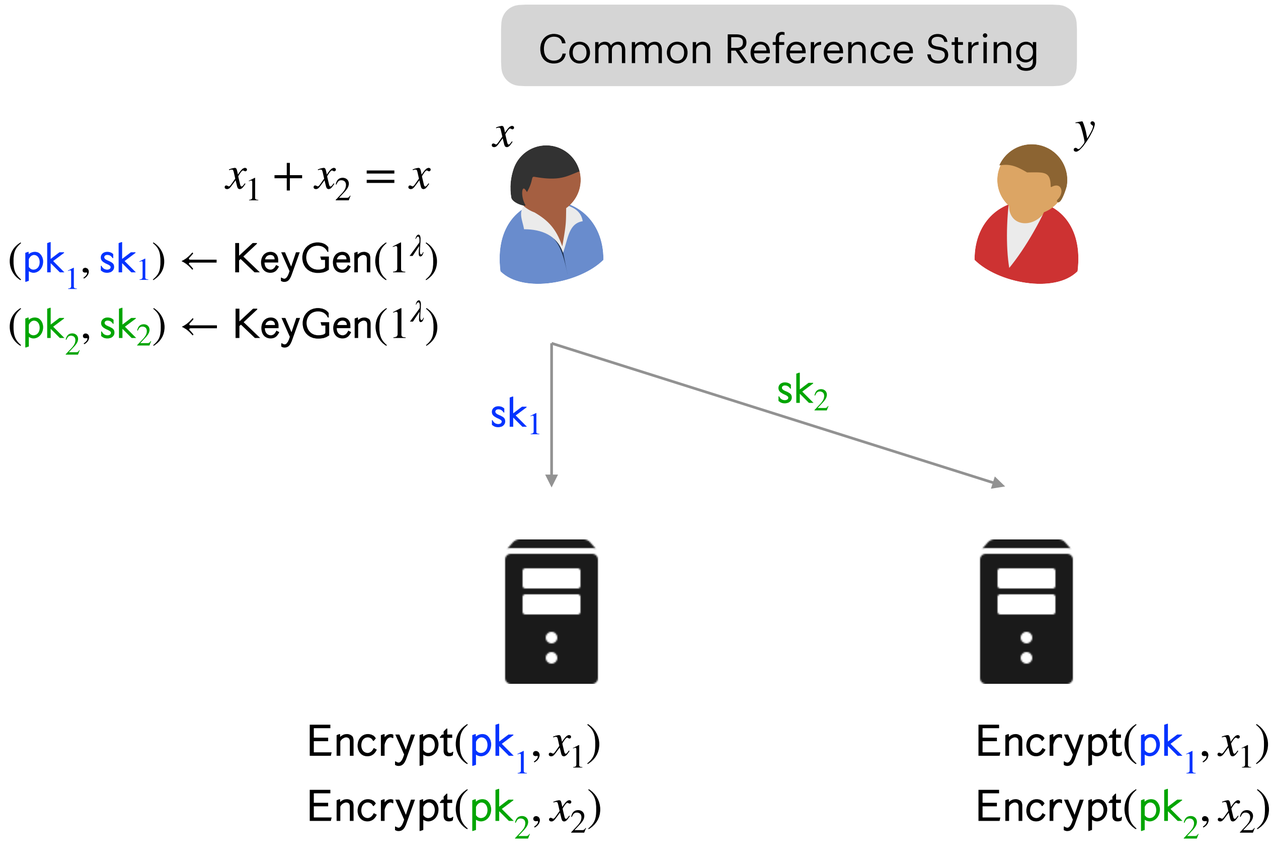
$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$(\mathsf{pk}_2, \mathsf{sk}_2) \leftarrow \mathsf{KeyGen}(1^\lambda)$

Barriers to **Delegating** Two-key HSS



Barriers to **Delegating** Two-key HSS



Barriers to **Delegating** Two-key HSS

Common Reference String

x

$x_1 + x_2 = x$

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$(\mathsf{pk}_2, \mathsf{sk}_2) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{Encrypt}(\mathsf{pk}_1, x_1)$

$\mathsf{Encrypt}(\mathsf{pk}_2, x_2)$

y

$y_1 + y_2 = y$

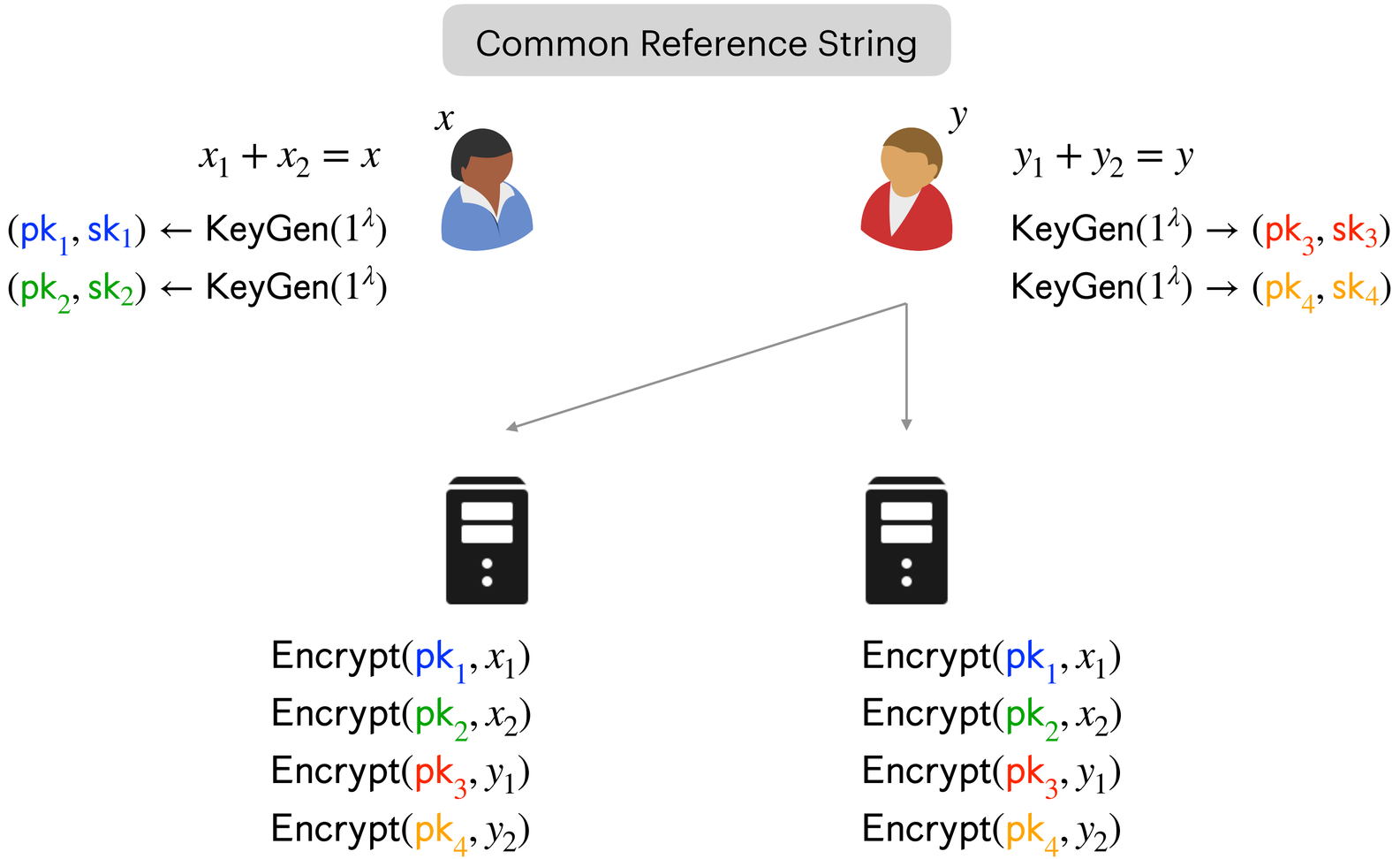
$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_3, \mathsf{sk}_3)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_4, \mathsf{sk}_4)$

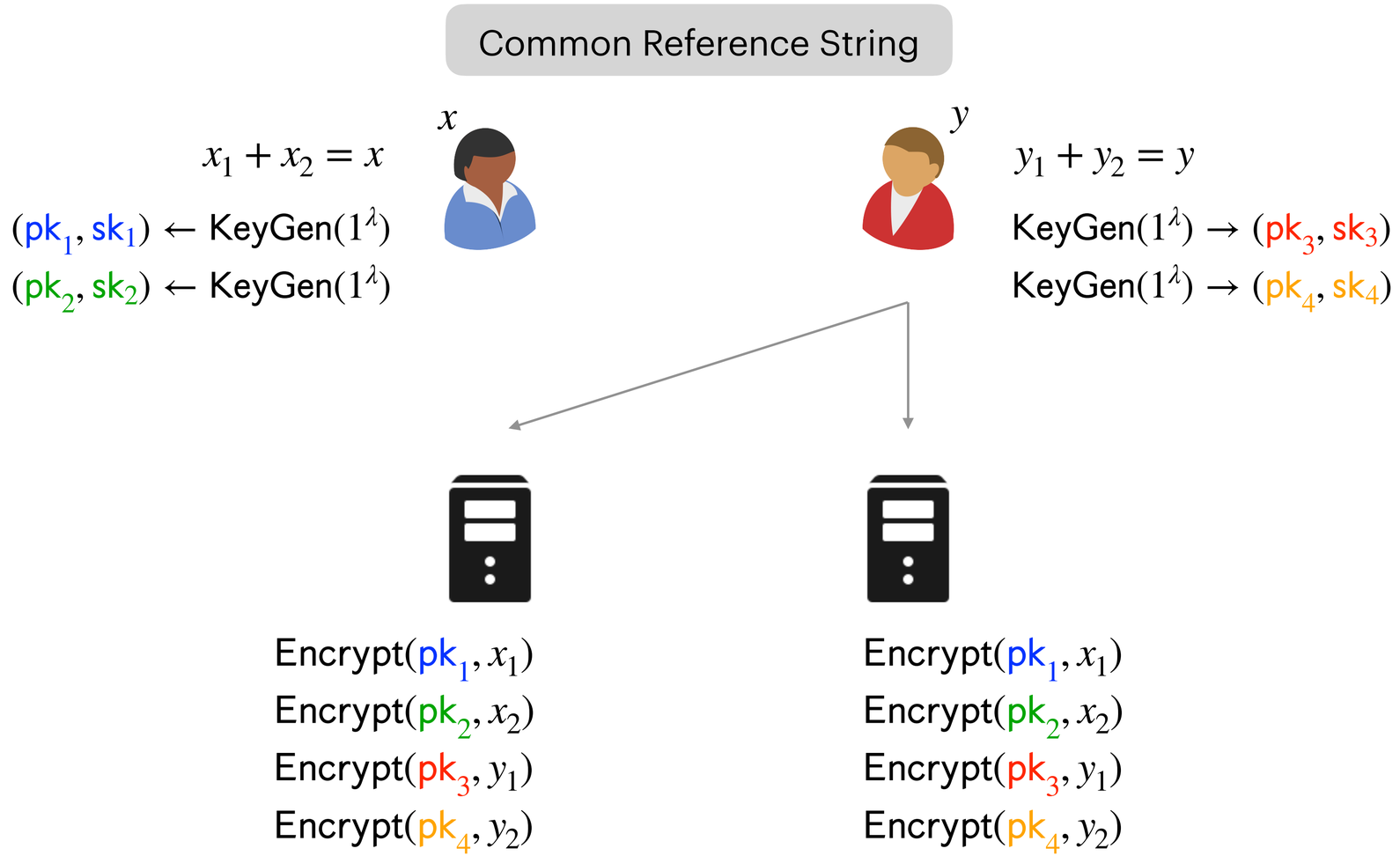
$\mathsf{Encrypt}(\mathsf{pk}_1, x_1)$

$\mathsf{Encrypt}(\mathsf{pk}_2, x_2)$

Barriers to **Delegating** Two-key HSS



Barriers to **Delegating** Two-key HSS



Requires evaluation under
four keys!

Barriers to **Delegating** Two-key HSS

Common Reference String

$x_1 + x_2 = x$

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$(\mathsf{pk}_2, \mathsf{sk}_2) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$y_1 + y_2 = y$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_3, \mathsf{sk}_3)$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_4, \mathsf{sk}_4)$

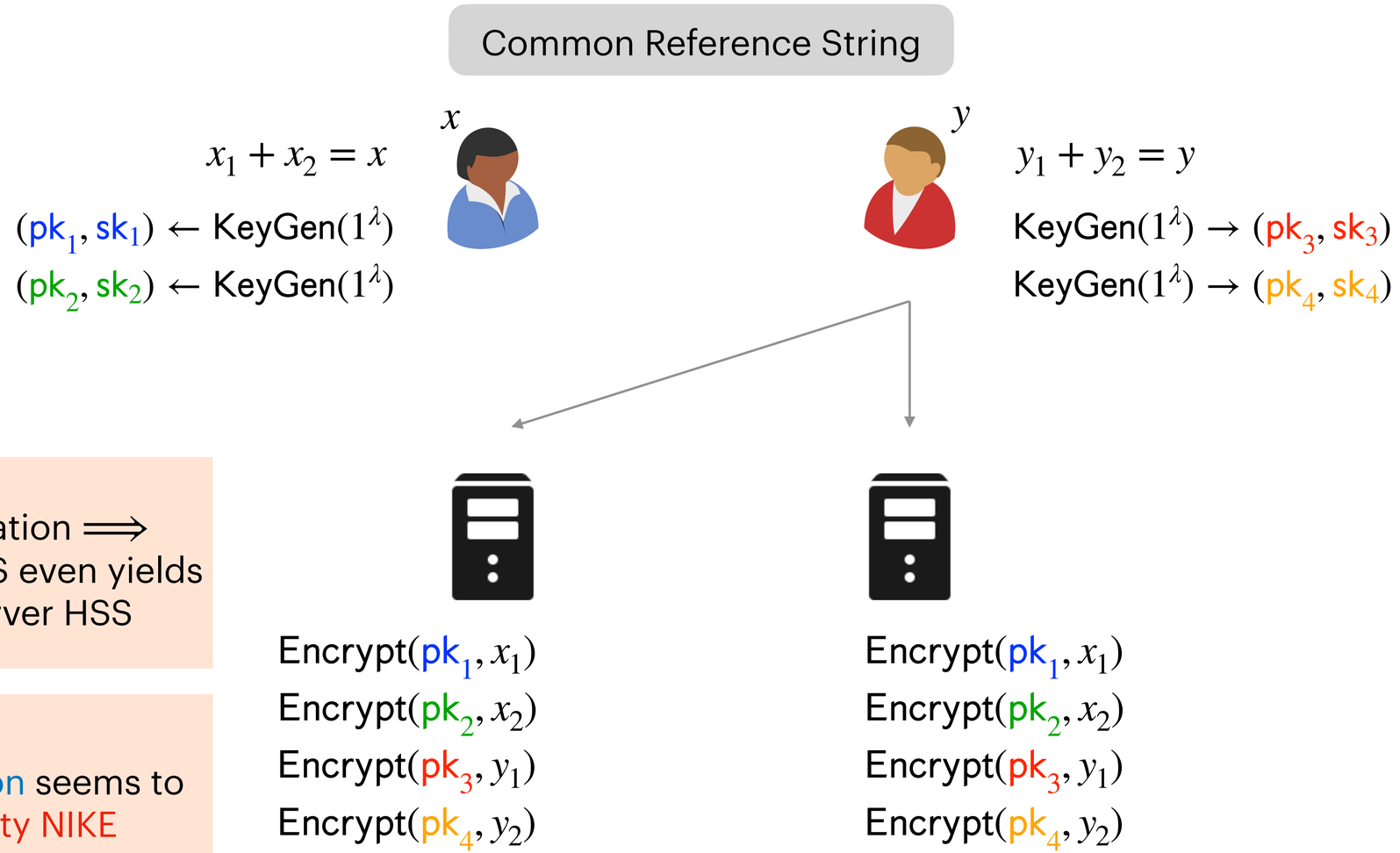
$\mathsf{Encrypt}(\mathsf{pk}_1, x_1)$
$\mathsf{Encrypt}(\mathsf{pk}_2, x_2)$
$\mathsf{Encrypt}(\mathsf{pk}_3, y_1)$
$\mathsf{Encrypt}(\mathsf{pk}_4, y_2)$

$\mathsf{Encrypt}(\mathsf{pk}_1, x_1)$
$\mathsf{Encrypt}(\mathsf{pk}_2, x_2)$
$\mathsf{Encrypt}(\mathsf{pk}_3, y_1)$
$\mathsf{Encrypt}(\mathsf{pk}_4, y_2)$

Private synchronization $\Longrightarrow$
Unclear if two-key HSS even yields two-client two-server HSS

Requires evaluation under four keys!

Barriers to **Delegating** Two-key HSS



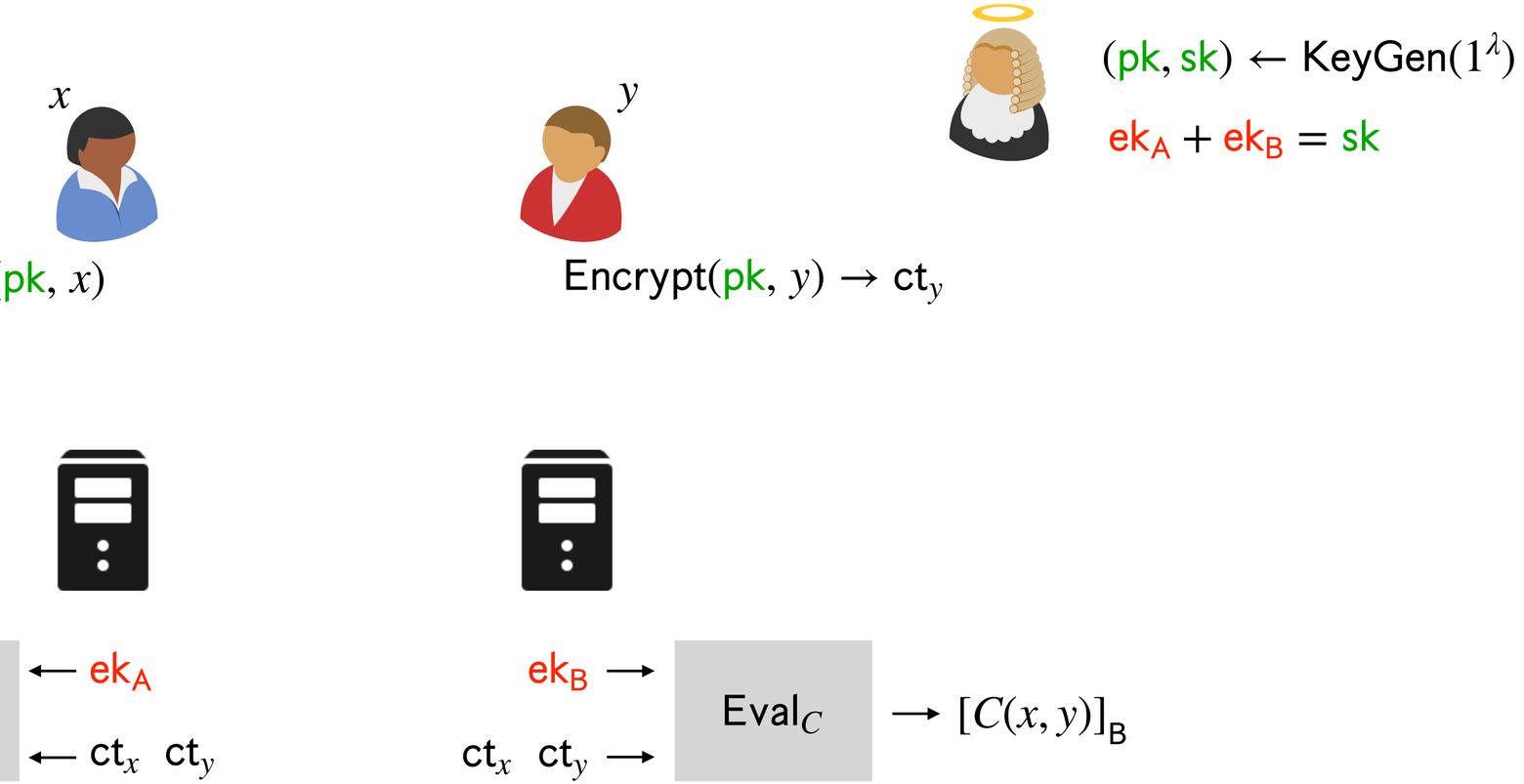
Private synchronization $\Longrightarrow$
Unclear if two-key HSS even yields two-client two-server HSS

Public synchronization seems to require three-party NIKE

Requires evaluation under four keys!

Client-Server HSS with **Correlated Setup**

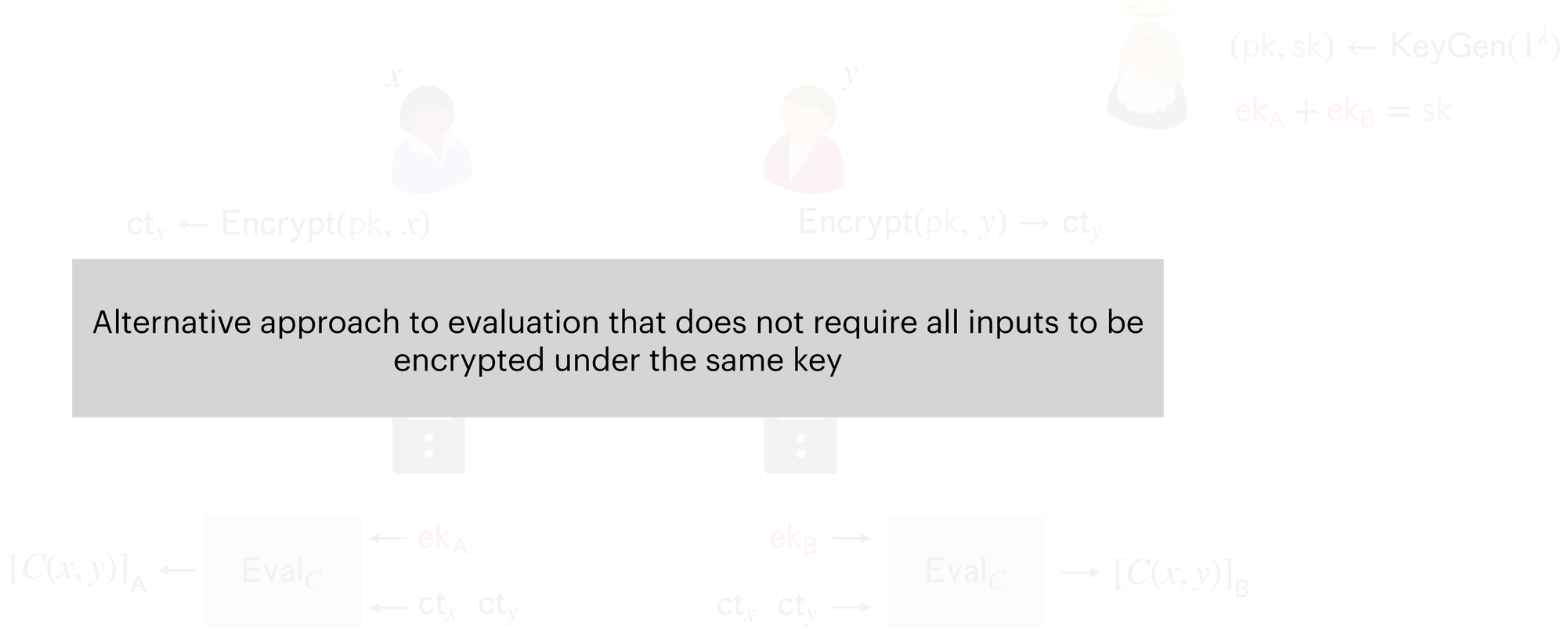
[Boyle-Gilboa-Ishai'16]



Barrier to Removing Correlated Setup: All inputs must be encrypted under a common key

Client-Server HSS with **Correlated Setup**

[Boyle-Gilboa-Ishai'16]



Barrier to Removing Correlated Setup: All inputs must be encrypted under a common key

Outline

Barriers to Removing Correlated Setup

Our Approach

Extensions

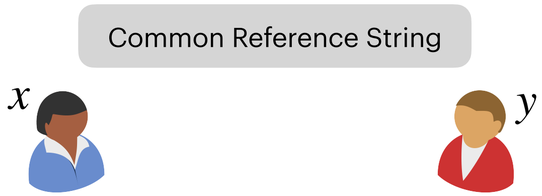
HSS for **Multiplication** is All You Need

HSS for **Multiplication** is All You Need

Two-party HSS for
multiplication in the CRS model

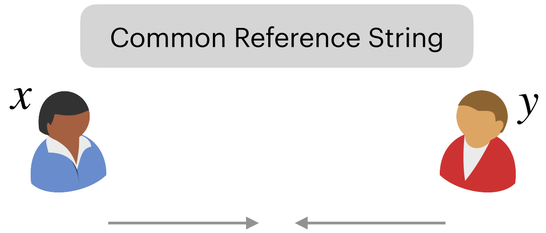
HSS for **Multiplication** is All You Need

Two-party HSS for
multiplication in the CRS model



HSS for **Multiplication** is All You Need

Two-party HSS for
multiplication in the CRS model



HSS for **Multiplication** is All You Need

Two-party HSS for
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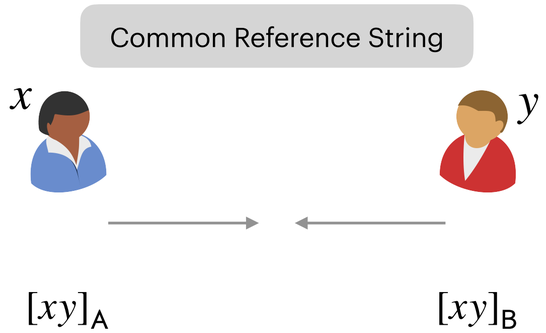
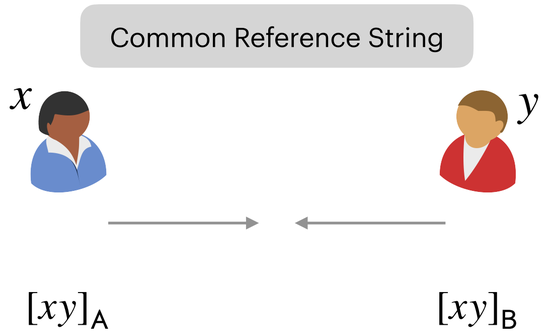
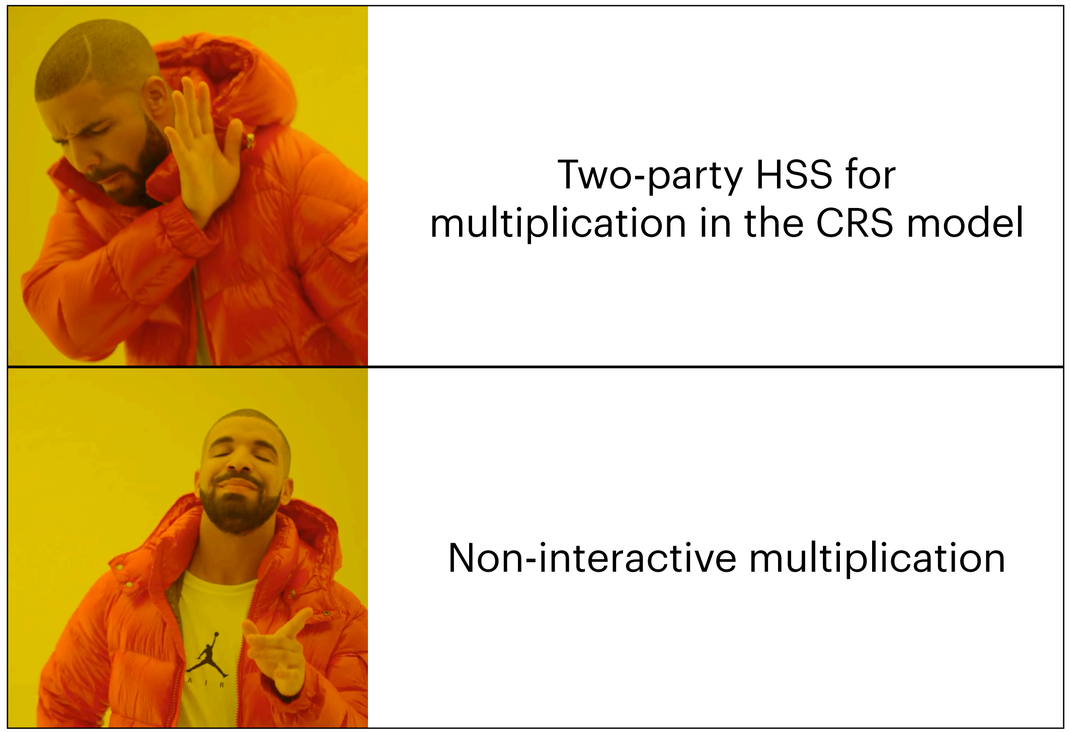


HSS for **Multiplication** is All You Need

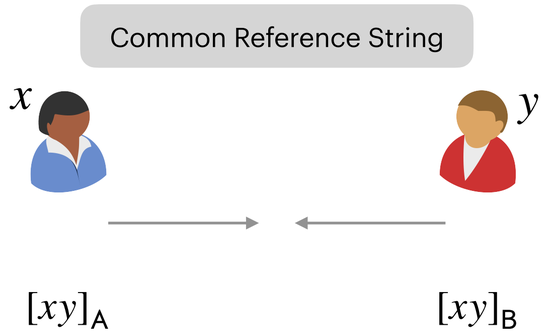
Two-party HSS for
multiplication in the CRS model

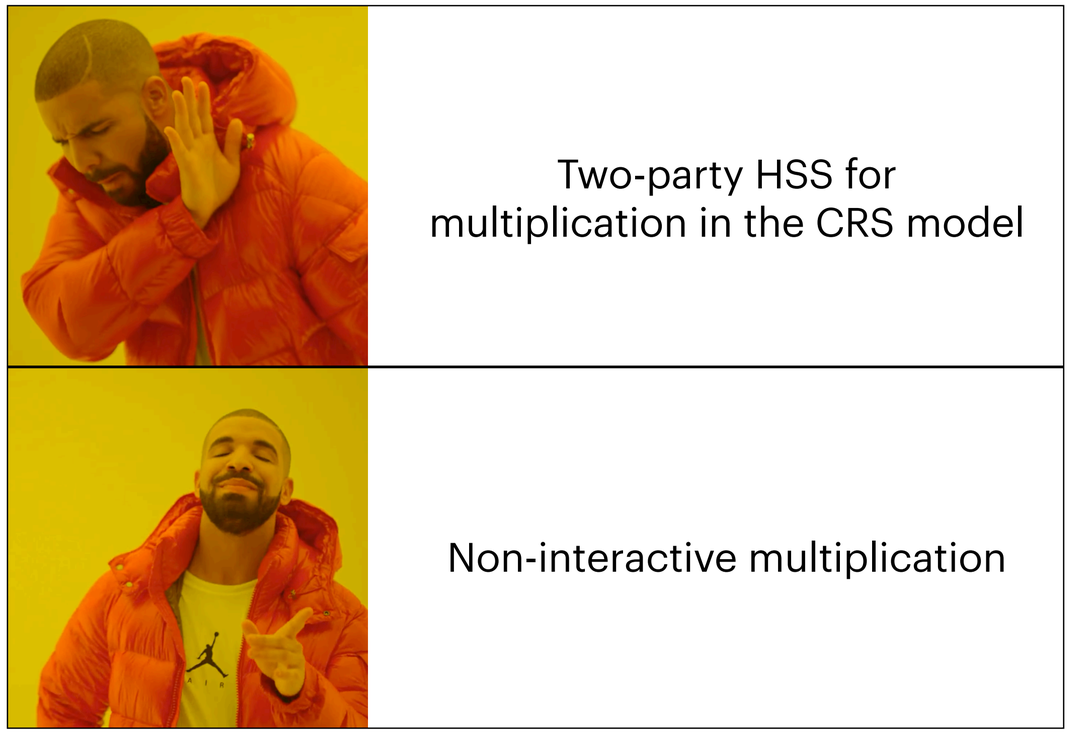




HSS for **Multiplication** is All You Need

Non-interactive multiplication

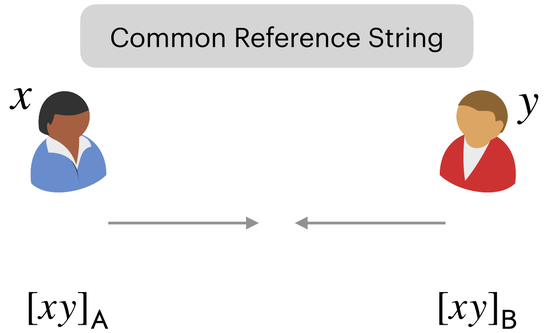




HSS for **Multiplication** is All You Need

Non-interactive multiplication

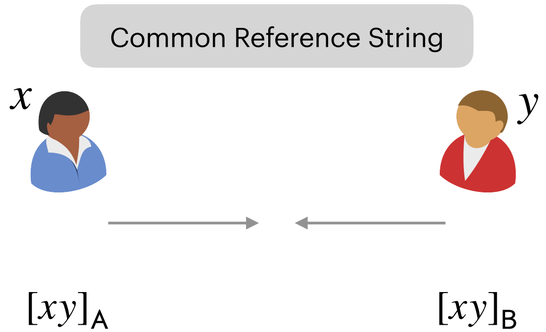
Two-client two-server HSS for multiplication in the CRS model

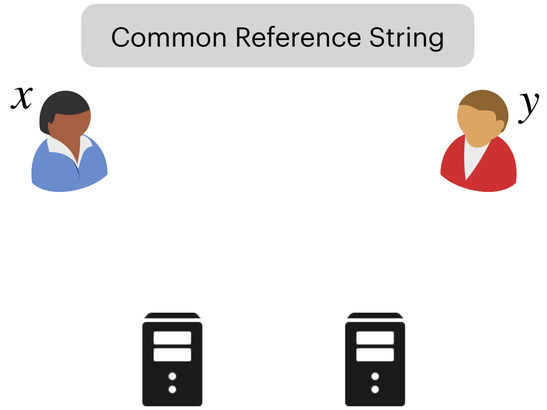


HSS for **Multiplication** is All You Need

Non-interactive multiplication

Two-client two-server HSS for multiplication in the CRS model

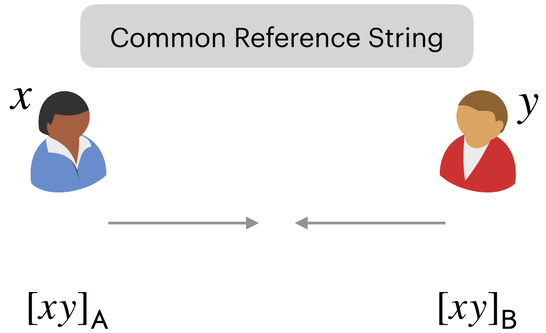


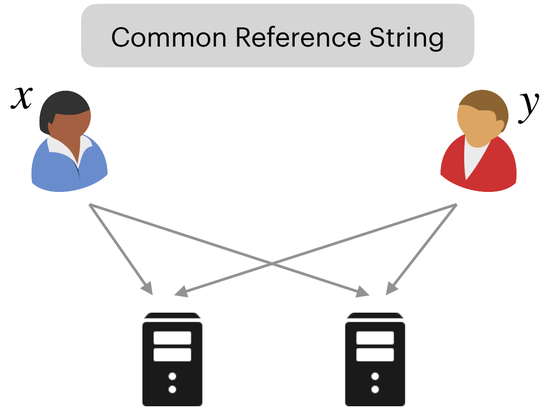


HSS for **Multiplication** is All You Need

Non-interactive multiplication

Two-client two-server HSS for multiplication in the CRS model

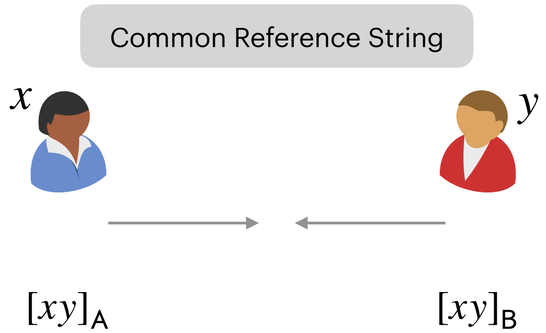


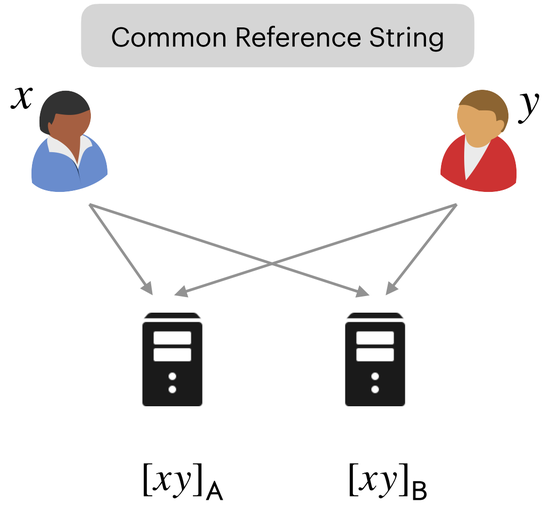


HSS for **Multiplication** is All You Need

Non-interactive multiplication

Two-client two-server HSS for multiplication in the CRS model

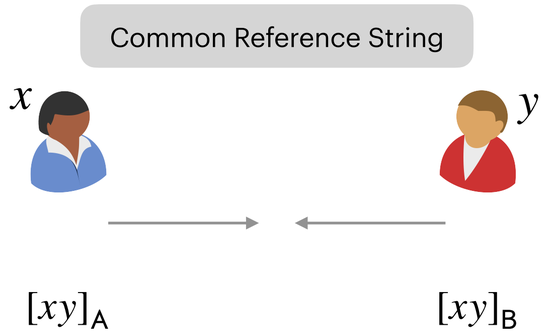


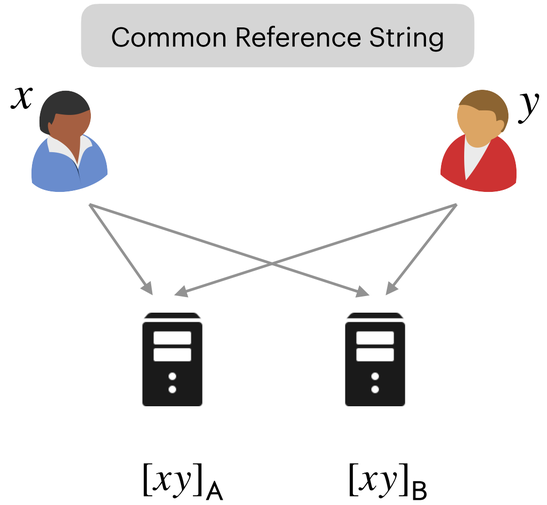


HSS for **Multiplication** is All You Need

Non-interactive multiplication

Delegatable non-interactive multiplication





HSS for **Multiplication** is All You Need

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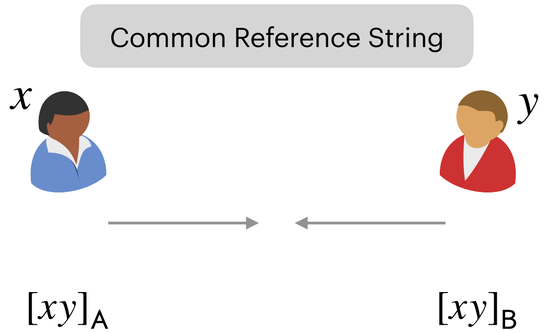

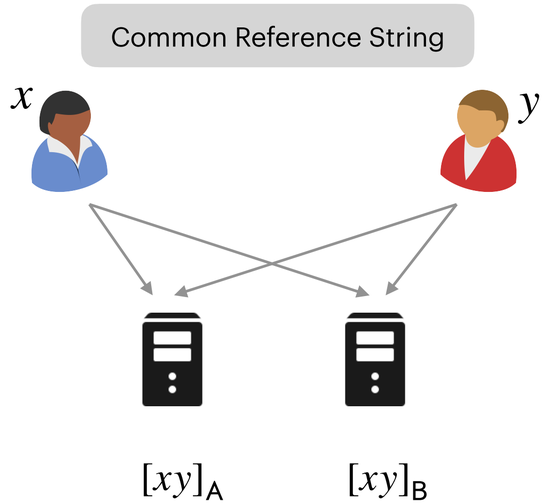

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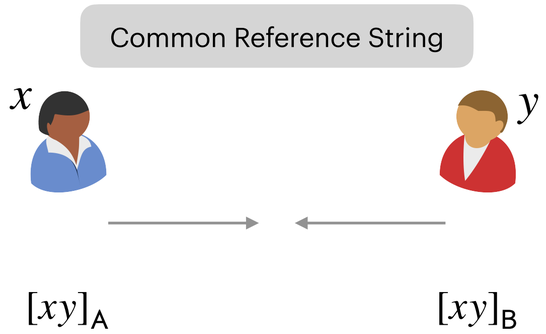

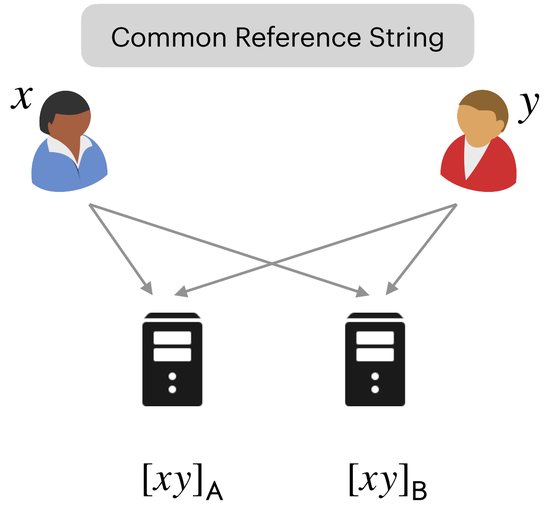

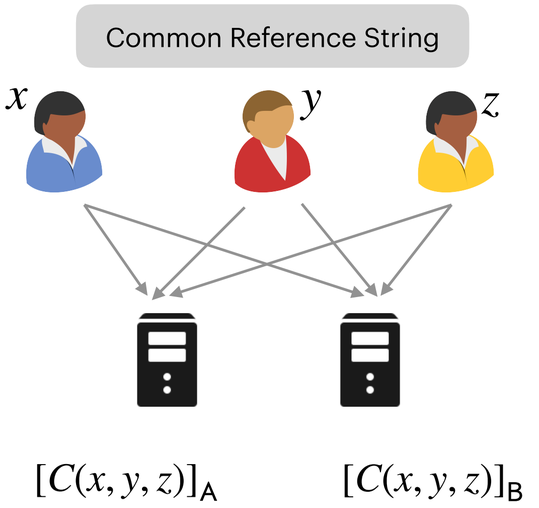

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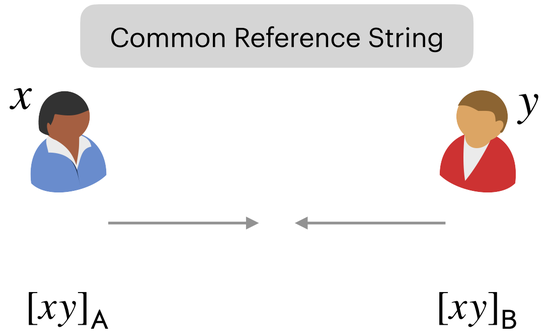

Non-Interactive Multiplication

[Döttling-Garg-Ishai-Malavolta-Mour-Ostrovsky'19] [Abram-Roy-Scholl'24]

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y

$u \leftarrow \mathbb{Z}_p$

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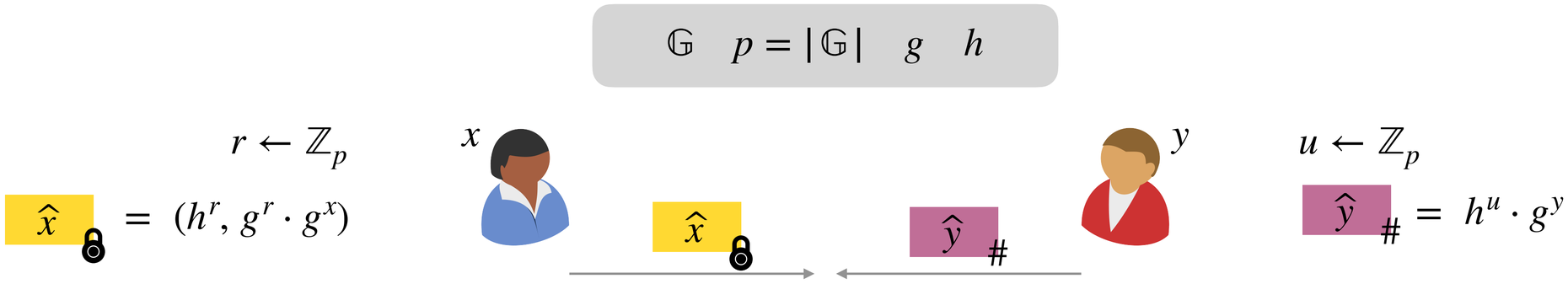

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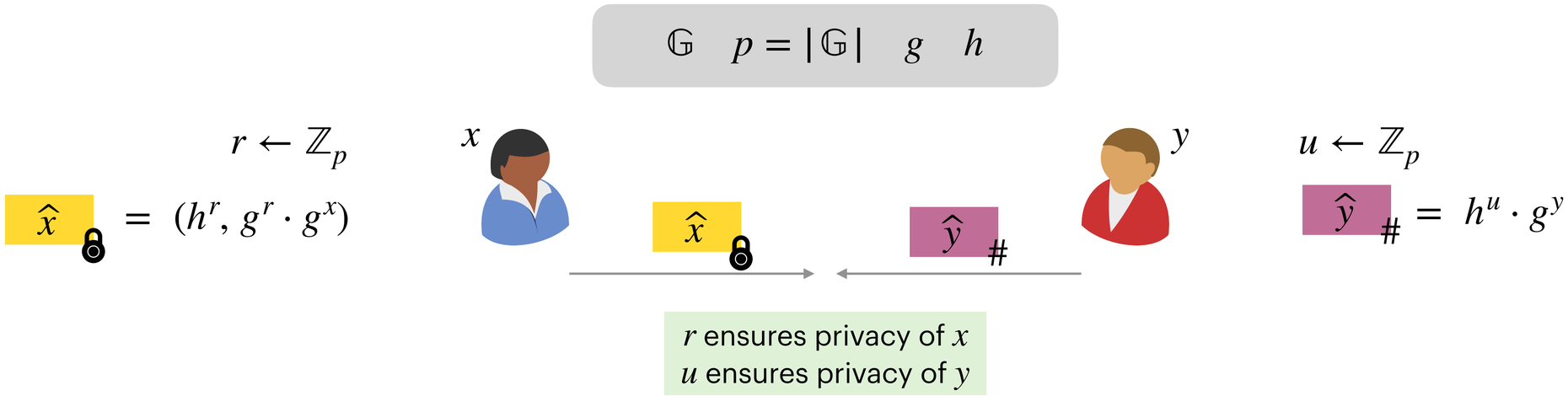

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$\hat{x}$

$\hat{y}_{\#}$

$\gamma_{\mathsf{A}} = \left(h^u \cdot g^y\right)^r$

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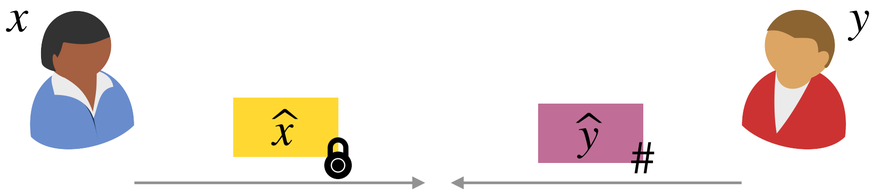

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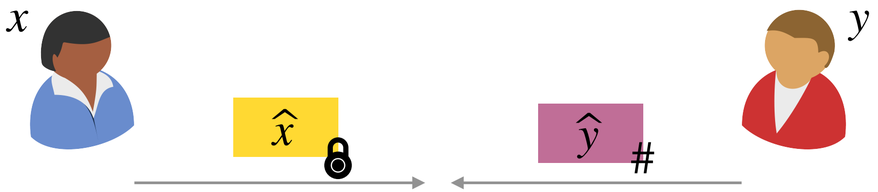

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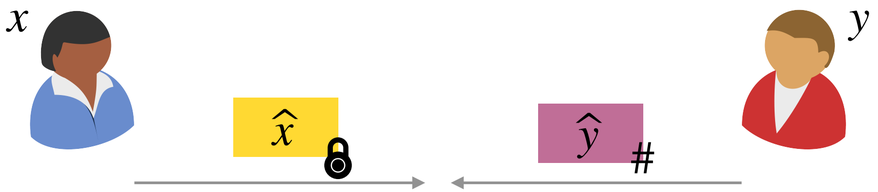

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DDLog ↓

$[xy]_A$

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DDLog ↓

$[xy]_B$

Distributed Discrete Log (DDLog): Non-interactively convert divisive shares into additive shares

[Boyle-Gilboa-Ishai'16]

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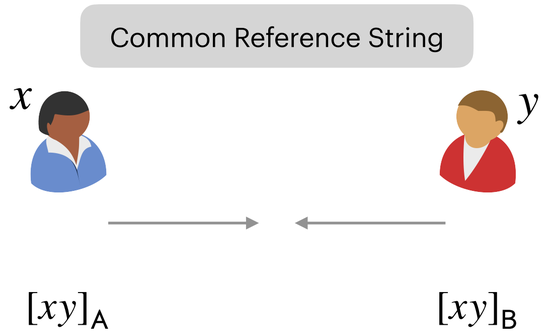

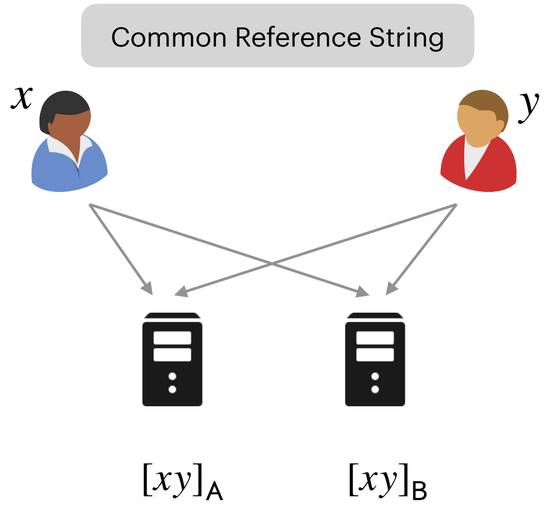

Delegating Non-Interactive Multiplication

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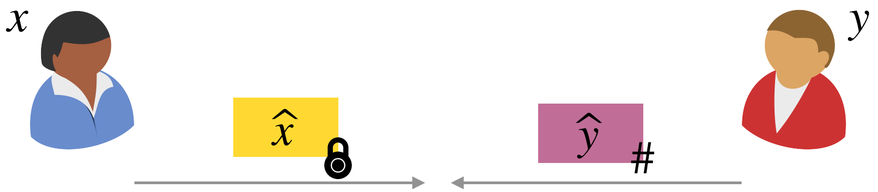

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DDLog ↓

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$$\frac{\gamma_B}{\gamma_A} = \frac{(h^r)^u \cdot (g^r \cdot g^x)^y}{(h^u \cdot g^y)^r} = g^{xy}$$

$\gamma_B = (h^r)^u \cdot (g^r \cdot g^x)^y$

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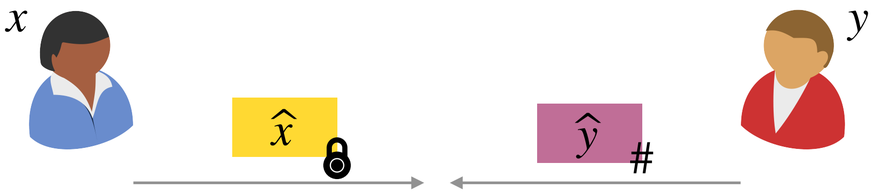

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Observation: Decoding is "linear" in the received messages

Delegating Non-Interactive Multiplication

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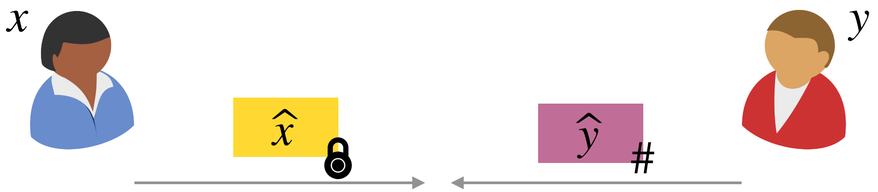

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DDLog

$[xy]_B$

Observation: Decoding is "linear" in the received messages

Using shares of r, u, and y to decode gives divisive shares of xy

Delegatable Non-Interactive Multiplication

$\mathbb{G} \quad p = |\mathbb{G}| \quad g \quad h$

$r \leftarrow \mathbb{Z}_p$

$\hat{x} = (h^r, g^r \cdot g^x)$

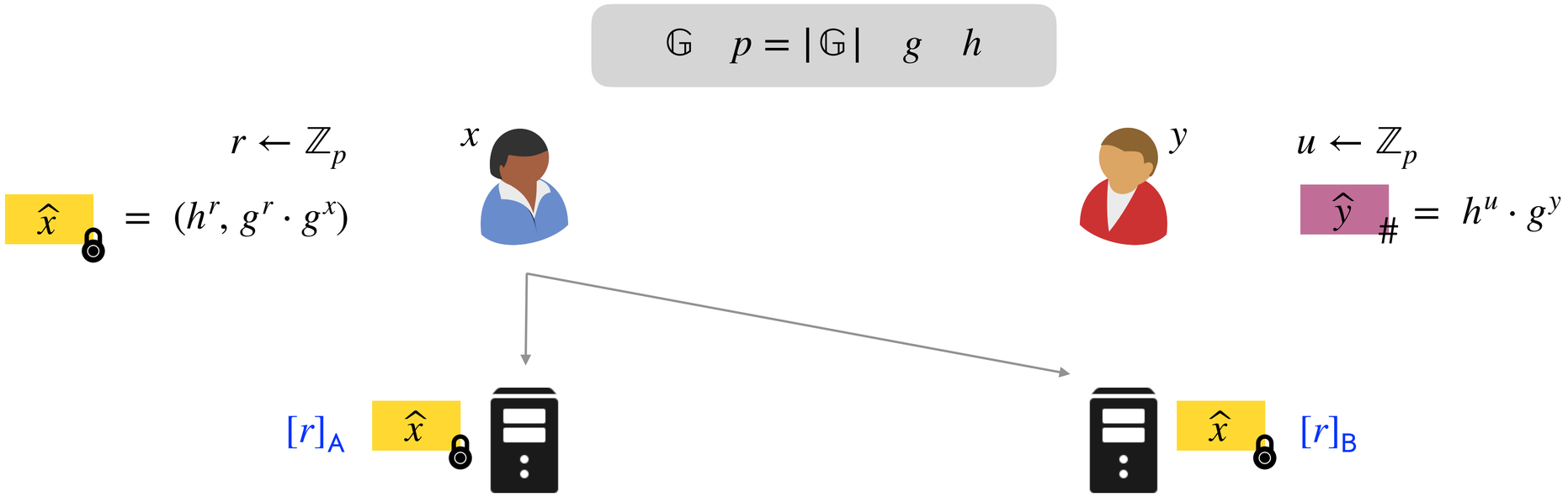
x

y

$u \leftarrow \mathbb{Z}_p$

$\hat{y}_{\#} = h^u \cdot g^y$

Delegatable Non-Interactive Multiplication



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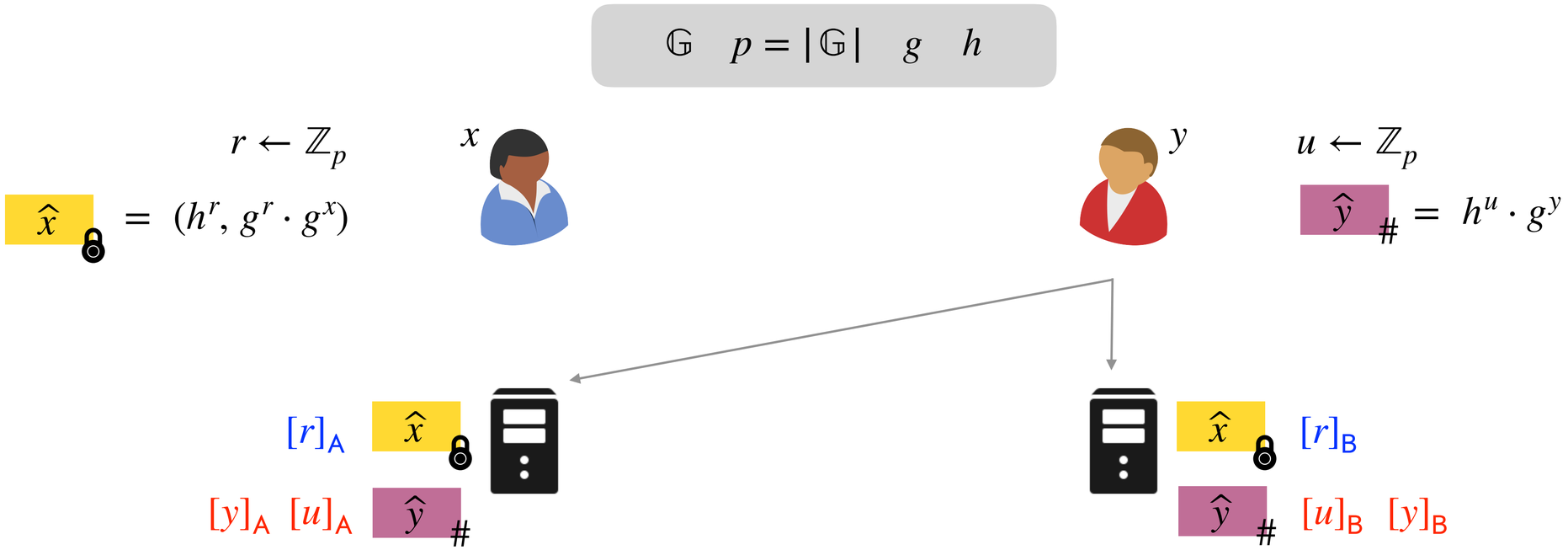

Delegatable Non-Interactive Multiplication

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x

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$[r]_A \quad \widehat{x}$

$[y]_A \quad [u]_A \quad \widehat{y}_{\#}$

$\widehat{x} \quad [r]_B$

$\widehat{y}_{\#} \quad [u]_B \quad [y]_B$

$$g^{-[xy]_A} = \frac{\left(h^u \cdot g^y\right)^{[r]_A}}{\left(h^r\right)^{[u]_A} \cdot \left(g^r \cdot g^x\right)^{[y]_A}}$$

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$[r]_A$ $\widehat{x}$

$[y]_A$ $[u]_A$ $\widehat{y}_{\#}$

$\widehat{x}$ $[r]_B$

$\widehat{y}_{\#}$ $[u]_B$ $[y]_B$

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$[y]_A \quad [u]_A \quad \hat{y}_{\#}$

$\hat{x} \quad [r]_B$

$\hat{y}_{\#} \quad [u]_B \quad [y]_B$

$$[xy]_A \xleftarrow{\text{DDLog}} g^{-[xy]_A} = \frac{\left(h^u \cdot g^y\right)^{[r]_A}}{\left(h^r\right)^{[u]_A} \cdot \left(g^r \cdot g^x\right)^{[y]_A}}$$

$$\frac{g^{[xy]_B}}{g^{-[xy]_A}} = g^{xy}$$

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Non-interactive multiplication

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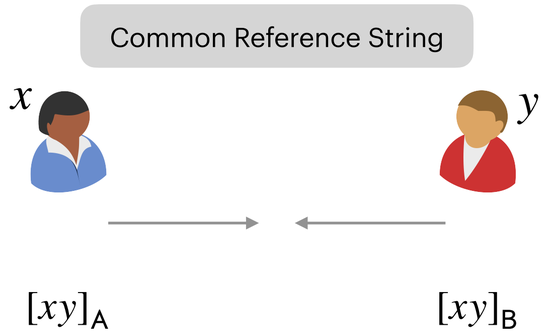

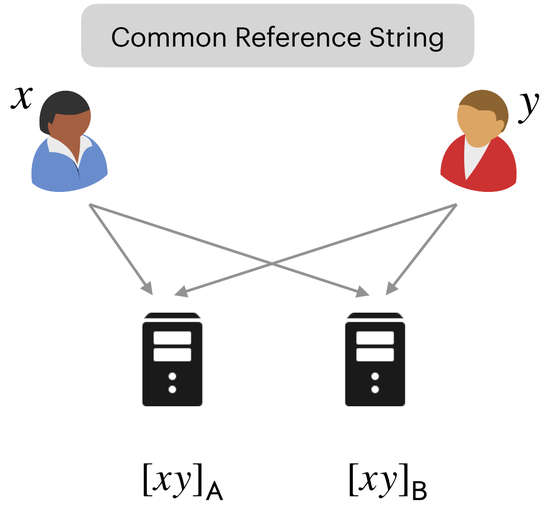

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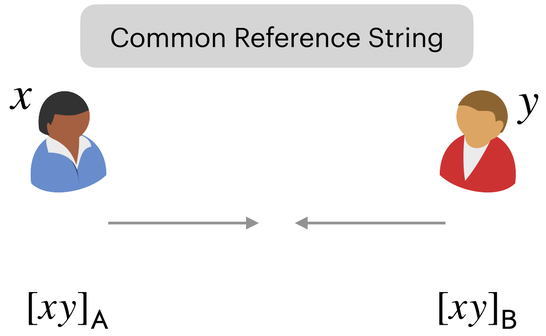
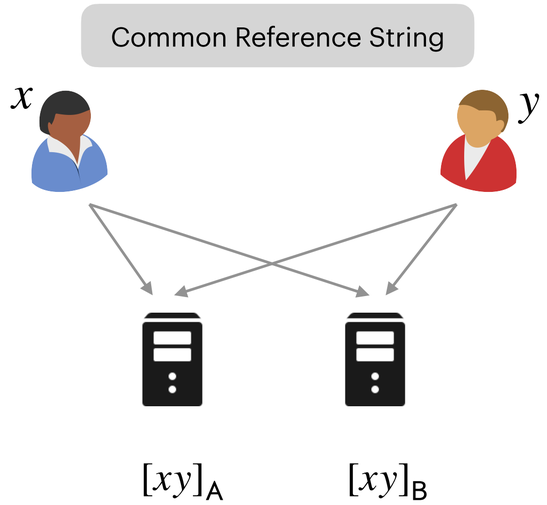
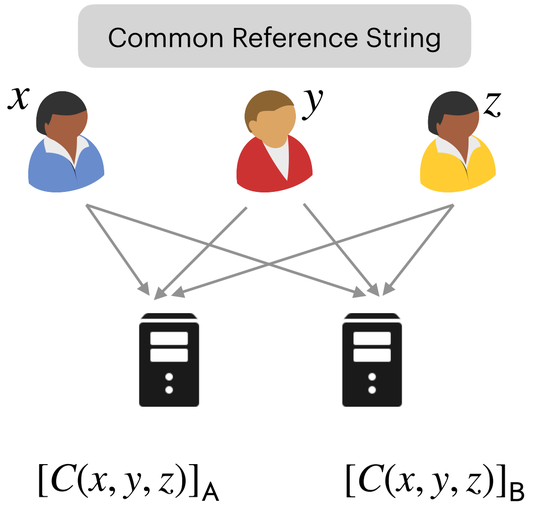






Towards Evaluating RMS Programs

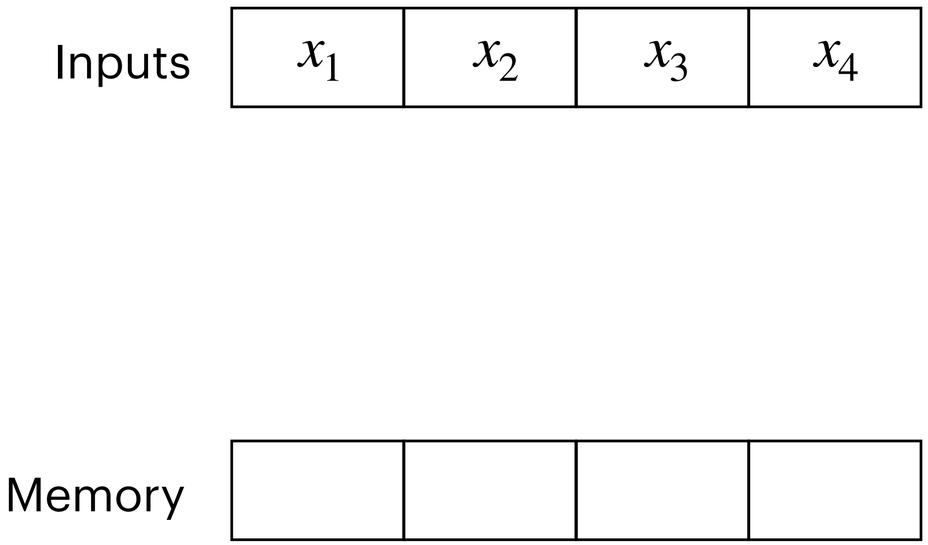
Multiplying inputs with intermediate values of the computation suffices to evaluate RMS programs

NIM can be used to multiply inputs with intermediate values

Restricted Multiplication Straight-line (RMS) Programs

[Boyle-Gilboa-Ishai'16]

RMS Programs



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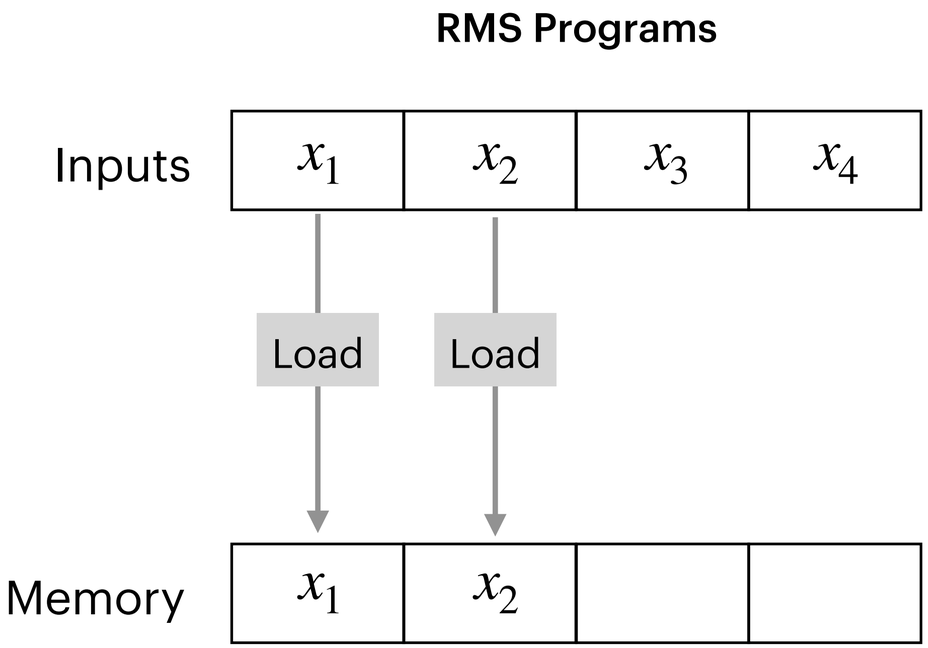

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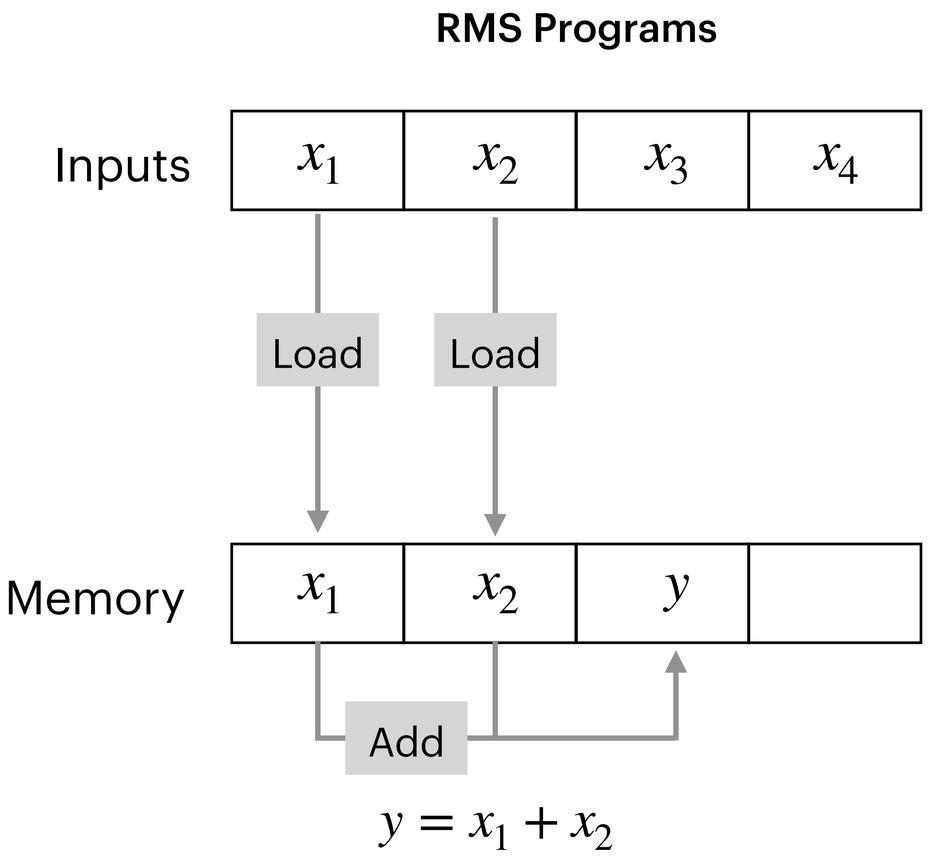

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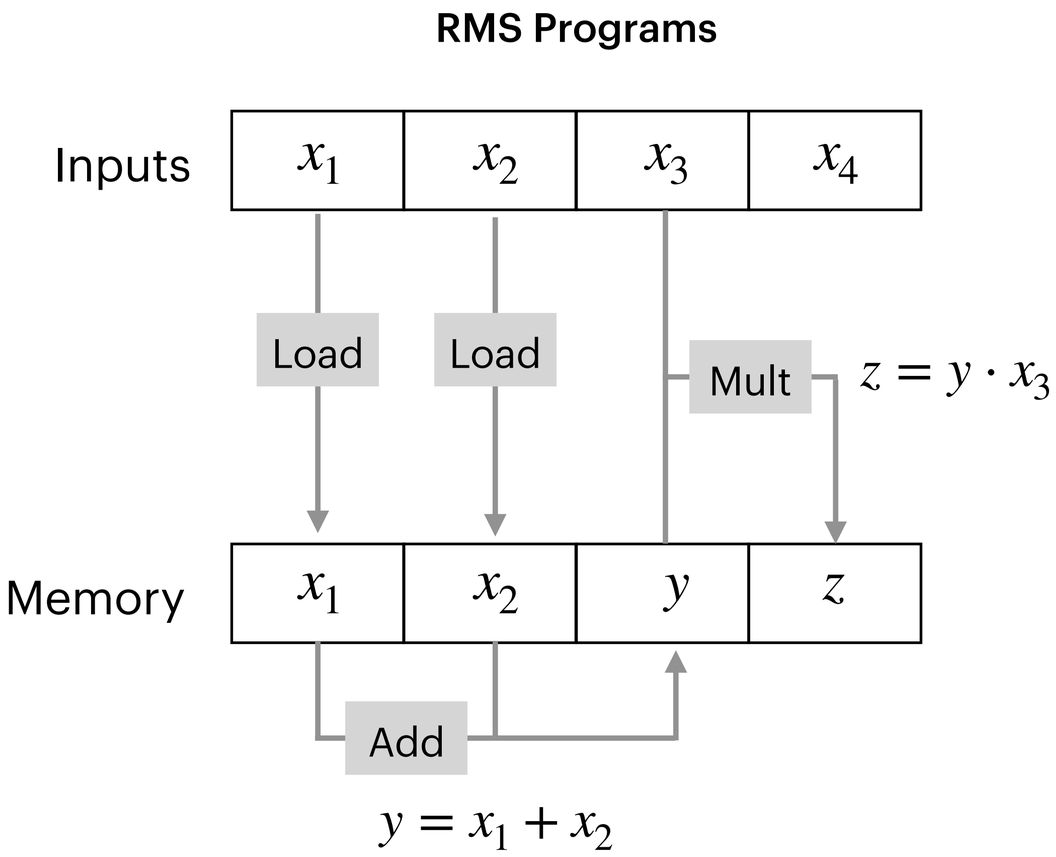

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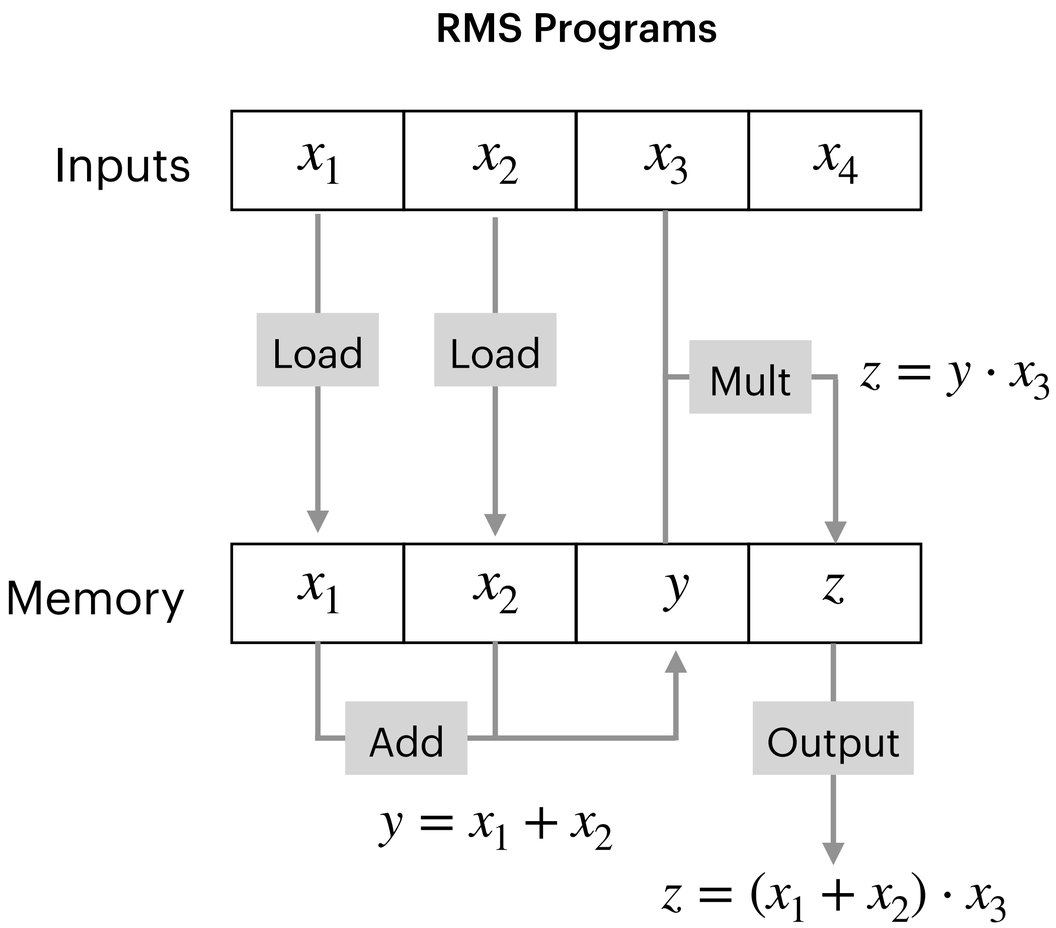

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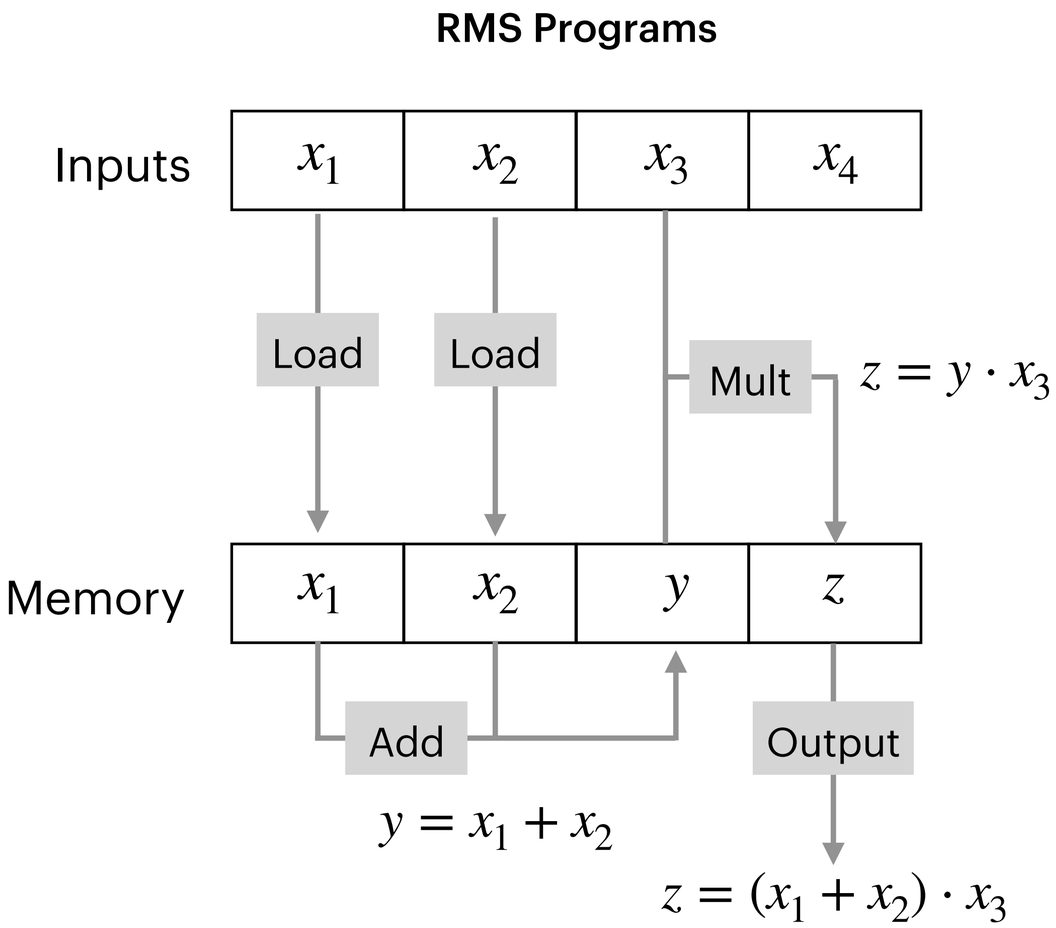
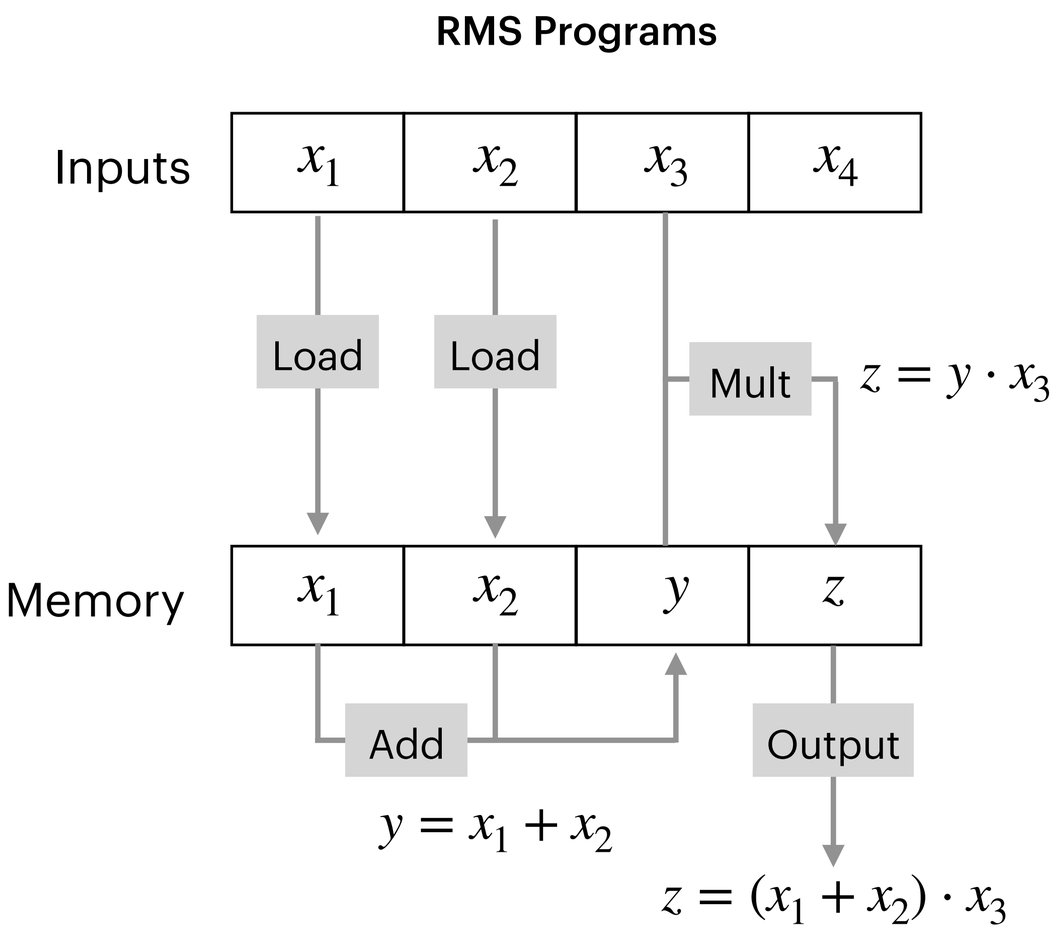
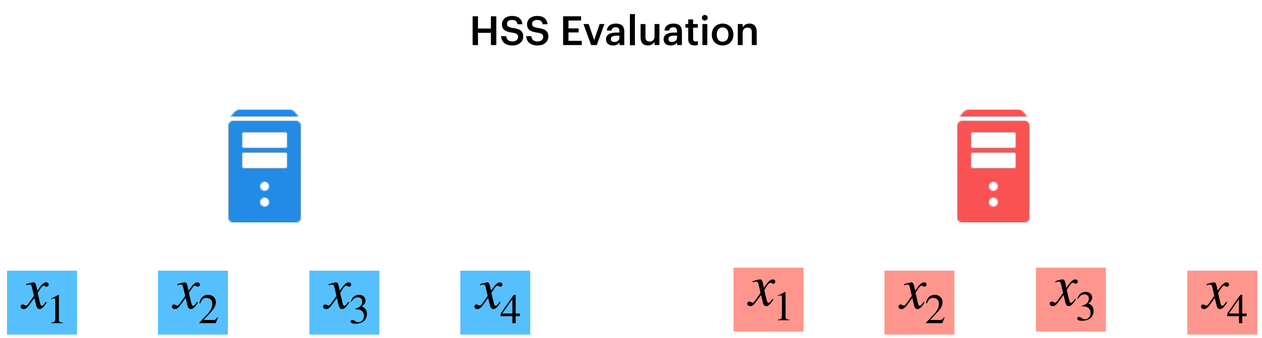


Cannot multiply two memory values

Distributed Evaluation of RMS Programs

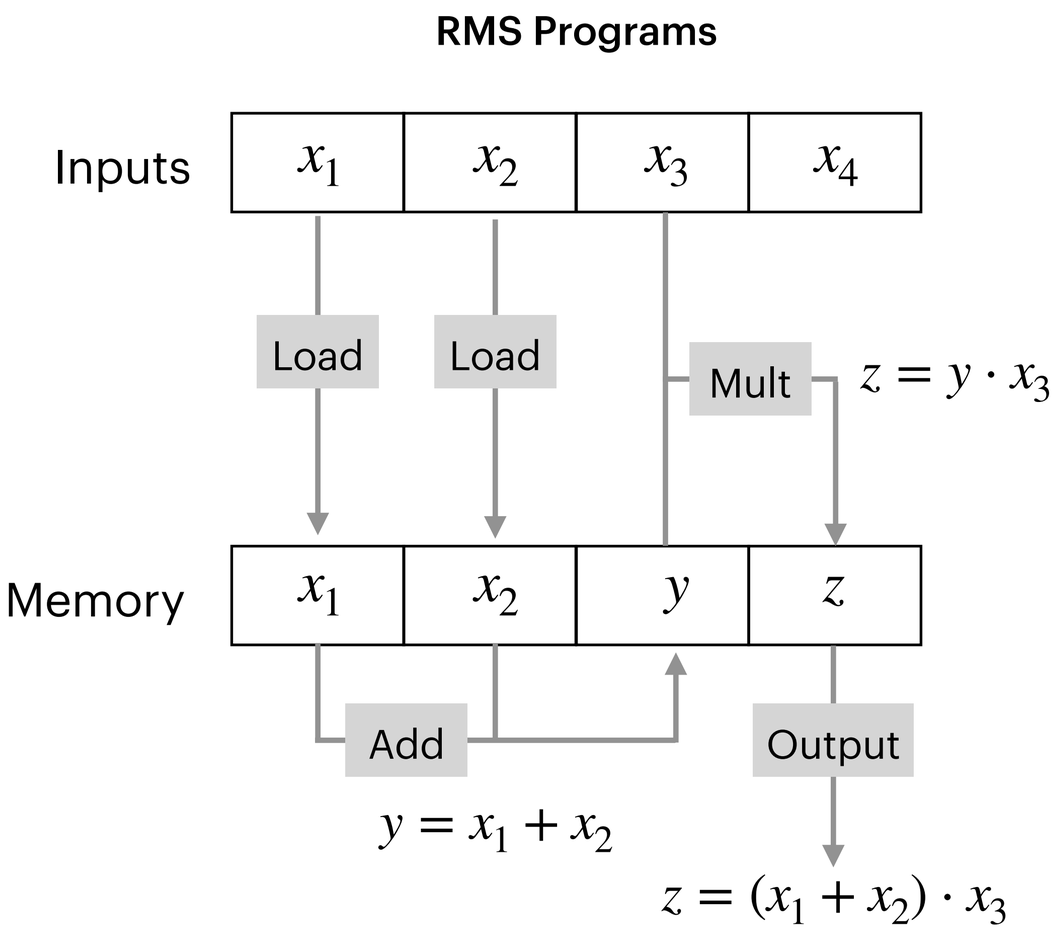
[Boyle-Gilboa-Ishai'16]

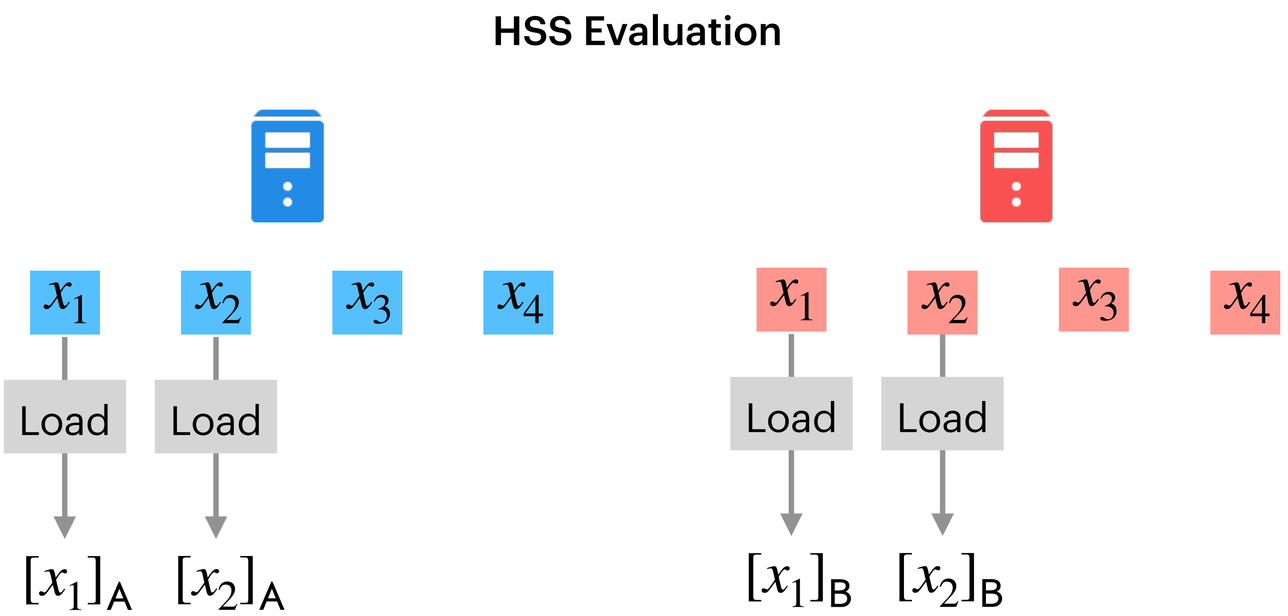




Distributed Evaluation of RMS Programs

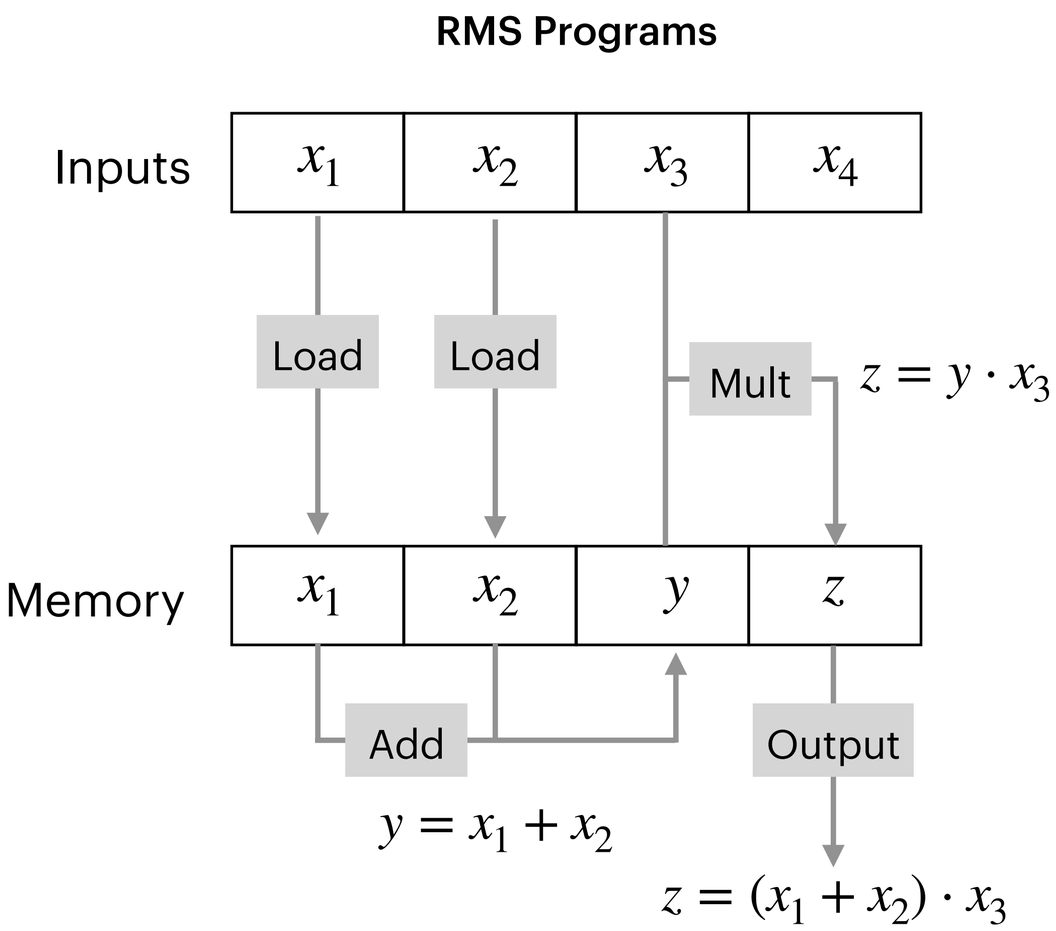
[Boyle-Gilboa-Ishai'16]

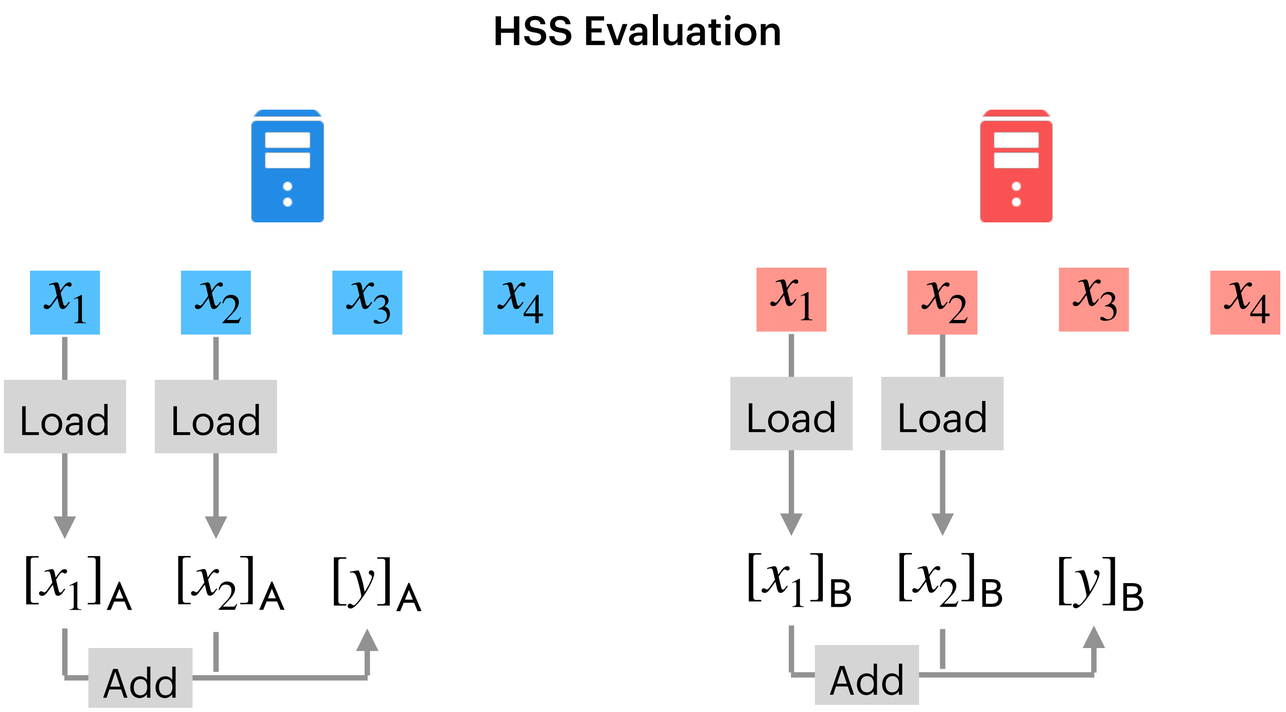




Distributed Evaluation of RMS Programs

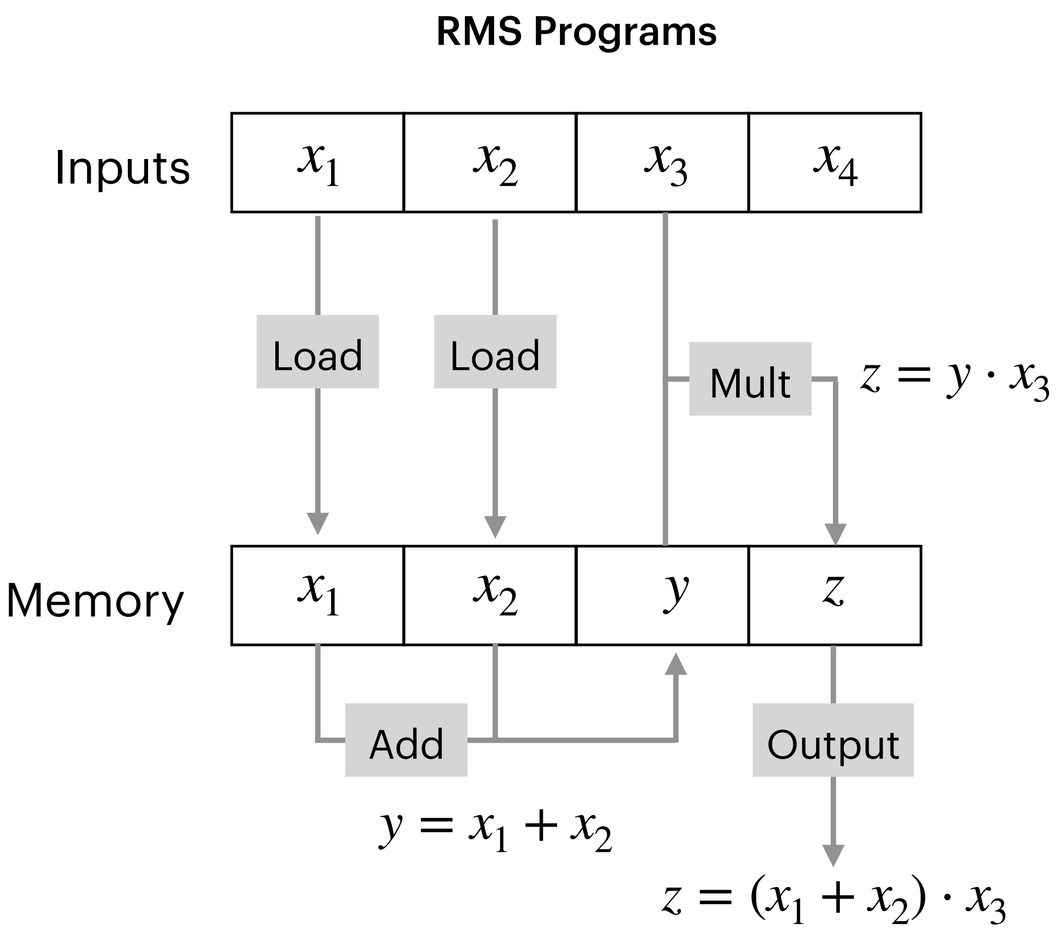
[Boyle-Gilboa-Ishai'16]

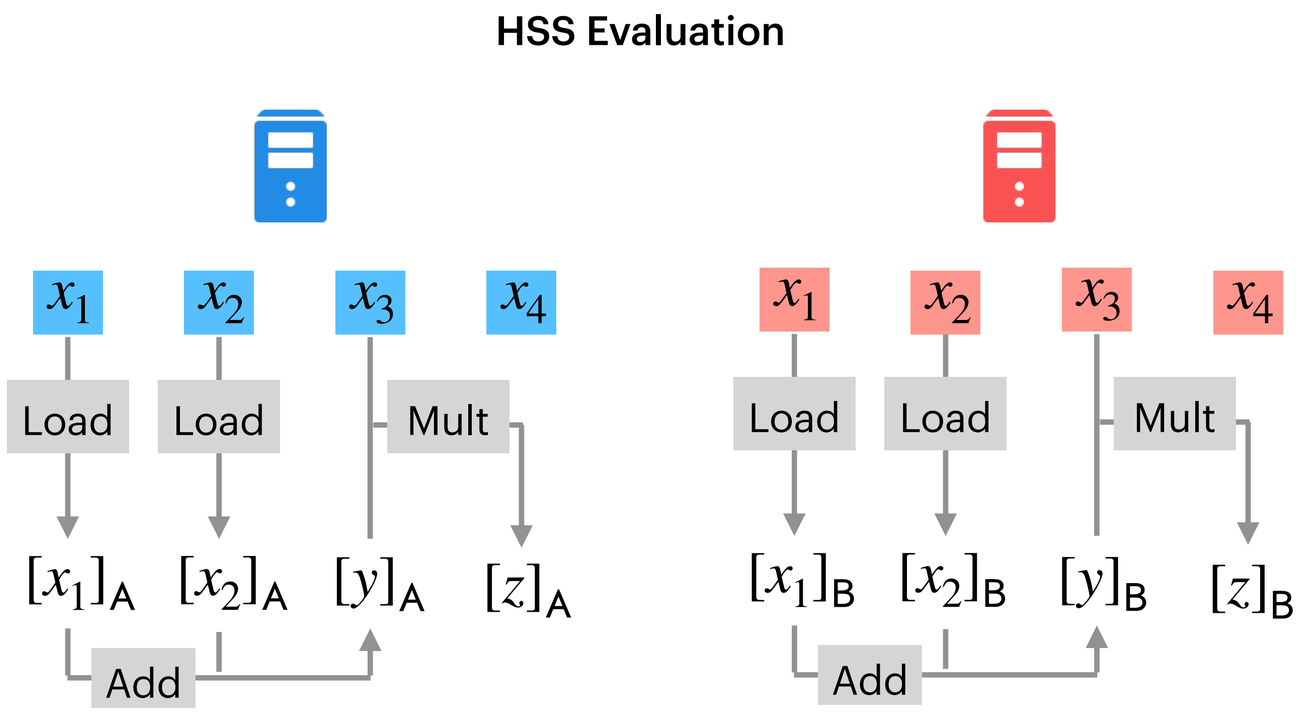




Distributed Evaluation of RMS Programs

[Boyle-Gilboa-Ishai'16]

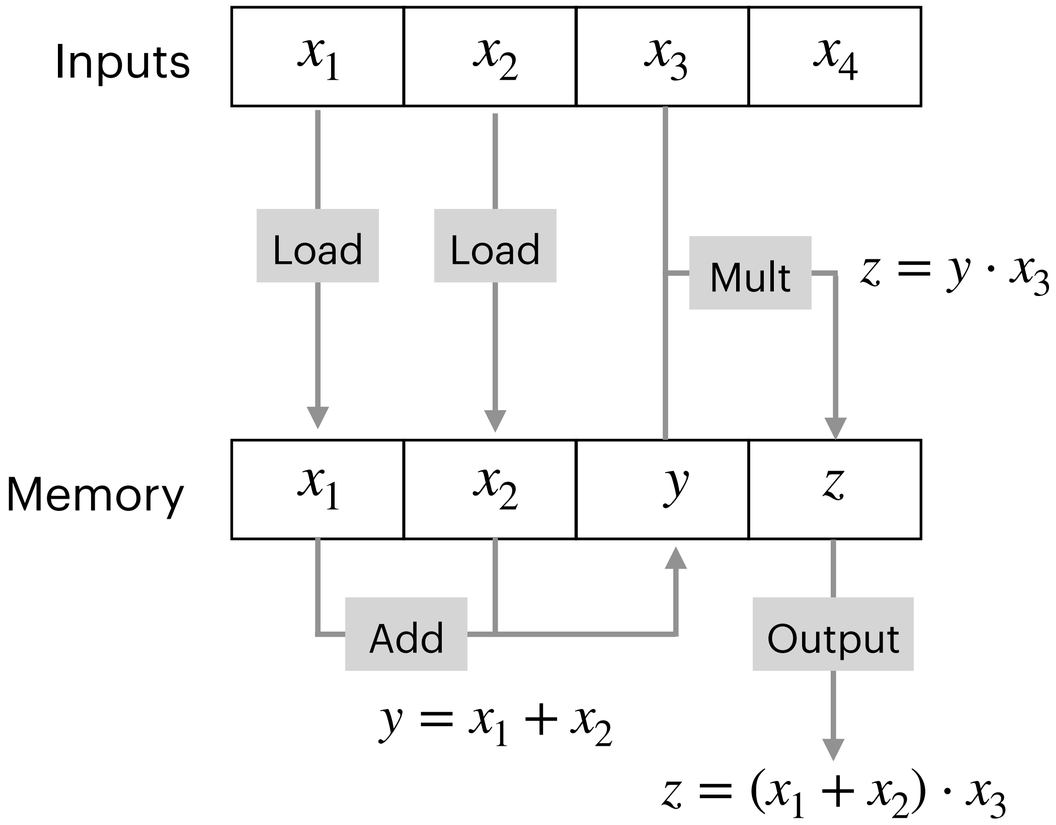




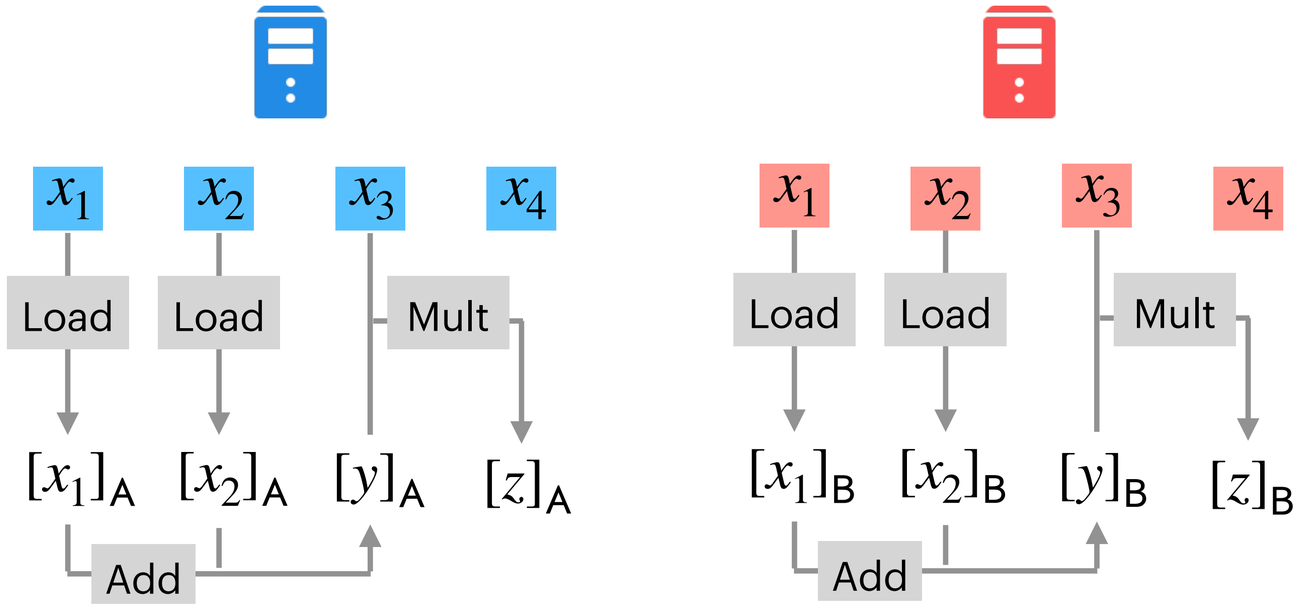
Distributed Evaluation of RMS Programs

[Boyle-Gilboa-Ishai'16]

RMS Programs



HSS Evaluation



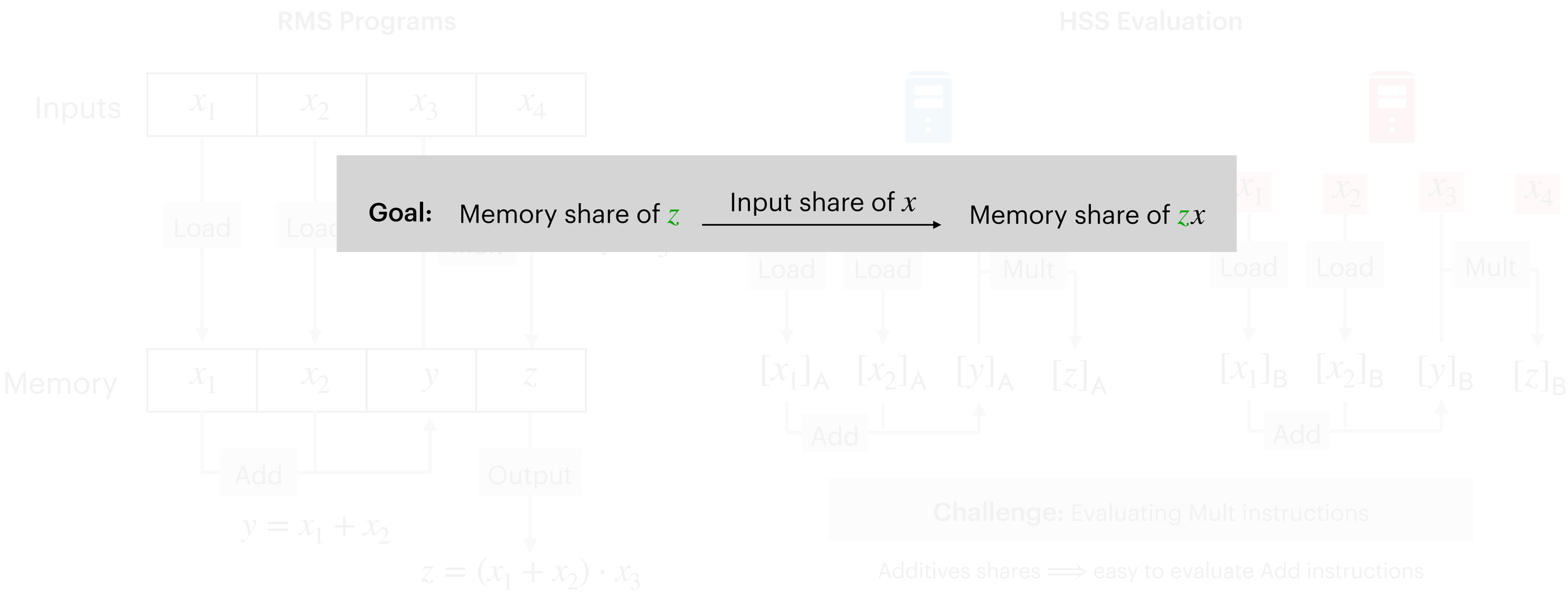
Challenge: Evaluating Mult instructions

Additives shares $\Longrightarrow$ easy to evaluate Add instructions

Load instructions will follow from Mult instructions

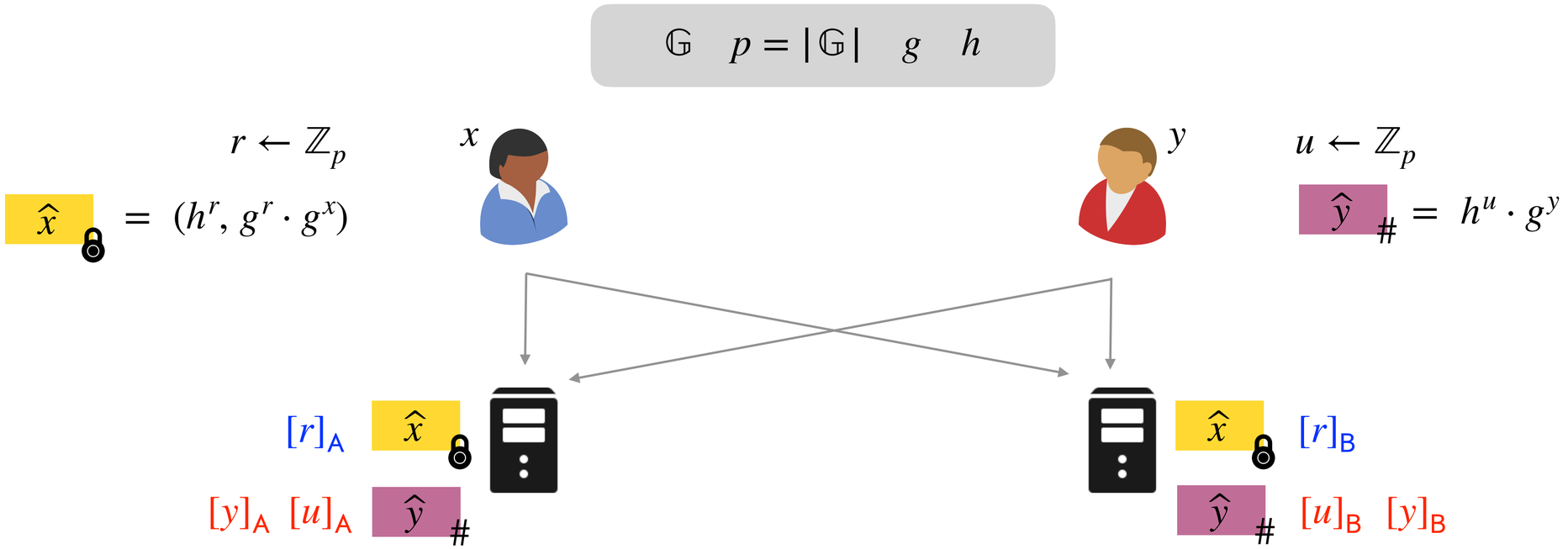
Distributed Evaluation of RMS Programs

[Boyle-Gilboa-Ishai'16]



Goal: Memory share of z —Input share of x→ Memory share of zx

Delegatable Non-Interactive Multiplication



$$[xy]_A \xleftarrow{\text{DDLog}} g^{-[xy]_A} = \frac{\left(h^u \cdot g^y\right)^{[r]_A}}{\left(h^r\right)^{[u]_A} \cdot \left(g^r \cdot g^x\right)^{[y]_A}} \qquad \frac{g^{[xy]_B}}{g^{-[xy]_A}} = g^{xy} \qquad \frac{\left(h^r\right)^{[u]_B} \cdot \left(g^r \cdot g^x\right)^{[y]_B}}{\left(h^u \cdot g^y\right)^{[r]_B}} = g^{[xy]_B} \xrightarrow{\text{DDLog}} [xy]_B$$

Extending Delegatable NIM

$\mathbb{G} \quad p = |\mathbb{G}| \quad g \quad h$

$r \leftarrow \mathbb{Z}_p$

$\hat{x} = (h^r, g^r \cdot g^x)$

$u \leftarrow \mathbb{Z}_p$

$\hat{y}_{\#} = h^u \cdot g^y$

$[z \cdot r]_A$ $\hat{x}$

$[z \cdot y]_A$ $[z \cdot u]_A$ $\hat{y}_{\#}$

$\hat{x}$ $[z \cdot r]_B$

$\hat{y}_{\#}$ $[z \cdot u]_B$ $[z \cdot y]_B$

Extending Delegatable NIM

$\mathbb{G} \quad p = |\mathbb{G}| \quad g \quad h$

$r \leftarrow \mathbb{Z}_p$

$\hat{x} = (h^r, g^r \cdot g^x)$

x

y

$u \leftarrow \mathbb{Z}_p$

$\hat{y}_{\#} = h^u \cdot g^y$

$[z \cdot r]_A \quad \hat{x}$

$[z \cdot y]_A \quad [z \cdot u]_A \quad \hat{y}_{\#}$

$\hat{x} \quad [z \cdot r]_B$

$\hat{y}_{\#} \quad [z \cdot u]_B \quad [z \cdot y]_B$

$$g^{-[z \cdot xy]_A} = \frac{\left(h^u \cdot g^y\right)^{[z \cdot r]_A}}{\left(h^r\right)^{[z \cdot u]_A} \cdot \left(g^r \cdot g^x\right)^{[z \cdot y]_A}}$$

Extending Delegatable NIM

$\mathbb{G} \quad p = |\mathbb{G}| \quad g \quad h$

$r \leftarrow \mathbb{Z}_p$

$\hat{x} = (h^r, g^r \cdot g^x)$

y

$u \leftarrow \mathbb{Z}_p$

$\hat{y}_{\#} = h^u \cdot g^y$

$[z \cdot r]_A \quad \hat{x}$

$[z \cdot y]_A \quad [z \cdot u]_A \quad \hat{y}_{\#}$

$\hat{x} \quad [z \cdot r]_B$

$\hat{y}_{\#} \quad [z \cdot u]_B \quad [z \cdot y]_B$

$$g^{-[z \cdot xy]_A} = \frac{\left(h^u \cdot g^y\right)^{[z \cdot r]_A}}{\left(h^r\right)^{[z \cdot u]_A} \cdot \left(g^r \cdot g^x\right)^{[z \cdot y]_A}}$$

$$\frac{\left(h^r\right)^{[z \cdot u]_B} \cdot \left(g^r \cdot g^x\right)^{[z \cdot y]_B}}{\left(h^u \cdot g^y\right)^{[z \cdot r]_B}} = g^{[z \cdot xy]_B}$$

Extending Delegatable NIM

$\mathbb{G} \quad p = |\mathbb{G}| \quad g \quad h$

$r \leftarrow \mathbb{Z}_p$

$\hat{x} = (h^r, g^r \cdot g^x)$

$u \leftarrow \mathbb{Z}_p$

$\hat{y}_{\#} = h^u \cdot g^y$

$[z \cdot r]_A \quad \hat{x}$

$[z \cdot y]_A \quad [z \cdot u]_A \quad \hat{y}_{\#}$

$\hat{x} \quad [z \cdot r]_B$

$\hat{y}_{\#} \quad [z \cdot u]_B \quad [z \cdot y]_B$

$$[z \cdot xy]_A \xleftarrow{\text{DDLog}} g^{-[z \cdot xy]_A} = \frac{\left(h^u \cdot g^y\right)^{[z \cdot r]_A}}{\left(h^r\right)^{[z \cdot u]_A} \cdot \left(g^r \cdot g^x\right)^{[z \cdot y]_A}}$$

$$\frac{\left(h^r\right)^{[z \cdot u]_B} \cdot \left(g^r \cdot g^x\right)^{[z \cdot y]_B}}{\left(h^u \cdot g^y\right)^{[z \cdot r]_B}} = g^{[z \cdot xy]_B} \xrightarrow{\text{DDLog}} [z \cdot xy]_B$$

Extending Delegatable NIM

$\mathbb{G} \quad p = |\mathbb{G}| \quad g \quad h$

$\hat{x}$ $r \leftarrow \text{Encode}(x)$  x

 y $u \leftarrow \mathbb{Z}_p$

$\hat{y}_{\#} = h^u \cdot g^y$

$[z \cdot r]_A$ $\hat{x}$ 

$[z \cdot y]_A$ $[z \cdot u]_A$ $\hat{y}_{\#}$

 $\hat{x}$ $[z \cdot r]_B$

$\hat{y}_{\#}$ $[z \cdot u]_B$ $[z \cdot y]_B$

$$[z \cdot xy]_A \xleftarrow{\text{DDLog}} g^{-[z \cdot xy]_A} = \frac{\left(h^u \cdot g^y\right)^{[z \cdot r]_A}}{\left(h^r\right)^{[z \cdot u]_A} \cdot \left(g^r \cdot g^x\right)^{[z \cdot y]_A}}$$

$$\frac{\left(h^r\right)^{[z \cdot u]_B} \cdot \left(g^r \cdot g^x\right)^{[z \cdot y]_B}}{\left(h^u \cdot g^y\right)^{[z \cdot r]_B}} = g^{[z \cdot xy]_B} \xrightarrow{\text{DDLog}} [z \cdot xy]_B$$

Extending Delegatable NIM

$\mathbb{G} \quad p = |\mathbb{G}| \quad g \quad h$

$\hat{x}$ $r \leftarrow \text{Encode}(x)$  x

 y $\text{Hash}(y) \rightarrow u$ $\hat{y}$ #

$[z \cdot r]_A$ $\hat{x}$

$[z \cdot y]_A$ $[z \cdot u]_A$ $\hat{y}$ #

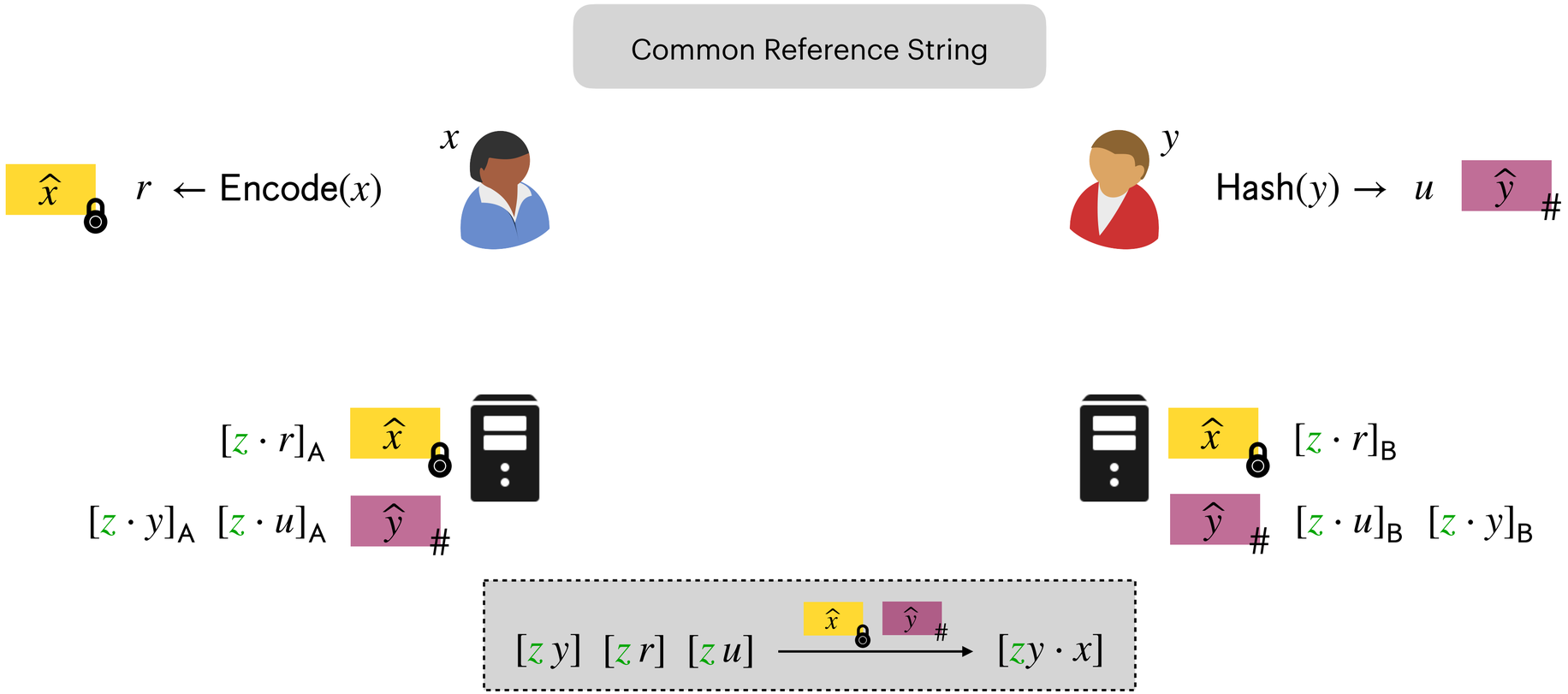
$\hat{x}$ $[z \cdot r]_B$

$\hat{y}$ # $[z \cdot u]_B$ $[z \cdot y]_B$

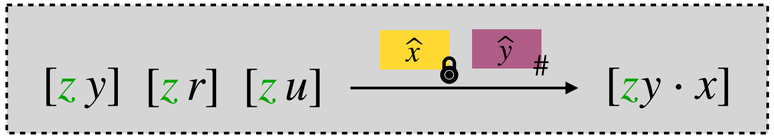
$$[z \cdot xy]_A \xleftarrow{\text{DDLog}} g^{-[z \cdot xy]_A} = \frac{\left(h^u \cdot g^y\right)^{[z \cdot r]_A}}{\left(h^r\right)^{[z \cdot u]_A} \cdot \left(g^r \cdot g^x\right)^{[z \cdot y]_A}}$$

$$\frac{\left(h^r\right)^{[z \cdot u]_B} \cdot \left(g^r \cdot g^x\right)^{[z \cdot y]_B}}{\left(h^u \cdot g^y\right)^{[z \cdot r]_B}} = g^{[z \cdot xy]_B} \xrightarrow{\text{DDLog}} [z \cdot xy]_B$$

Extending Delegatable NIM



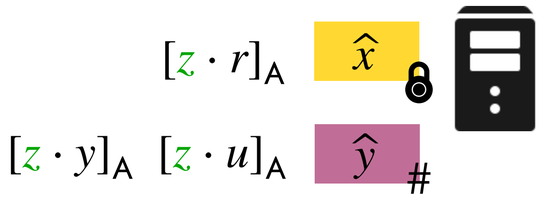
Extending Delegatable NIM

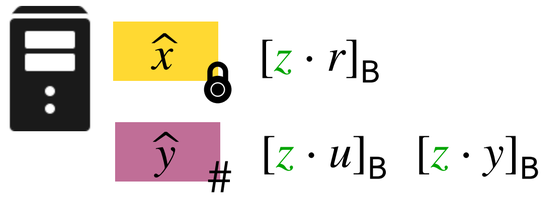


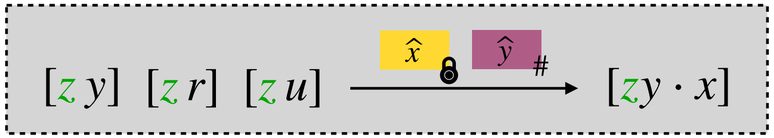
Common Reference String

$\hat{x}$ $r \leftarrow \text{Encode}(x)$

$\text{Hash}(y) \rightarrow u$ $\hat{y}$ #







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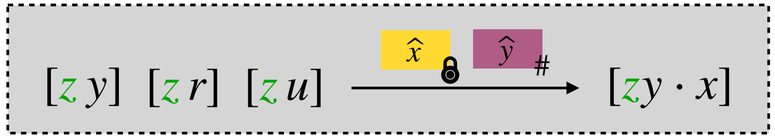

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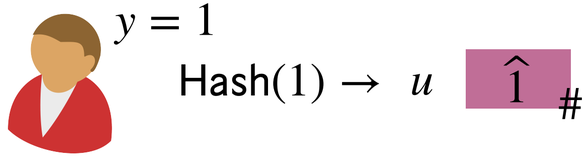

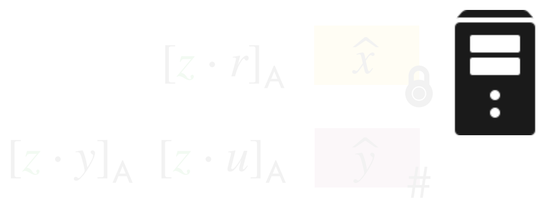

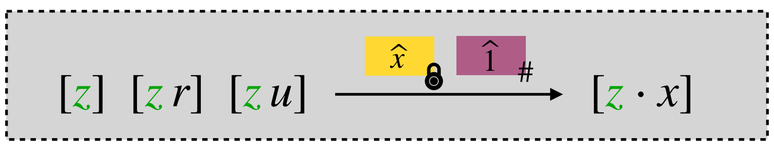

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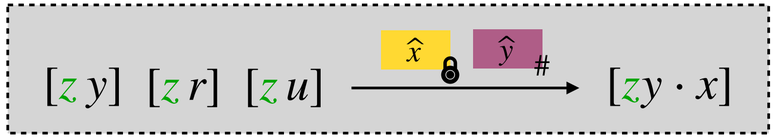

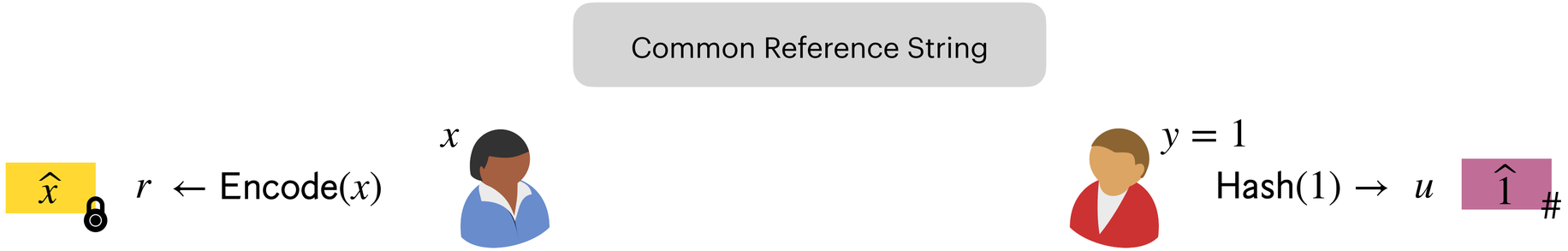

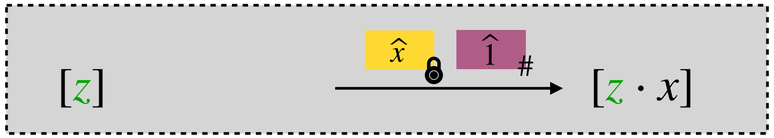

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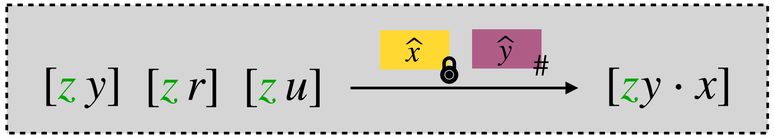

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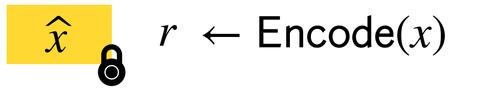
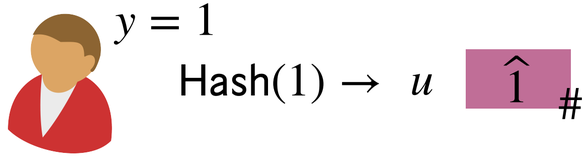
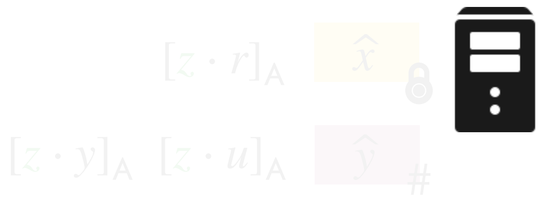
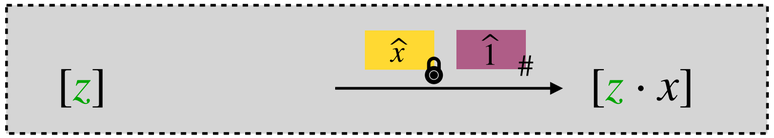
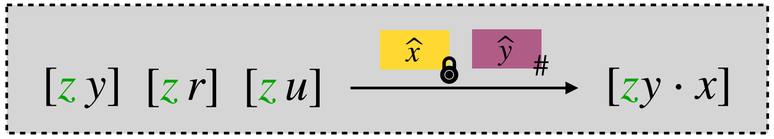






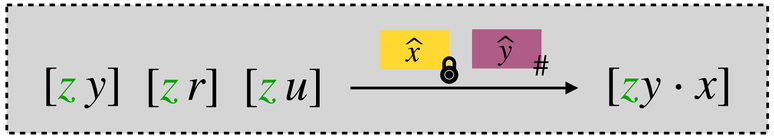


Goal: Memory share of z —Input share of x→ Memory share of zx



Attempt at Evaluating RMS Programs

Common Reference String

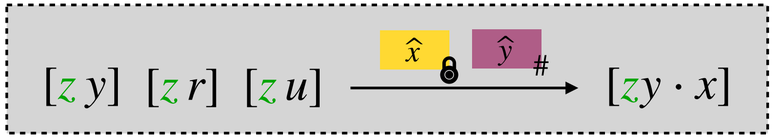


Attempt at Evaluating RMS Programs

Common Reference String

Defined in CRS

$\text{Hash}(1) \rightarrow u$ $\hat{1}$ #



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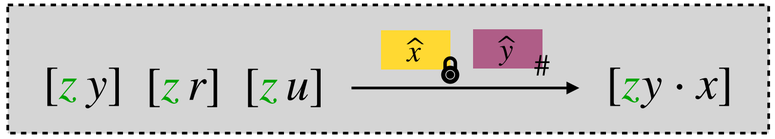

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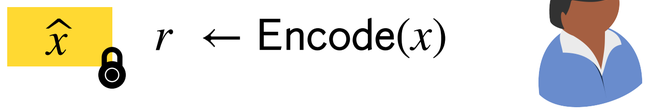

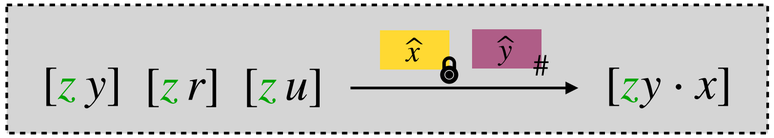

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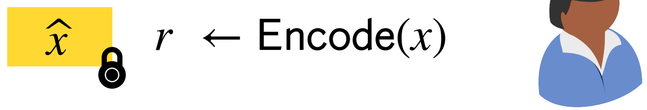
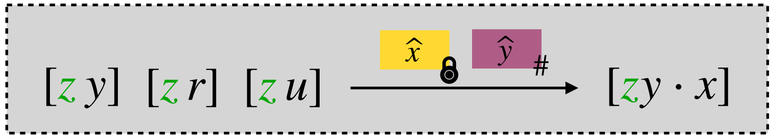


Memory share of z: $[z]$ $[z \cdot r]$



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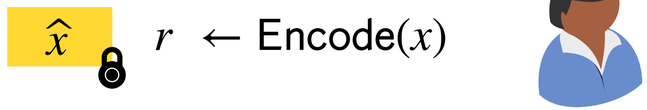
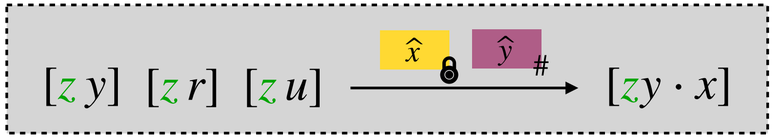


Memory share of z: $[z]$ $[z \cdot r]$



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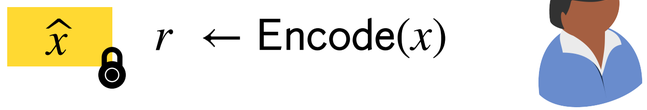
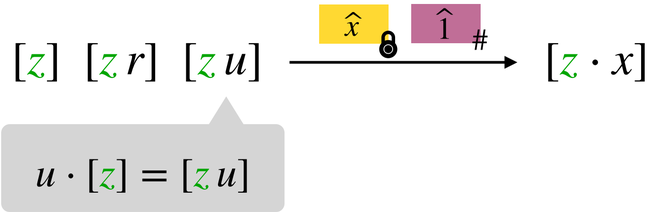
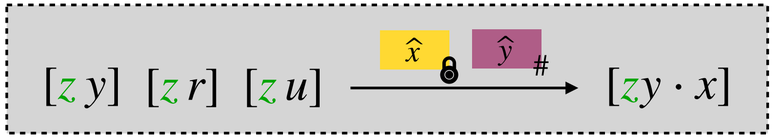


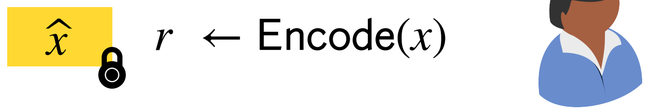
Memory share of z: $[z]$ $[z \cdot r]$





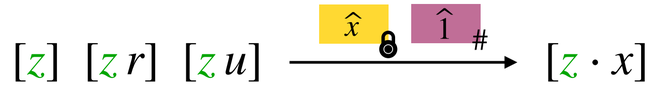
Attempt at Evaluating RMS Programs

Common Reference String

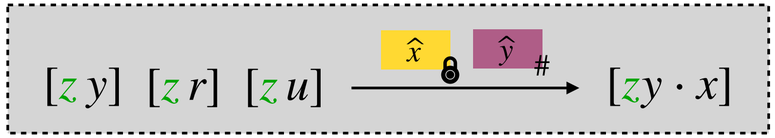


Memory share of z: $[z]$ $[z \cdot r]$

Memory share of zx: $[zx]$



$u \cdot [z] = [z\,u]$



Attempt at Evaluating RMS Programs

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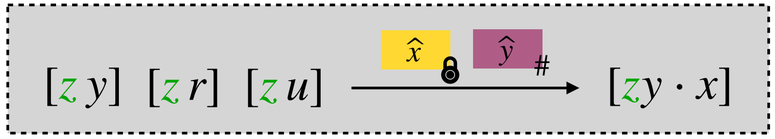
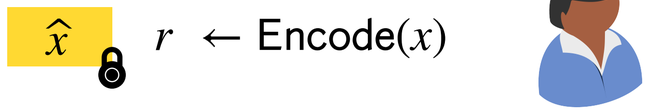


Attempt at Evaluating RMS Programs

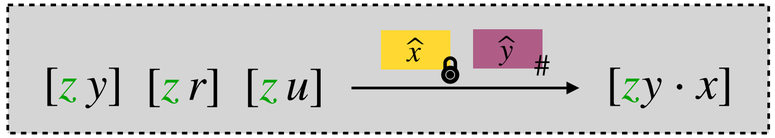
Common Reference String



Memory share of z: $[z]$ $[z \cdot r]$

Memory share of zx: $[zx]$ $[zx \cdot r]$

Need $[zx \cdot r]$ for subsequent multiplications



Attempt at Evaluating RMS Programs

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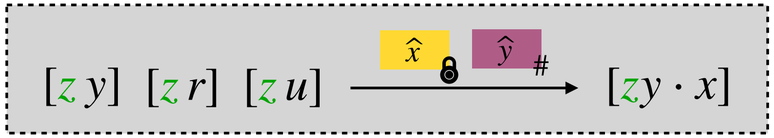


Attempt at Evaluating RMS Programs

Common Reference String

$\widehat{x}$ $r \leftarrow \text{Encode}(x)$

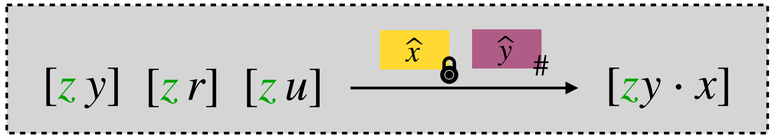
$\widehat{r}$ # $s \leftarrow \text{Hash}(r)$

u $\widehat{1}$ #

$\widehat{1}$ # u

Memory share of z: $[z]$ $[z \cdot r]$

Memory share of zx: $[zx]$ $[zx \cdot r]$



Attempt at Evaluating RMS Programs

Common Reference String

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$\widehat{r}$ # $s \leftarrow \text{Hash}(r)$

Memory share of z: $[z]$ $[z \cdot r]$

Memory share of zx: $[zx]$ $[zx \cdot r]$

Attempt at Evaluating RMS Programs

$[z\,y]\ \ [z\,r]\ \ [z\,u] \xrightarrow{\widehat{x}\ \ \widehat{y}\ \#} [zy \cdot x]$

Common Reference String

$\widehat{x}$ $r \leftarrow \text{Encode}(x)$

$\widehat{r}_{\#}$ $s \leftarrow \text{Hash}(r)$

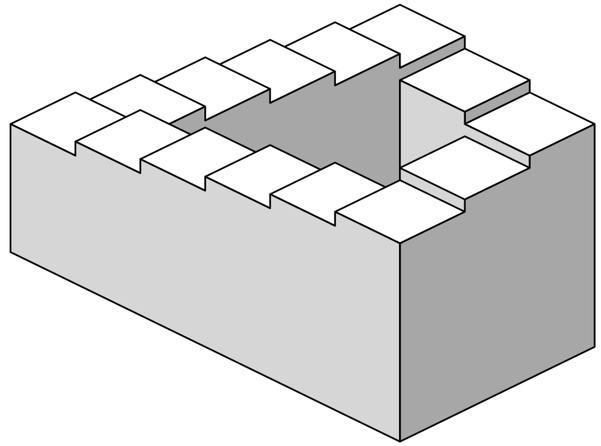
u $\widehat{1}_{\#}$

$\widehat{1}_{\#}$ u

Memory share of z: $[z]$ $[z \cdot r]$

Memory share of zx: $[zx]$ $[zx \cdot r]$

$[z\,r]\ \ [z\,r]\ \ [z\,s] \xrightarrow{\widehat{x}\ \ \widehat{r}\ \#} [zr \cdot x]$



$[z\,y]\ \ [z\,r]\ \ [z\,u] \xrightarrow{\widehat{x}\ \ \widehat{y}^{\#}} [zy \cdot x]$

Attempt at Evaluating RMS Programs

Common Reference String

$\widehat{x}$ $r \leftarrow \text{Encode}(x)$

$\widehat{r}^{\#}$ $s \leftarrow \text{Hash}(r)$

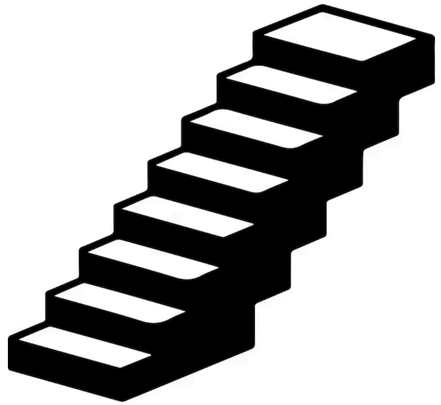
u $\widehat{1}^{\#}$

$\widehat{1}^{\#}$ u

Memory share of z: $[z]$ $[z \cdot r]$

Memory share of zx: $[zx]$ $[zx \cdot r]$

$[z\,r]\ \ [z\,r]\ \ [z\,s] \xrightarrow{\widehat{x}\ \ \widehat{r}^{\#}} [zr \cdot x]$



Solution: Encryption scheme with linear decryption

Encryption with Linear Decryption

$\mathbb{G} \quad p = |\mathbb{G}| \quad g$

Encryption with Linear Decryption

$\mathbb{G} \quad p = |\mathbb{G}| \quad g$

$\mathsf{pk} = g^{-\mathsf{sk}}$ $\mathsf{sk} \leftarrow \mathbb{Z}_p$

x

Encryption with Linear Decryption

$\mathbb{G} \quad p = |\mathbb{G}| \quad g$

$\mathsf{pk} = g^{-\mathsf{sk}} \qquad \mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{ct}_x = \left(g^r, \mathsf{pk}^r \cdot g^x\right) \qquad r \leftarrow \mathbb{Z}_p$

x

Encryption with Linear Decryption

$\mathbb{G}$ $p = |\mathbb{G}|$ g

$\mathsf{pk} = g^{-\mathsf{sk}}$ $\quad \mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{ct}_x = \left(g^r, \mathsf{pk}^r \cdot g^x\right)$ $\quad r \leftarrow \mathbb{Z}_p$

Decryption is "linear": $\left(g^r\right)^{\mathsf{sk}} \cdot \mathsf{pk}^r \cdot g^x = g^{r \cdot \mathsf{sk}} \cdot g^{-r \cdot \mathsf{sk}} \cdot g^x = g^x$

Encryption with Linear Decryption

$\mathbb{G} \quad p = |\mathbb{G}| \quad g$

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$[z]_A \quad [z \cdot \mathsf{sk}]_A \quad \mathsf{ct}_x$

$\mathsf{ct}_x \quad [z \cdot \mathsf{sk}]_B \quad [z]_B$

Encryption with Linear Decryption

$\mathbb{G} \quad p = |\mathbb{G}| \quad g$

$\mathsf{pk} = g^{-\mathsf{sk}} \qquad \mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{ct}_x = \left(g^r, \mathsf{pk}^r \cdot g^x\right) \qquad r \leftarrow \mathbb{Z}_p$

Decryption is "linear": $\left(g^r\right)^{\mathsf{sk}} \cdot \mathsf{pk}^r \cdot g^x = g^{r \cdot \mathsf{sk}} \cdot g^{-r \cdot \mathsf{sk}} \cdot g^x = g^x$

$[z]_\mathsf{A} \quad [z \cdot \mathsf{sk}]_\mathsf{A} \quad \mathsf{ct}_x$

$\mathsf{ct}_x \quad [z \cdot \mathsf{sk}]_\mathsf{B} \quad [z]_\mathsf{B}$

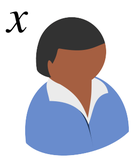
$g^{[z \cdot x]_\mathsf{A}} = \left(g^r\right)^{[z \cdot \mathsf{sk}]_\mathsf{A}} \cdot \left(\mathsf{pk}^r \cdot g^x\right)^{[z]_\mathsf{A}}$

Encryption with Linear Decryption

$\mathbb{G} \quad p = |\mathbb{G}| \quad g$

$\mathsf{pk} = g^{-\mathsf{sk}} \qquad \mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{ct}_x = \left(g^r, \mathsf{pk}^r \cdot g^x\right) \qquad r \leftarrow \mathbb{Z}_p$



Decryption is "linear": $\left(g^r\right)^{\mathsf{sk}} \cdot \mathsf{pk}^r \cdot g^x = g^{r \cdot \mathsf{sk}} \cdot g^{-r \cdot \mathsf{sk}} \cdot g^x = g^x$

$[z]_\mathsf{A} \quad [z \cdot \mathsf{sk}]_\mathsf{A} \quad \mathsf{ct}_x$

$g^{[z \cdot x]_\mathsf{A}} = \left(g^r\right)^{[z \cdot \mathsf{sk}]_\mathsf{A}} \cdot \left(\mathsf{pk}^r \cdot g^x\right)^{[z]_\mathsf{A}}$

$\mathsf{ct}_x \quad [z \cdot \mathsf{sk}]_\mathsf{B} \quad [z]_\mathsf{B}$

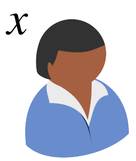
$\left(g^r\right)^{-[z \cdot \mathsf{sk}]_\mathsf{B}} \cdot \left(\mathsf{pk}^r \cdot g^x\right)^{-[z]_\mathsf{B}} = g^{-[z \cdot x]_\mathsf{B}}$

Encryption with Linear Decryption

$\mathbb{G} \quad p = |\mathbb{G}| \quad g$

$\mathsf{pk} = g^{-\mathsf{sk}} \qquad \mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{ct}_x = \left(g^r, \mathsf{pk}^r \cdot g^x\right) \qquad r \leftarrow \mathbb{Z}_p$



Decryption is "linear": $\left(g^r\right)^{\mathsf{sk}} \cdot \mathsf{pk}^r \cdot g^x = g^{r \cdot \mathsf{sk}} \cdot g^{-r \cdot \mathsf{sk}} \cdot g^x = g^x$

$[z]_\mathsf{A} \quad [z \cdot \mathsf{sk}]_\mathsf{A} \quad \mathsf{ct}_x$

$\mathsf{ct}_x \quad [z \cdot \mathsf{sk}]_\mathsf{B} \quad [z]_\mathsf{B}$

$[z \cdot x]_\mathsf{A} \xleftarrow{\text{DDLog}} g^{[z \cdot x]_\mathsf{A}} = \left(g^r\right)^{[z \cdot \mathsf{sk}]_\mathsf{A}} \cdot \left(\mathsf{pk}^r \cdot g^x\right)^{[z]_\mathsf{A}}$

$\left(g^r\right)^{-[z \cdot \mathsf{sk}]_\mathsf{B}} \cdot \left(\mathsf{pk}^r \cdot g^x\right)^{-[z]_\mathsf{B}} = g^{-[z \cdot x]_\mathsf{B}} \xrightarrow{\text{DDLog}} [z \cdot x]_\mathsf{B}$

Encryption with Linear Decryption

$\mathbb{G} \quad p = |\mathbb{G}| \quad g$

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{ct}_x \leftarrow \mathsf{Encrypt}(\mathsf{pk}, x)$

$[z]_\mathsf{A} \quad [z \cdot \mathsf{sk}]_\mathsf{A} \quad \mathsf{ct}_x$

$\mathsf{ct}_x \quad [z \cdot \mathsf{sk}]_\mathsf{B} \quad [z]_\mathsf{B}$

$[z] \quad [z\,\mathsf{sk}] \xrightarrow{\mathsf{ct}_x} [z\,x]$

Encryption with Linear Decryption

$\mathbb{G} \quad p = |\mathbb{G}| \quad g$

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\mathsf{ct}_x \leftarrow \mathsf{Encrypt}(\mathsf{pk}, x)$

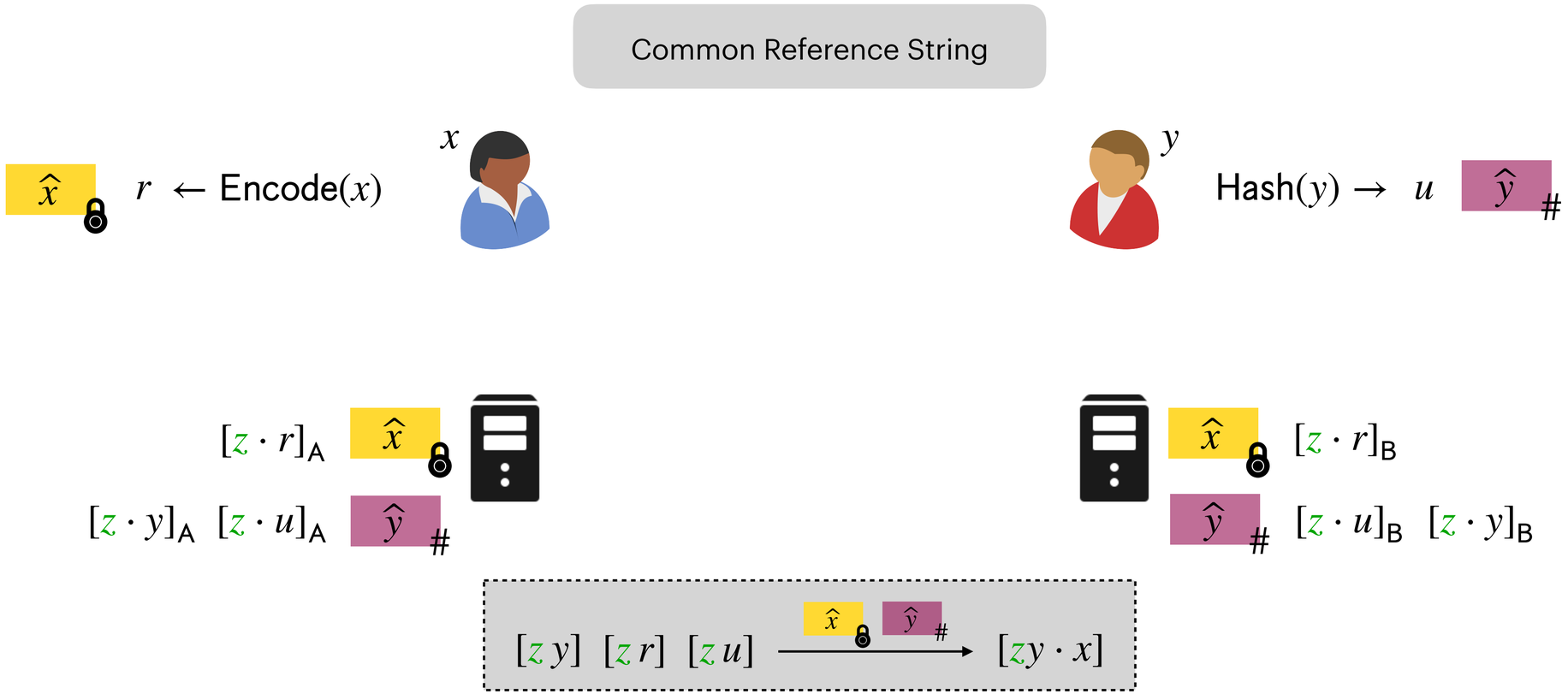
$[z]_A \quad [z \cdot \mathsf{sk}]_A \quad \mathsf{ct}_x$

$\mathsf{ct}_x \quad [z \cdot \mathsf{sk}]_B \quad [z]_B$

$[z] \quad [z\,\mathsf{sk}] \xrightarrow{\mathsf{ct}_x} [z\,x]$

Switch from $[z\,\mathsf{sk}]$ to $[z\,x]$

A Simplification of Delegatable NIM



A Simplification of Delegatable NIM

Simplification: Hash random values or public constants

Common Reference String

$\hat{x}$ $r \leftarrow \text{Encode}(x)$ x

y $\text{Hash}(y) \rightarrow u$ $\hat{y}$ #

$[z \cdot r]_A$ $\hat{x}$

$[z \cdot y]_A$ $[z \cdot u]_A$ $\hat{y}$ #

$\hat{x}$ $[z \cdot r]_B$

$\hat{y}$ # $[z \cdot u]_B$ $[z \cdot y]_B$

$[z\,y]$ $[z\,r]$ $[z\,u]$ $\xrightarrow{\hat{x}\ \ \hat{y}\ \#}$ $[zy \cdot x]$

A Simplification of Delegatable NIM

Simplification: Hash random values or public constants

Common Reference String

$\hat{x}$ $r \leftarrow \text{Encode}(x)$ x

y $\text{Hash}(y) \rightarrow$ ~~u~~ $\hat{y}$ #

$[z \cdot r]_A$ $\hat{x}$

$[z \cdot y]_A$ $[z \cdot u]_A$ $\hat{y}$ #

$\hat{x}$ $[z \cdot r]_B$

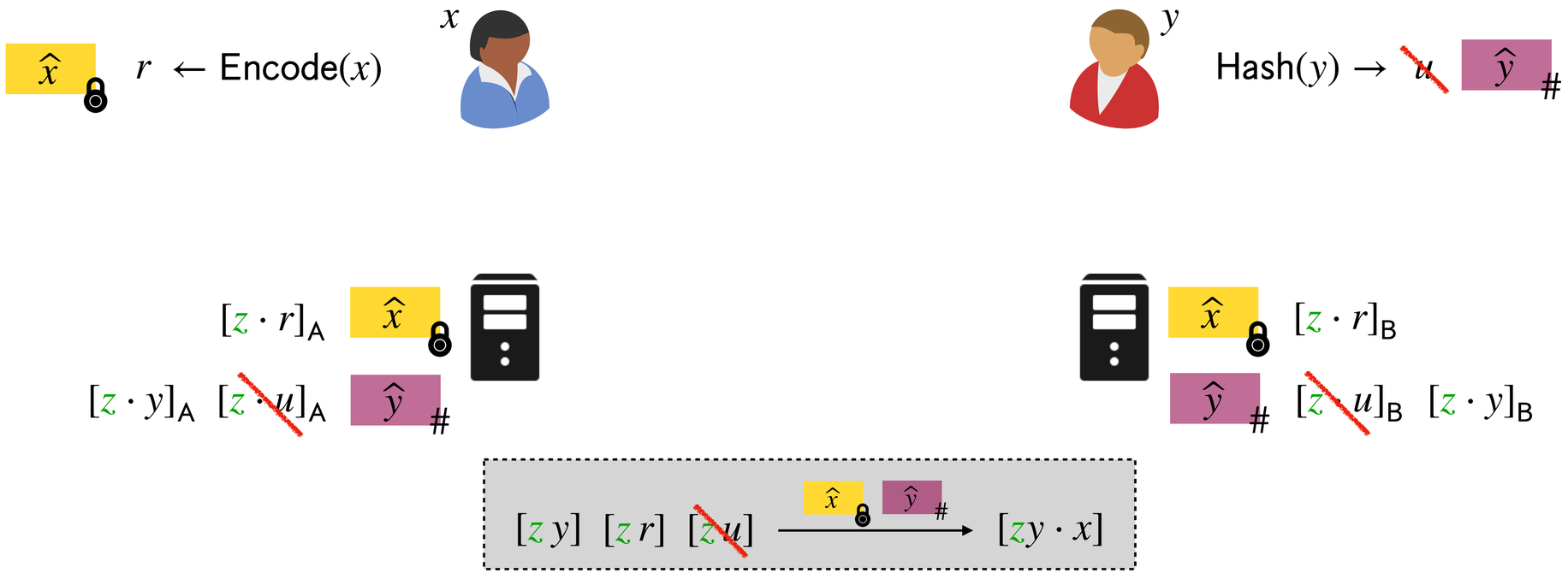
$\hat{y}$ # $[z \cdot u]_B$ $[z \cdot y]_B$

$[z\,y]$ $[z\,r]$ $[z\,u]$ $\xrightarrow{\hat{x}\ \hat{y}\ \#}$ $[zy \cdot x]$

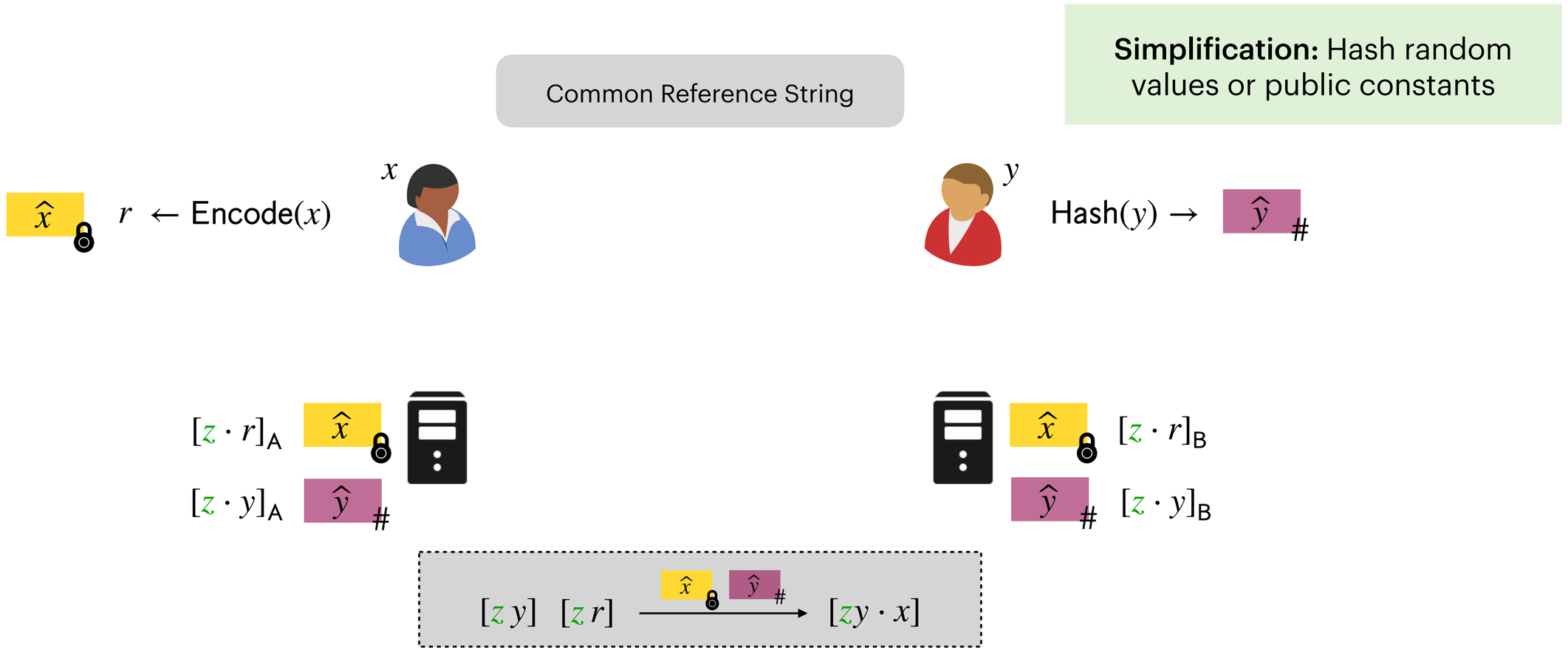
A Simplification of Delegatable NIM

Simplification: Hash random values or public constants

Common Reference String



A Simplification of Delegatable NIM



A Simplification of Delegatable NIM

Simplification: Hash random values or public constants

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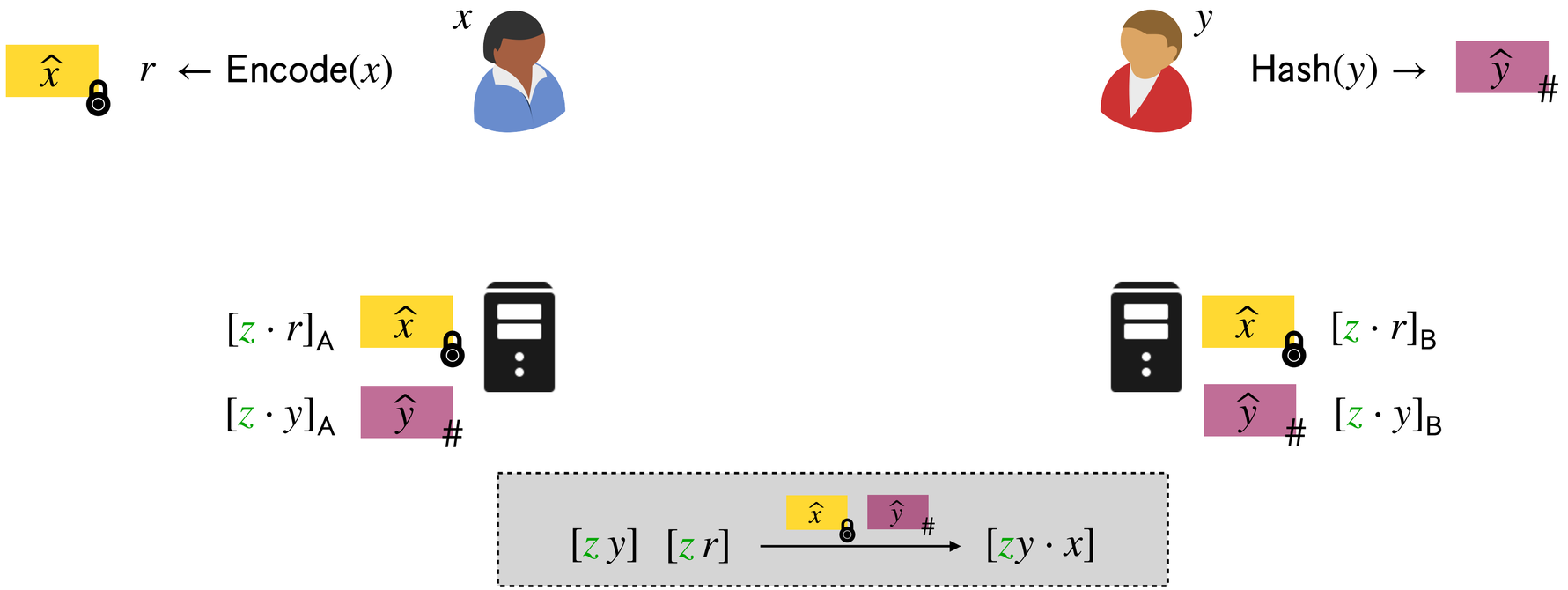
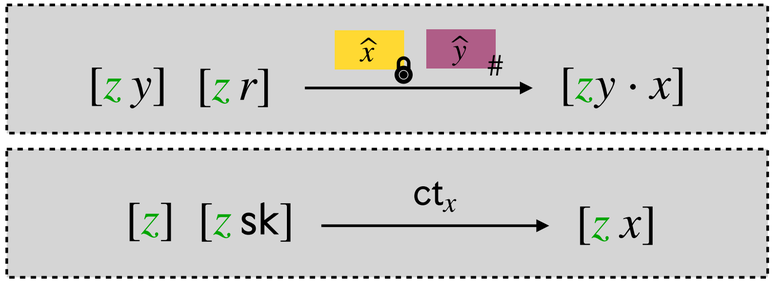


Multiply $[z\,y]$ and x

(using $[z\,r]$)

Evaluating RMS Programs



Common Reference String

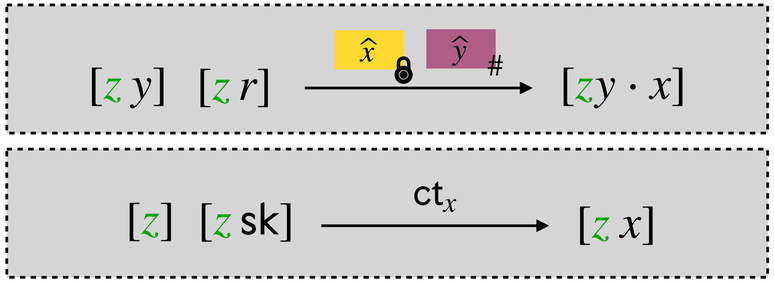
Evaluating RMS Programs

$[z\,y]\ \ [z\,r] \xrightarrow{\hat{x}\ \ \hat{y}\ \#} [zy \cdot x]$

$[z]\ \ [z\,\mathsf{sk}] \xrightarrow{\mathsf{ct}_x} [z\,x]$

Common Reference String

Evaluating RMS Programs



Common Reference String

$\widehat{x_1}$ $r_1 \leftarrow \mathsf{Encode}(x_1)$

Evaluating RMS Programs

$[z\ y]\ \ [z\ r] \xrightarrow{\widehat{x}\ \ \widehat{y}\ \#} [zy \cdot x]$

$[z]\ \ [z\ \mathsf{sk}] \xrightarrow{\mathsf{ct}_x} [z\ x]$

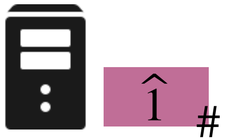
Common Reference String

$\widehat{x_1}$ $r_1 \leftarrow \mathsf{Encode}(x_1)$

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\widehat{\mathsf{sk}_1}\ \#$ $\leftarrow \mathsf{Hash}(\mathsf{sk}_1)$

$\mathsf{ct}_{r_1} \leftarrow \mathsf{Encrypt}(\mathsf{pk}_1, r_1)$



Evaluating RMS Programs

$[z\,y]\ \ [z\,r] \xrightarrow{\widehat{x}\ \ \widehat{y}\ \#} [zy \cdot x]$

$[z]\ \ [z\,\mathsf{sk}] \xrightarrow{\mathsf{ct}_x} [z\,x]$

Common Reference String

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$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\widehat{\mathsf{sk}_1}\ \#$ $\leftarrow \mathsf{Hash}(\mathsf{sk}_1)$

$\mathsf{ct}_{r_1} \leftarrow \mathsf{Encrypt}(\mathsf{pk}_1, r_1)$

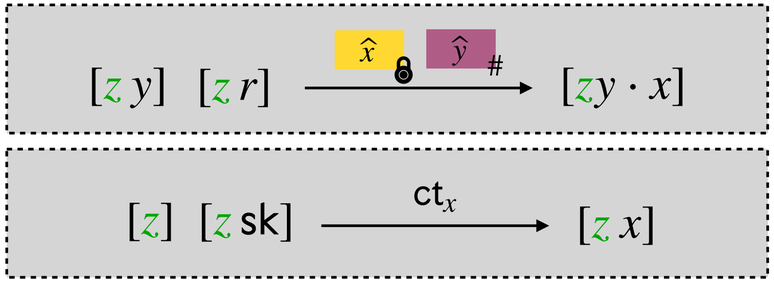
$\mathsf{Encode}(x_2) \rightarrow r_2$ $\widehat{x_2}$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

$\mathsf{Hash}(\mathsf{sk}_2) \rightarrow \widehat{\mathsf{sk}_2}\ \#$

$\mathsf{Encrypt}(\mathsf{pk}_2, r_2) \rightarrow \mathsf{ct}_{r_2}$

Evaluating RMS Programs



Common Reference String

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$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\widehat{\mathsf{sk}_1}$ # $\leftarrow \mathsf{Hash}(\mathsf{sk}_1)$

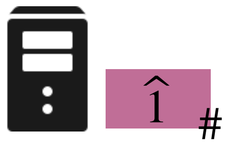
$\mathsf{ct}_{r_1} \leftarrow \mathsf{Encrypt}(\mathsf{pk}_1, r_1)$

$\mathsf{Encode}(x_2) \rightarrow r_2$ $\widehat{x_2}$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

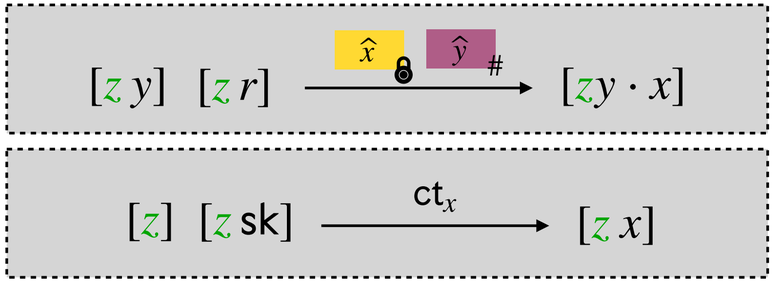
$\mathsf{Hash}(\mathsf{sk}_2) \rightarrow$ $\widehat{\mathsf{sk}_2}$ #

$\mathsf{Encrypt}(\mathsf{pk}_2, r_2) \rightarrow \mathsf{ct}_{r_2}$



Memory share of z: $[z]$ $[z\,\mathsf{sk}_1]$ $[z\,\mathsf{sk}_2]$

Evaluating RMS Programs



Common Reference String

$\widehat{x_1}$ $r_1 \leftarrow \mathsf{Encode}(x_1)$

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\widehat{\mathsf{sk}_1}_{\#} \leftarrow \mathsf{Hash}(\mathsf{sk}_1)$

$\mathsf{ct}_{r_1} \leftarrow \mathsf{Encrypt}(\mathsf{pk}_1, r_1)$

$\mathsf{Encode}(x_2) \rightarrow r_2$ $\widehat{x_2}$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

$\mathsf{Hash}(\mathsf{sk}_2) \rightarrow \widehat{\mathsf{sk}_2}_{\#}$

$\mathsf{Encrypt}(\mathsf{pk}_2, r_2) \rightarrow \mathsf{ct}_{r_2}$

Memory share of z: $[z]$ $[z\,\mathsf{sk}_1]$ $[z\,\mathsf{sk}_2]$

Memory share of $z\,x_1$:

Evaluating RMS Programs

$[z\,y]\ \ [z\,r] \xrightarrow{\widehat{x}\ \ \widehat{y}\ \#} [zy \cdot x]$

$[z]\ \ [z\,\mathsf{sk}] \xrightarrow{\mathsf{ct}_x} [z\,x]$

Common Reference String

$\widehat{x_1}$ $r_1 \leftarrow \mathsf{Encode}(x_1)$

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\widehat{\mathsf{sk}_1}\ \#$ $\leftarrow \mathsf{Hash}(\mathsf{sk}_1)$

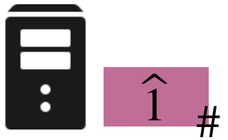
$\mathsf{ct}_{r_1} \leftarrow \mathsf{Encrypt}(\mathsf{pk}_1, r_1)$

$\mathsf{Encode}(x_2) \rightarrow r_2$ $\widehat{x_2}$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

$\mathsf{Hash}(\mathsf{sk}_2) \rightarrow \widehat{\mathsf{sk}_2}\ \#$

$\mathsf{Encrypt}(\mathsf{pk}_2, r_2) \rightarrow \mathsf{ct}_{r_2}$



Memory share of z: $[z]$ $[z\,\mathsf{sk}_1]$ $[z\,\mathsf{sk}_2]$

Memory share of $z\,x_1$:

1) Switch to $[z\,r_1]$: $[z]\ \ [z\,\mathsf{sk}_1] \xrightarrow{\mathsf{ct}_{r_1}} [z\,r_1]$

Evaluating RMS Programs

$[z\,y]\ \ [z\,r] \xrightarrow{\widehat{x}\ \ \widehat{y}\ \#} [zy \cdot x]$

$[z]\ \ [z\,\mathsf{sk}] \xrightarrow{\mathsf{ct}_x} [z\,x]$

Common Reference String

$\widehat{x_1}$ $r_1 \leftarrow \mathsf{Encode}(x_1)$

$(\mathsf{pk}_1, \mathsf{sk}_1) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$\widehat{\mathsf{sk}_1}\ \#$ $\leftarrow \mathsf{Hash}(\mathsf{sk}_1)$

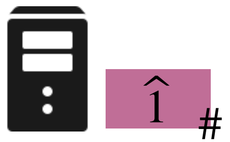
$\mathsf{ct}_{r_1} \leftarrow \mathsf{Encrypt}(\mathsf{pk}_1, r_1)$

$\mathsf{Encode}(x_2) \rightarrow r_2$ $\widehat{x_2}$

$\mathsf{KeyGen}(1^\lambda) \rightarrow (\mathsf{pk}_2, \mathsf{sk}_2)$

$\mathsf{Hash}(\mathsf{sk}_2) \rightarrow \widehat{\mathsf{sk}_2}\ \#$

$\mathsf{Encrypt}(\mathsf{pk}_2, r_2) \rightarrow \mathsf{ct}_{r_2}$



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Memory share of $z\,x_1$: $[zx_1]$

1) Switch to $[z\,r_1]$: $[z]\ \ [z\,\mathsf{sk}_1] \xrightarrow{\mathsf{ct}_{r_1}} [z\,r_1]$

2) Multiply $[z]$ with x_1: $[z]\ \ [z\,r_1] \xrightarrow{\widehat{x_1}\ \ \widehat{1}\ \#} [z \cdot x_1]$

Evaluating RMS Programs

$[z\,y]\ \ [z\,r] \xrightarrow{\widehat{x}\ \ \widehat{y}\ \#} [zy \cdot x]$

$[z]\ \ [z\,\mathsf{sk}] \xrightarrow{\mathsf{ct}_x} [z\,x]$

Common Reference String

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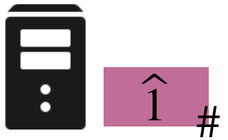

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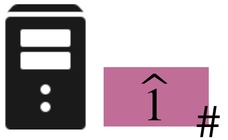

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Invariant preserved!

Evaluating RMS Programs

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Invariant preserved!

- Similar approach to multiply with x_2
- Extends naturally to arbitrary number of clients

HSS for **Multiplication** is All You Need

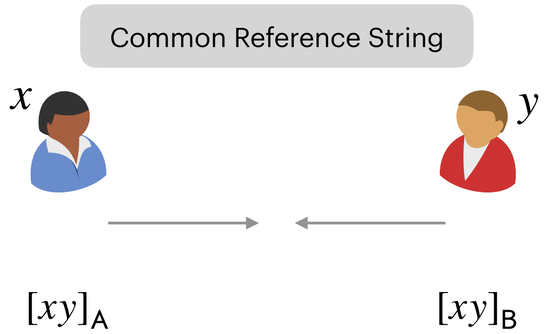
Linear Decoding

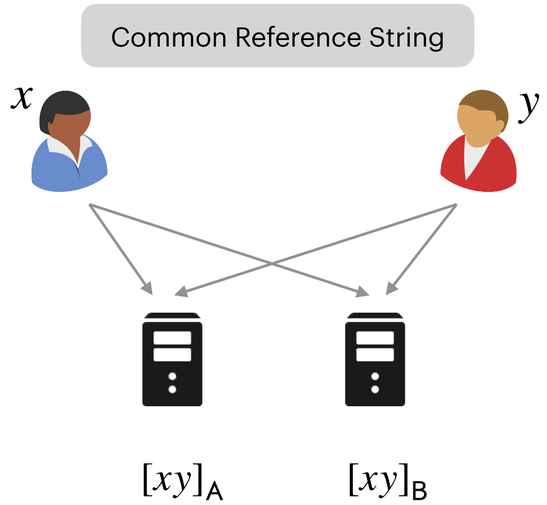
Encryption with
Linear Decryption

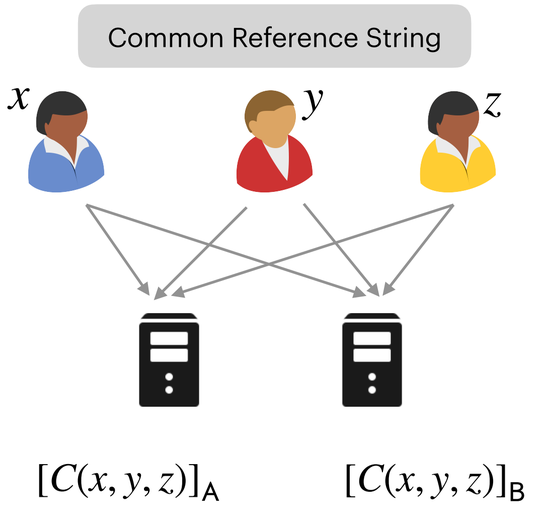
Non-interactive multiplication

Delegatable non-interactive multiplication

Client-server HSS for RMS programs







HSS for **Multiplication** is All You Need

Linear Decoding

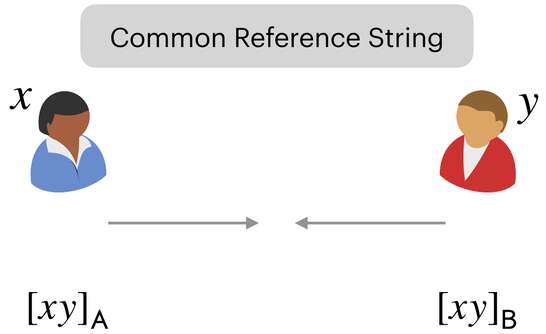
Encryption with Linear Decryption

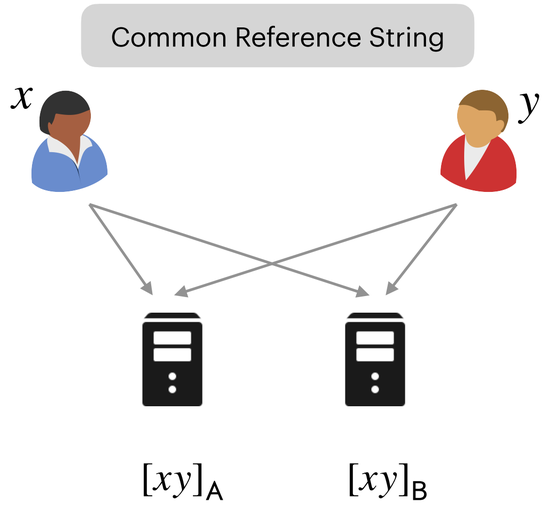
Implied by NIM with linear decoding

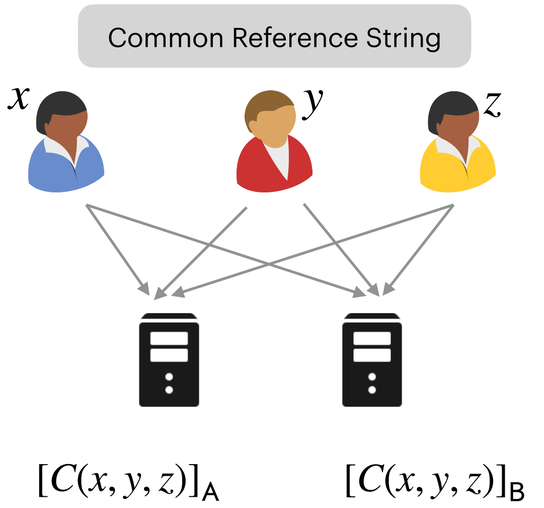
Non-interactive multiplication

Delegatable non-interactive multiplication

Client-server HSS for RMS programs







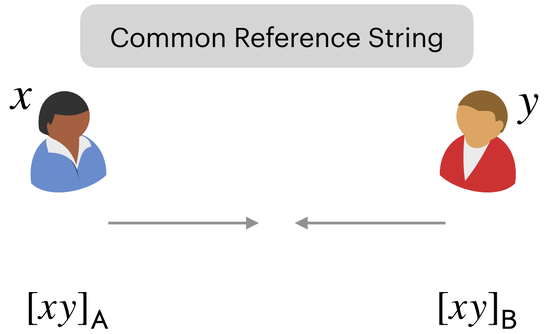
NIM with Linear Decoding is All You Need

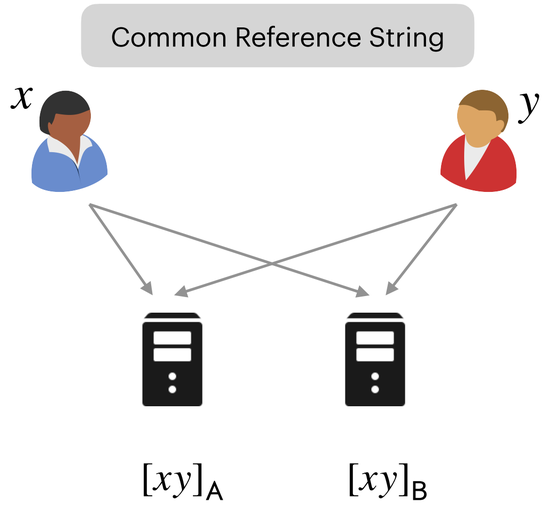
Linear Decoding

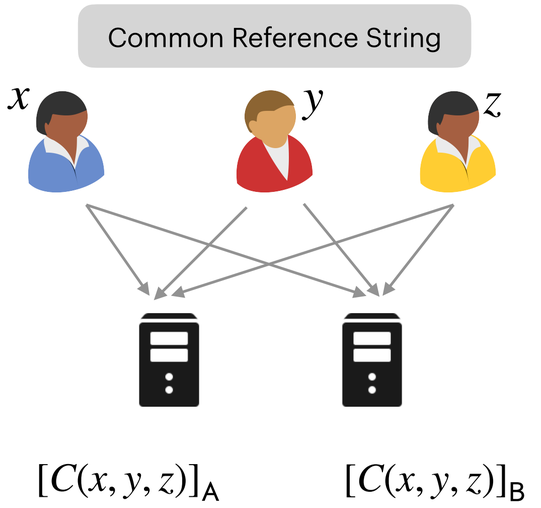
Non-interactive multiplication

Delegatable non-interactive multiplication

Client-server HSS for RMS programs







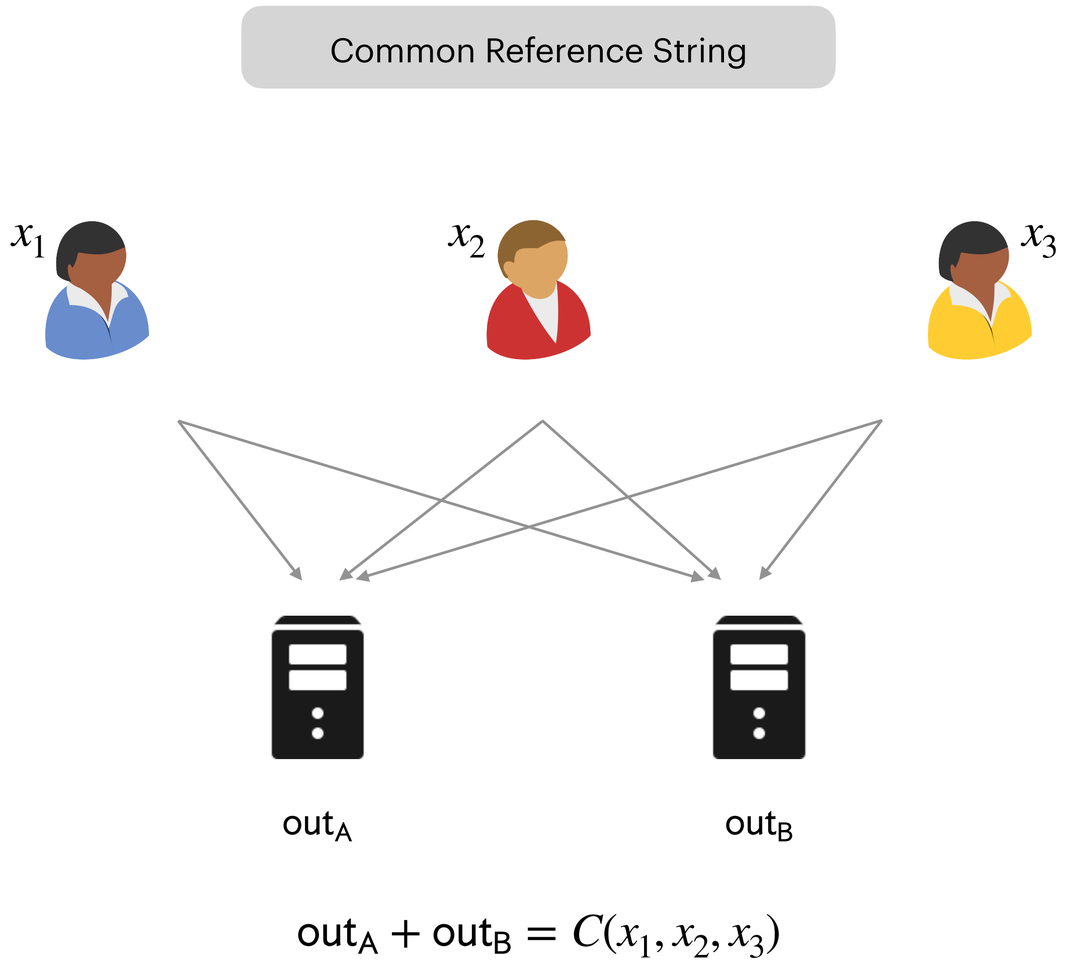
Outline

Barriers to Removing Correlated Setup

Our Approach

Extensions

Succinct Client-Server HSS



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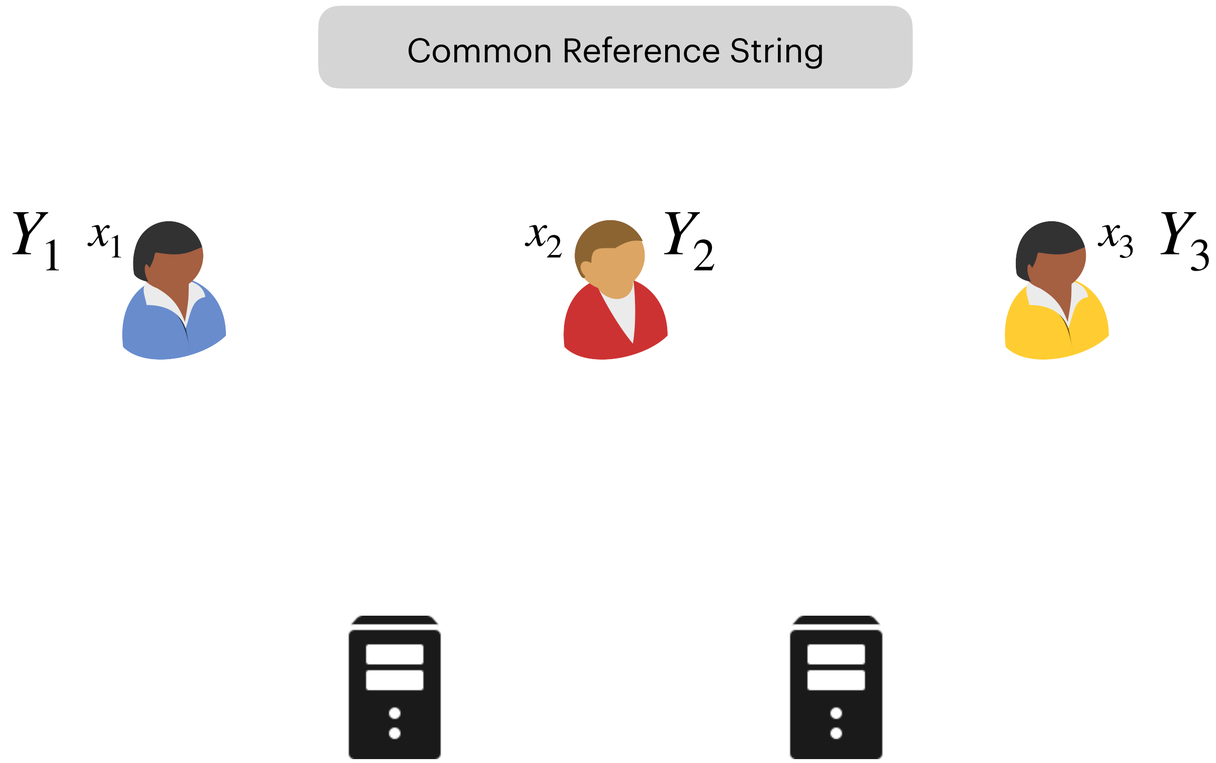

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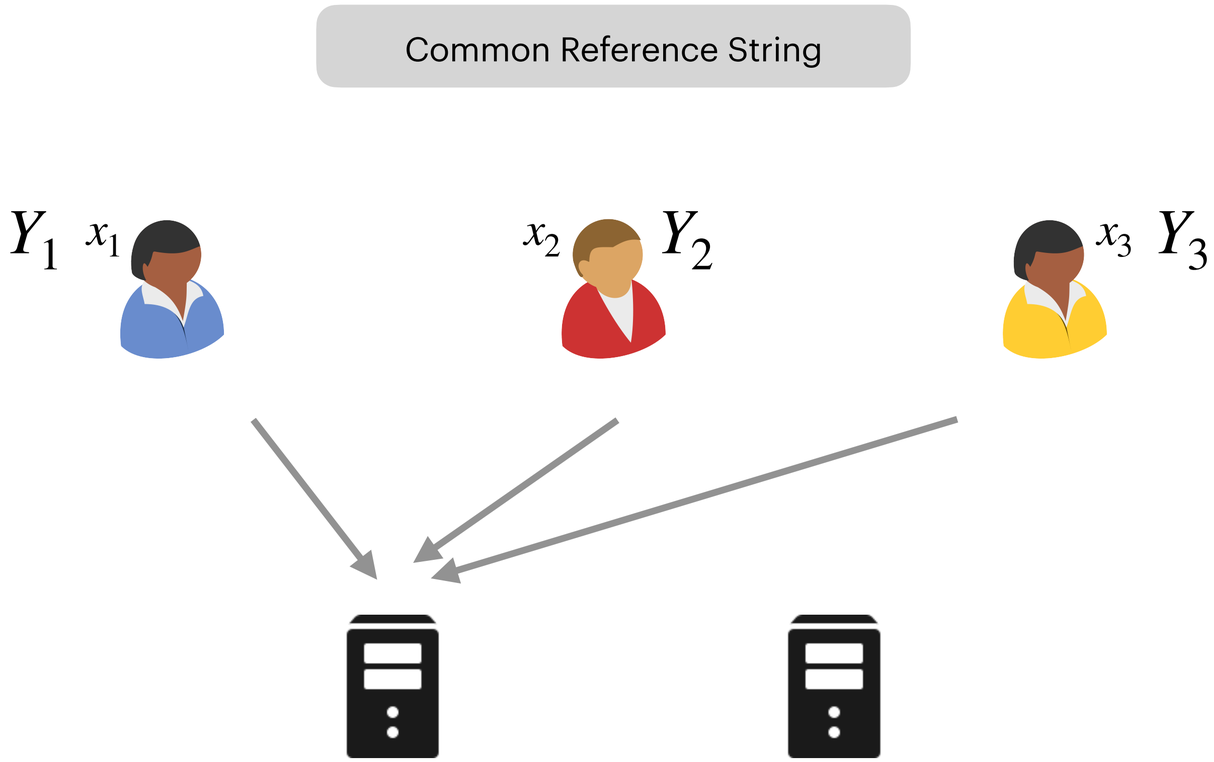

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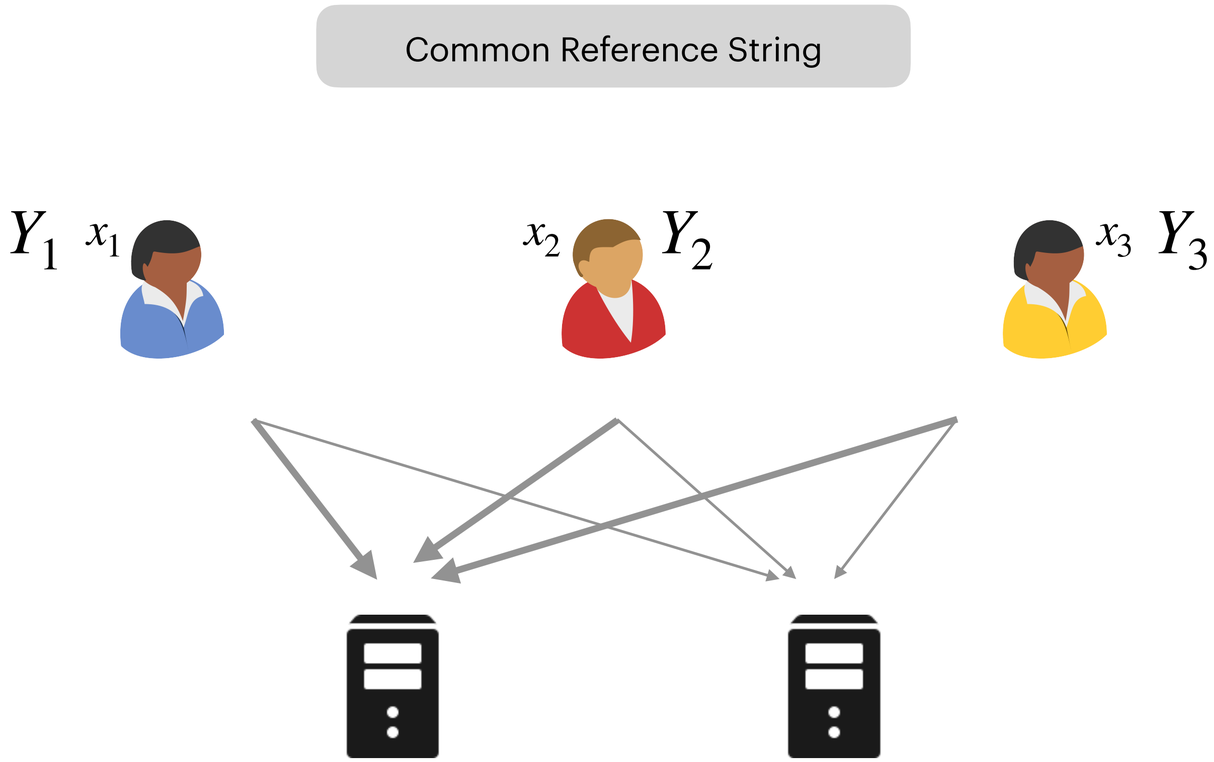

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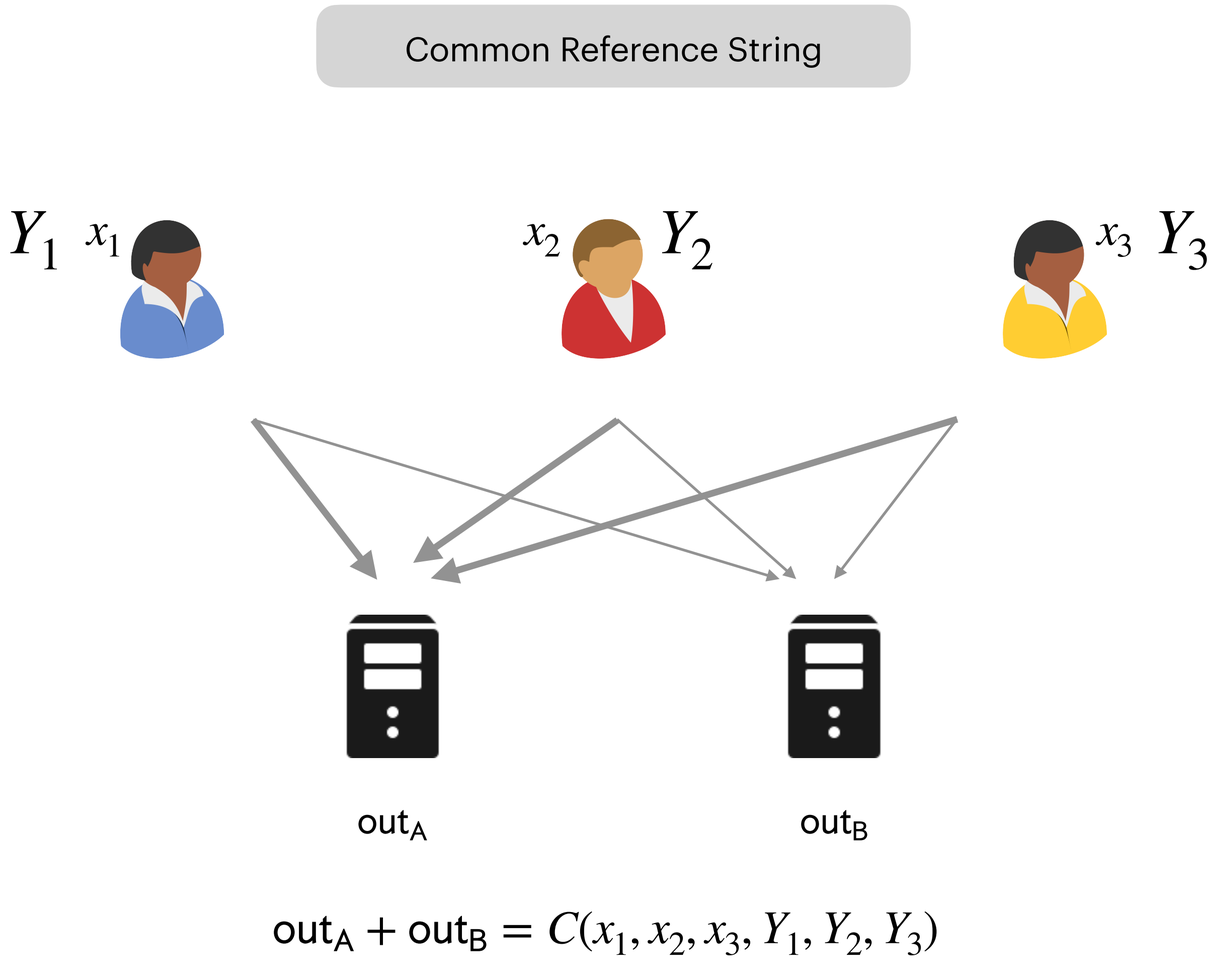
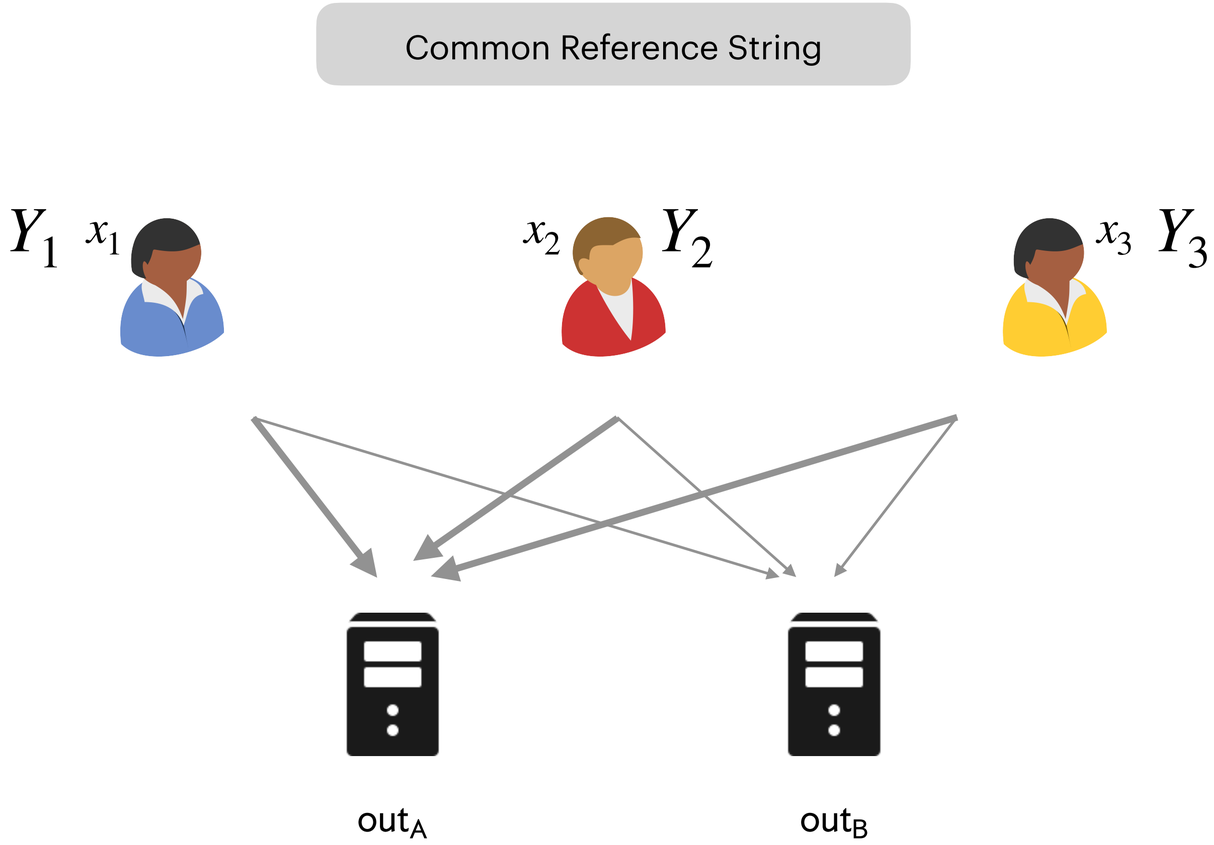


Succinct Client-Server HSS

Succinct multi-client two-server HSS in the CRS model for RMS programs



$$\text{out}_A + \text{out}_B = C(x_1, x_2, x_3, Y_1, Y_2, Y_3)$$

Succinct Client-Server HSS

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DDH, DCR, and class groups

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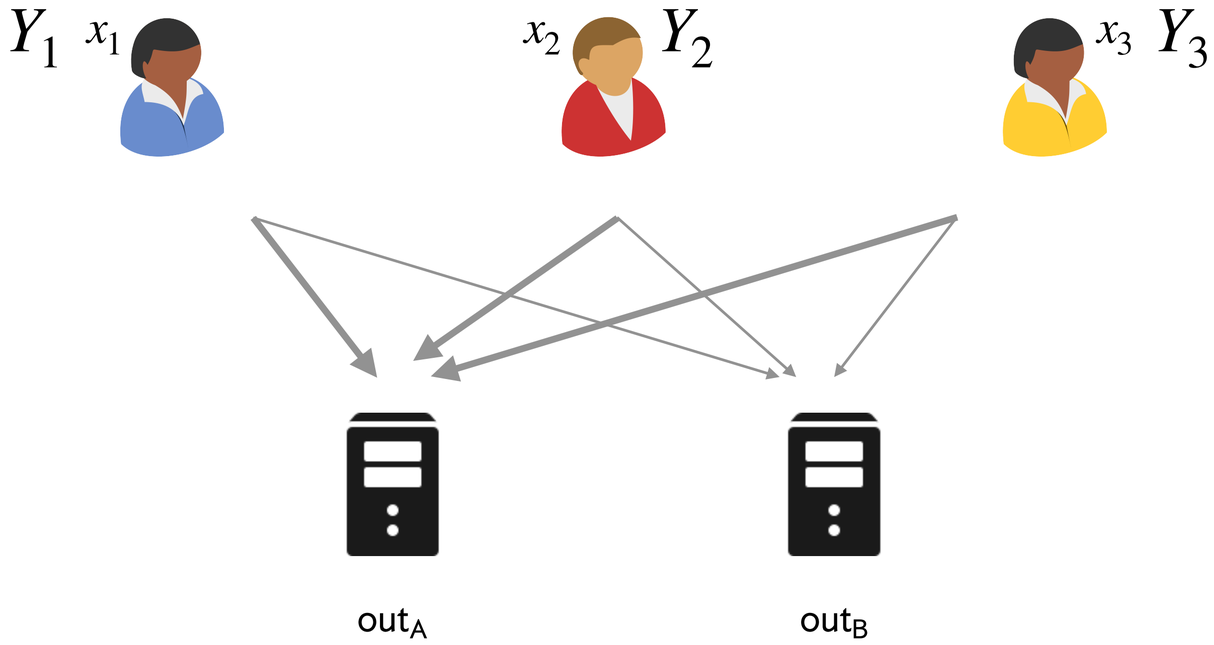

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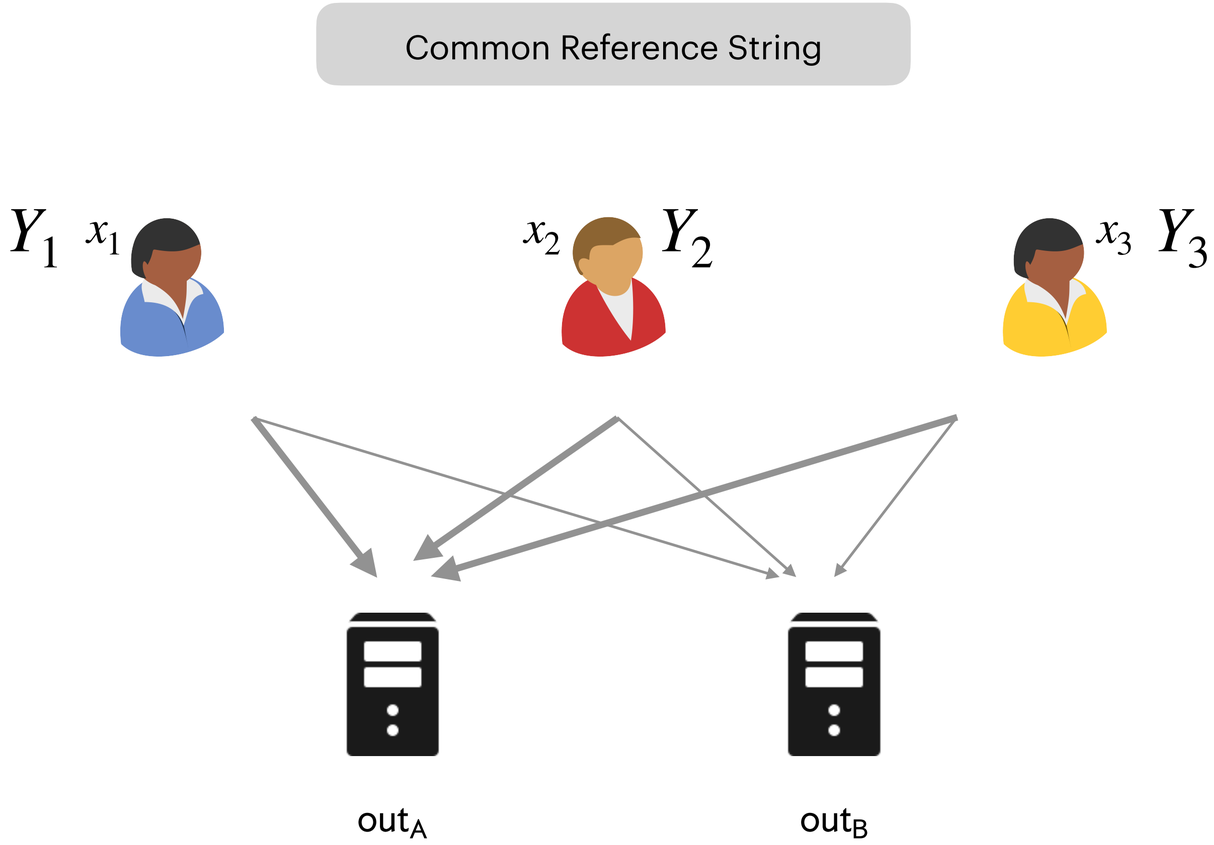
Previously required correlated setup or supported only two parties

[Abram-Roy-Scholl'24]
[Couteau-H-Pu'24]



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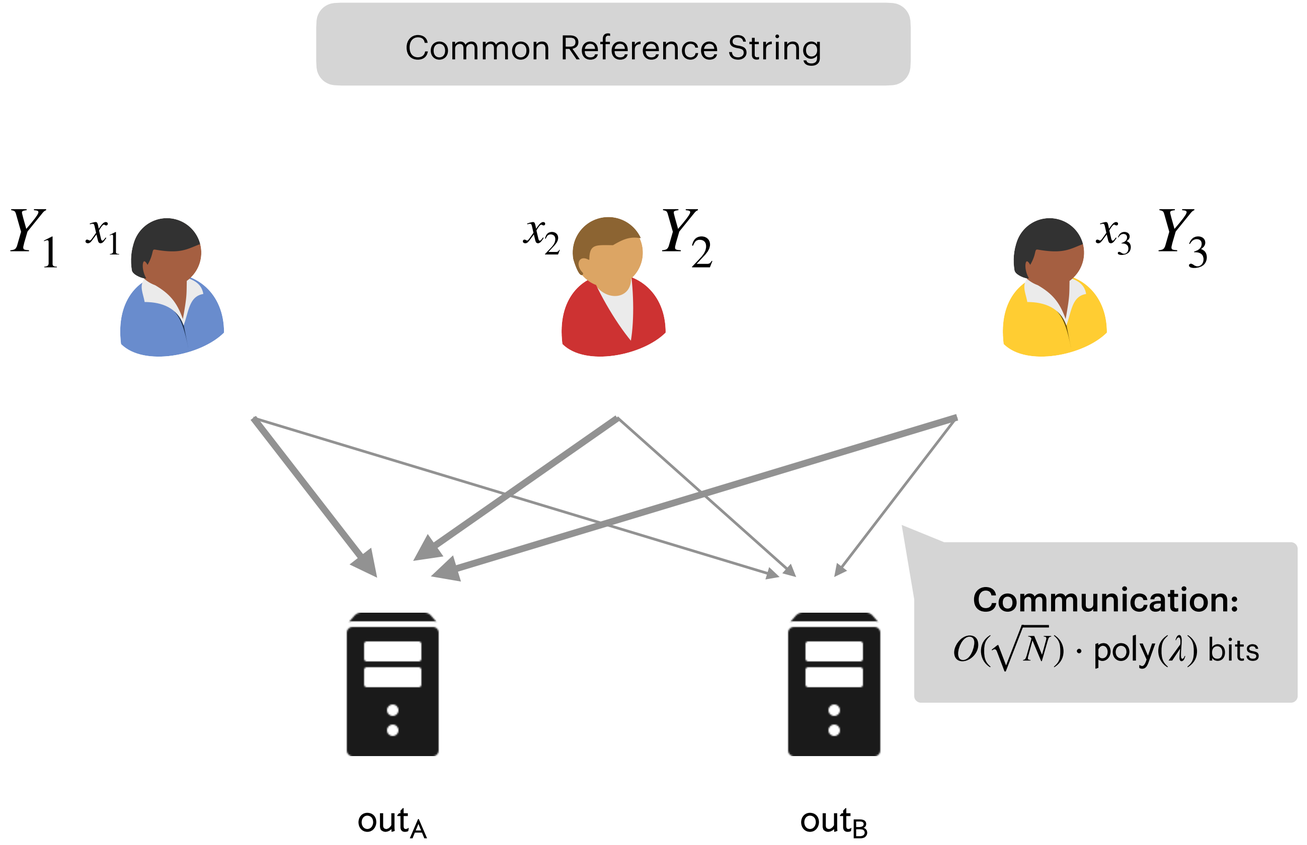

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Private long inputs

$$C \equiv \sum_{i,j} \mathsf{RMS}(x_1, \ldots, x_m) \cdot Y_i^{(j)}$$

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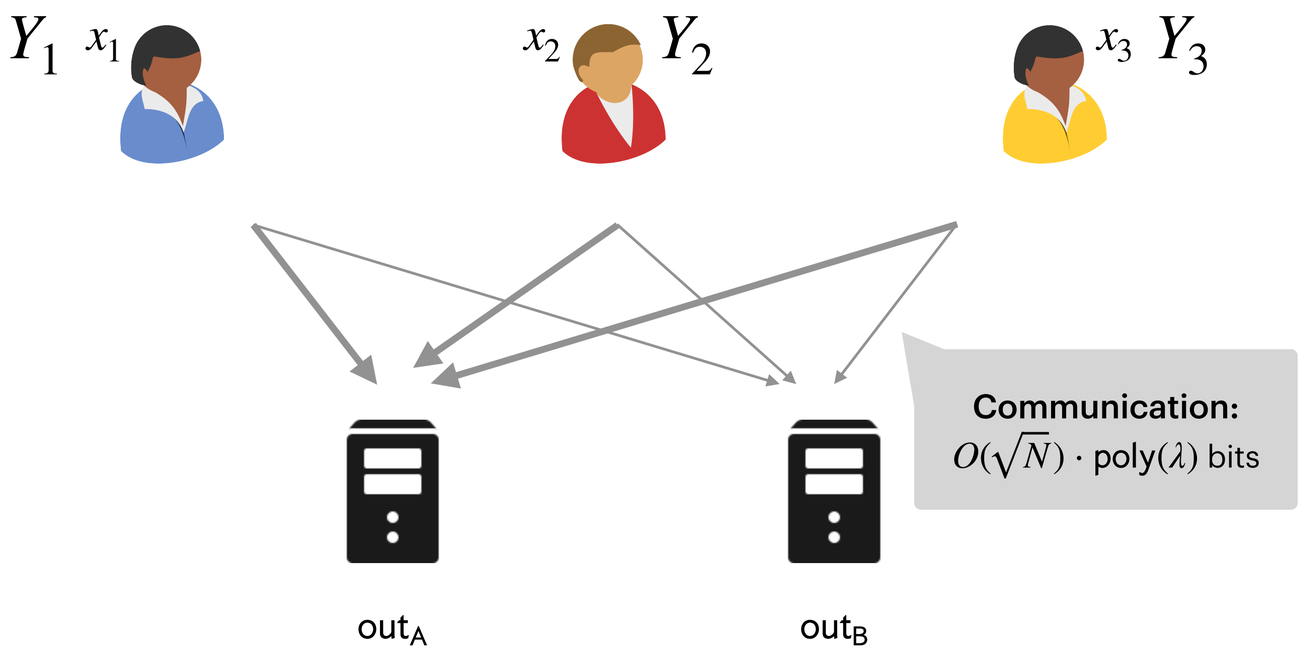

Communication:
$O(\sqrt{N}) \cdot \mathsf{poly}(\lambda)$ bits

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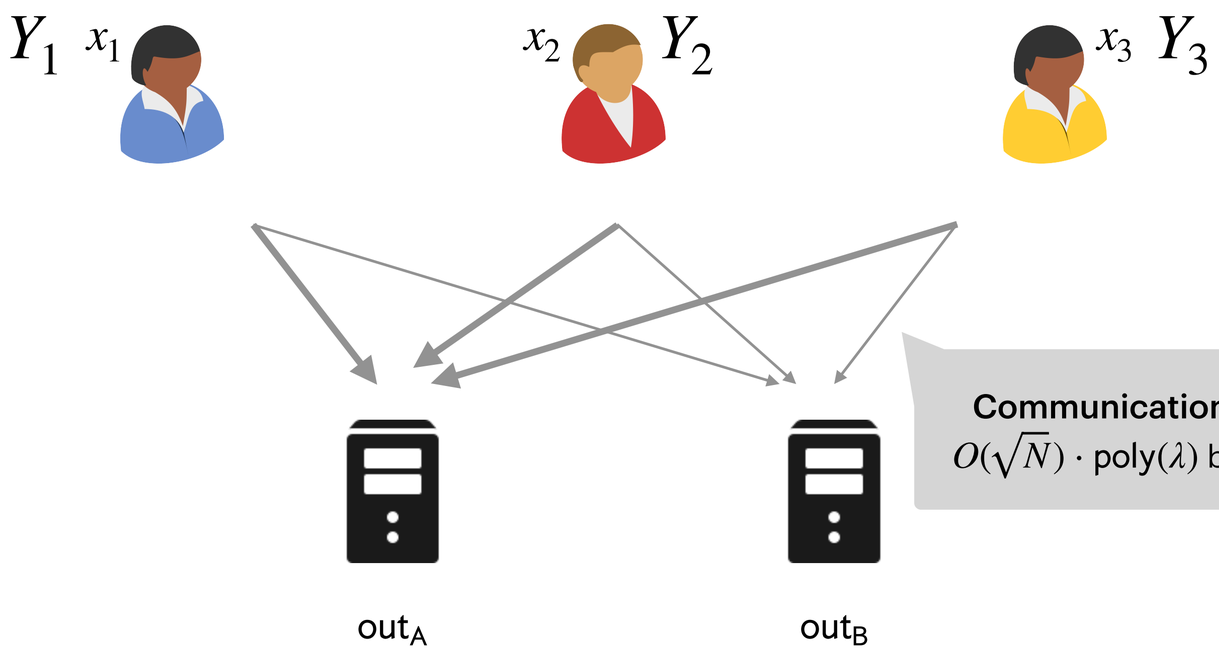

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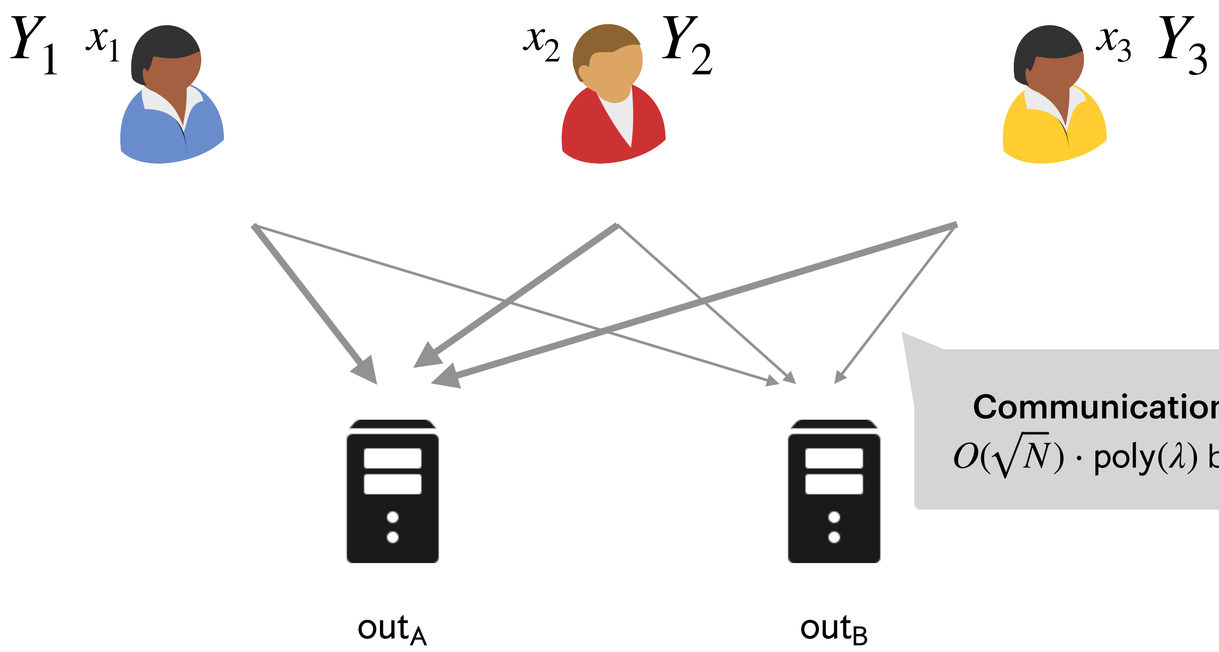

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Key Ingredient: Combine delegation and input-succinctness properties of NIM

Thank You