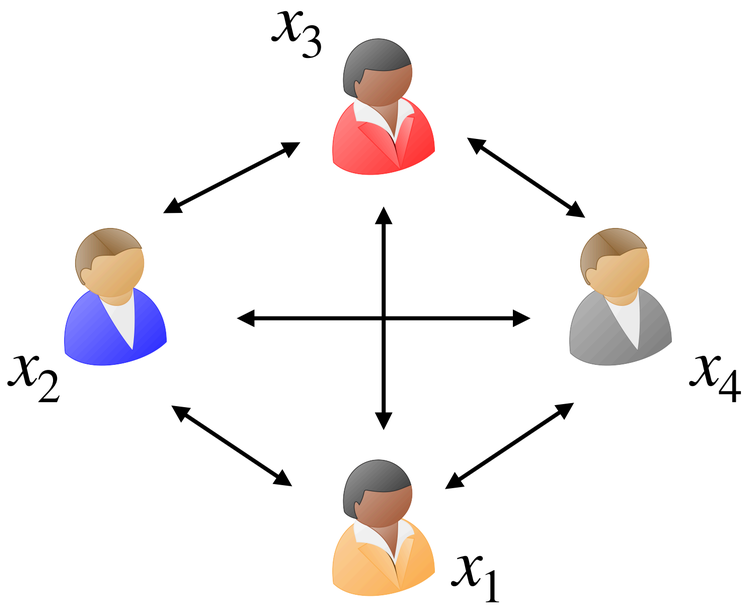
Scalable Multiparty Garbling

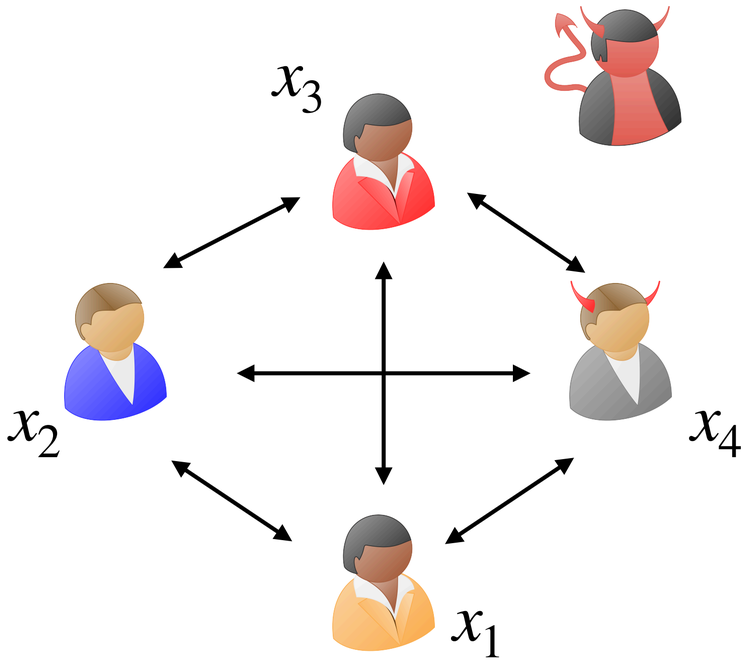
Gabrielle Beck Aarushi Goel **Aditya Hegde** Abhishek Jain Zhengzhong Jin Gabriel Kaptchuk

Secure Multiparty Computation



$$y = f(x_1, x_2, x_3, x_4)$$

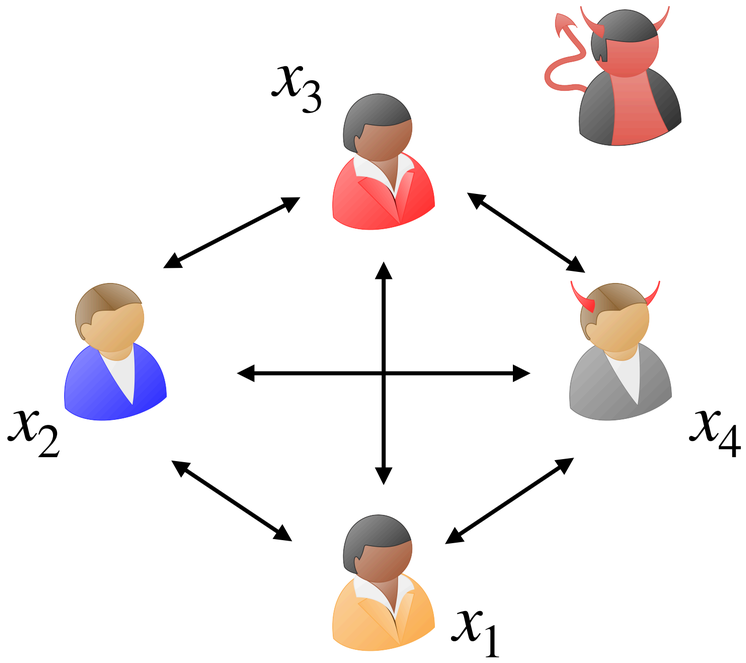
Secure Multiparty Computation



$$y = f(x_1, x_2, x_3, x_4)$$

Secure Multiparty Computation

- Applications
  - Machine learning on distributed datasets.
  - Data as a service: secure access to shared and private data.
  - Securing digital assets and key management.



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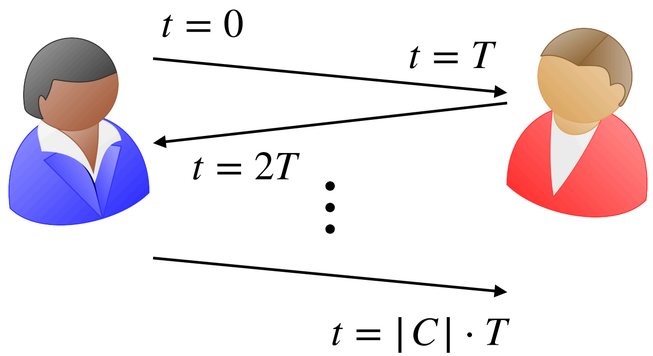
MPC Over The Internet

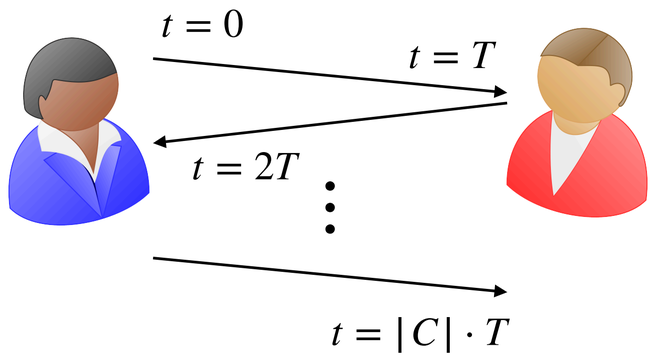
- Over the internet, network latency limits protocol runtime [WRK17b].



MPC Over The Internet

- Over the internet, network latency limits protocol runtime [WRK17b].

- Constant round MPC: Parties interact constant number of times.
  - Supports evaluating large-depth circuits over the internet.
  - 2 Approaches: FHE and Multiparty Garbling.



Constant Round MPC

- Best known constant round MPC has a total communication cost of

$$O(n^2 \cdot |C|)$$

Number of parties

Size of circuit

[HSS17,WRK17b,BCOOSS21].

- In the honest majority setting, non-constant round MPC with a total communication cost of

$$O(|C|)$$

[BGJK21,GSY21,GPS21,GPS22].

Problem: Bringing The Same Efficiency To Multiparty Garbling

Does there exist a multiparty garbling protocol with $O(|C|)$ communication in the honest majority setting?

MPC Protocols With $O(|C|)$ Communication

 Per party communication decreases as the number of parties increases.

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MPC that scales to hundreds and thousands of parties.

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 MPC that scales to hundreds and thousands of parties.

 Honest majority is a more plausible assumption.

Our Contributions

- Scalable multiparty garbling
  - $O(|C|)$ communication complexity
  - Malicious security with abort
  - $t < n\left(\frac{1}{2} - \epsilon\right)$
  - Based on the LPN assumption over large fields
  - Practical: ~700 MB for garbling AES-128 with 250 parties.

Agenda

- Structure Of Garbled Circuit
- Multiparty Garbling Protocol
- Efficiency Analysis

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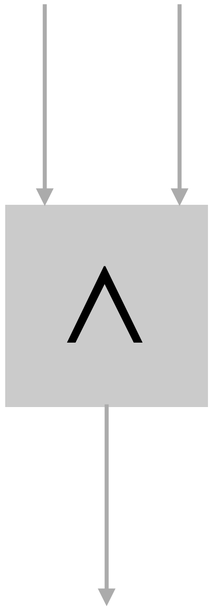
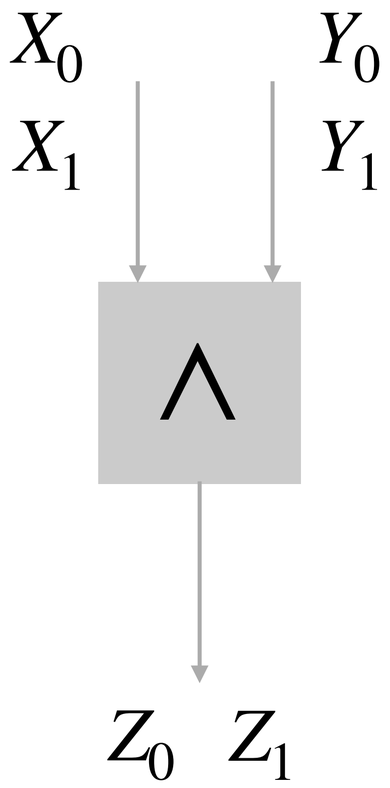
Two Party Garbling: Garbler



Truth Table

0	0	0
0	1	0
1	0	0
1	1	1

Two Party Garbling: Garbler



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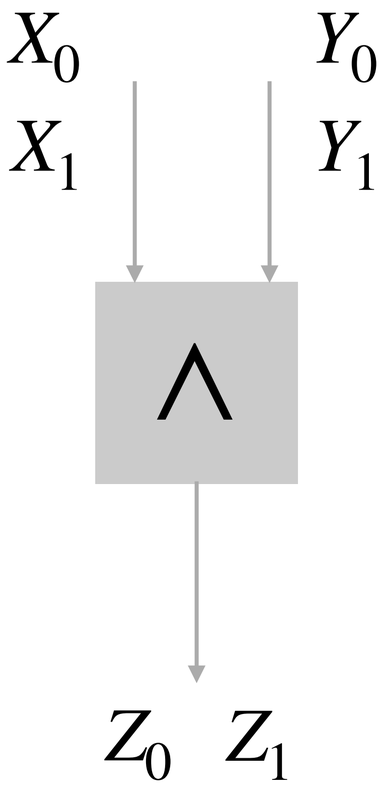

Encrypting The Truth Table

X_0	Y_0	Z_0
X_0	Y_1	Z_0
X_1	Y_0	Z_0
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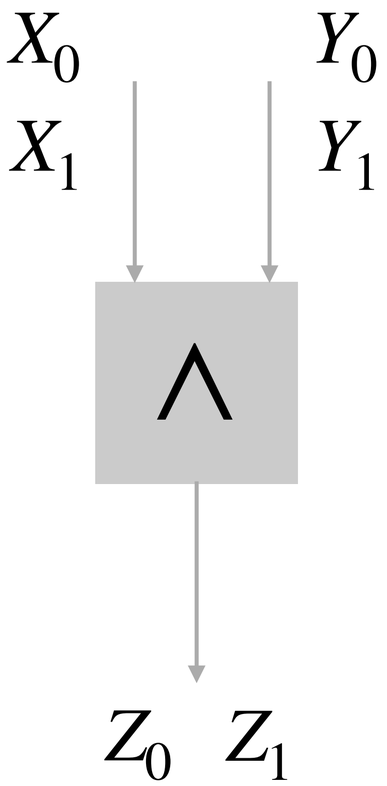

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X_0	Y_0	Z_0	$\text{Enc}_{X_0,Y_0}(Z_0)$
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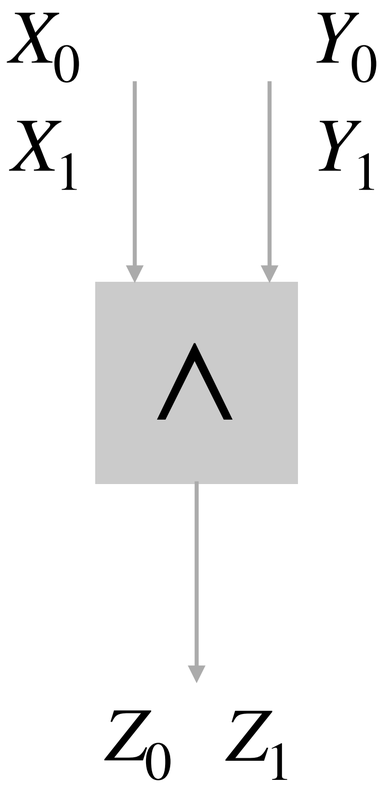

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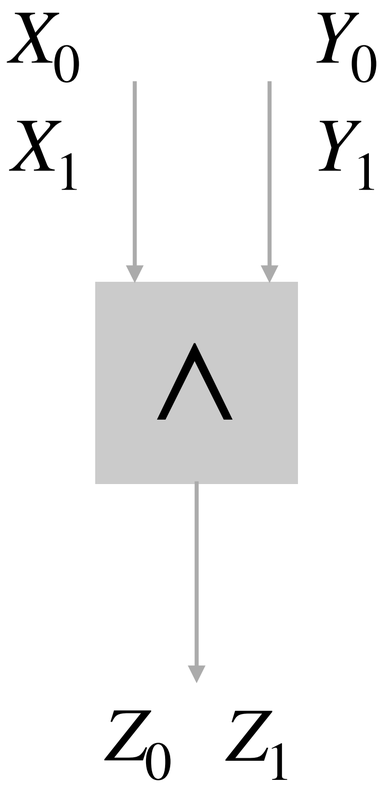

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Shuffled

$\mathscr{G}$

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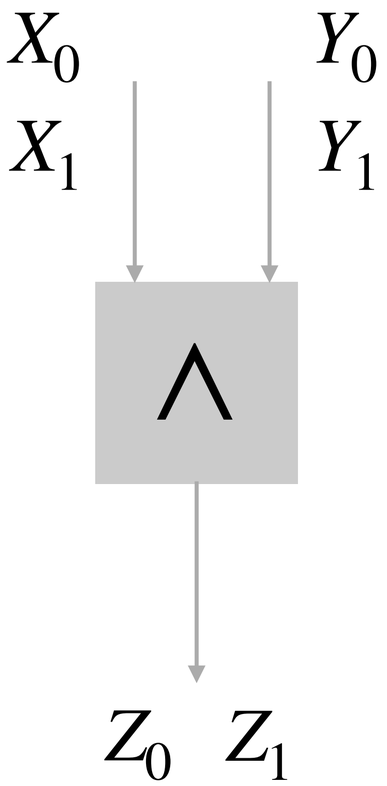

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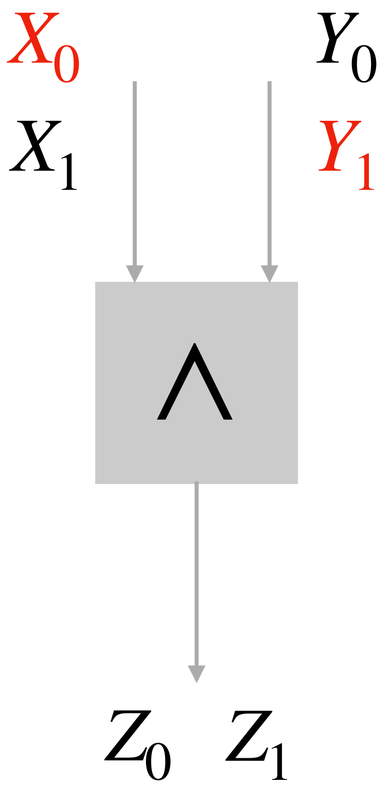
X_1	Y_1	Z_1	$\text{Enc}_{X_1,Y_1}(Z_1)$

Shuffled

Computed and sent to evaluator. $\mathcal{G}$

Two Party Garbling: Evaluator



Garbled Table

X_0	Y_0	Z_0	$\text{Enc}_{X_0,Y_0}(Z_0)$
X_0	Y_1	Z_0	$\text{Enc}_{X_0,Y_1}(Z_0)$
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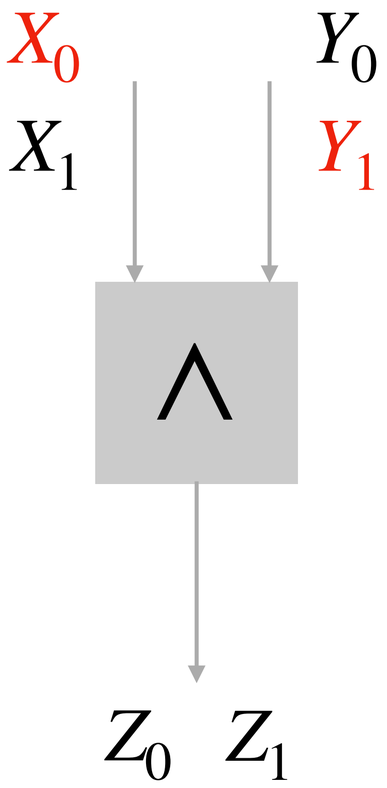

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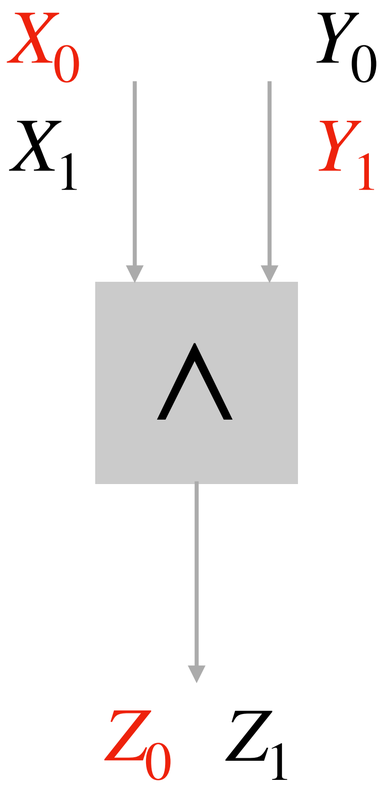

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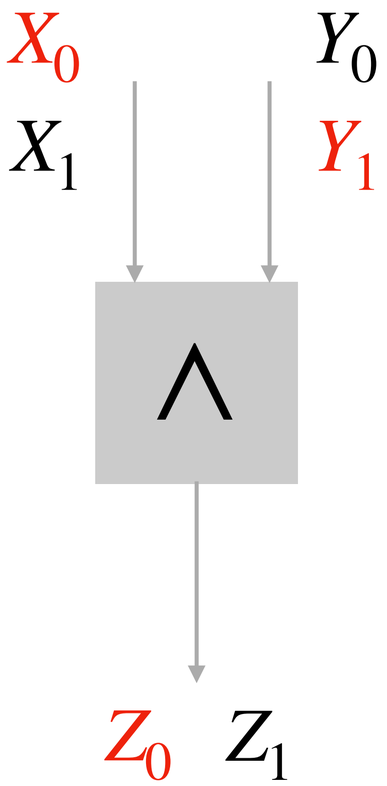

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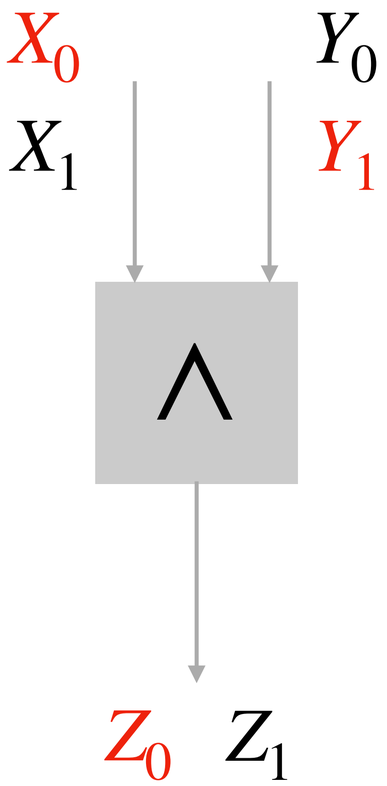
Shuffled

$\mathscr{G}$

Evaluator should learn **exactly one** wire label.

Two Party Garbling: Evaluator



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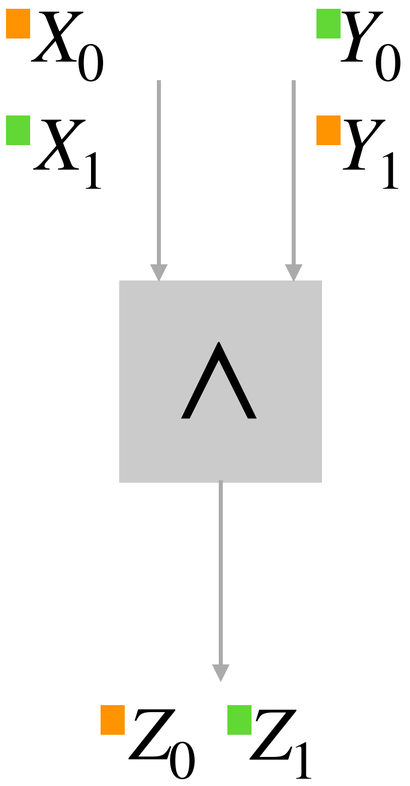
Evaluator should learn **exactly one** wire label.

Which ciphertext to decrypt?

Two Party Garbling: Point And Permute

Color wire labels to indicate which ciphertext to decrypt.



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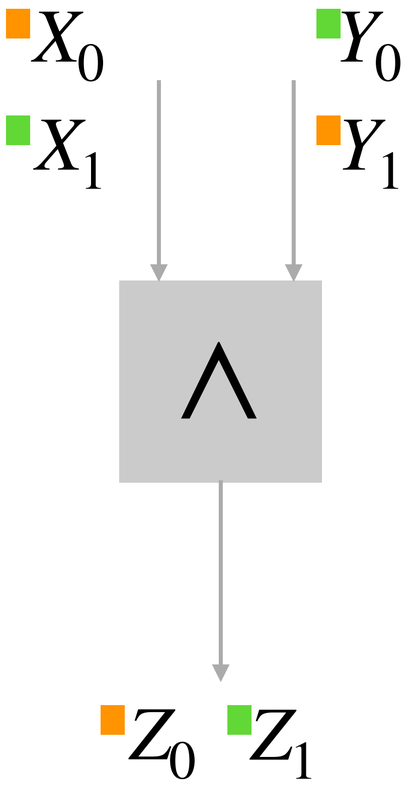

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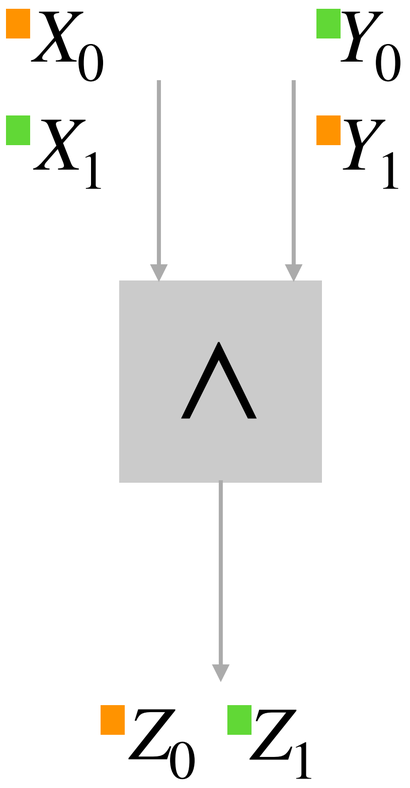

$\rightarrow 0$

$\rightarrow 1$

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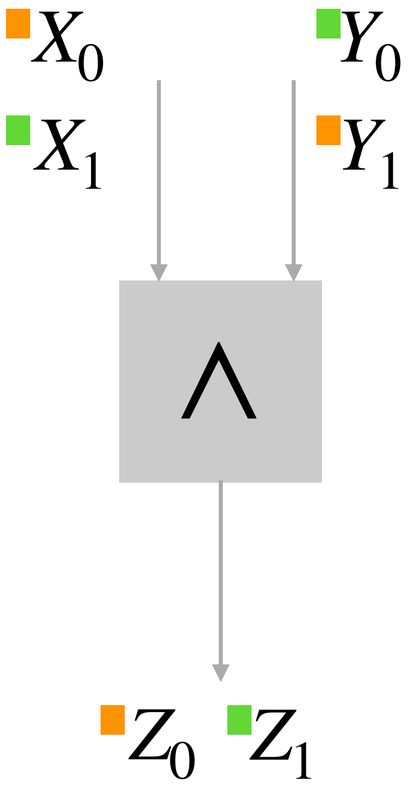

$\blacksquare \rightarrow 0$

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$\mathcal{G}$

Coloring is independent of wire label assignment.

A Template For Multiparty Garbling [BMR90]

Non-constant round MPC → Constant round MPC

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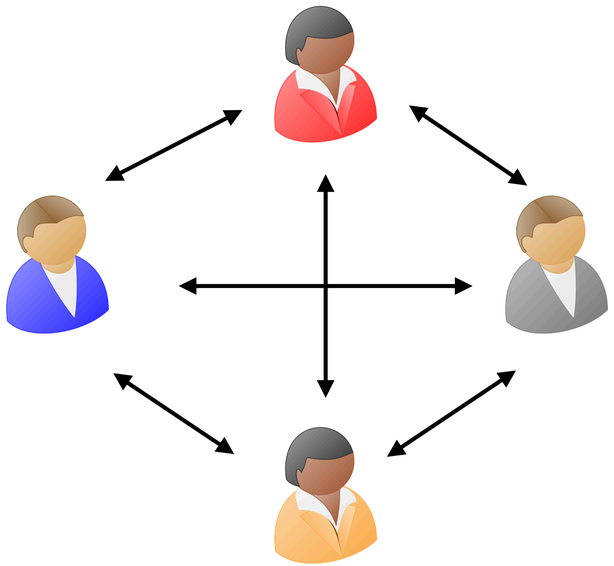
Garbling Phase



$\mathscr{G} \quad X_0, Y_1$

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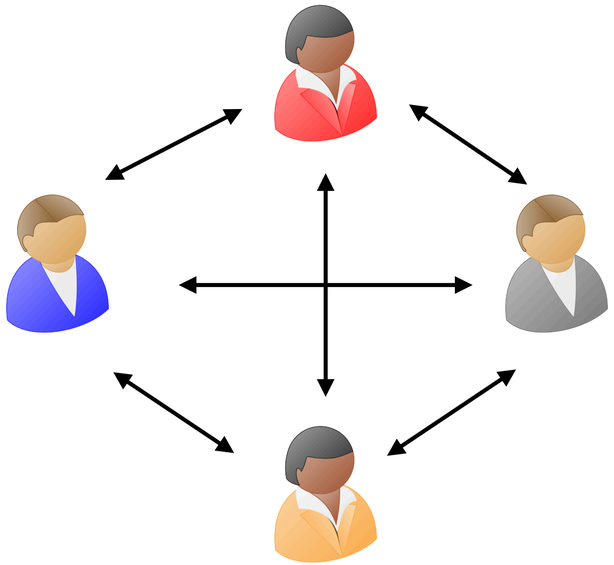

$\mathscr{G} \quad X_0, Y_1$

Evaluation Phase

$\mathsf{Eval}(\mathscr{G}, X_0, Y_1)$

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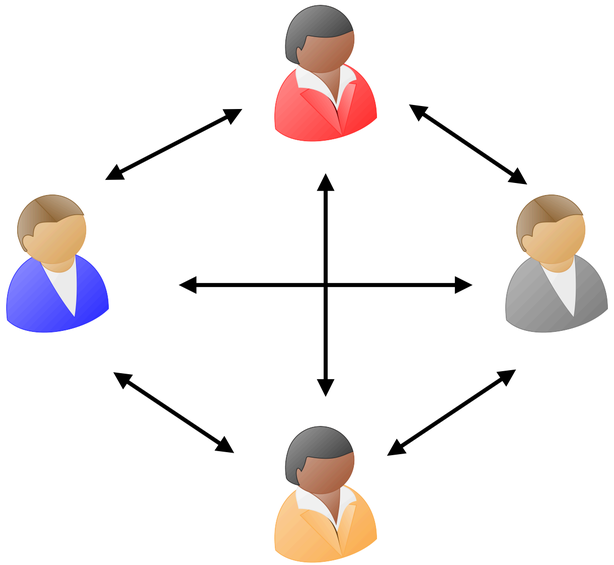

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- MPC ensures wire labels are private.
- All gates can be garbled in parallel $\Longrightarrow$ constant round MPC.

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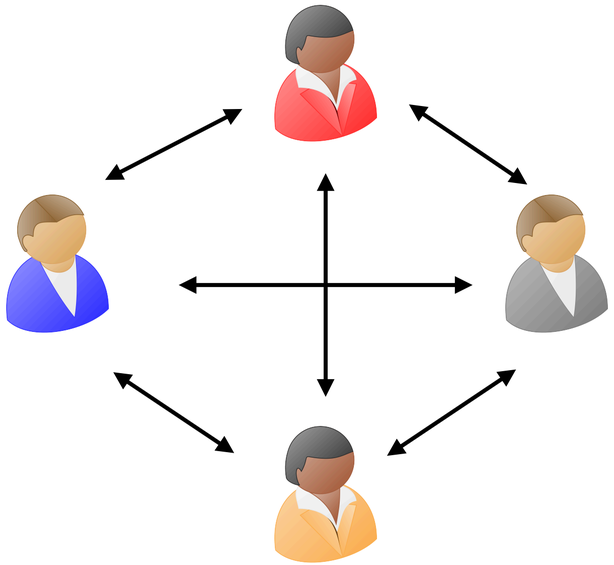

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- MPC ensures wire labels are private.
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$O(|C|)$ MPC for garbling $\Longrightarrow$ $O(|C|)$ constant round MPC?

Non-black box use of encryption.

Moving Away From Non-black Box Constructions

Garbling g^{th} gate: $\text{Enc}_{X,Y}(Z) = \text{PRF}_X(g) \oplus \text{PRF}_Y(g) \oplus Z$

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Evaluate the PRF outside the MPC [DI05] $\rightarrow$ $|\mathcal{G}|$ is linear in n.

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X_1 Y_1 X_2 Y_2 X_3 Y_3

$X = X_1 \| X_2 \| X_3$ $Y = Y_1 \| Y_2 \| Y_3$

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X_1 Y_1 X_2 Y_2 X_3 Y_3

$X = X_1 \| X_2 \| X_3 \qquad Y = Y_1 \| Y_2 \| Y_3$

$\text{PRF}_X(g) = PRF_{X_1}(g) \oplus PRF_{X_2}(g) \oplus PRF_{X_3}(g)$

Each party can locally evaluate PRF.

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X_1 Y_1 X_2 Y_2 X_3 Y_3

$X = X_1 \| X_2 \| X_3$ $Y = Y_1 \| Y_2 \| Y_3$

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Each party can locally evaluate PRF.

Wire labels and hence encryptions are linear in n.

Moving Away From Non-black Box Constructions

Garbling g^{th} gate: $\text{Enc}_{X,Y}(Z) = \text{PRF}_X(g) \oplus \text{PRF}_Y(g) \oplus Z$

Evaluate the PRF outside the MPC [DI05] $\rightarrow$ $|\mathcal{G}|$ is linear in n.

Use key-homomorphic PRFs [BLO17] $\rightarrow$ $|\mathcal{G}|$ independent of n.

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X_1 Y_1 X_2 Y_2 X_3 Y_3

$X = X_1 + X_2 + X_3$ $Y = Y_1 + Y_2 + Y_3$

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X_1 Y_1 X_2 Y_2 X_3 Y_3

Homomorphism operator

$X = X_1 + X_2 + X_3$ $Y = Y_1 + Y_2 + Y_3$

$\text{PRF}_{X_1}(g) \tilde{+} \text{PRF}_{X_2}(g) \tilde{+} \text{PRF}_{X_3}(g) = \text{PRF}_X(g)$

Each party can locally evaluate PRF on its share.

Moving Away From Non-black Box Constructions

Garbling g^{th} gate: $\text{Enc}_{X,Y}(Z) = \text{PRF}_X(g) \oplus \text{PRF}_Y(g) \oplus Z$

Evaluate the PRF outside the MPC [DI05] $\rightarrow$ $|\mathcal{G}|$ is linear in n.

Use key-homomorphic PRFs [BLO17] $\rightarrow$ $|\mathcal{G}|$ independent of n.



X_1 Y_1 X_2 Y_2 X_3 Y_3

$X = X_1 + X_2 + X_3$ $Y = Y_1 + Y_2 + Y_3$

Wire labels independent of n.

Homomorphism operator

$\text{PRF}_{X_1}(g) \tilde{+} \text{PRF}_{X_2}(g) \tilde{+} \text{PRF}_{X_3}(g) = \text{PRF}_X(g)$

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Does not extend to threshold setting.

Garbling Using Doubly Linear Homomorphic PRFs

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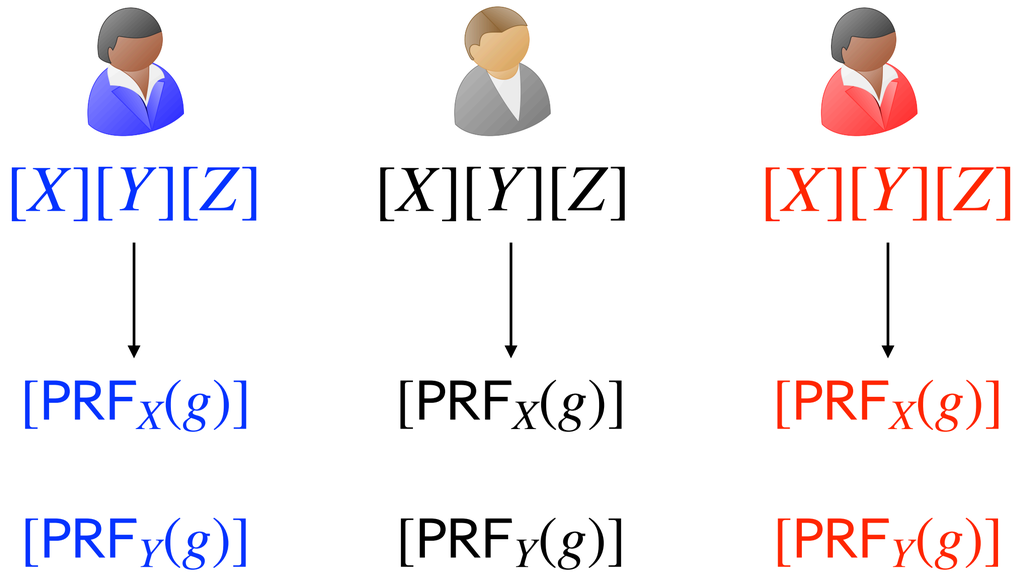

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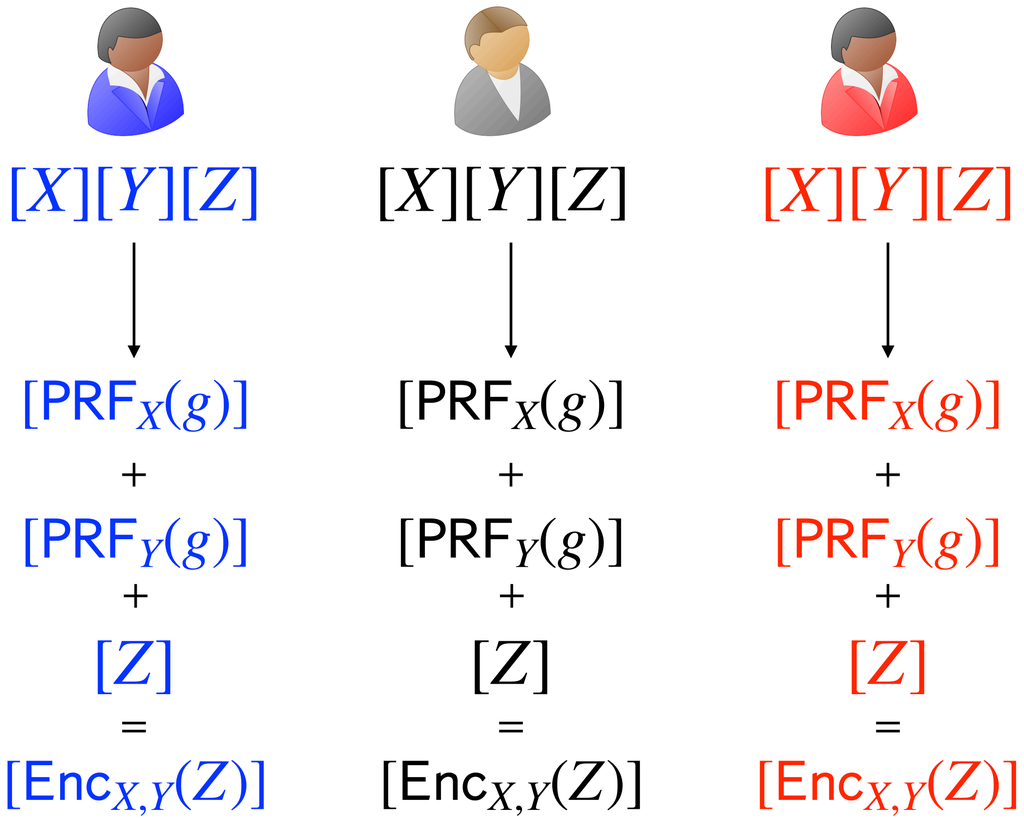

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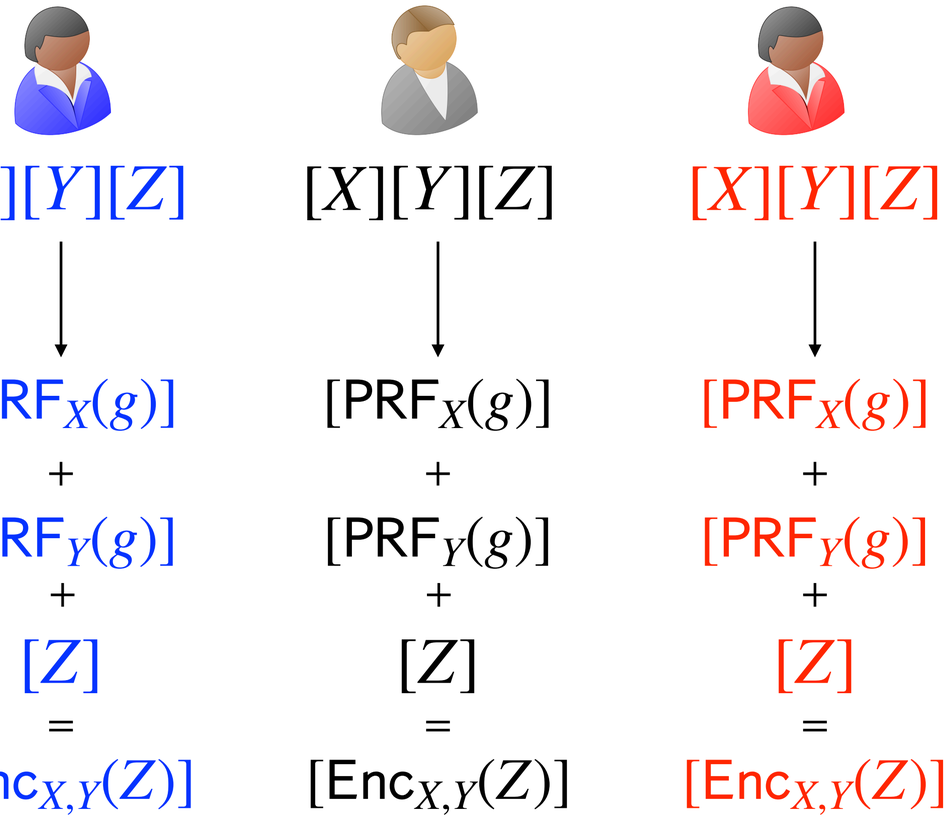

No known constructions of DLH-PRFs.

Garbling Using LPN

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[BCOOSS21] used LPN for garbling in the dishonest majority setting.

Agenda

- Structure Of Garbled Circuit
  - Secret sharing of LPN keys and errors → secret sharing of ciphertext.
- Multiparty Garbling Protocol
- Efficiency analysis

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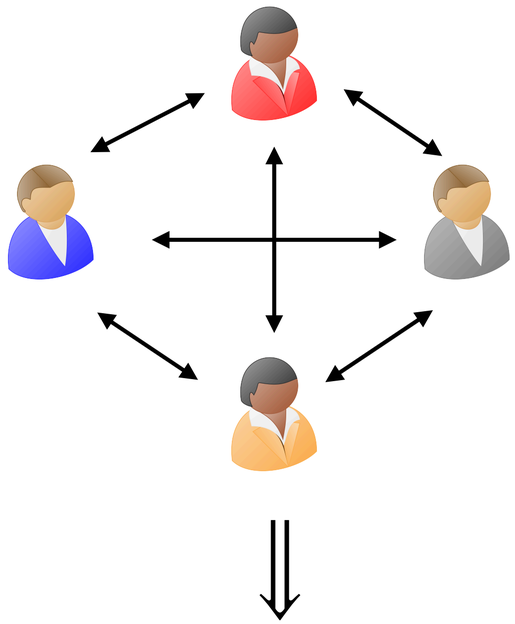
Computing The Garbled Circuit

Pre-processing Phase



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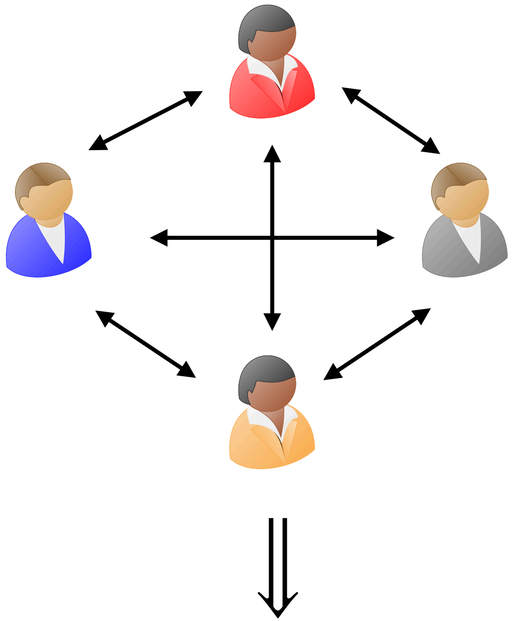

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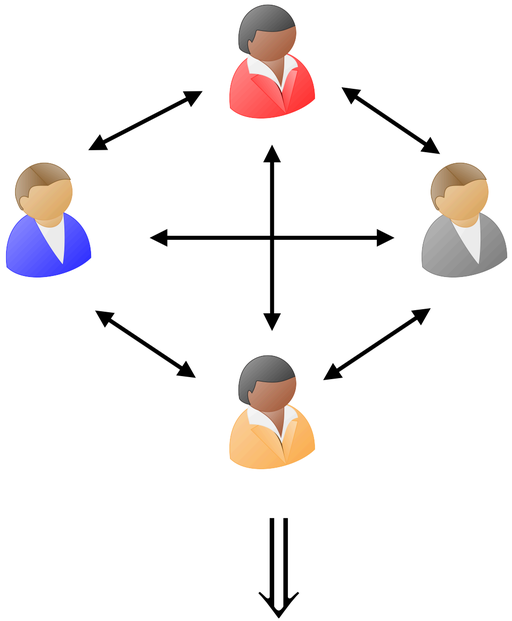

$\mathcal{G}$ - Garbled circuit

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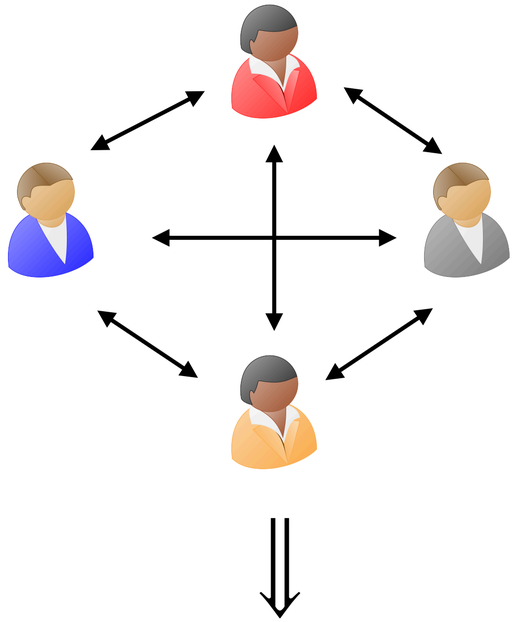

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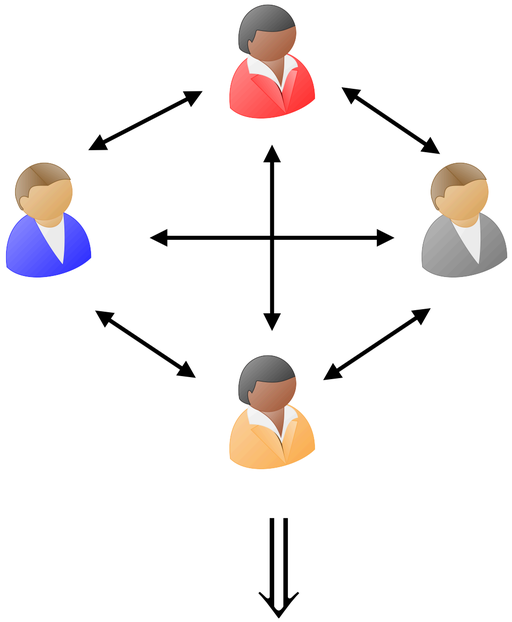

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Packed Secret Sharing [FY92]

Regular Secret Sharing

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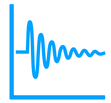
v 

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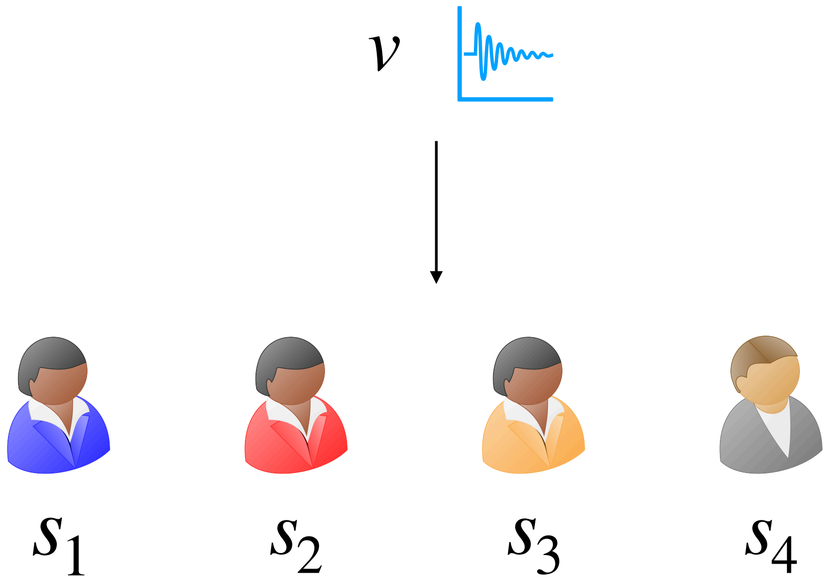

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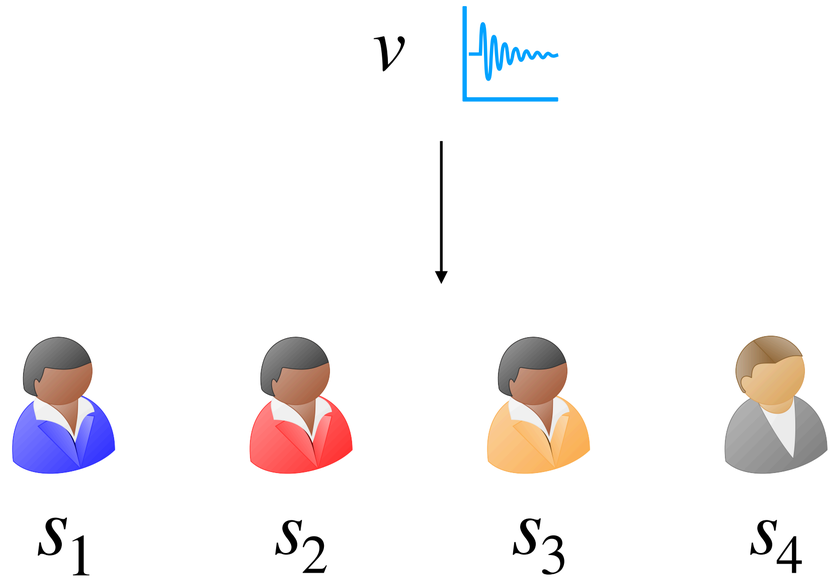

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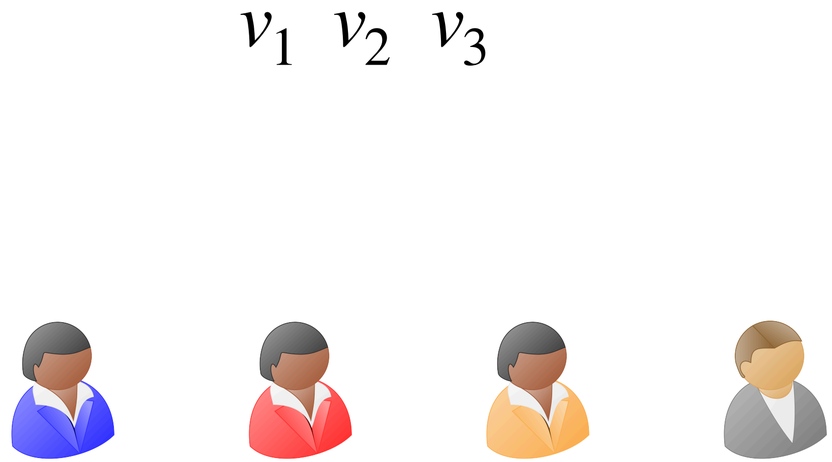

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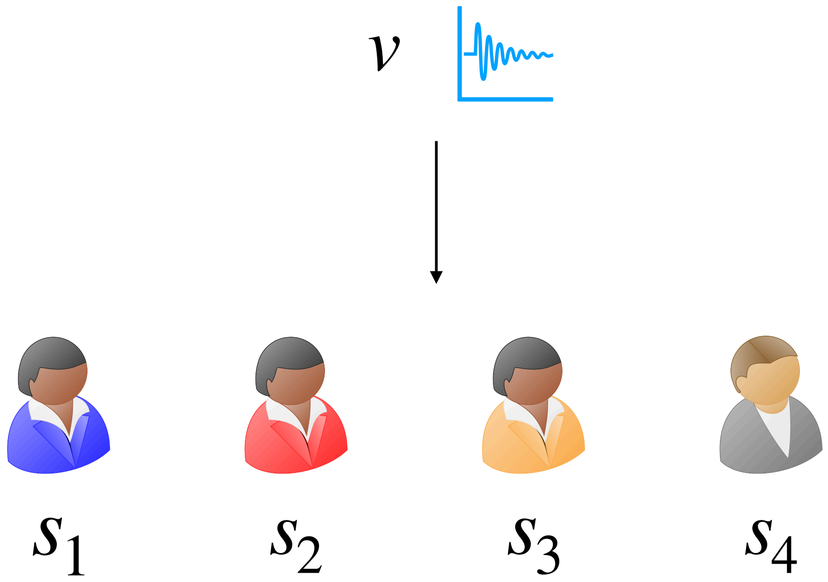

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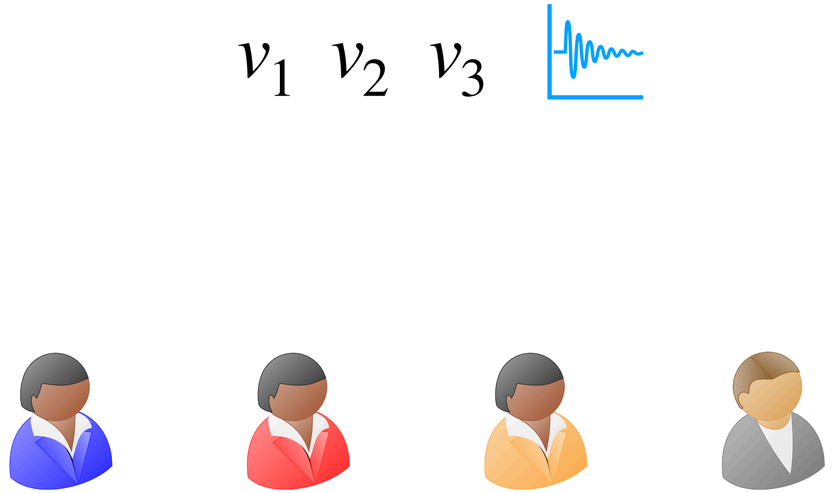

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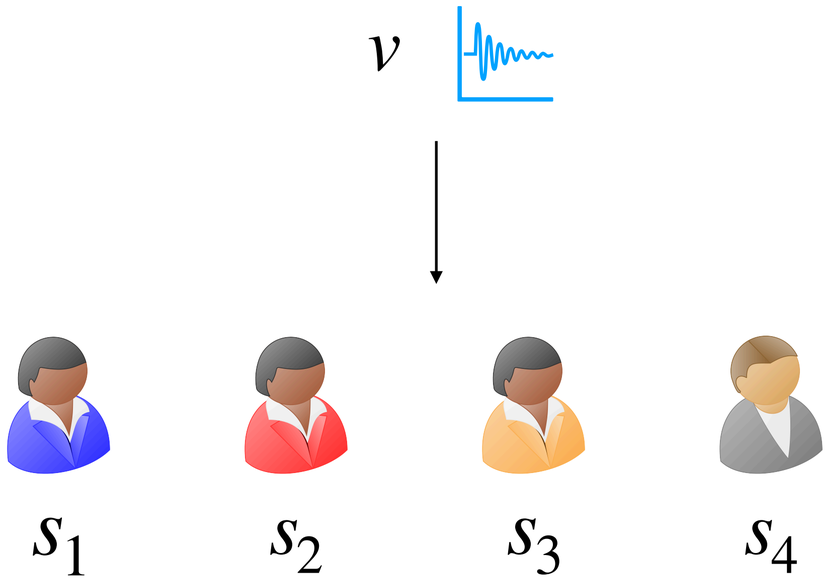

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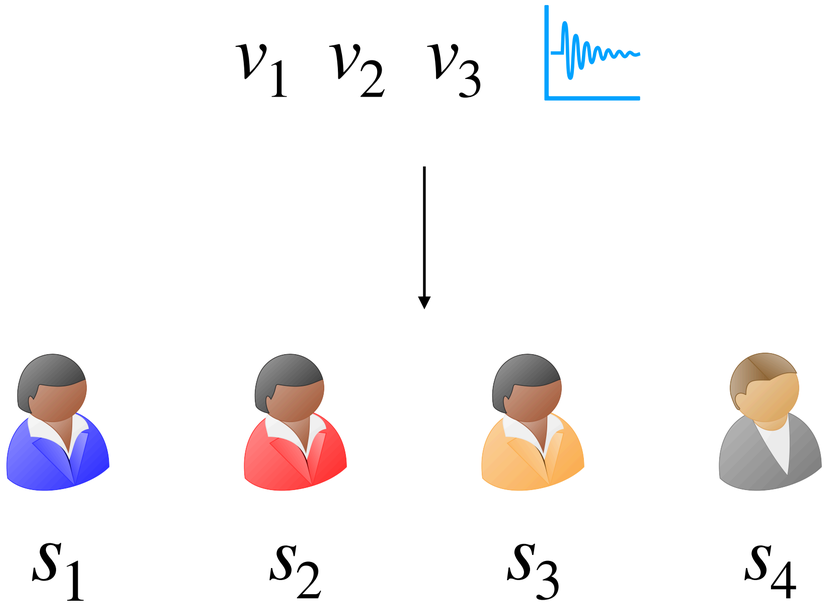

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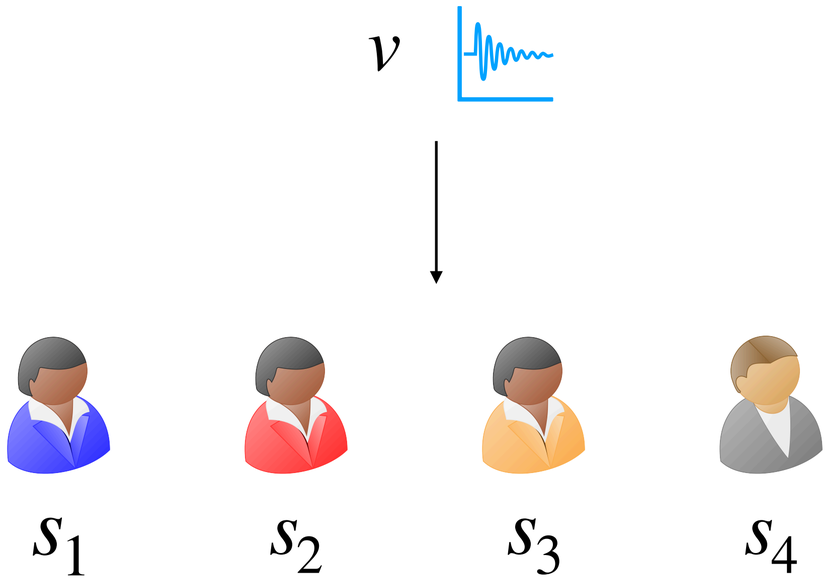

n shares → 1 secret

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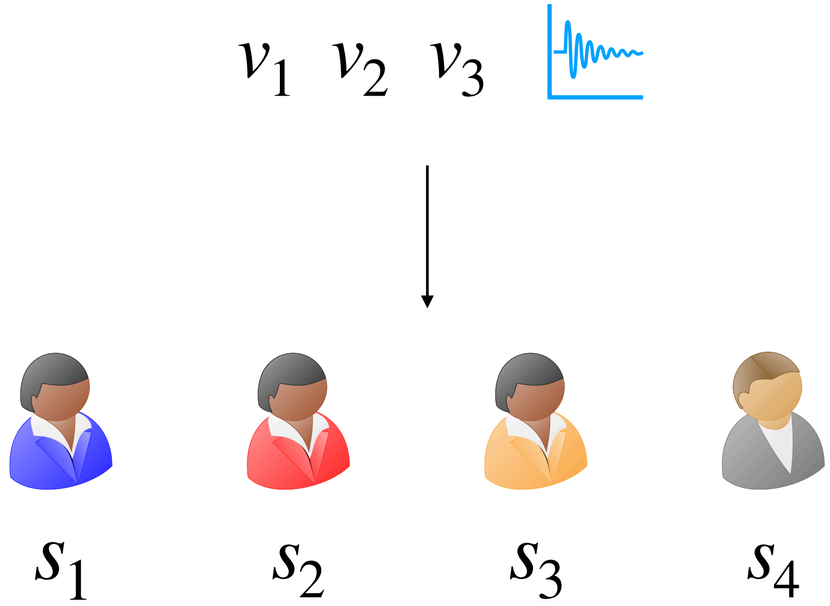

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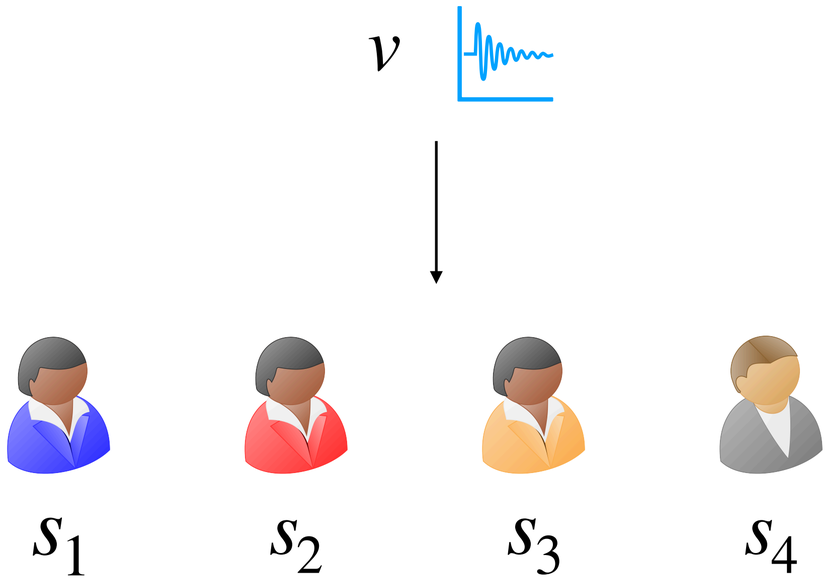

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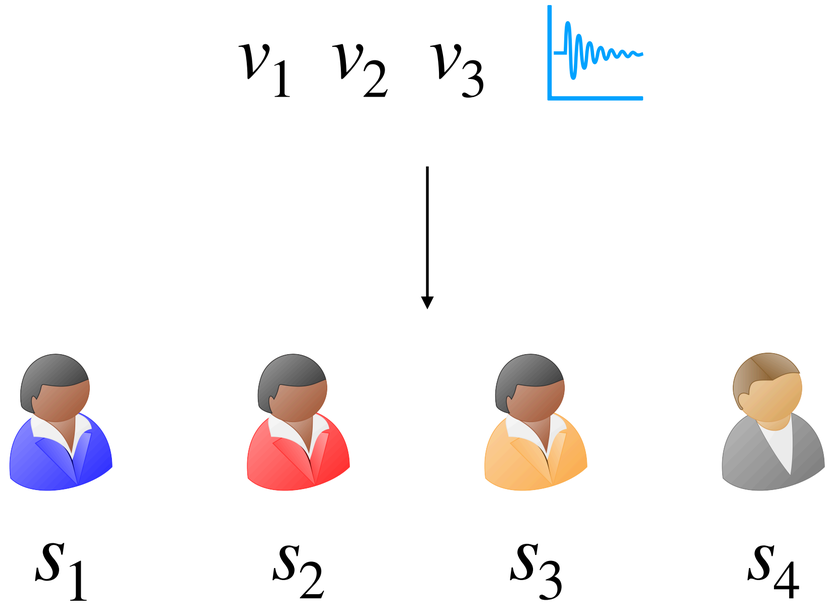

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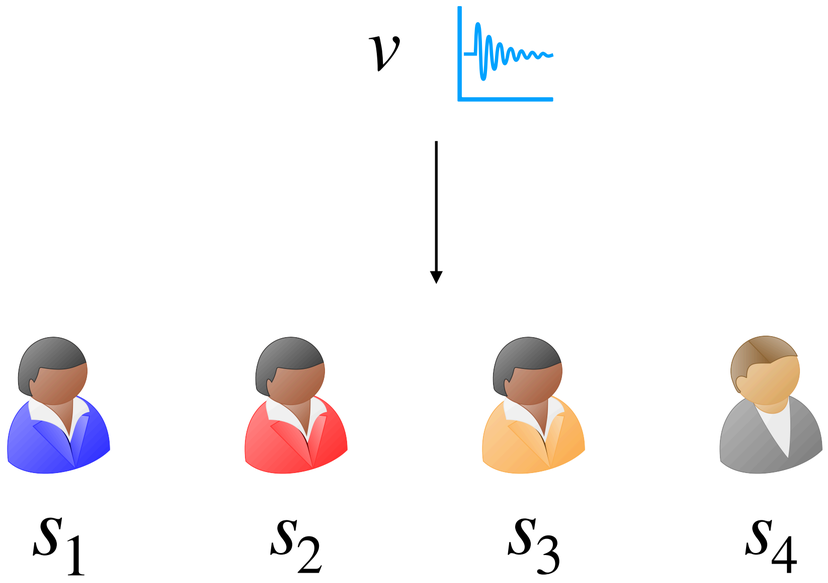

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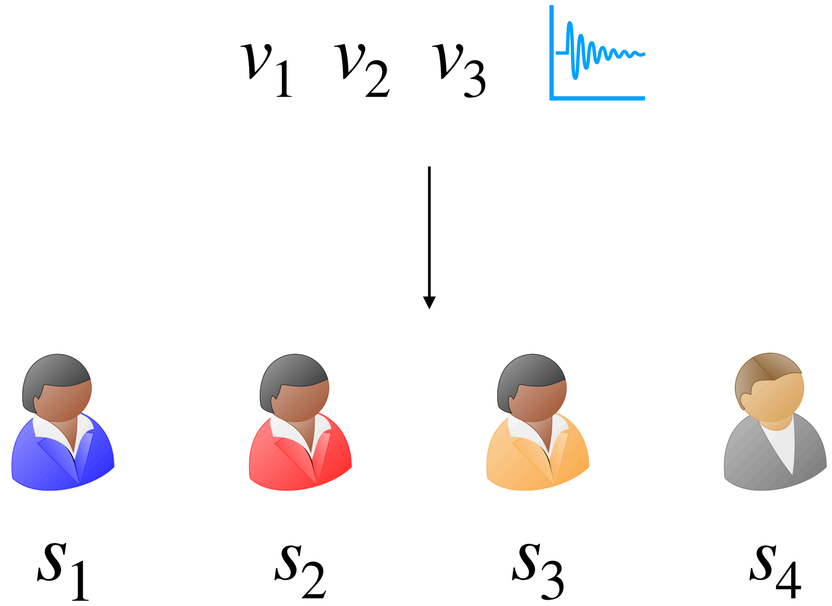

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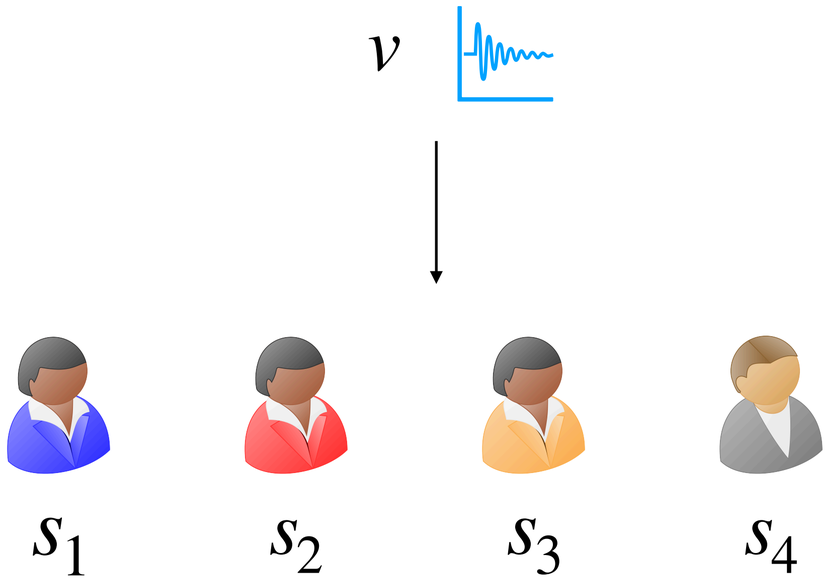

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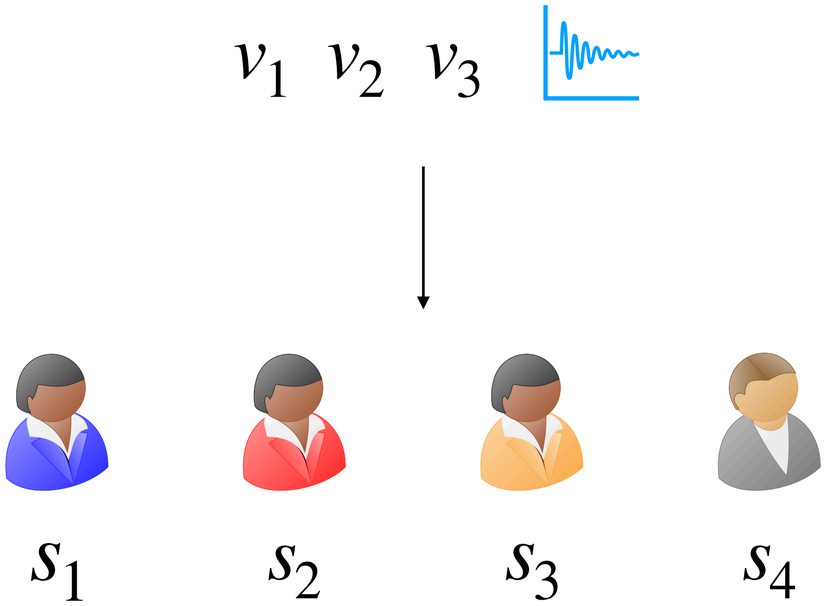

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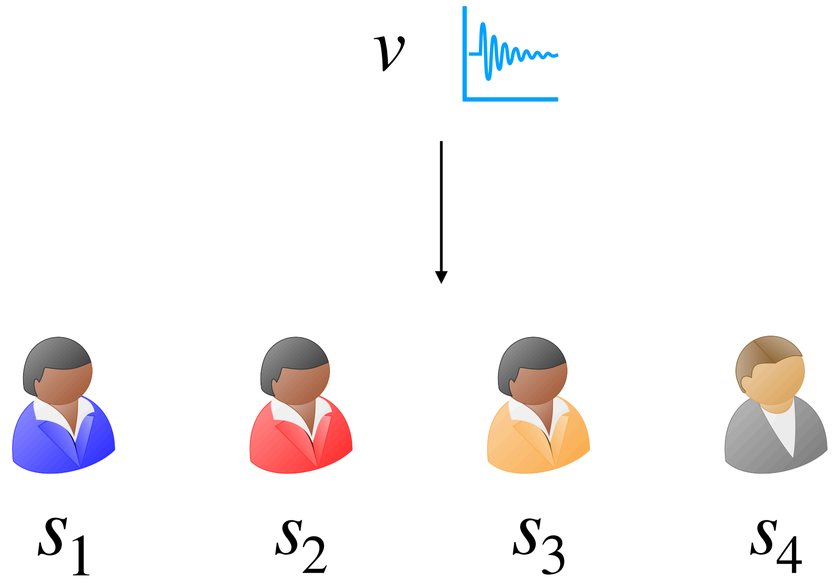

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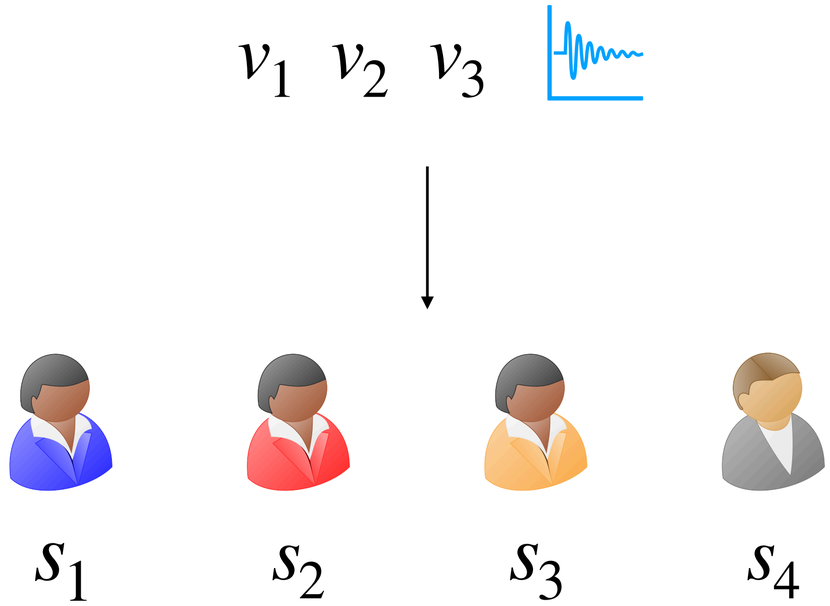

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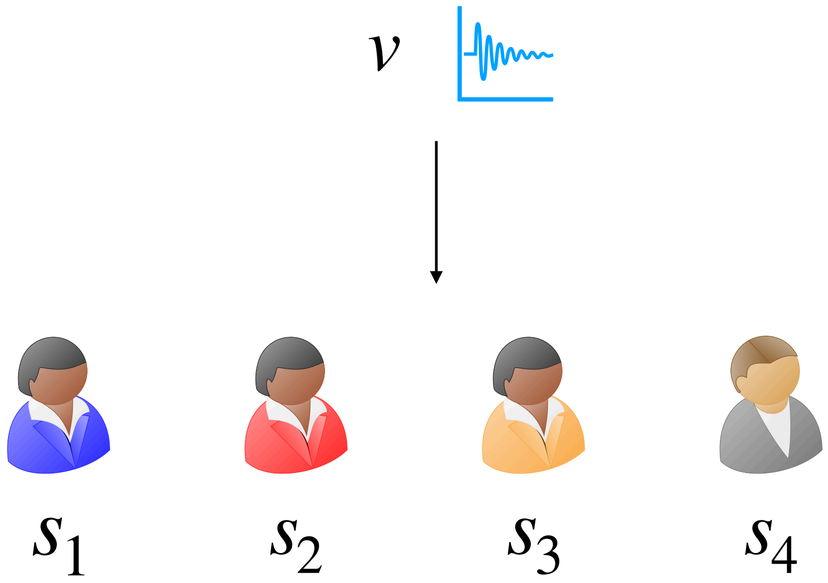

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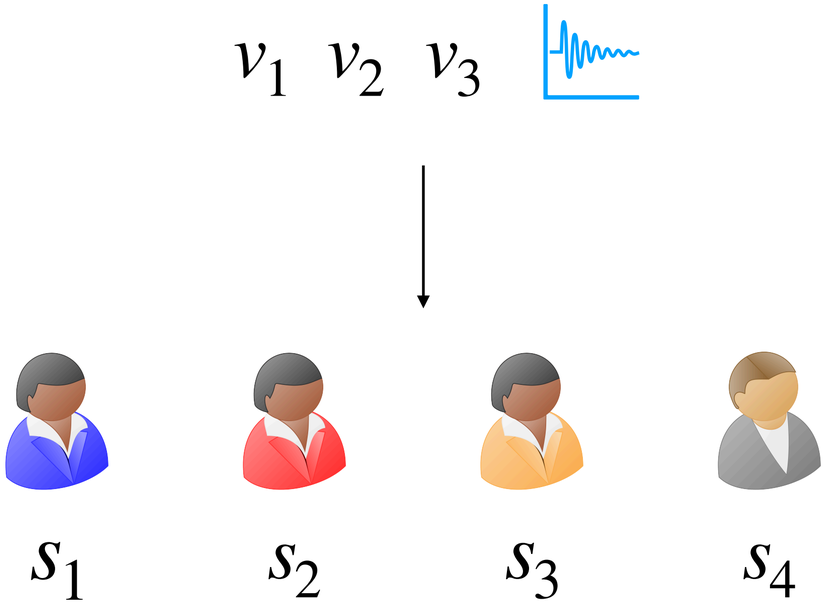

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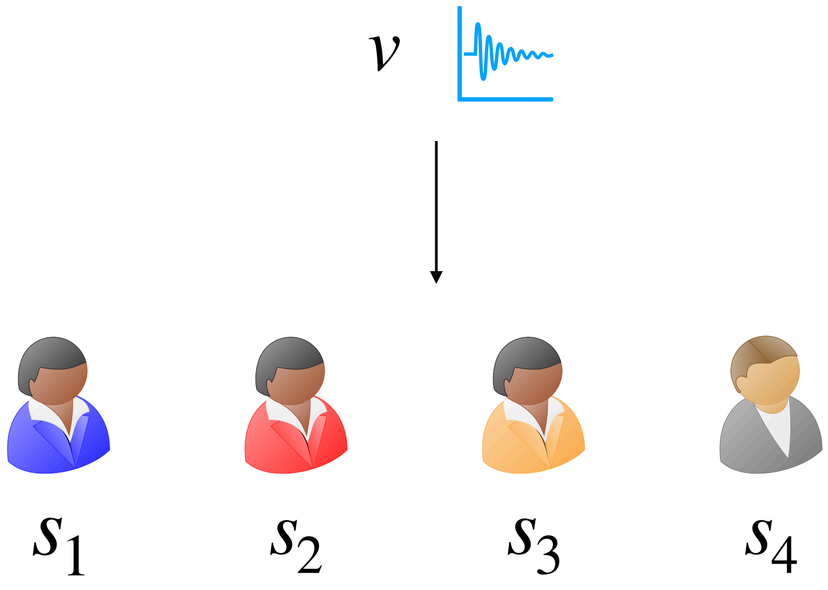

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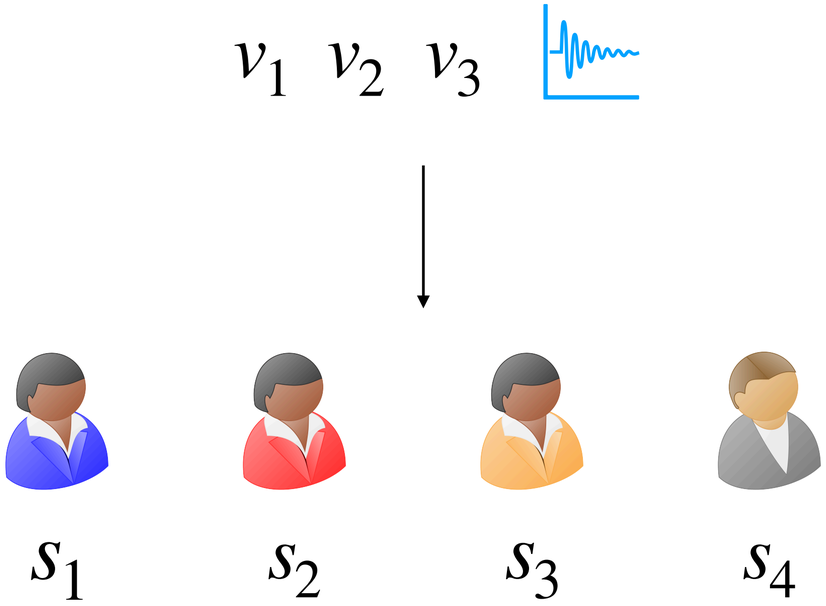

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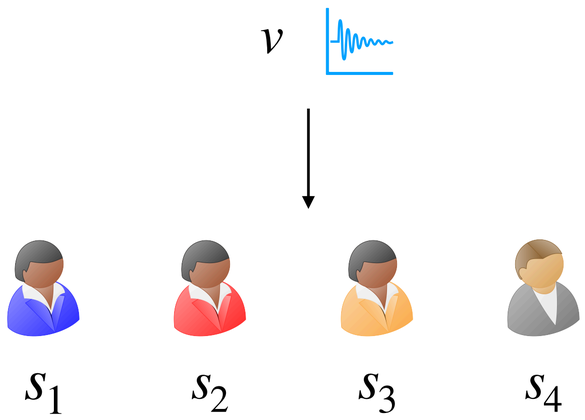
Cannot reconstruct $\leq t$ $\quad t + \ell \leq$ Can reconstruct

$$|\mathbb{F}| > n + \ell$$

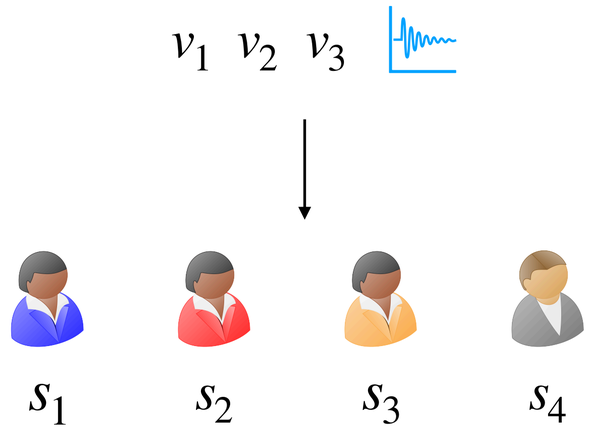
MPC Using Packed Secret Sharing

Regular Sharing



$O(n\,|\,C\,|\,)$ communication MPC

Packed Sharing



$\ell = O(n) \Longrightarrow O(\,|\,C\,|\,)$ communication MPC

MPC Using Packed Secret Sharing

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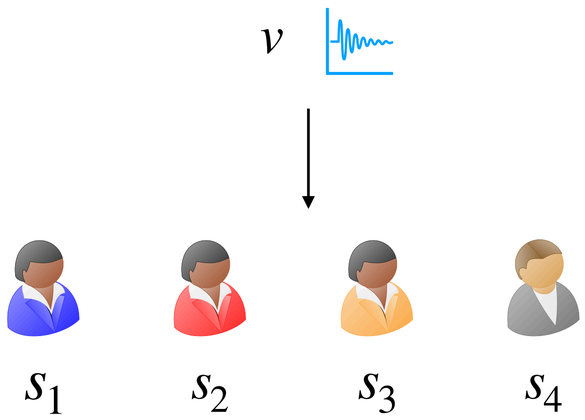
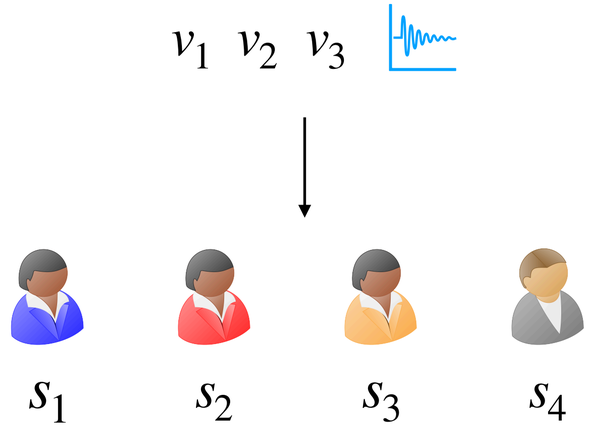


$O(n\,|\,C\,|\,)$ communication MPC

$$t = \frac{n-1}{2}$$

Packed Sharing

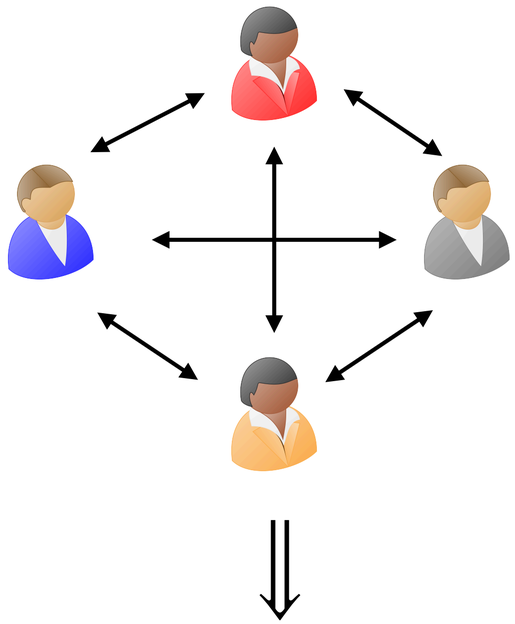


$\ell = O(n) \Longrightarrow O(\,|\,C\,|\,)$ communication MPC

$$t = \left(\frac{1}{2} - \epsilon\right) \cdot n$$

Computing The Garbled Circuit

Pre-processing Phase



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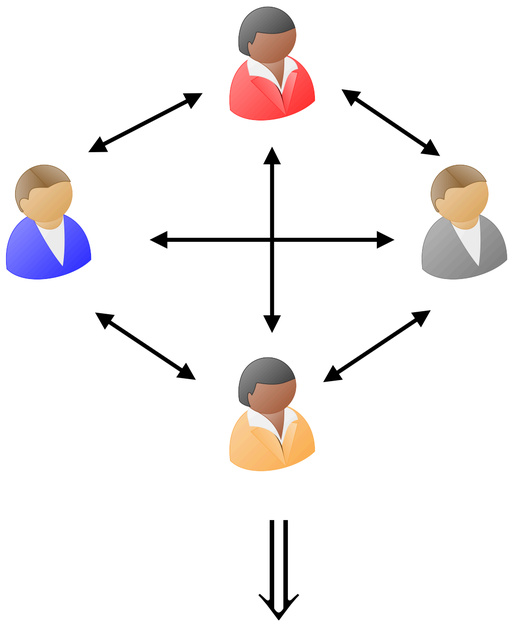
Garbling Phase



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$\{X_{\text{inp}}\}$ - Wire labels for inputs

Evaluation Phase

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Computing Shares of Wire Labels

- Wire labels are LPN keys.
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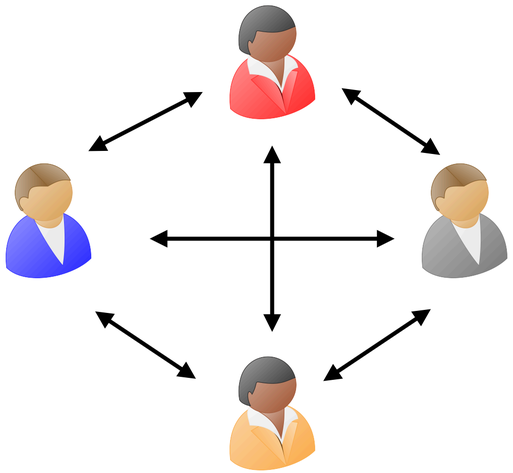

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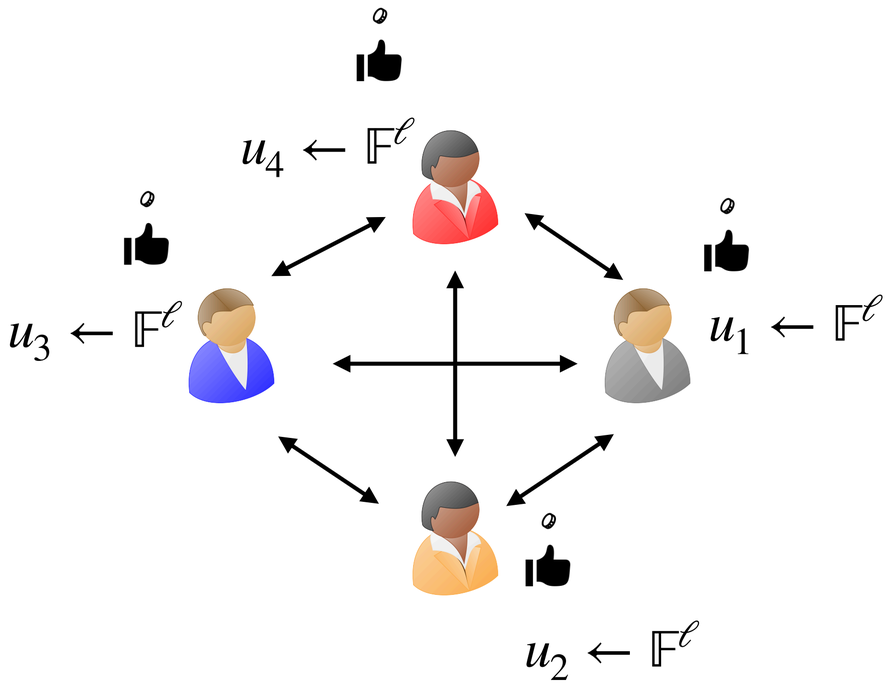

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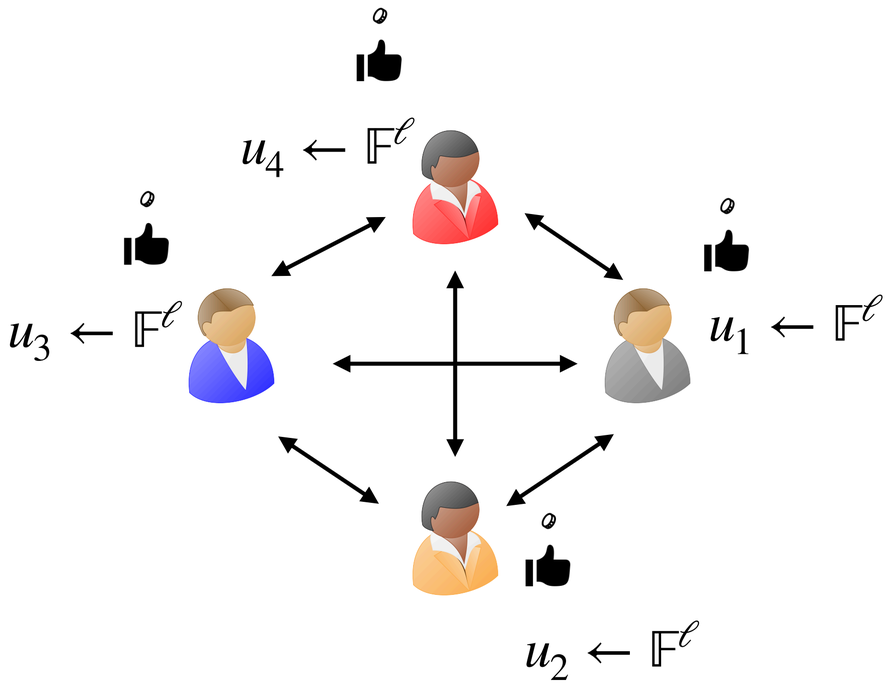

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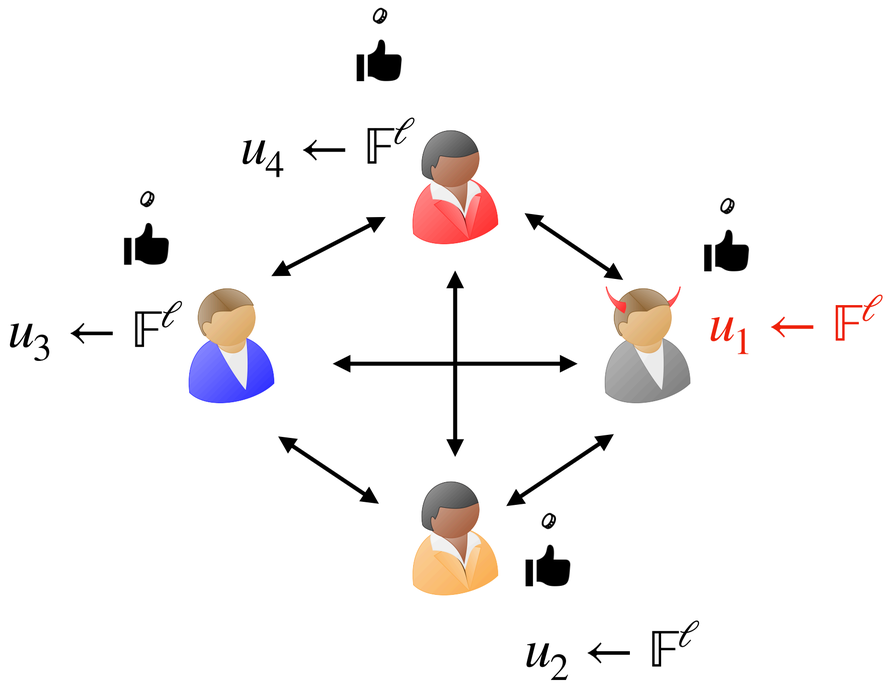

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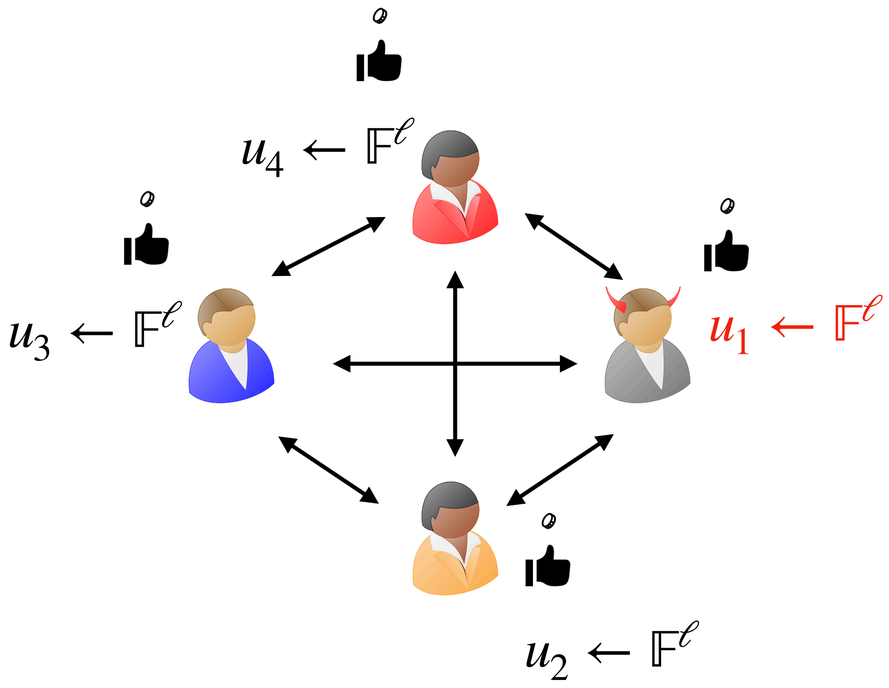

Super-invertible matrix over $\mathbb{F}$

$$\begin{bmatrix} [u_1] \\ [u_2] \\ [u_3] \\ [u_4] \end{bmatrix} = \begin{bmatrix} [r_1] \\ [r_2] \\ [r_3] \end{bmatrix}$$

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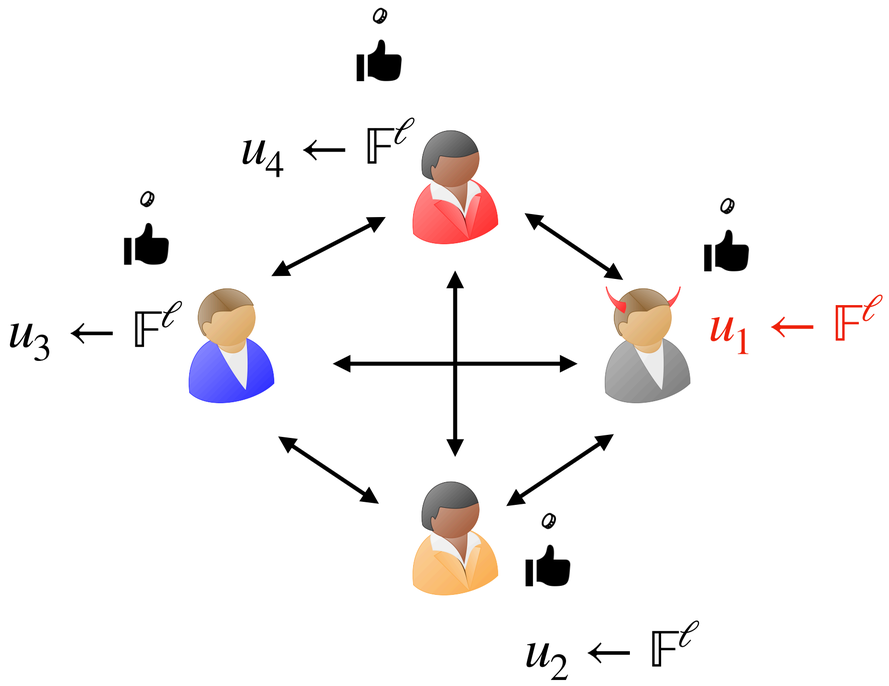

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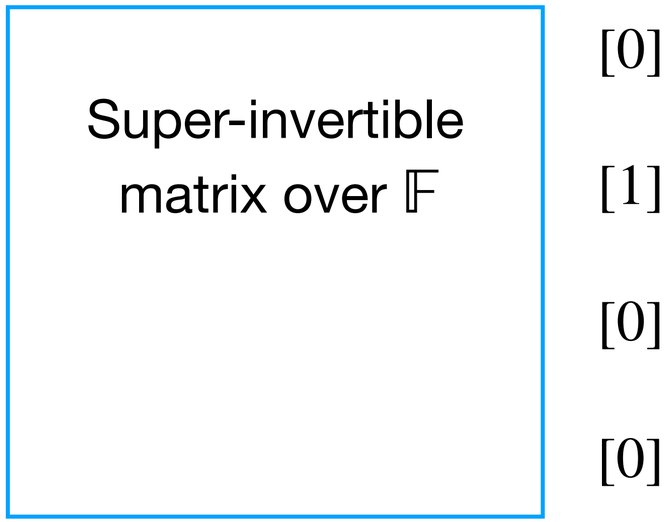

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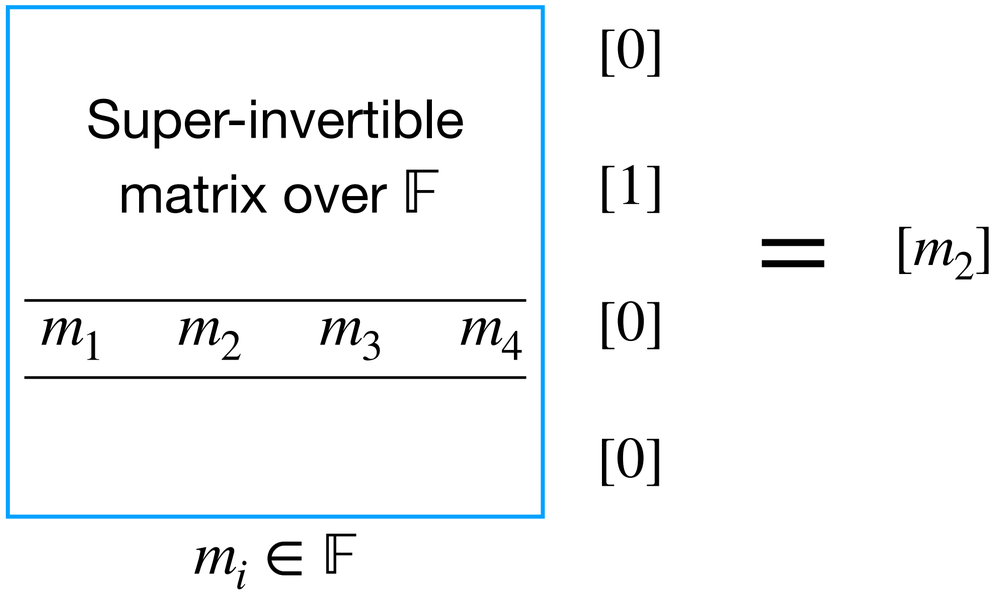

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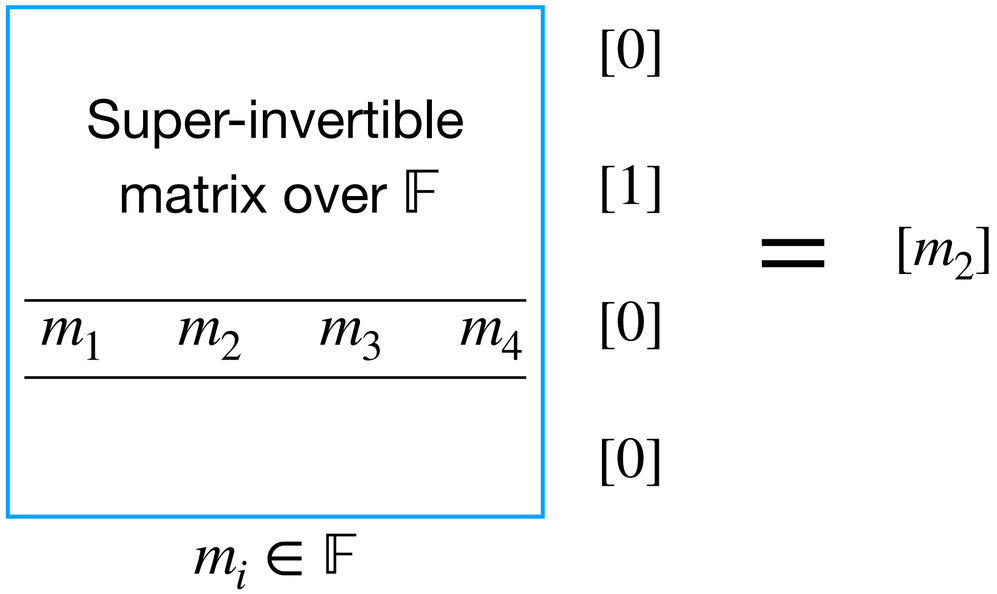

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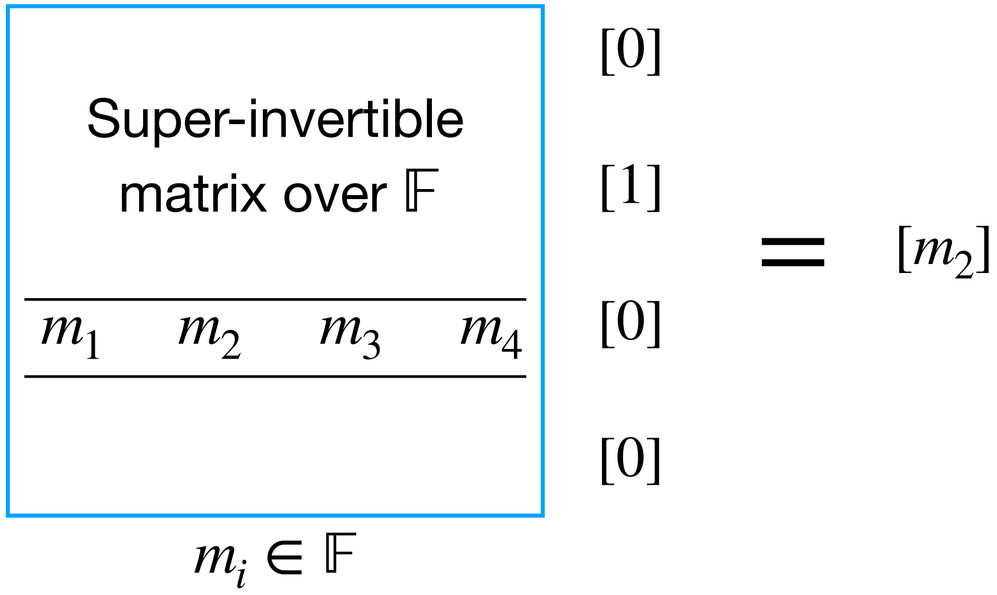

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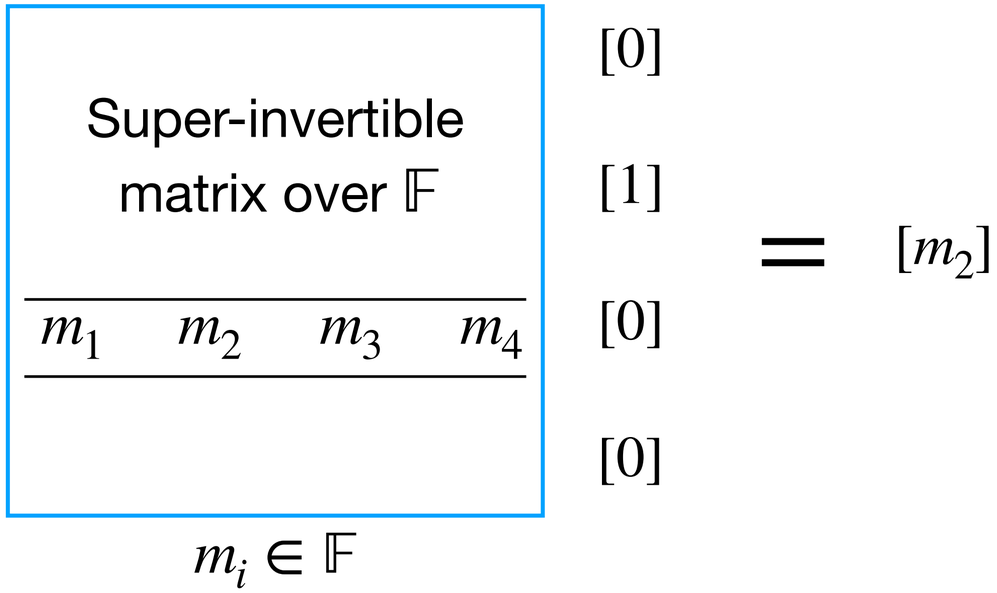

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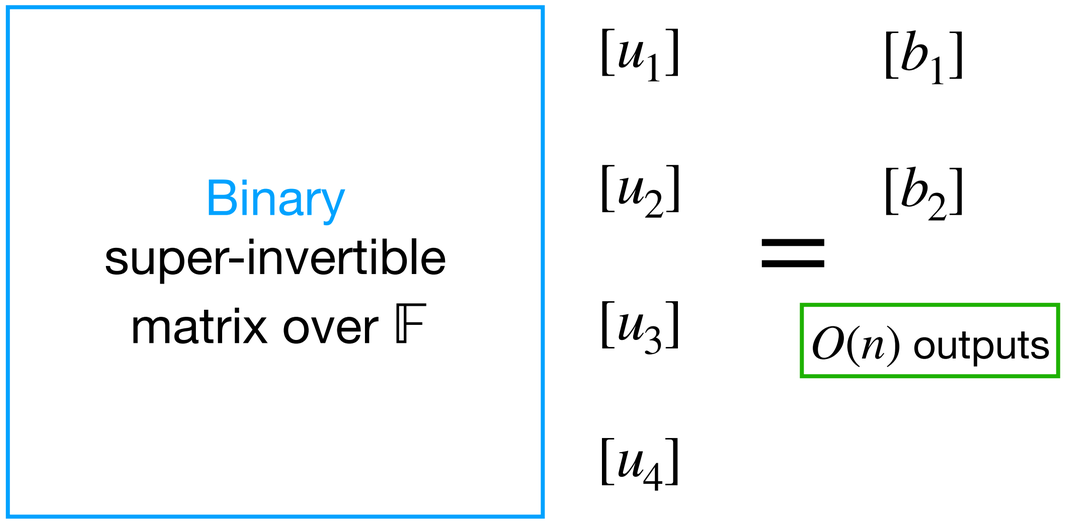

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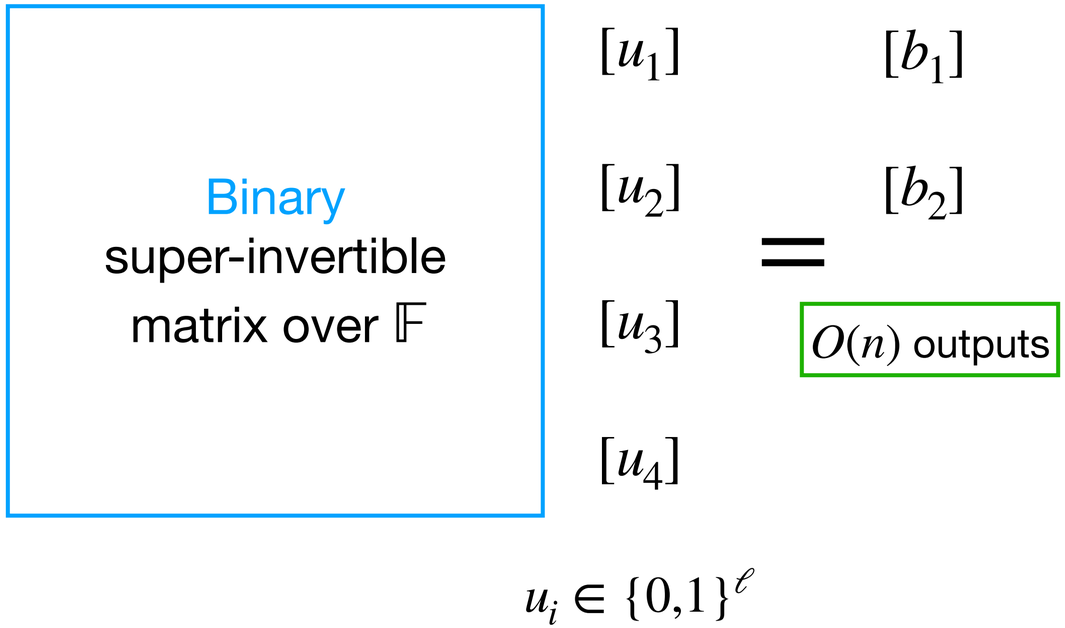

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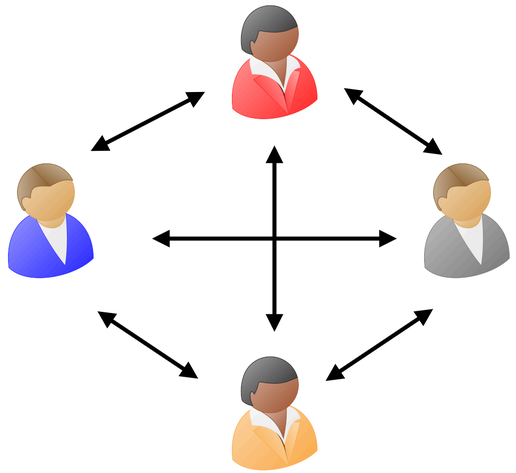

$$[b_1] \quad [b_2] \quad \ldots \quad [b_p]$$

Packed sharing of random bits

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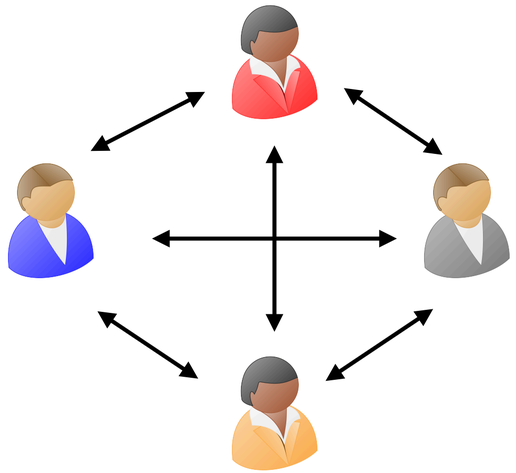

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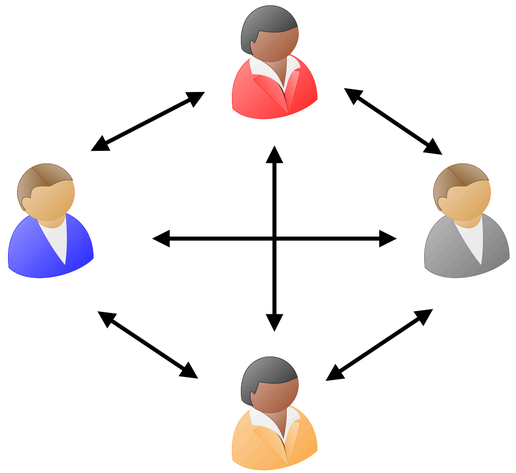

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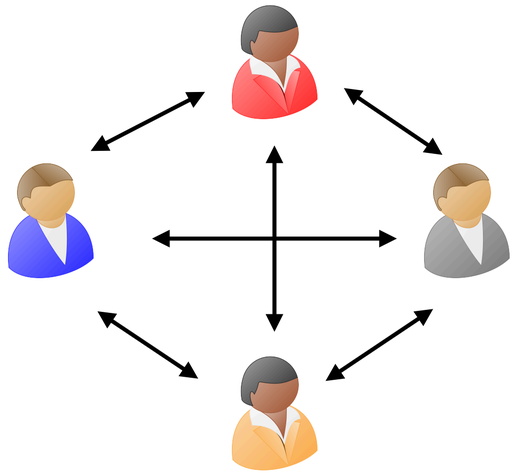

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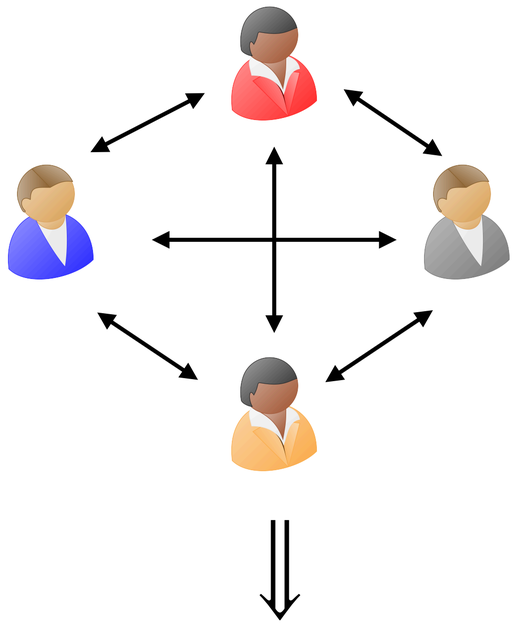
$O(1)$ communication

Computing The Garbled Circuit

Pre-processing Phase



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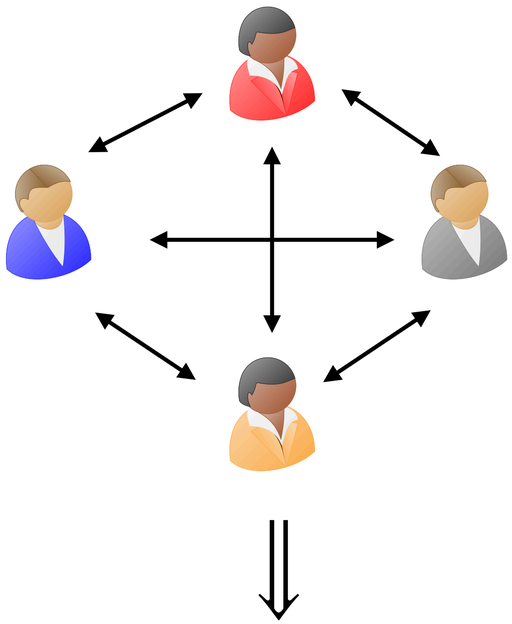

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Garbling Phase

Pre-processing phase

$[X][Y][Z]$
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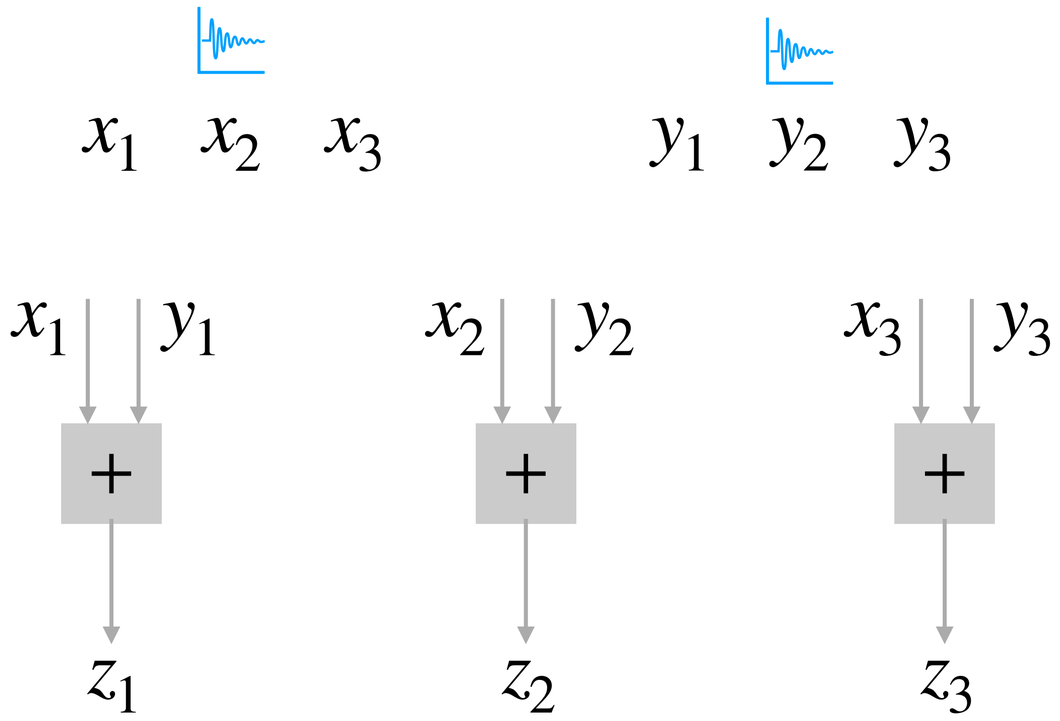
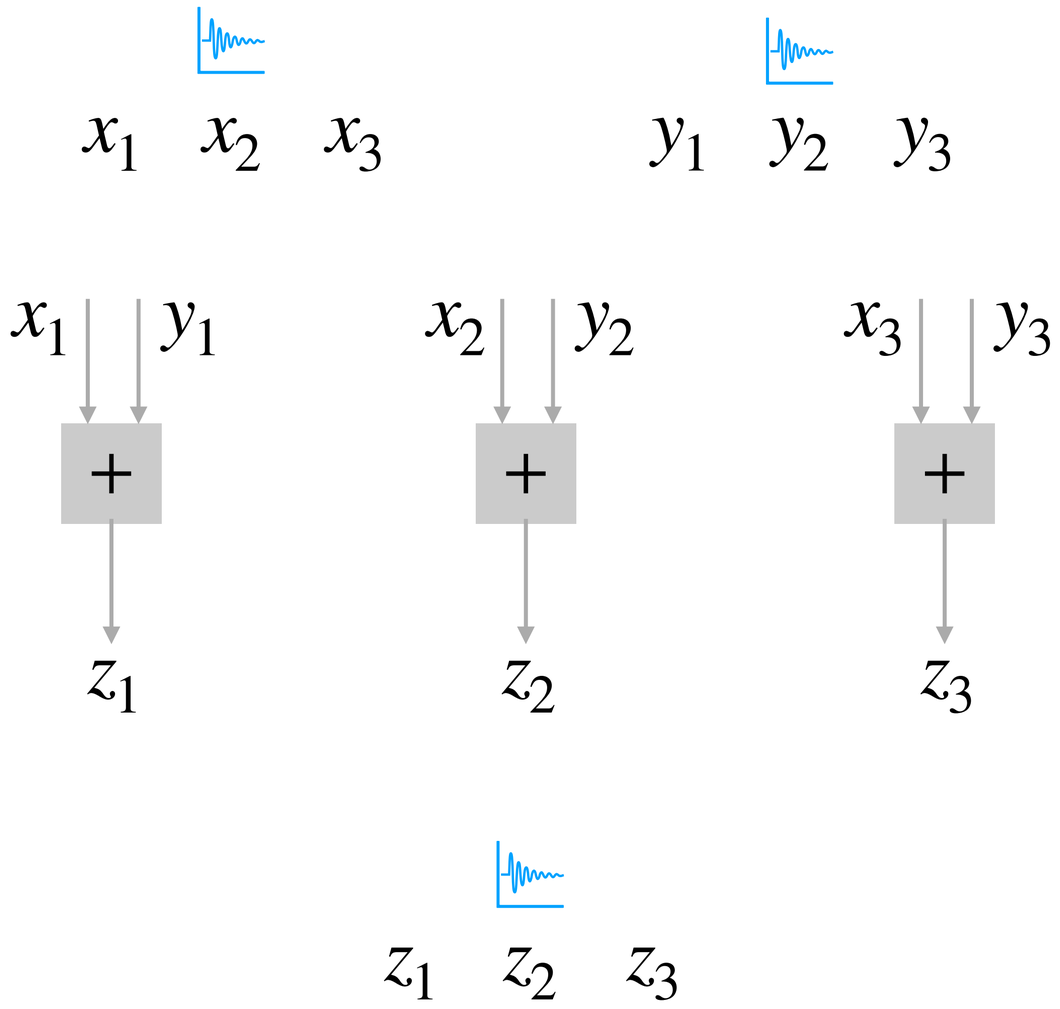


Alignment Of Secrets In Packed Sharing

Computation is straightforward when secrets are aligned.



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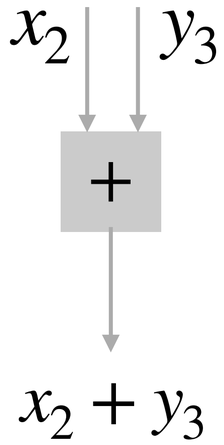

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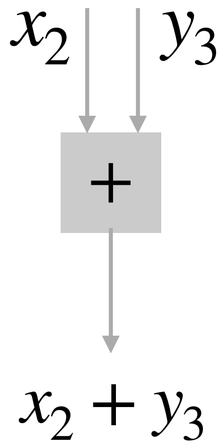

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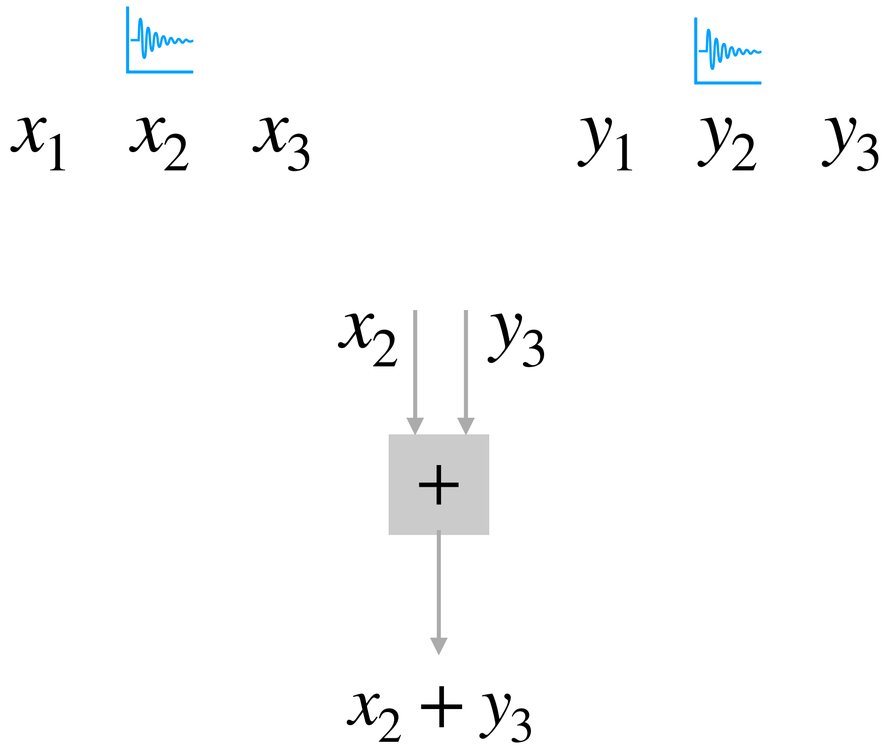

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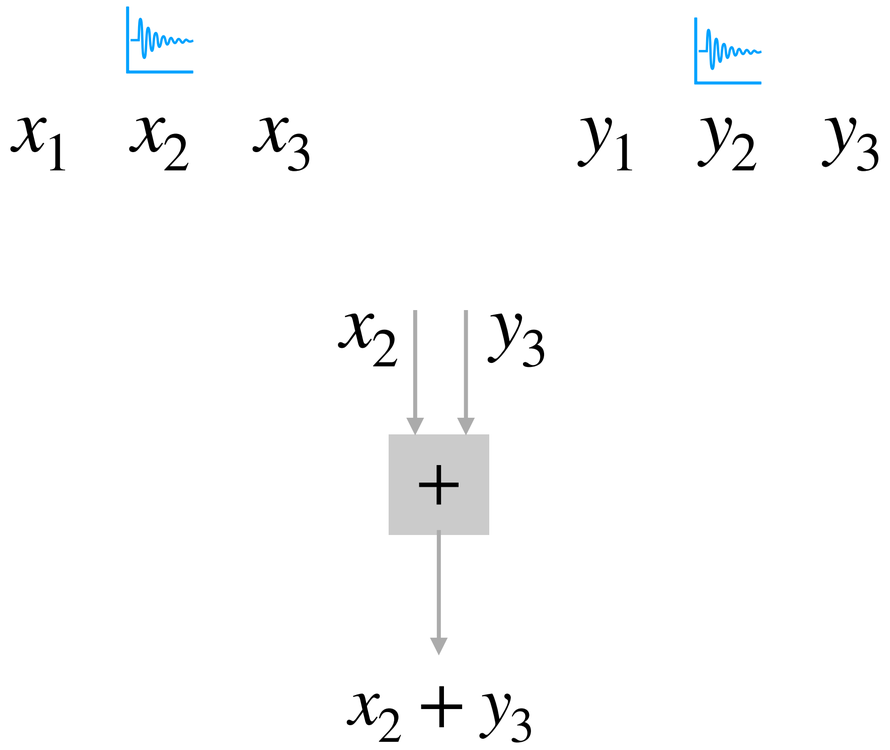

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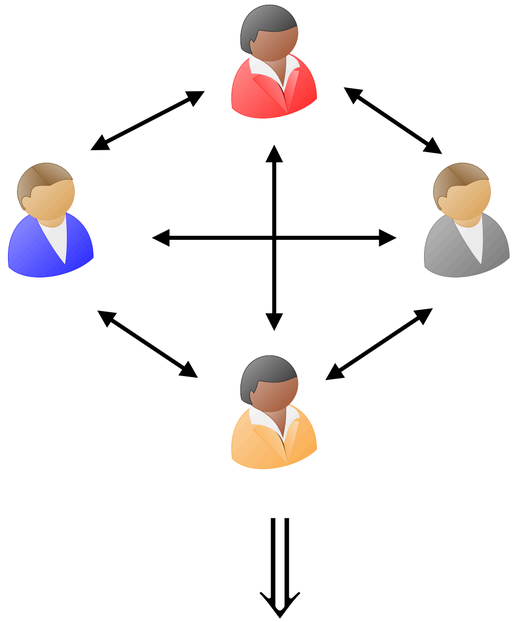

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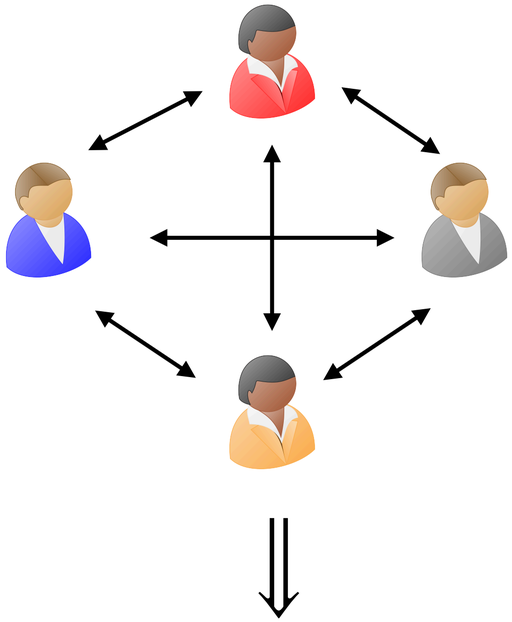

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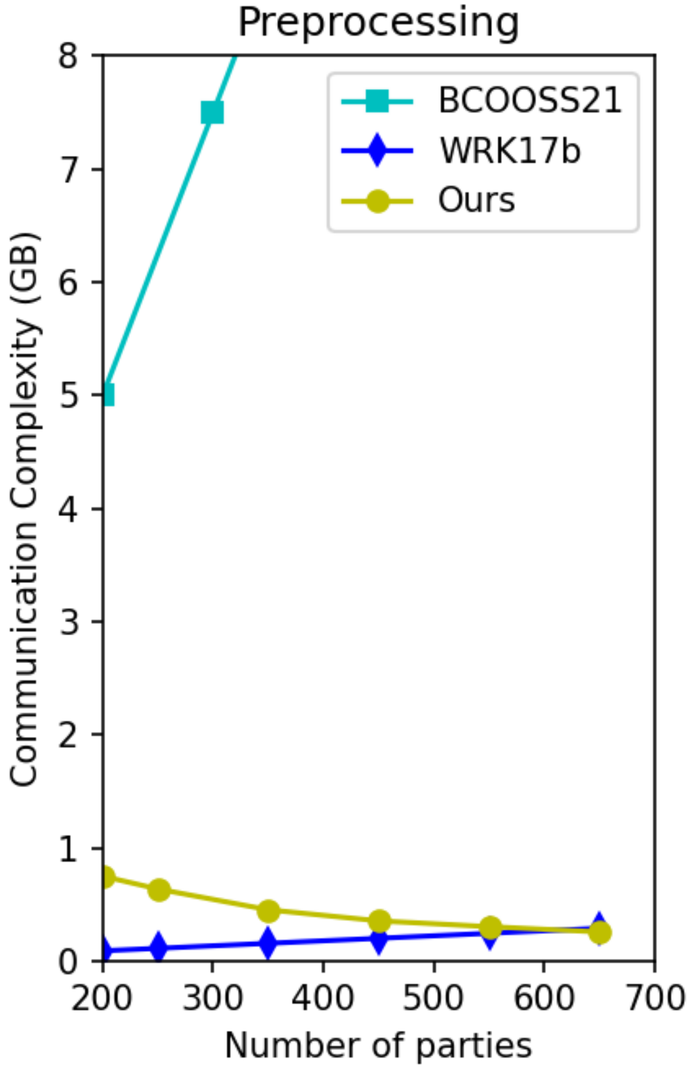
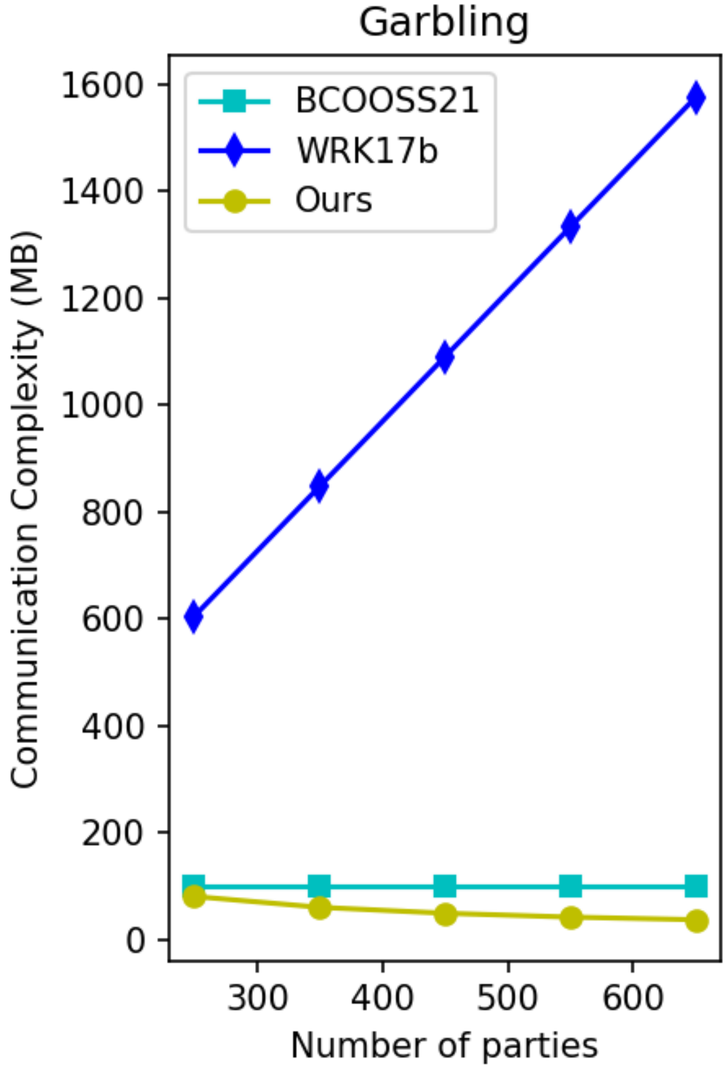
Agenda

- Structure Of Garbled Circuit
- Multiparty Garbling Protocol
- Efficiency Analysis

Communication Cost Estimates For Evaluating AES-128





Maliciously secure protocol

Per party costs

$$t \approx \frac{n}{4}$$

$$\ell \approx \frac{n}{4}$$

Bottlenecks To Concrete Efficiency

Semi-honest Protocol

- AES-128
- 8 threads per party
- $n = 250$
- $t = 50$
- $\ell = 50$

	Computation	Communication
Pre-processing Phase		
Total	1.94 min	240 MB

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Error Generation	1.73 min	196 MB

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Low rate of binary error correcting codes.

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Garbling Phase		
Total	1.64 min	43 MB

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- AES-128
- 8 threads per party
- $n = 250$
- $t = 50$
- $\ell = 50$

	Computation	Communication
Pre-processing Phase		
Total	1.94 min	240 MB
Error Generation	1.73 min	196 MB
Garbling Phase		
Total	1.64 min	43 MB
Re-aligning Secrets	27.66 sec	28 MB

Future Directions

- Improving concrete efficiency
  - Concretely efficient protocol for error generation.
  - Incorporating existing GC optimizations.
- Constructing $O(|C|)$ communication multiparty garbling from weaker assumptions.

Thank you

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