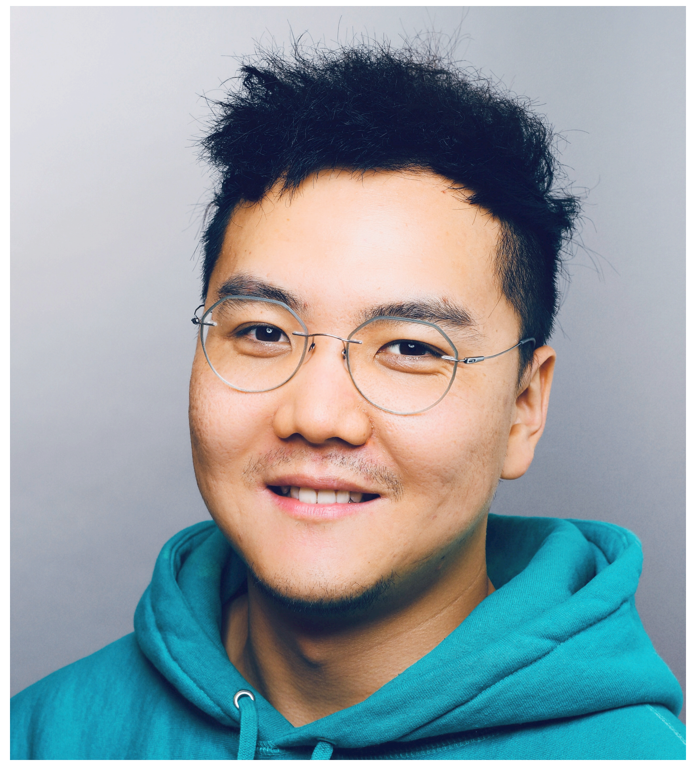
Enhanced Trapdoor Hashing from DDH and DCR

EUROCRYPT 2025



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CNRS, IRIF
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Setting: Two-Round Sender-Receiver Computation

Public function F

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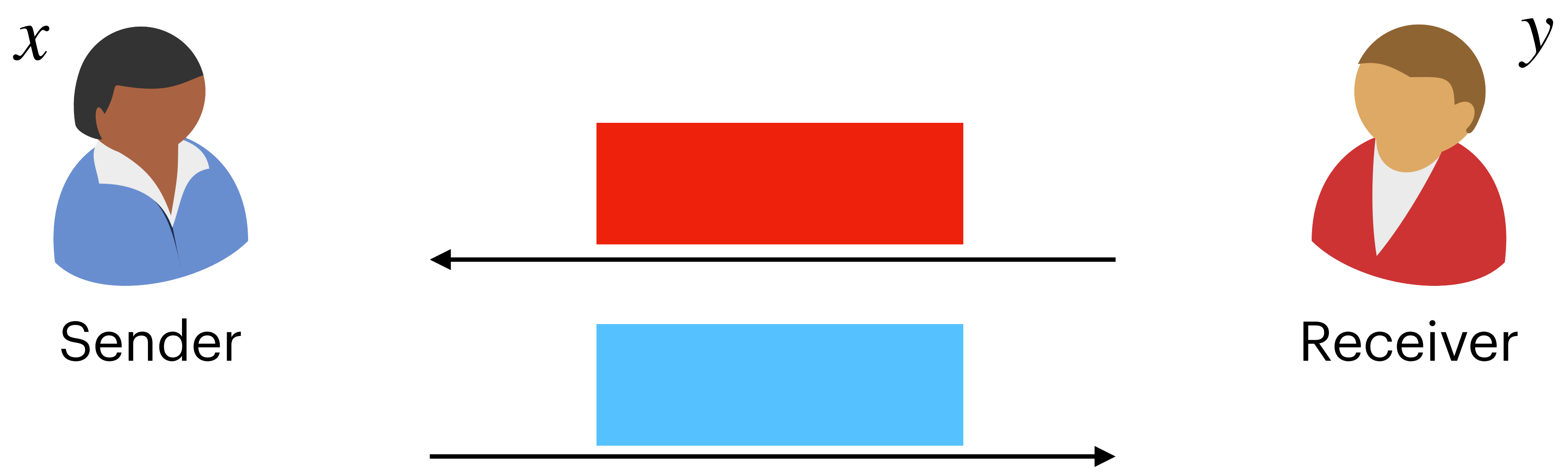

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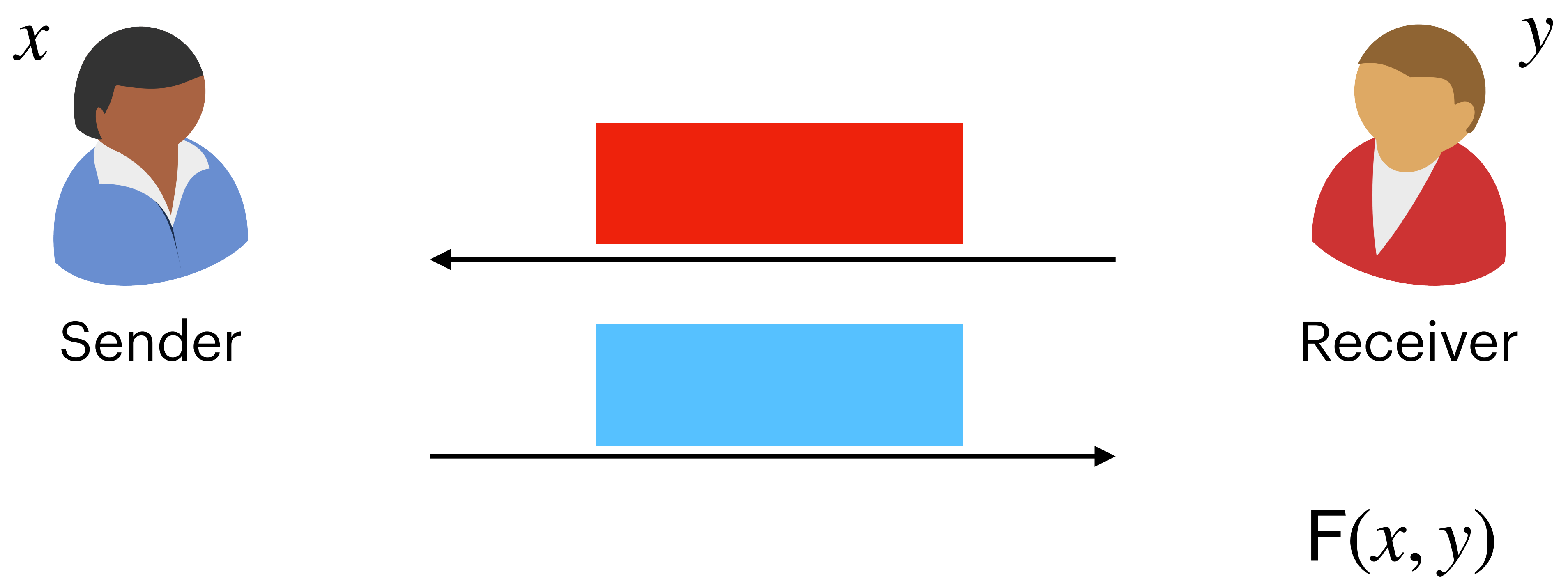

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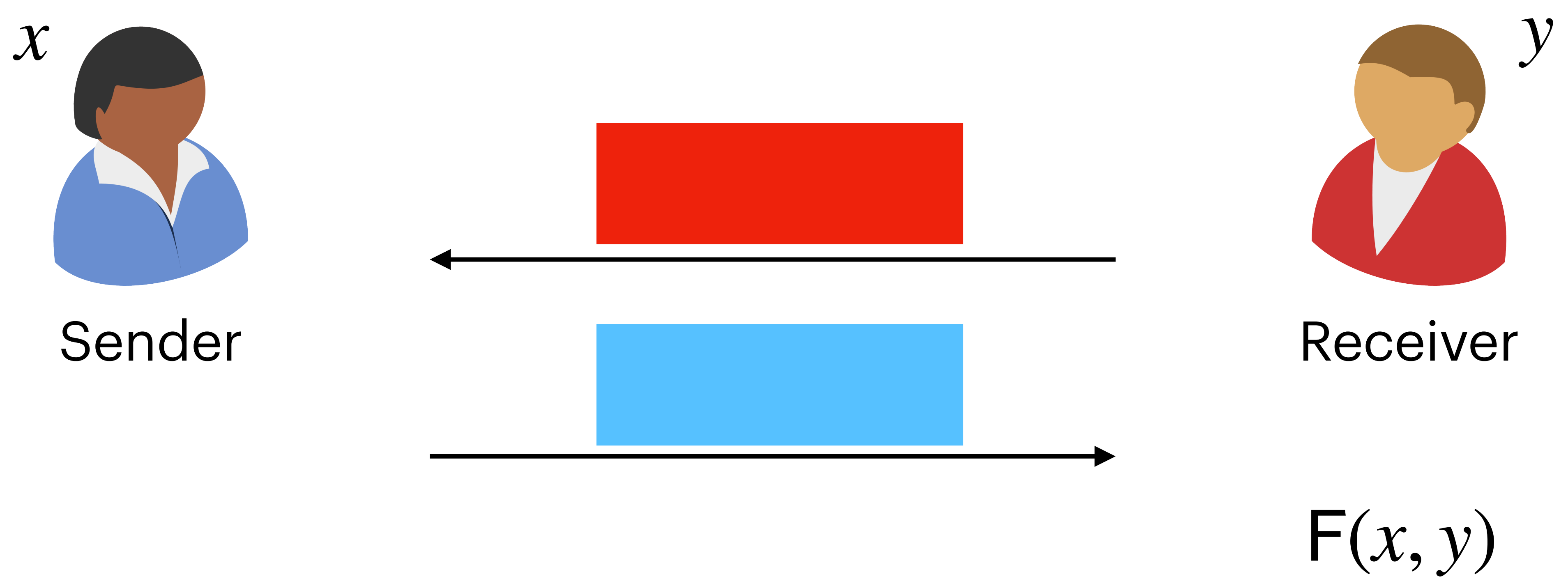

Privacy:  does not learn y

 does not learn x

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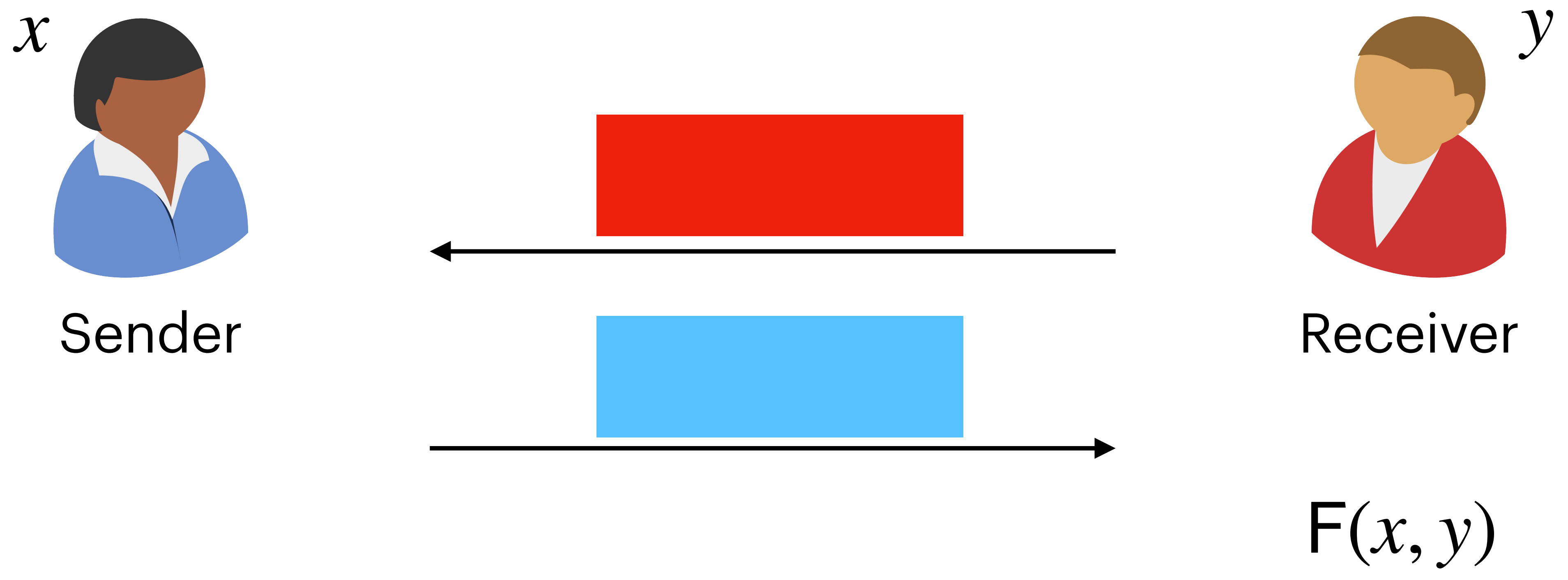

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What is the **minimum communication** cost of **semi-honest** secure protocols?

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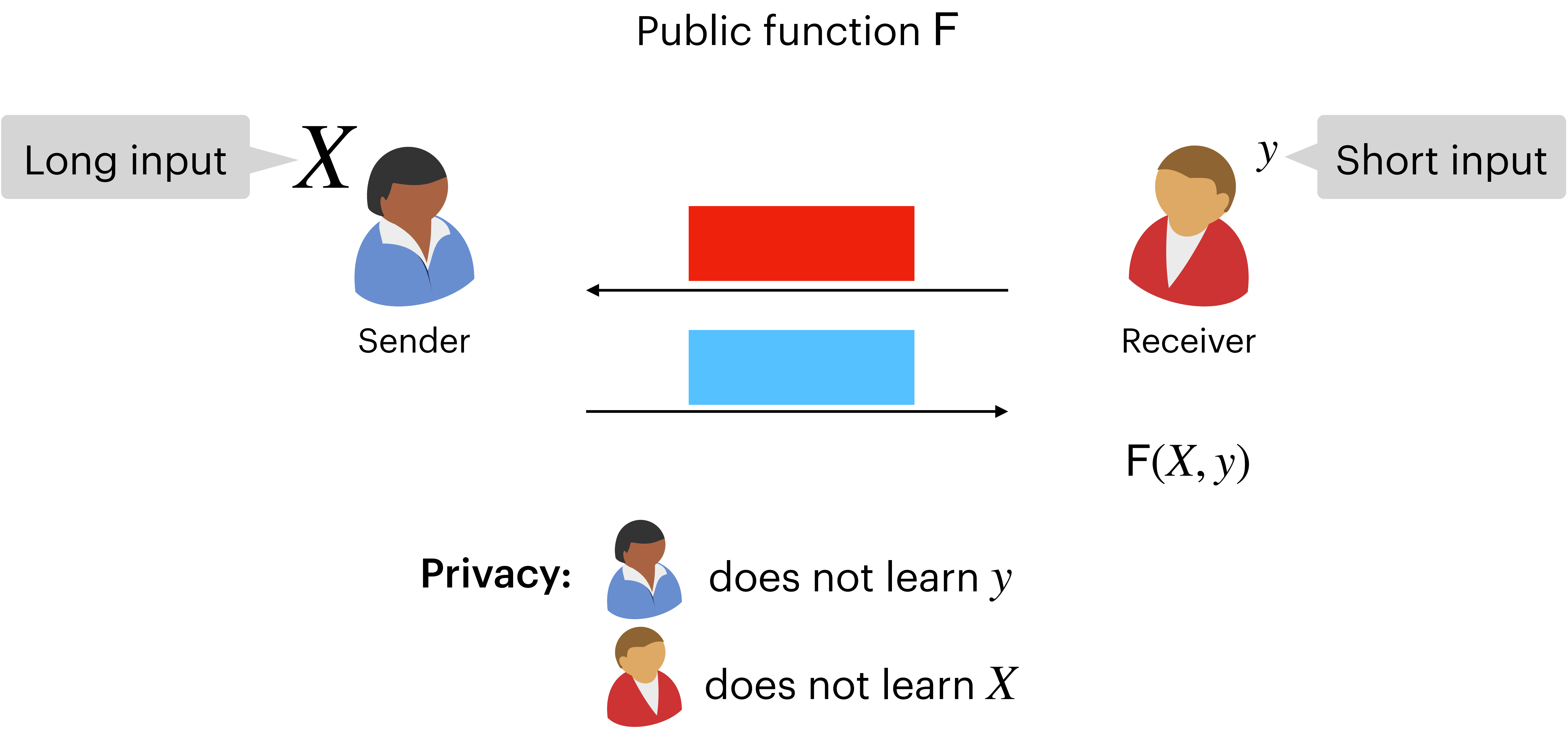

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Ideal World Two-Round Sender-Receiver Computation

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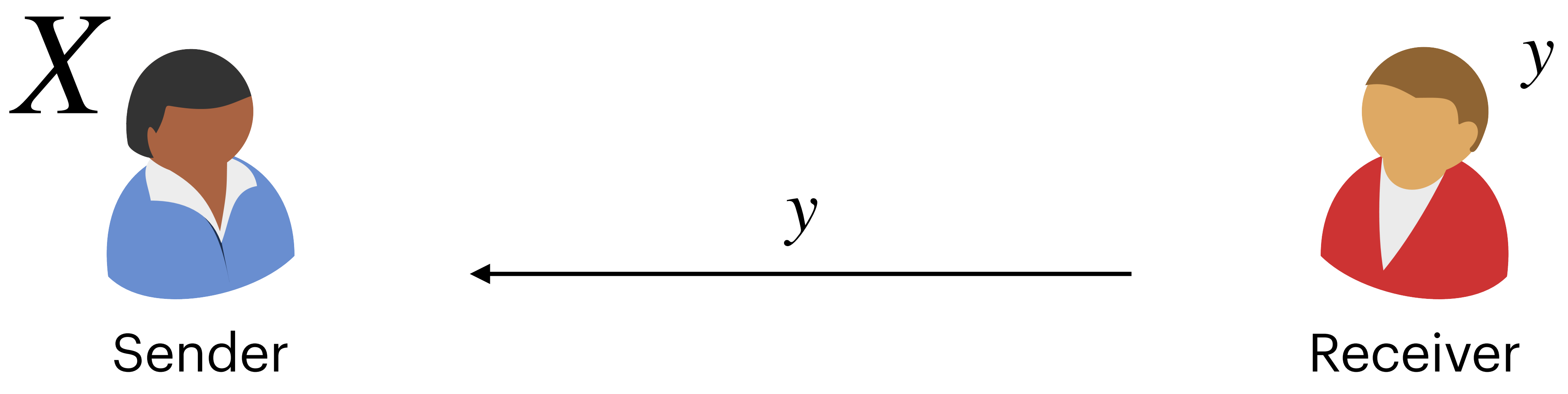

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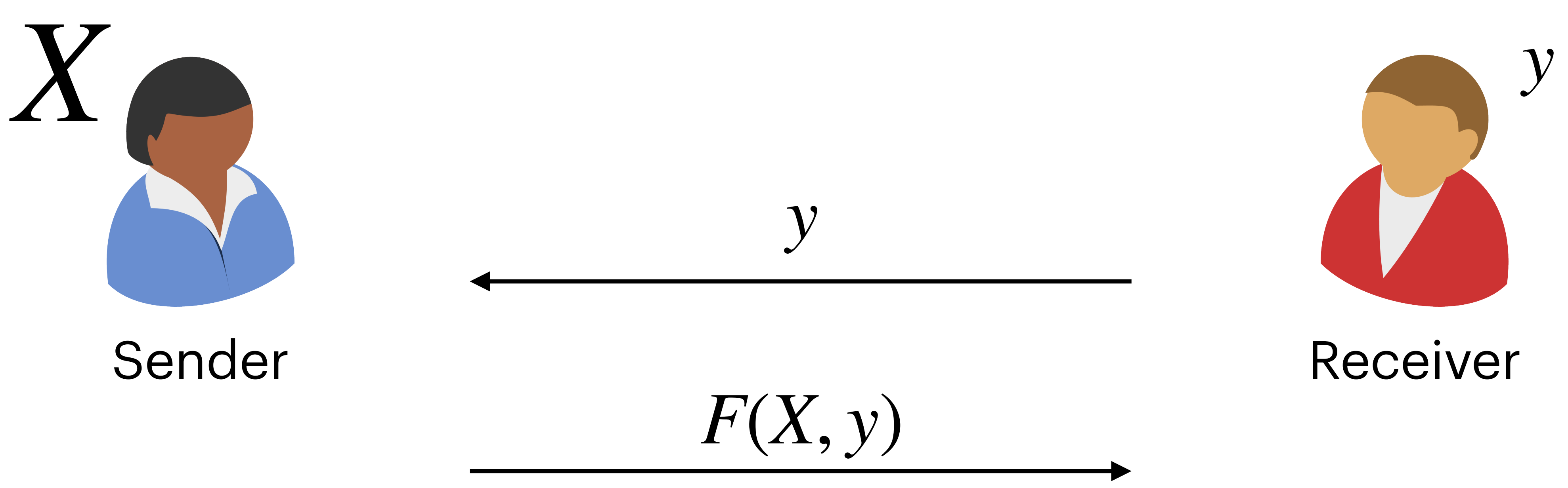

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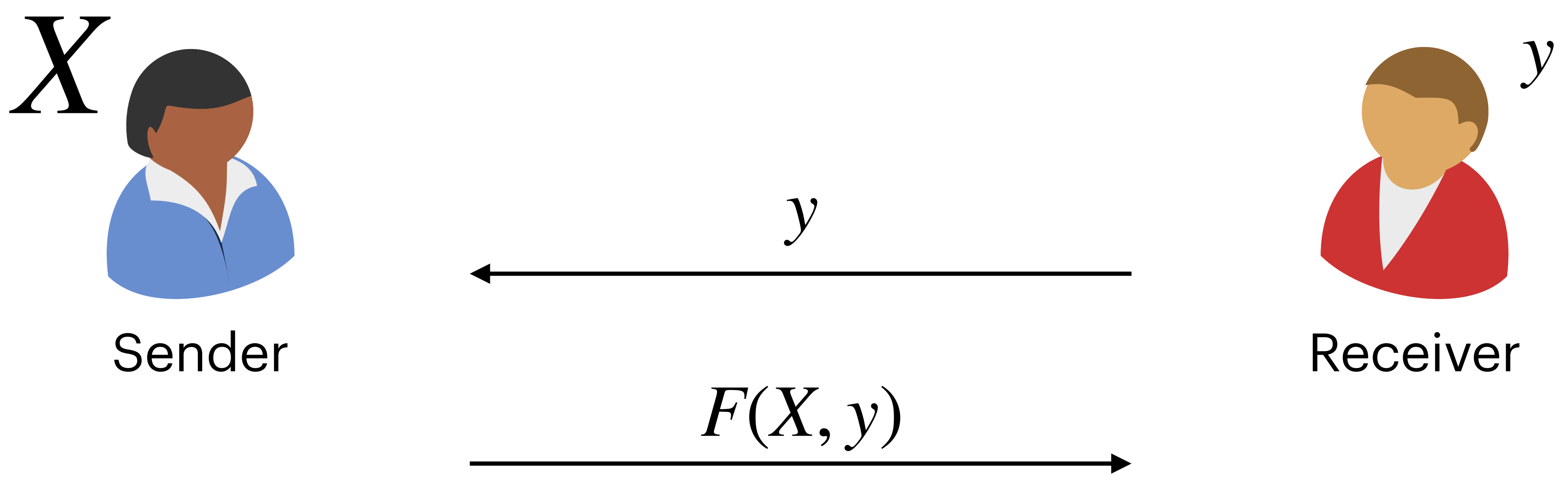

Total communication: $|y| + |F(X, y)|$

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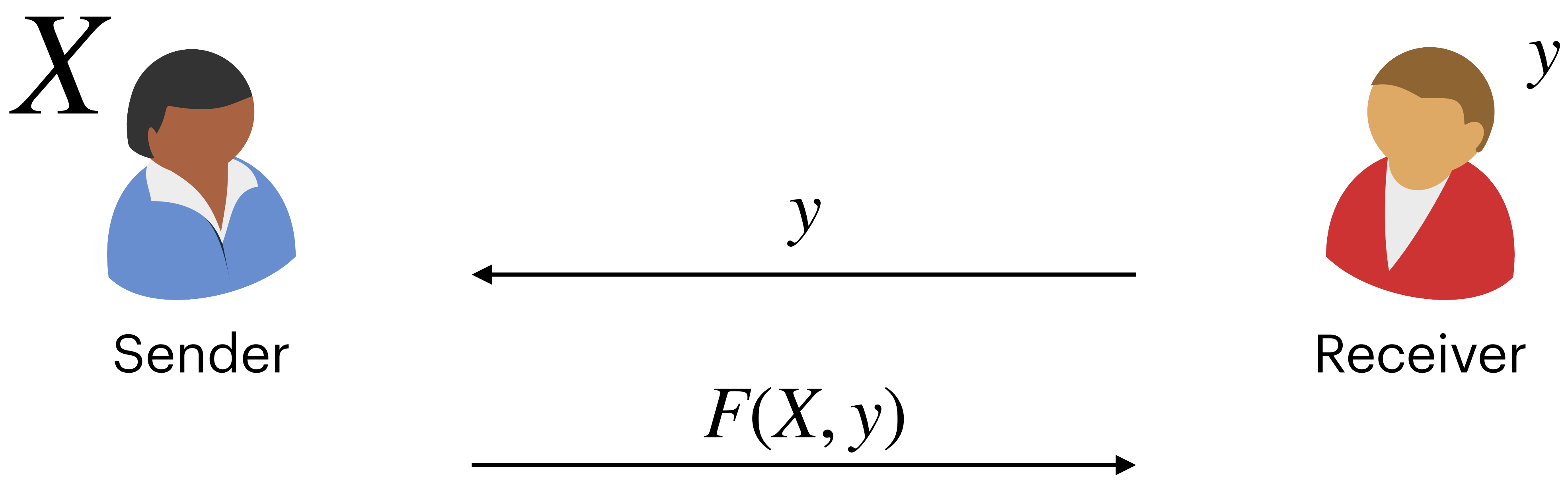

Total communication: $|y| + |F(X, y)|$

Can secure protocols achieve similar efficiency?

Trapdoor Hash Functions (TDH)

[Döttling-Garg-Ishai-Malavolta-Mour-Ostrovsky'19]

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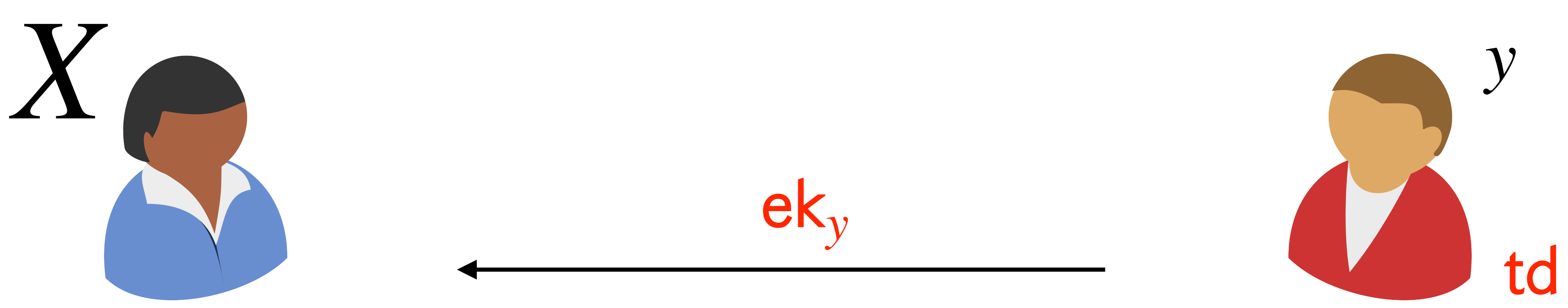

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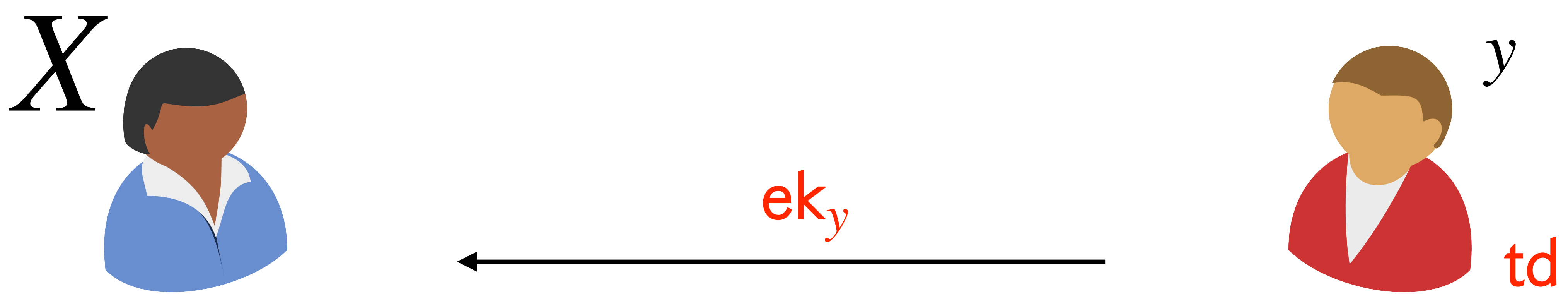

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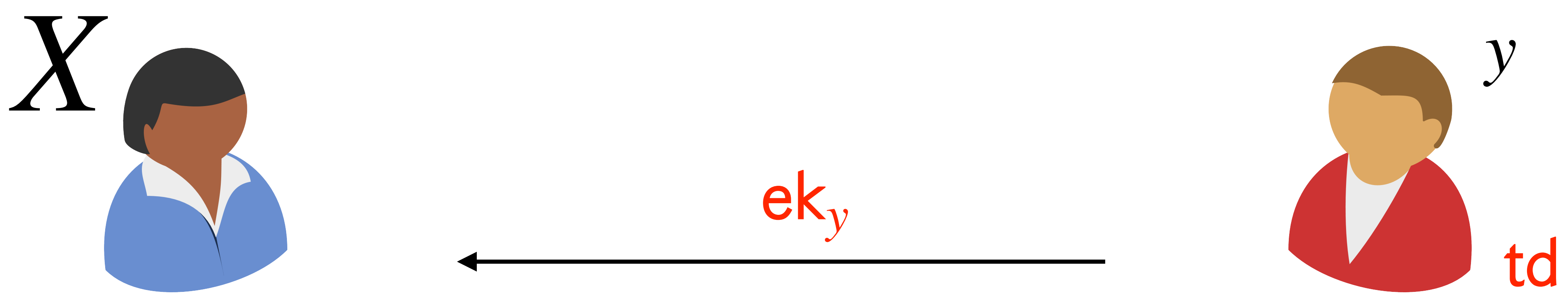

$h \leftarrow \mathsf{Hash}(X)$

$e \leftarrow \mathsf{Encode}(\mathsf{ek}_y, X)$

Trapdoor Hash Functions (TDH)

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Public function F

X 

 y

td

$\xleftarrow{\mathsf{ek}_y}$

$h \leftarrow \mathsf{Hash}(X)$

$e \leftarrow \mathsf{Encode}(\mathsf{ek}_y, X)$

$\xrightarrow{h,\ e}$

$F(X, y) \leftarrow \mathsf{Decode}(\mathsf{td}, h, e)$

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Privacy: ek_y hides y

(h, e) hides X

Trapdoor Hash Functions (TDH)

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Public function F

X 

ek_y $\longleftarrow$

 y td

$h \leftarrow \mathsf{Hash}(X)$

$e \leftarrow \mathsf{Encode}(\mathsf{ek}_y, X)$

h, e $\longrightarrow$

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Efficiency: h is small i.e., $|h| = o(|X|) \cdot \mathsf{poly}(\lambda)$

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ek_y

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h, e

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Privacy: ek_y hides y

(h, e) hides X

Efficiency: h is small i.e., $|h| = o(|X|) \cdot \mathsf{poly}(\lambda)$

e has high rate i.e., $|e| \approx |F(X, y)|$

Rate: $\dfrac{|F(X, y)|}{|e|}$

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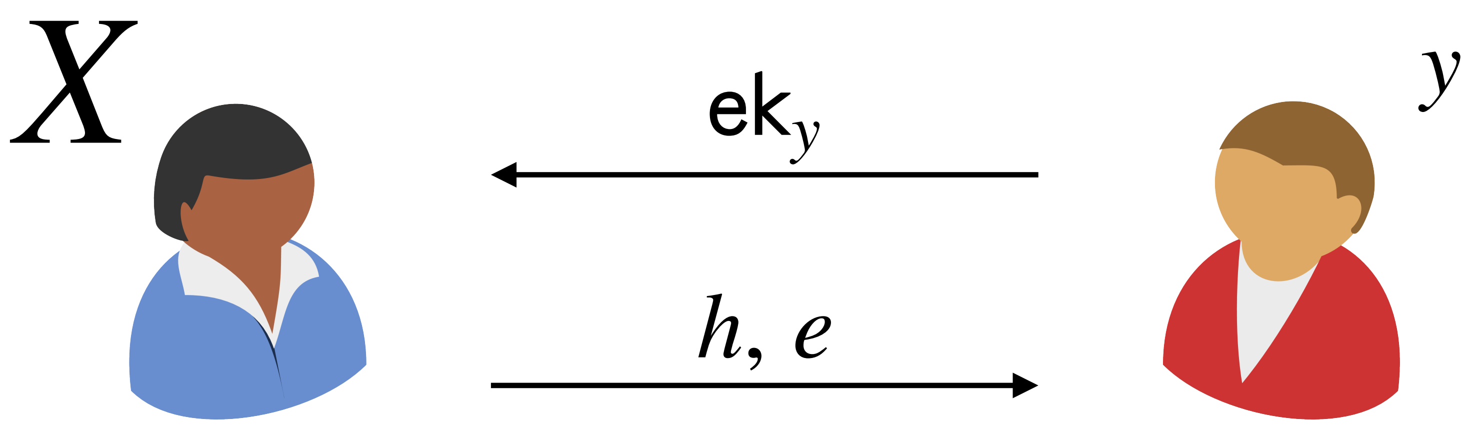

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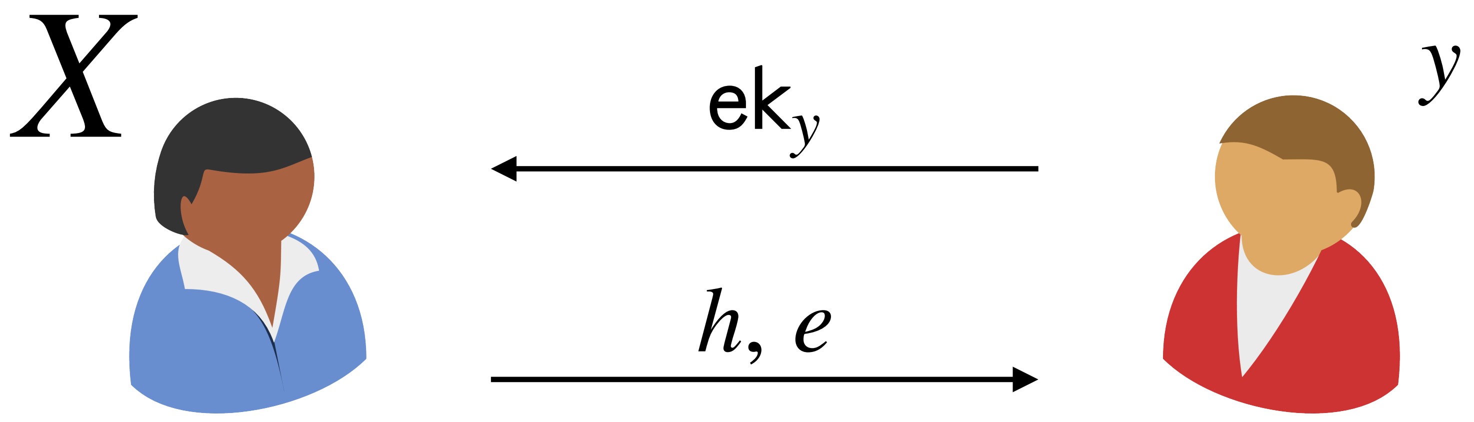
$$F(X, y) = \sum_i x_i \cdot y_i$$



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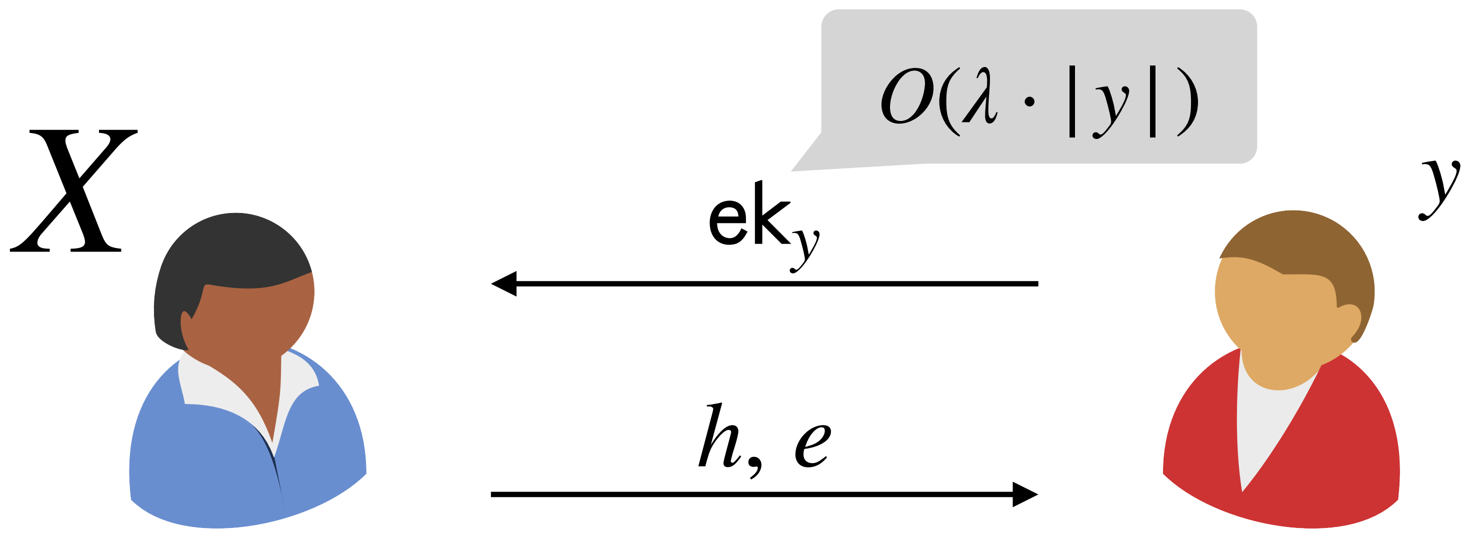

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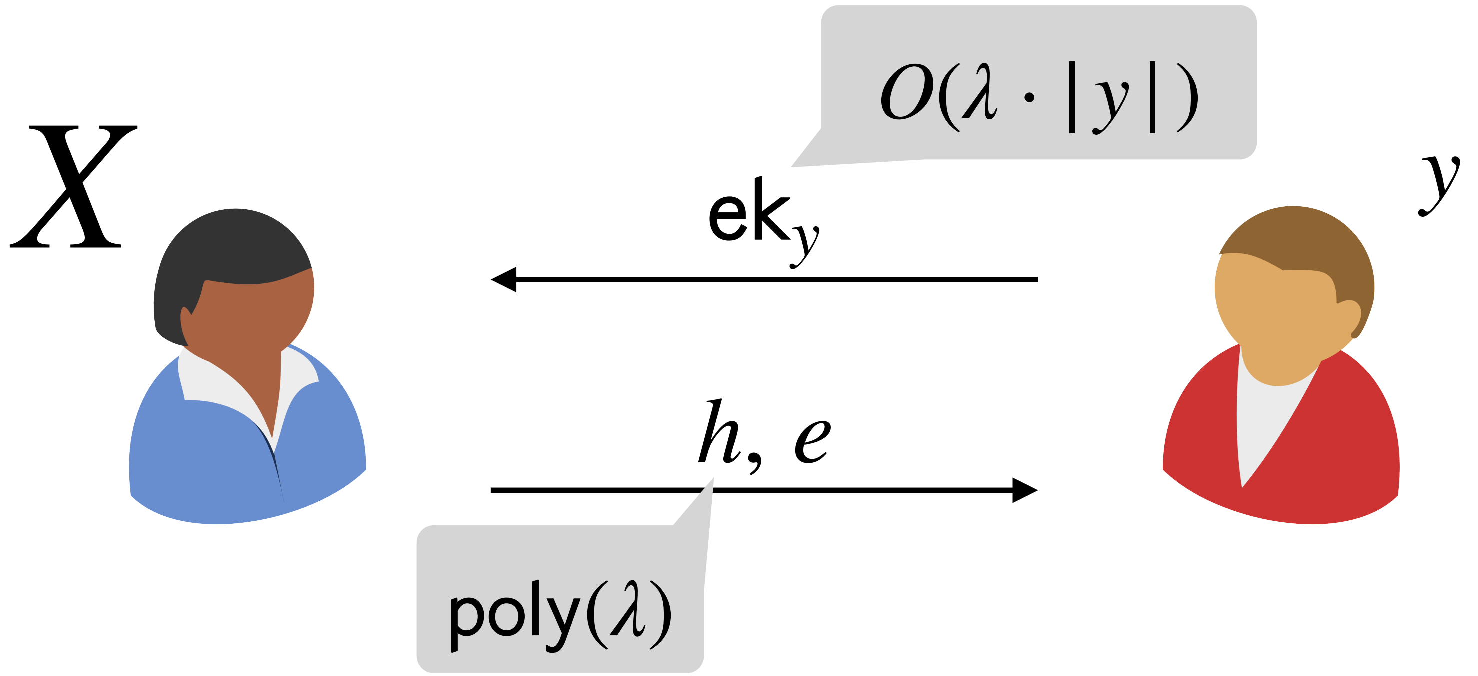

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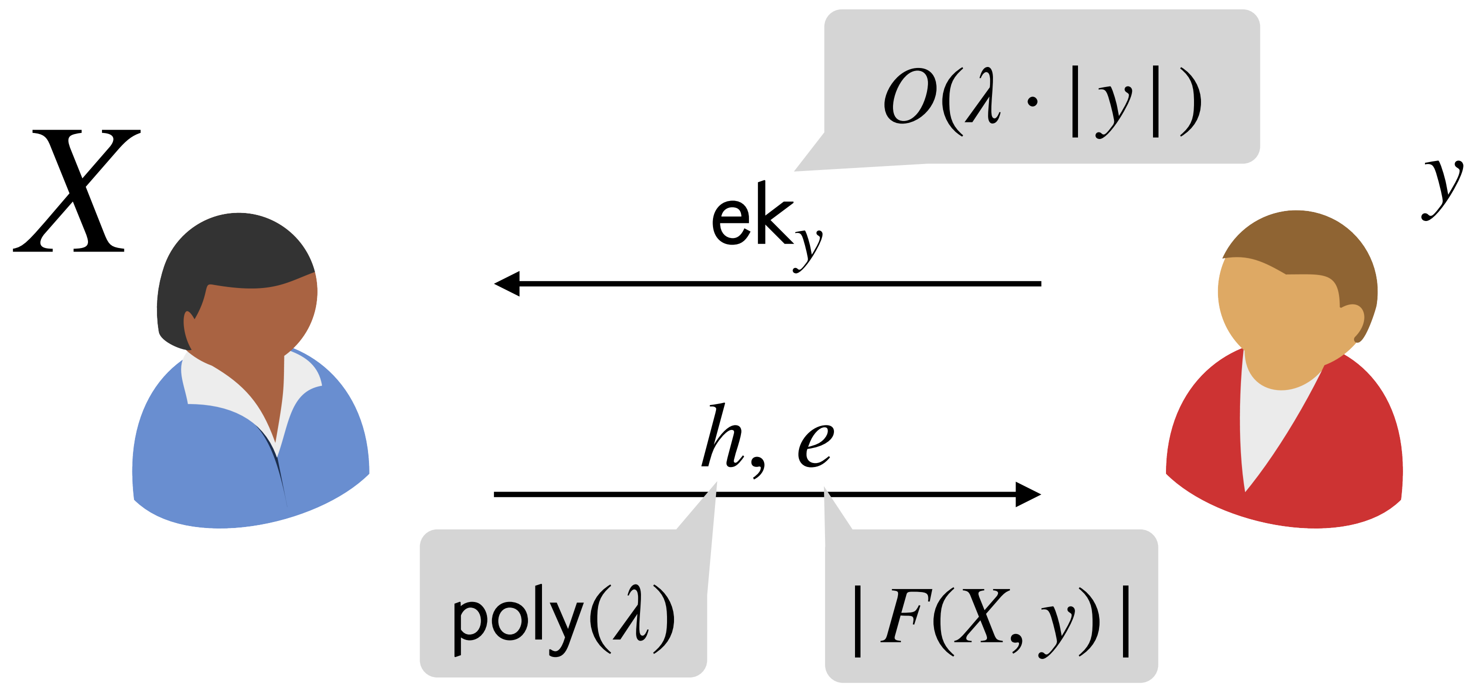

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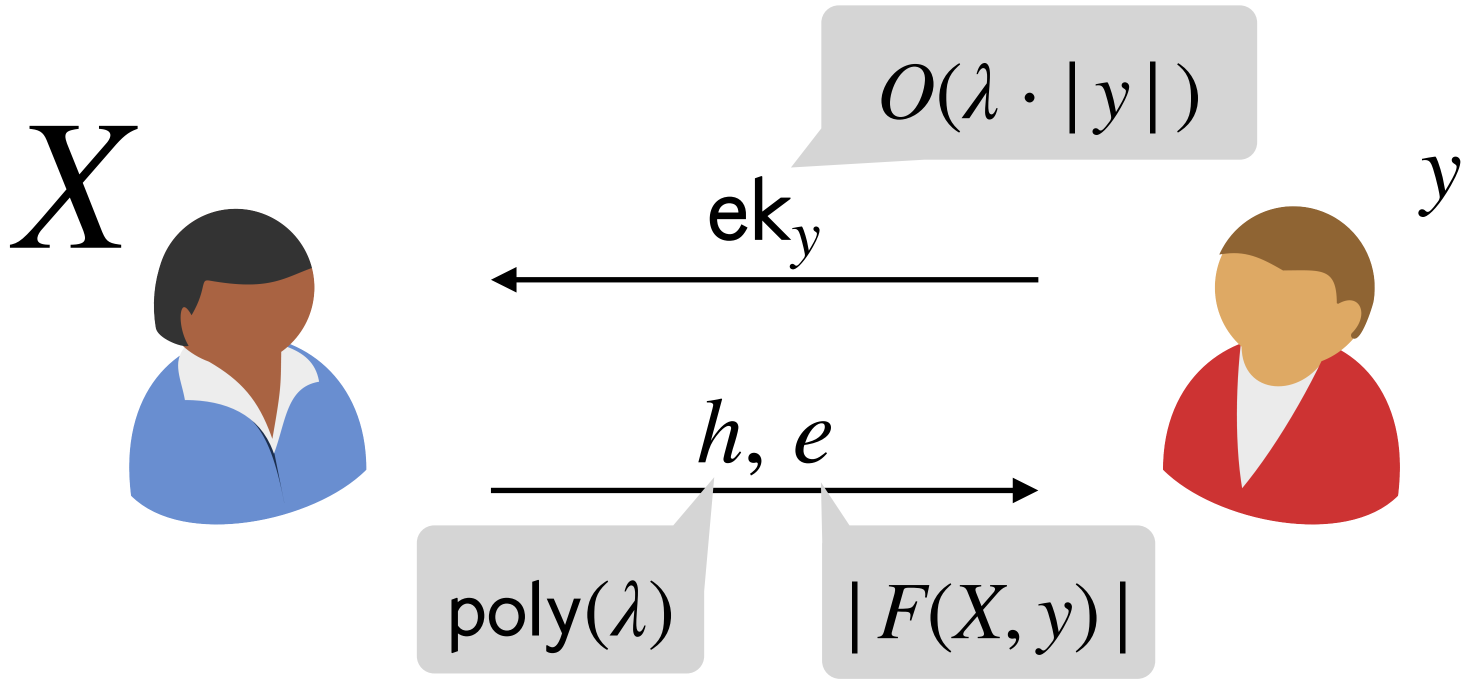

Assumptions: DCR, DDH, QR, LWE

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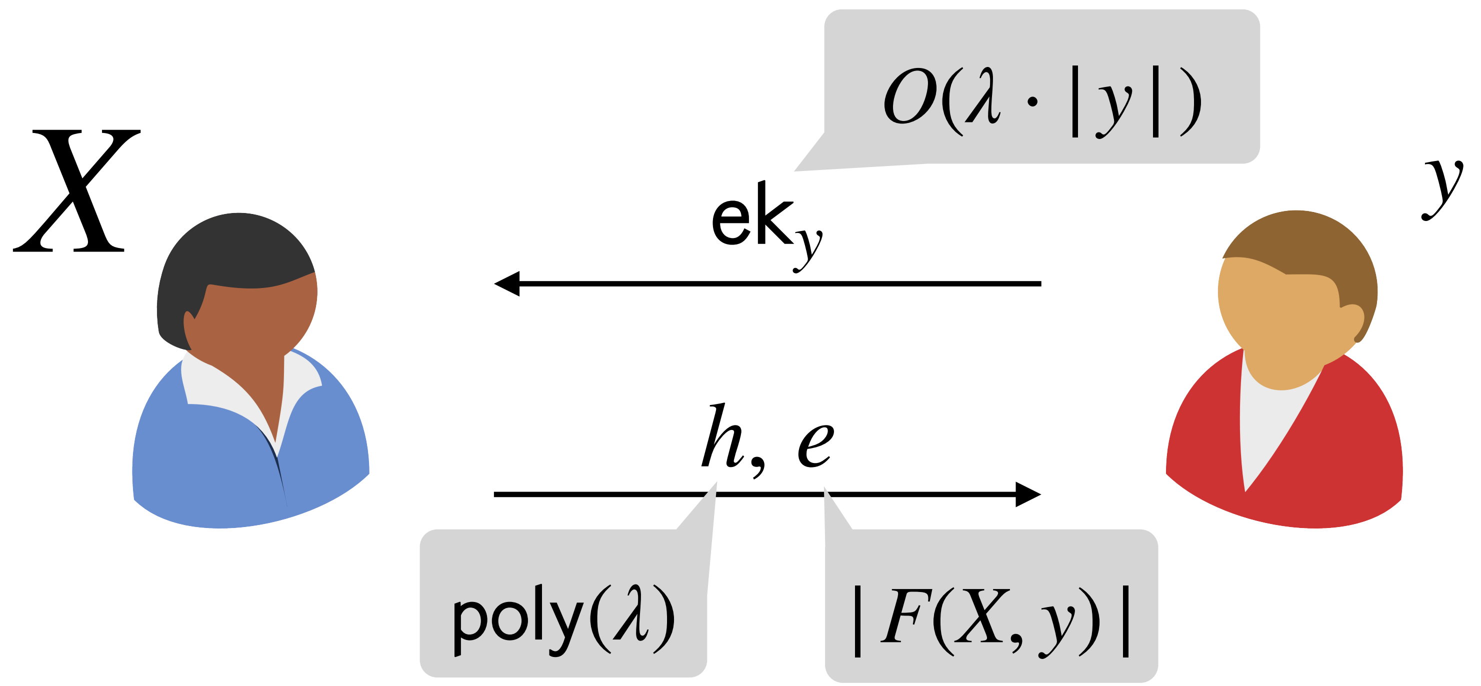

Assumptions: DCR, DDH, QR, LWE

Can we improve the functionality of TDH from group-based assumptions?

Enhanced Trapdoor Hash Functions from DDH and DCR

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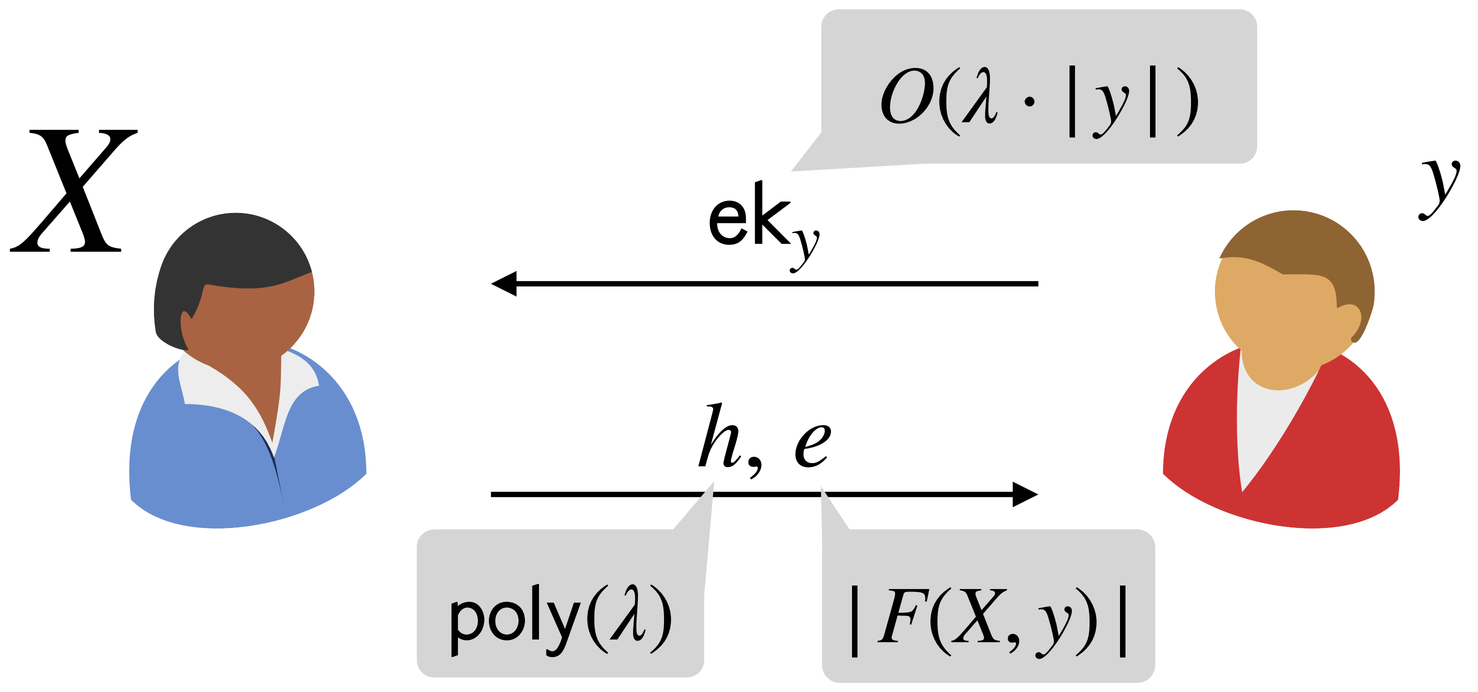
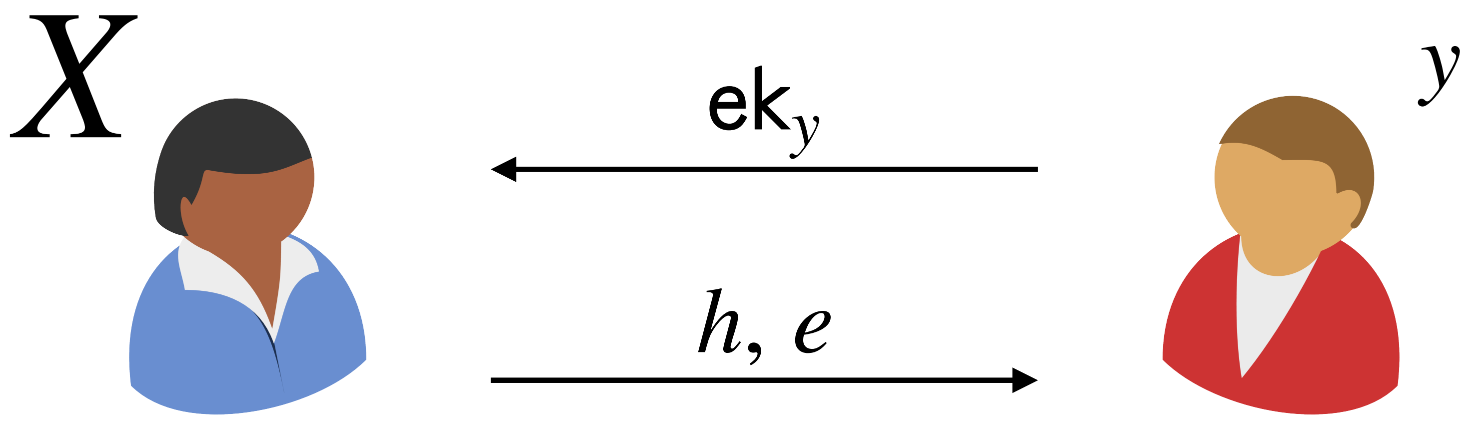


This work



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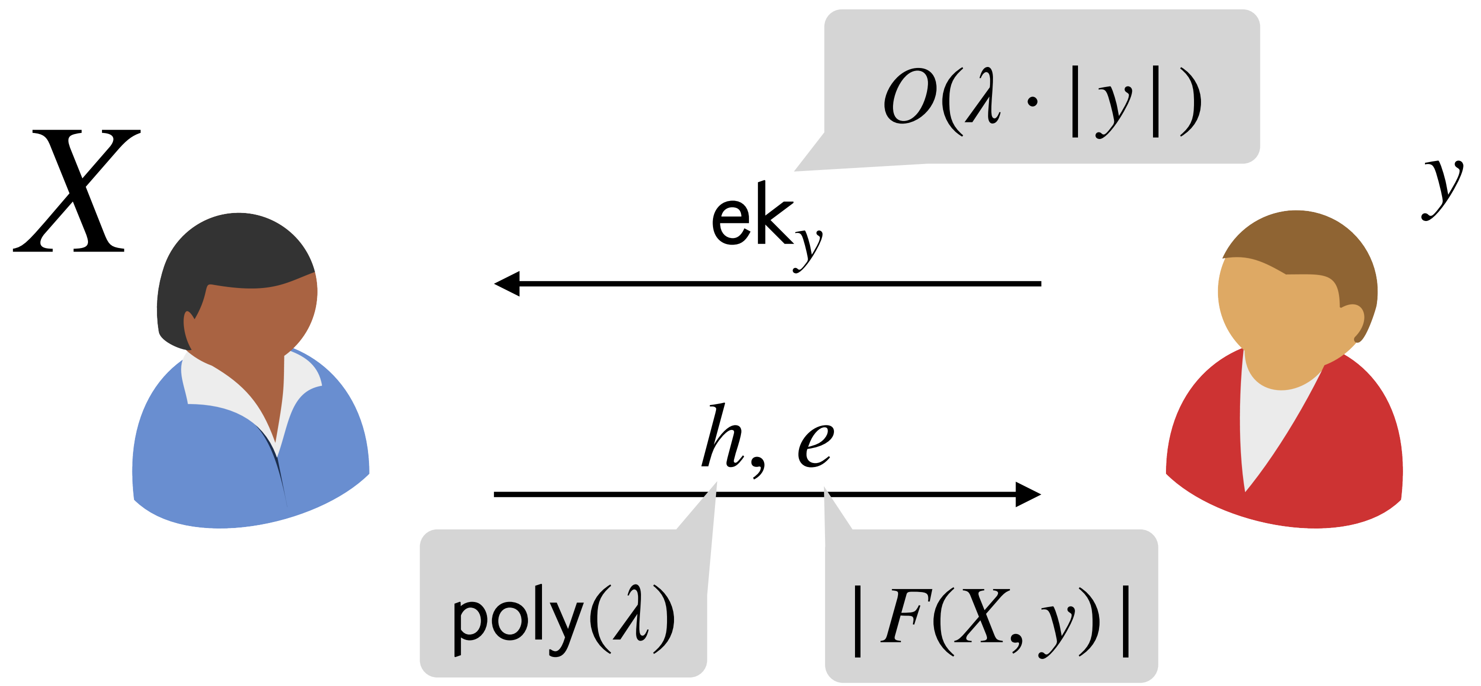


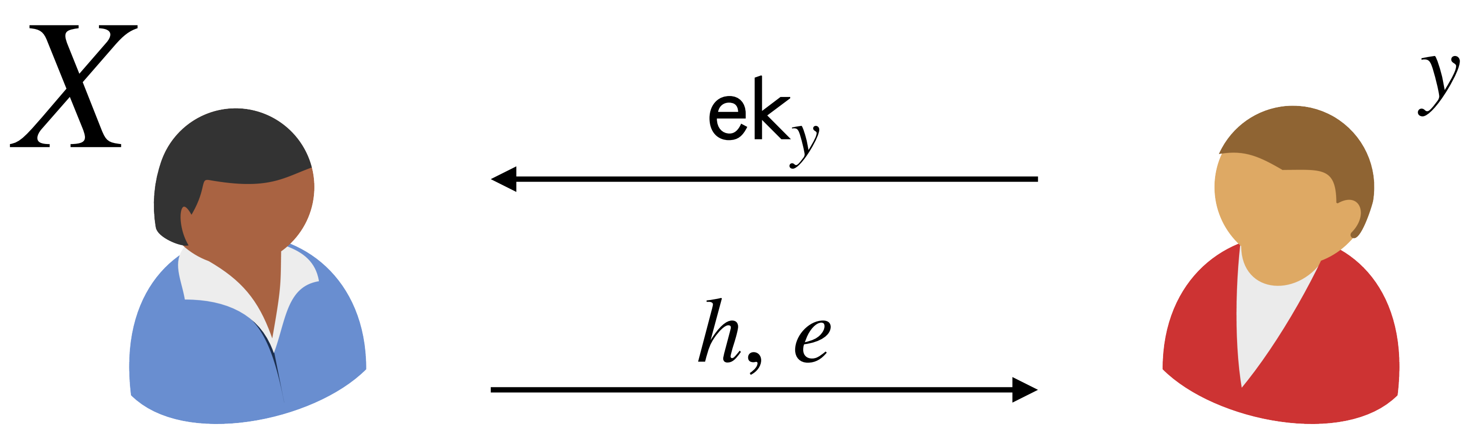
Expressivity

Supports computing Bilinear-NC^1 programs

This work

$$F(X, y) = \sum_{i=1}^{n} f_i(X) \cdot g_i(y)$$

NC^1



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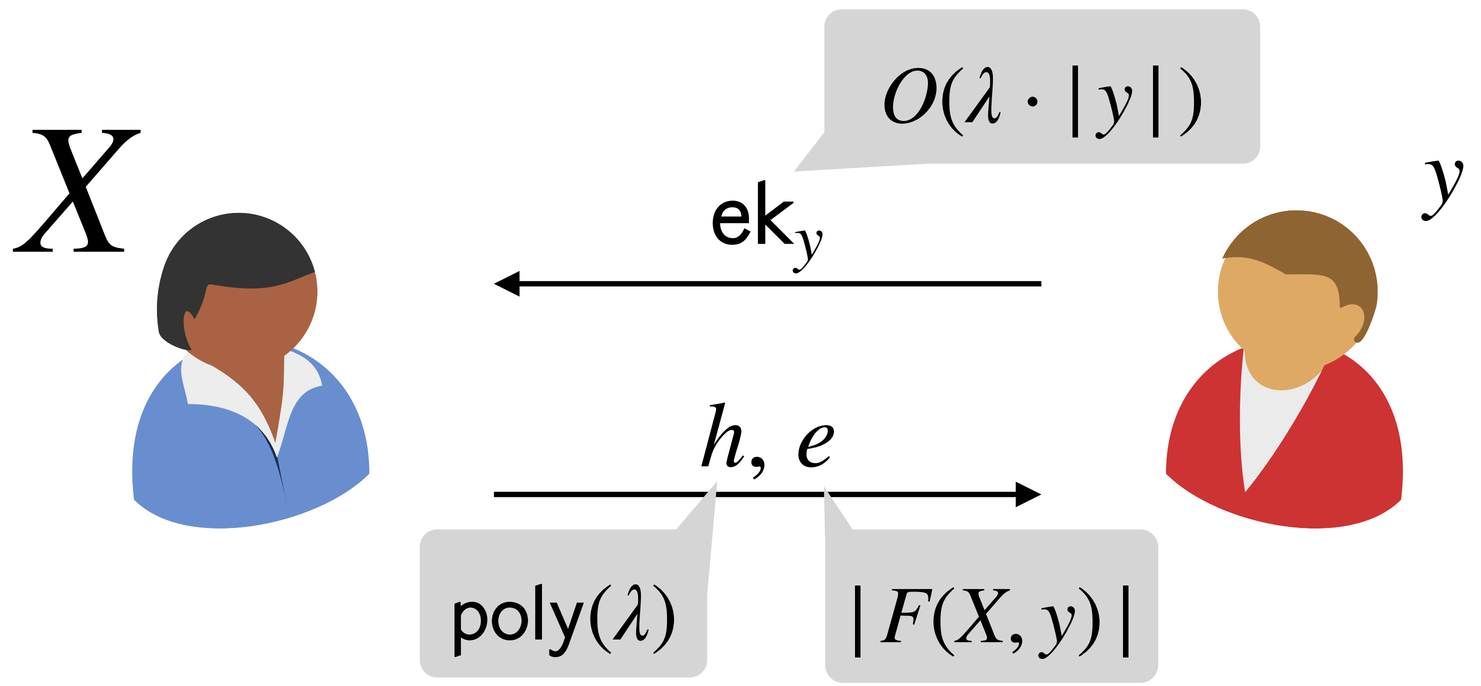

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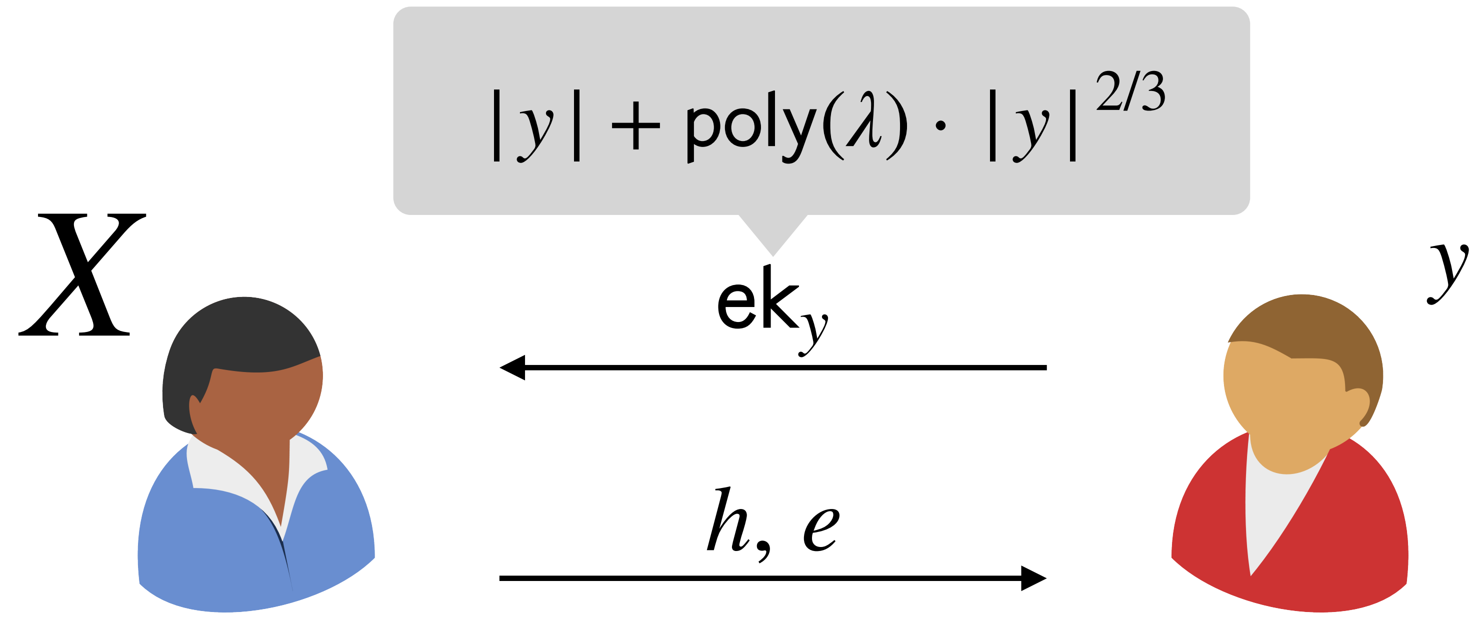

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Compactness

Encoding keys of size $|y|(1 + o(1))$

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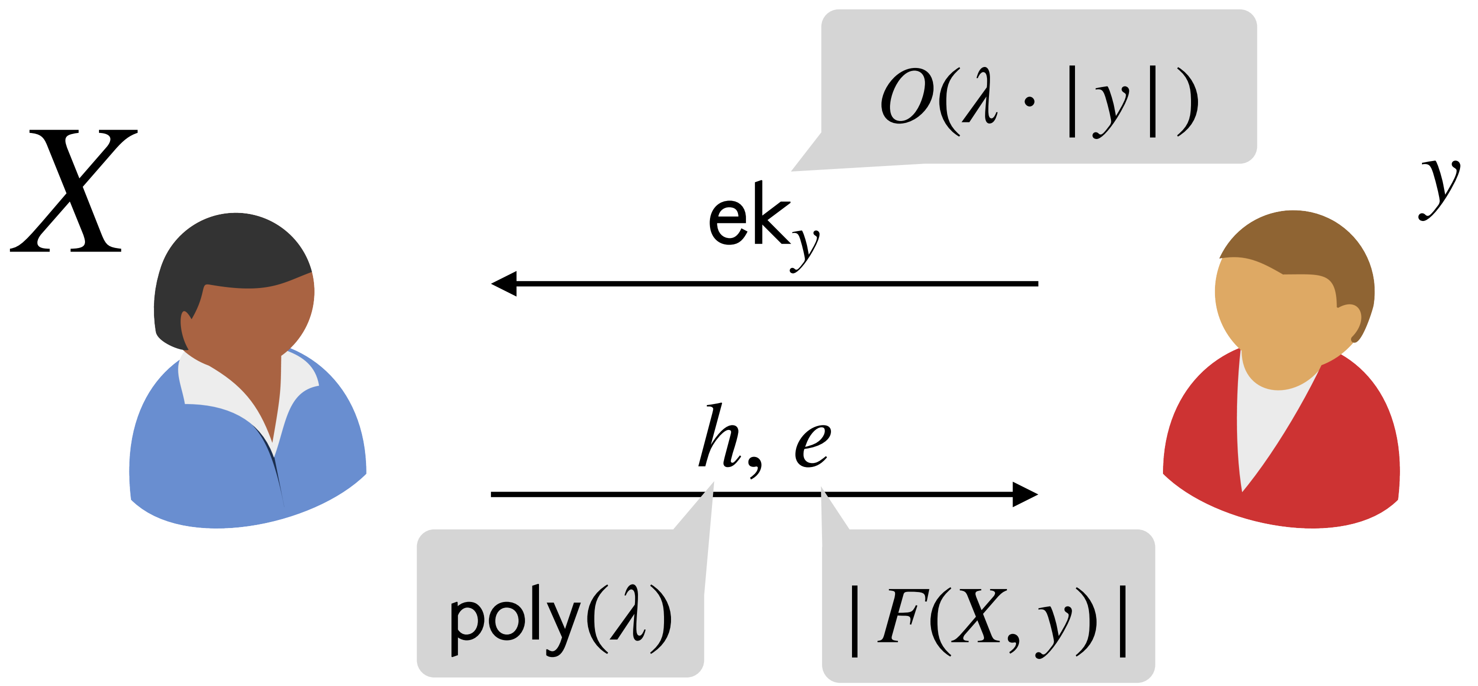

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Reusable encoding key with functions chosen on-the-fly

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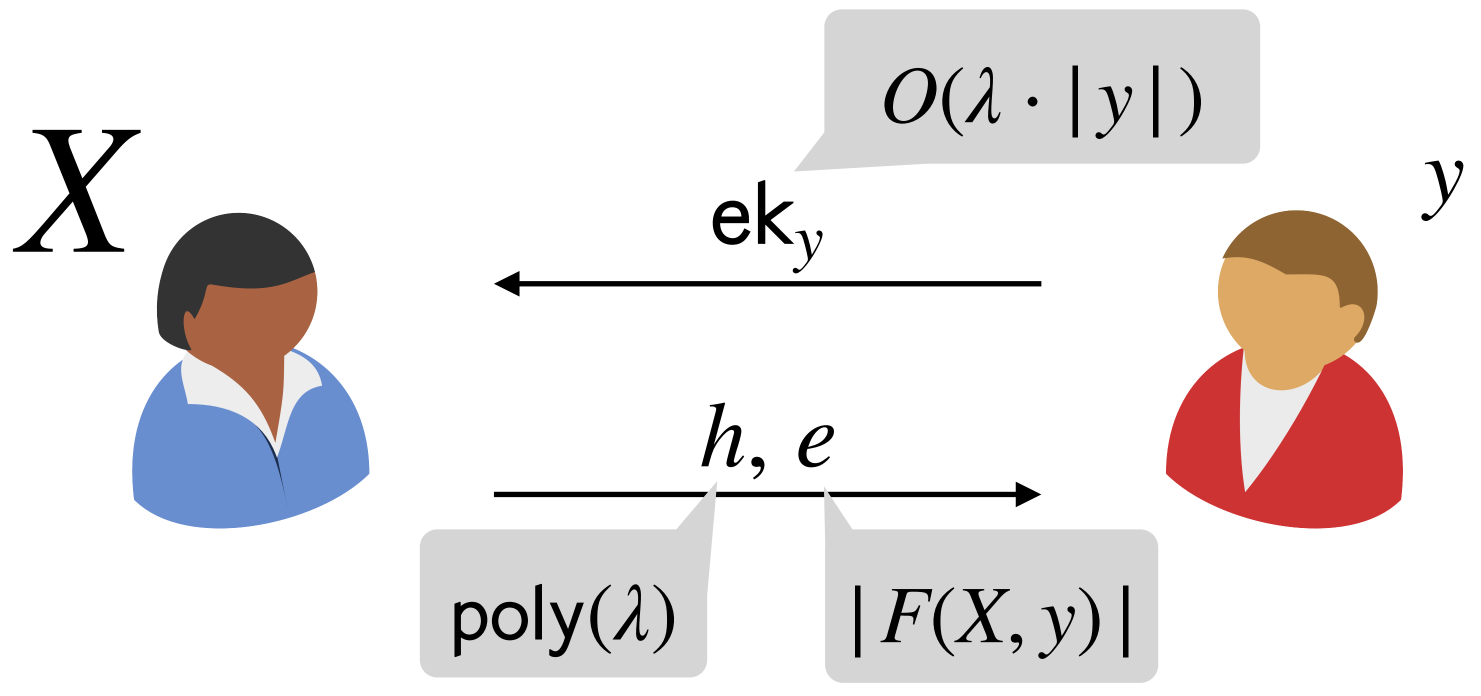

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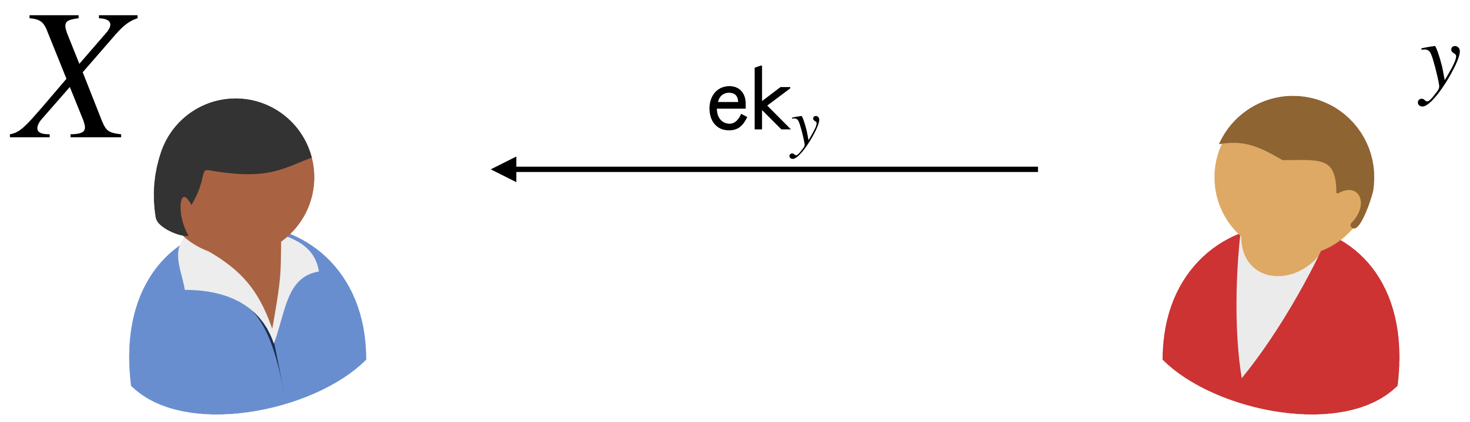

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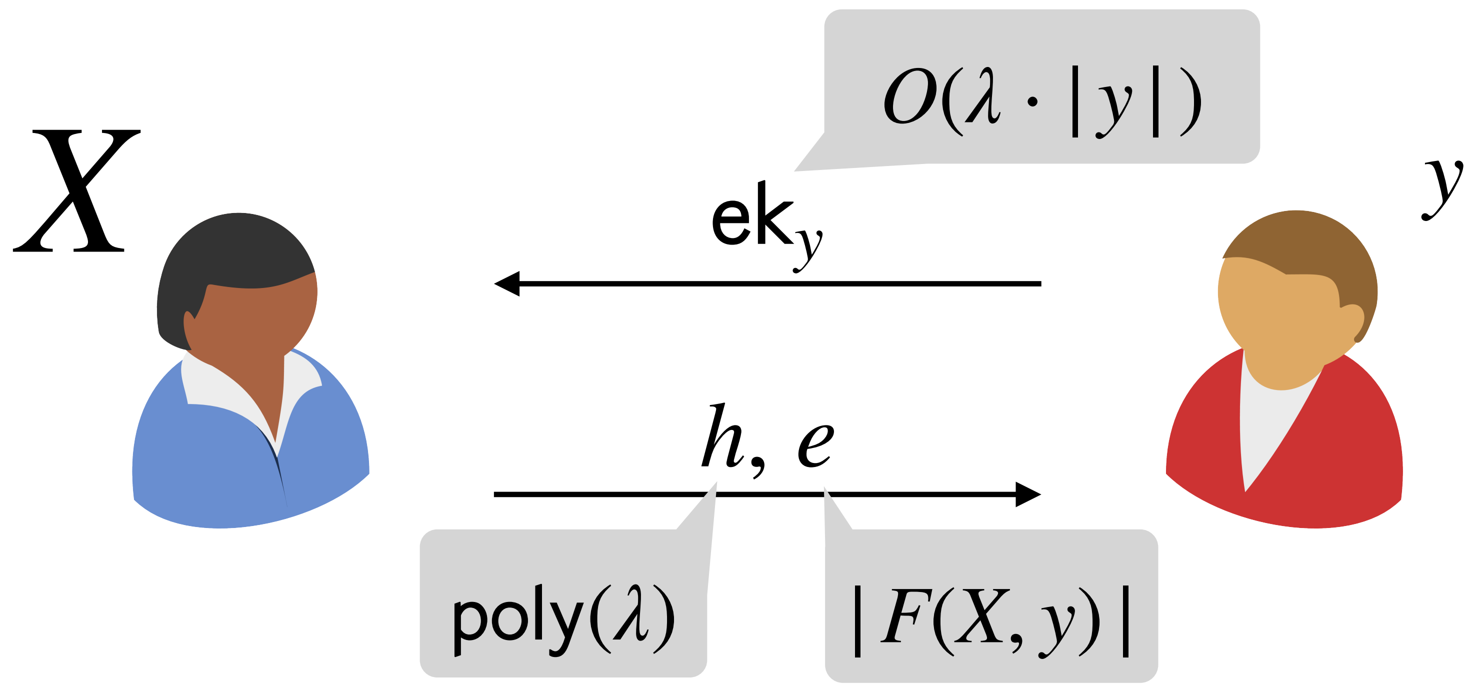
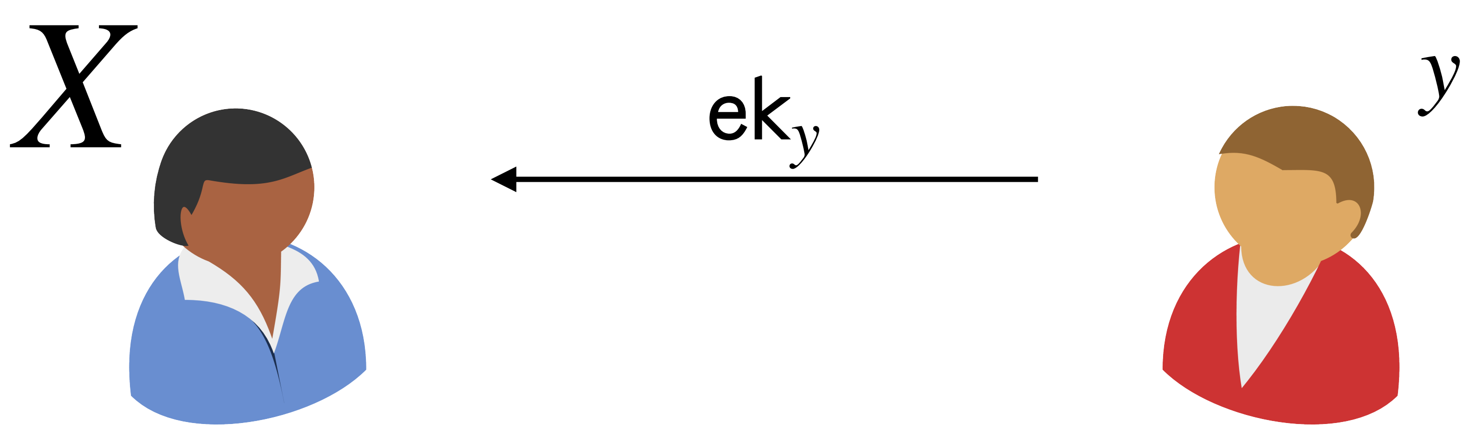


This work

$F_1 \quad F_2 \quad F_3$



Expressivity	**Compactness**	**Reusability**
Supports computing Bilinear-NC^1 programs	Encoding keys of size $\lvert y \rvert(1 + o(1))$	Reusable encoding key with functions chosen on-the-fly

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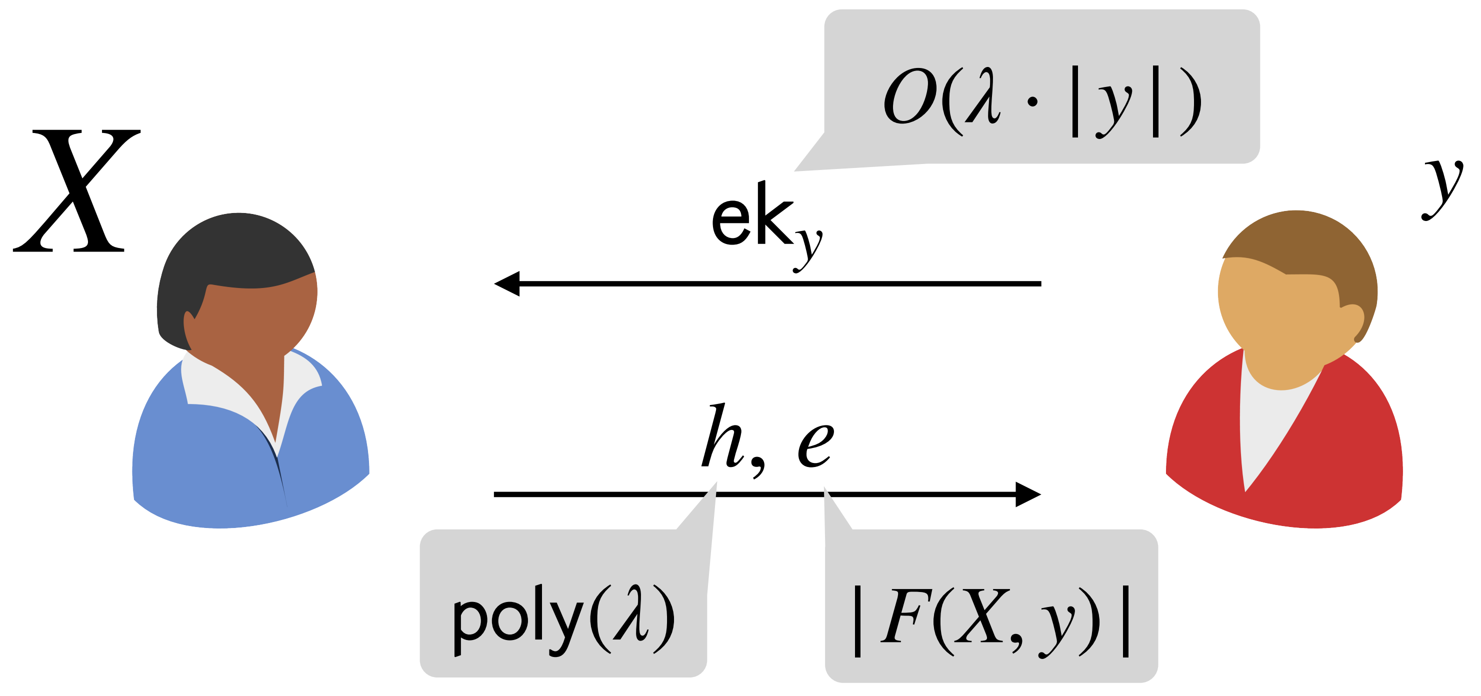

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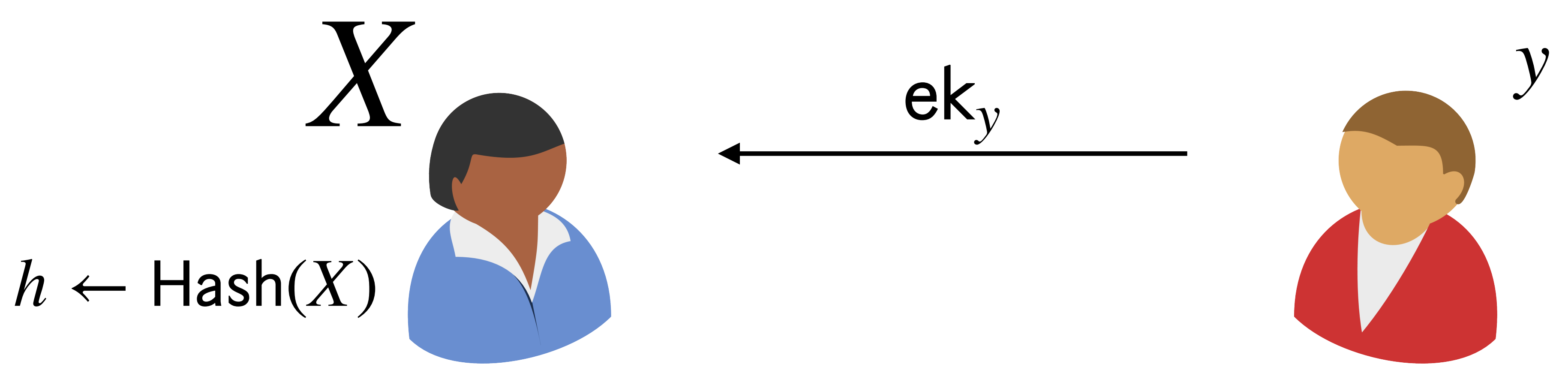
F_1 F_2 F_3



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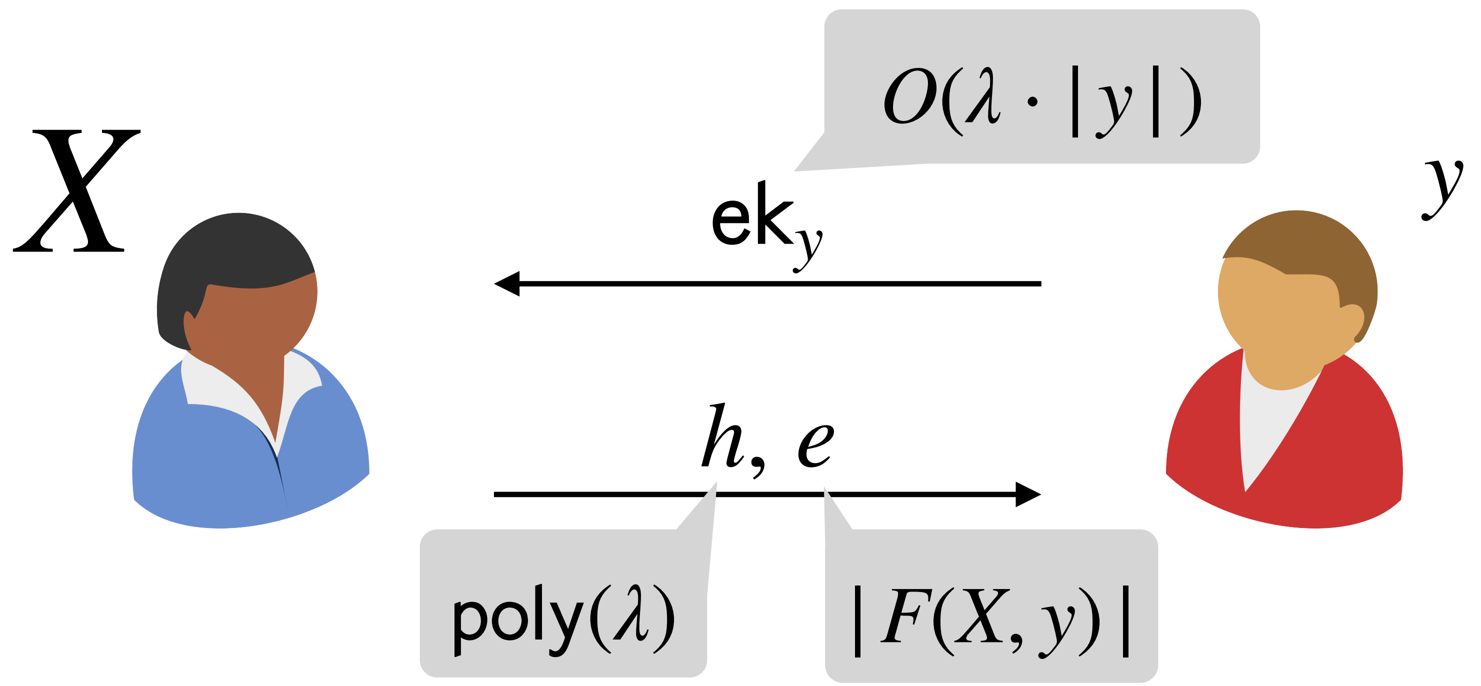
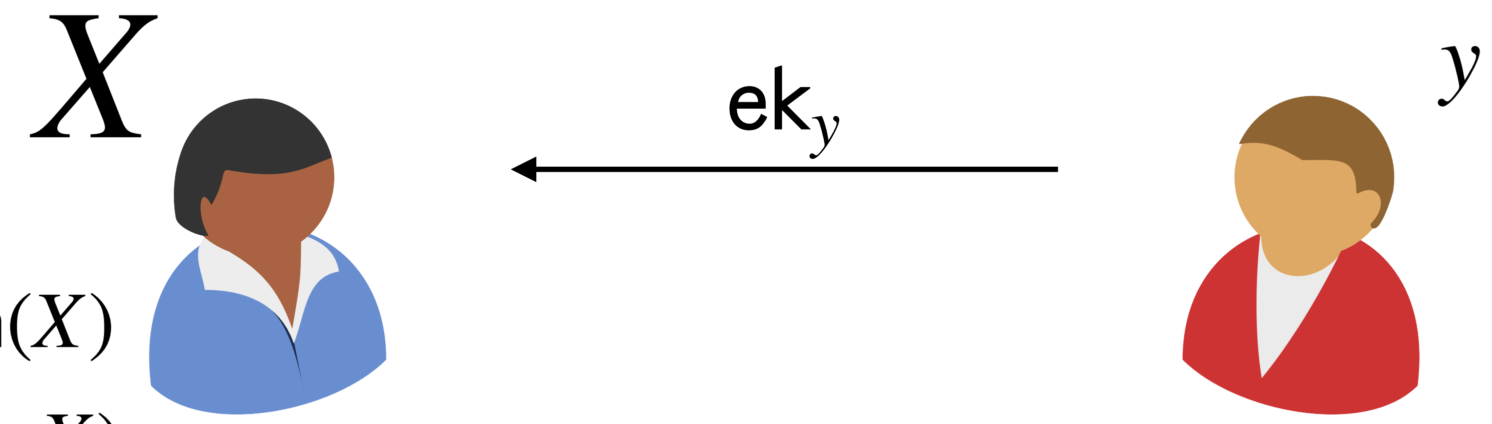


This work

F_1 $\color{blue}{F_2}$ $\color{red}{F_3}$



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$e_1 \leftarrow \mathsf{Encode}(\mathsf{F}_1, \mathsf{ek}_y, X)$

$\color{blue}{e_2} \leftarrow \mathsf{Encode}(\color{blue}{\mathsf{F}_2}, \mathsf{ek}_y, X)$

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Expressivity	**Compactness**	**Reusability**
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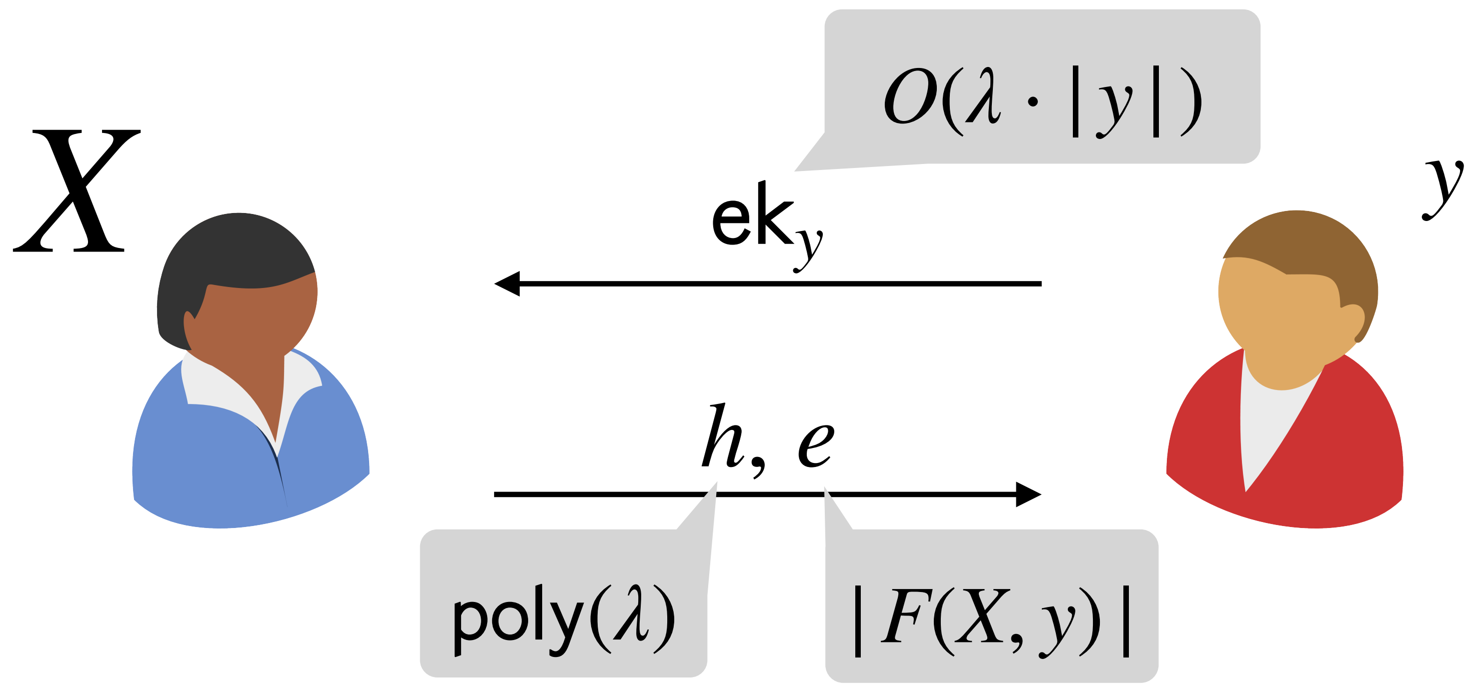

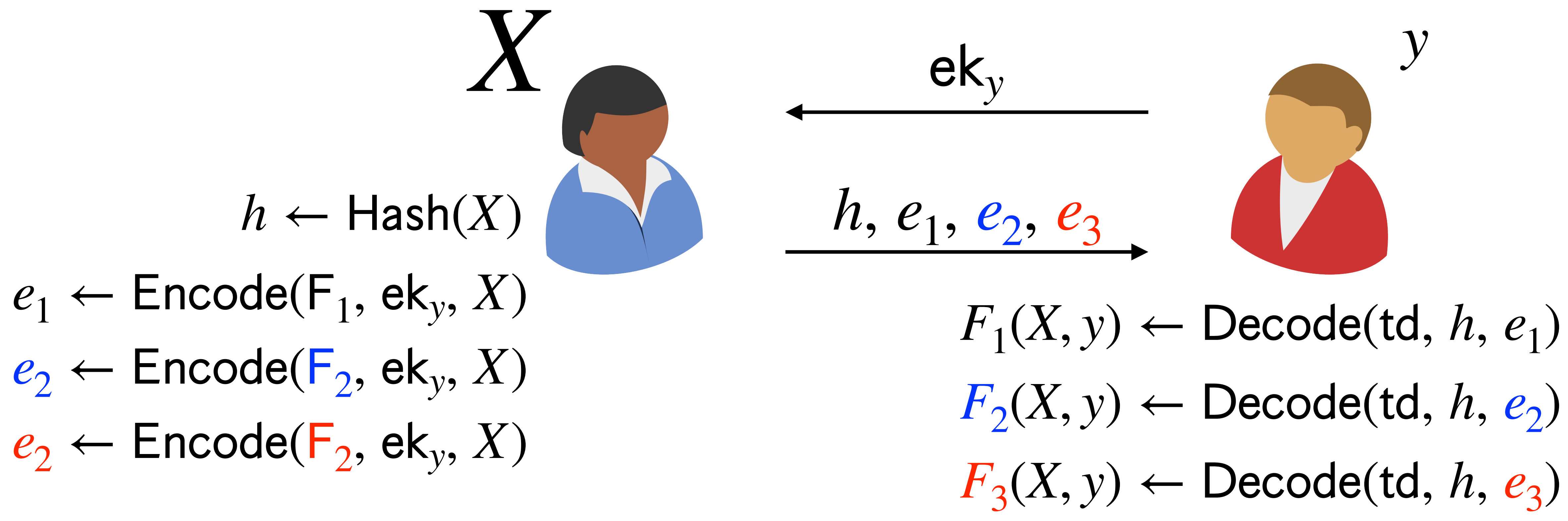

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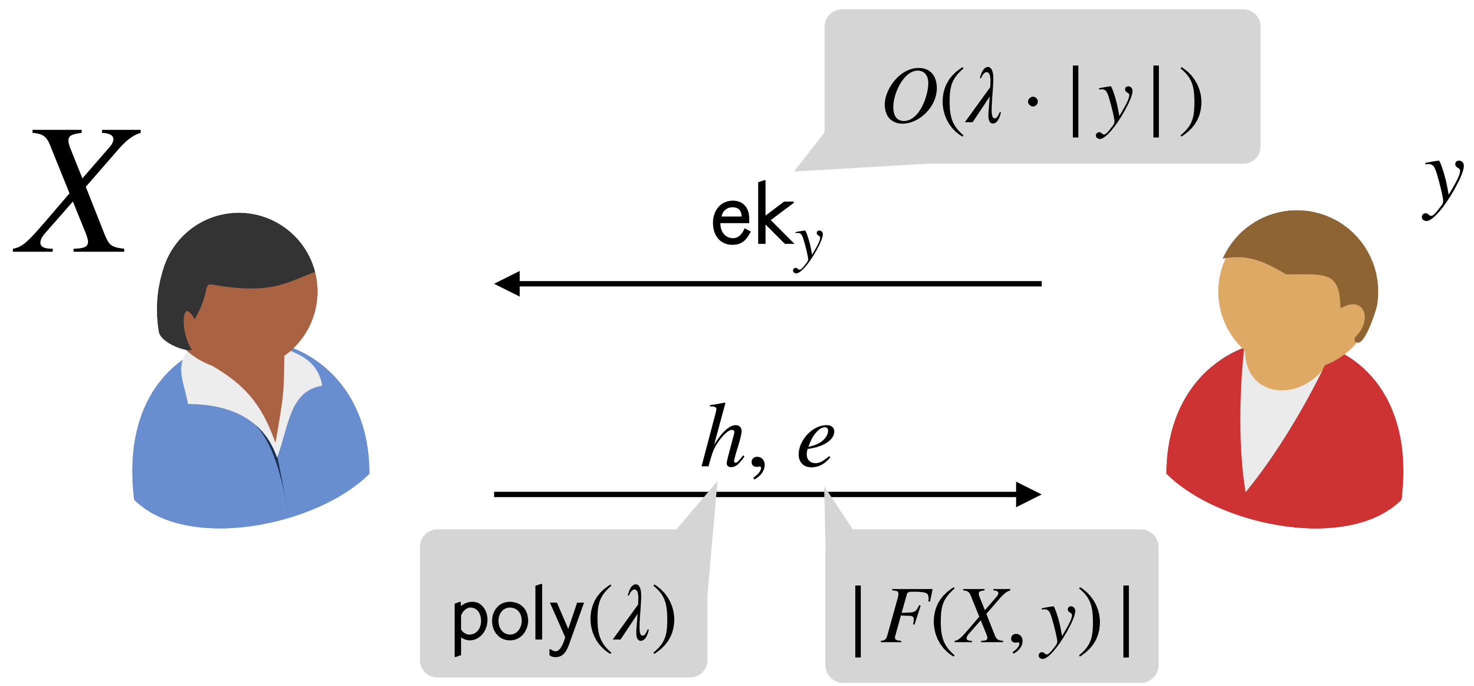

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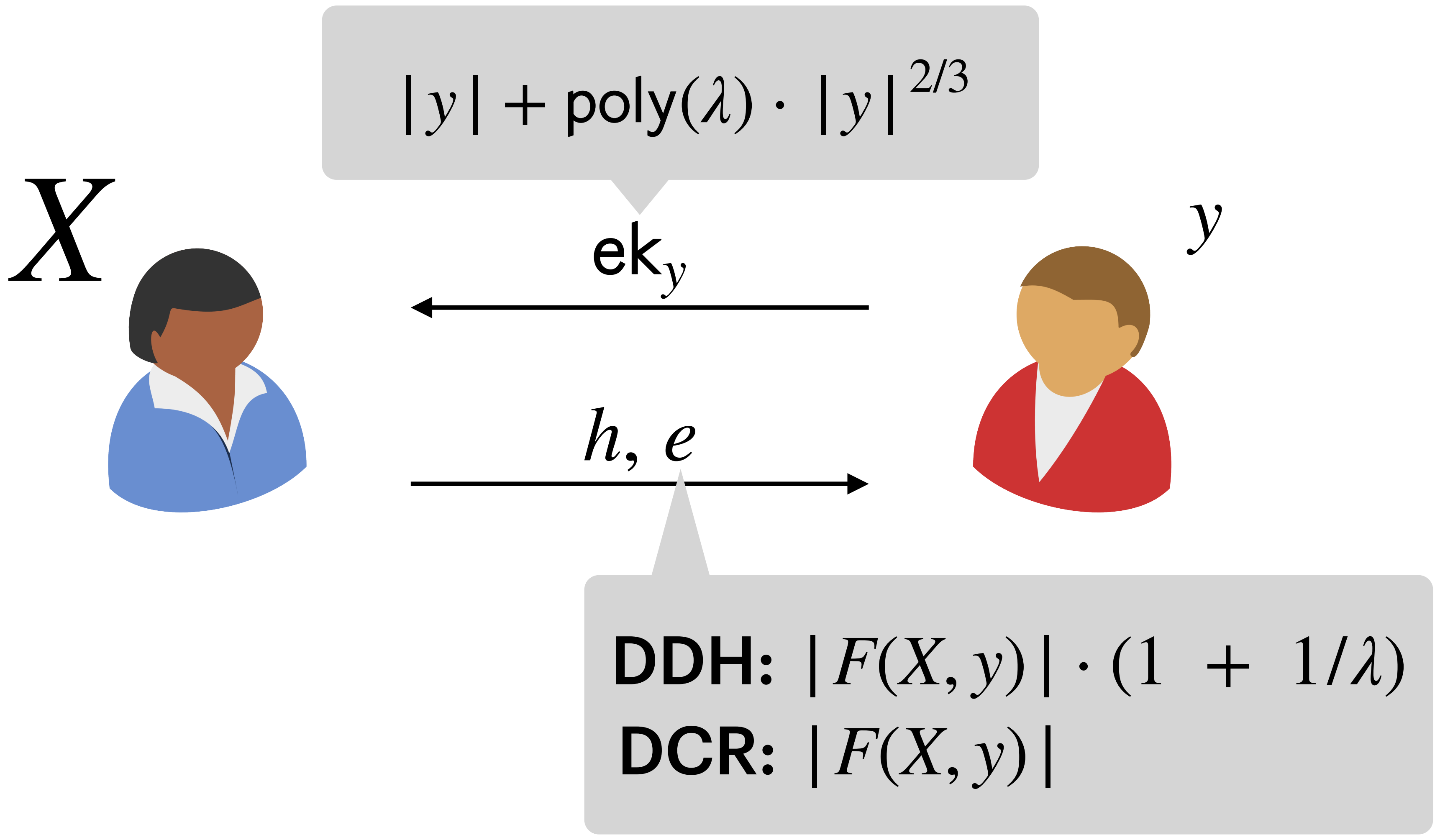

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Batch-OT with optimal rate from DDH

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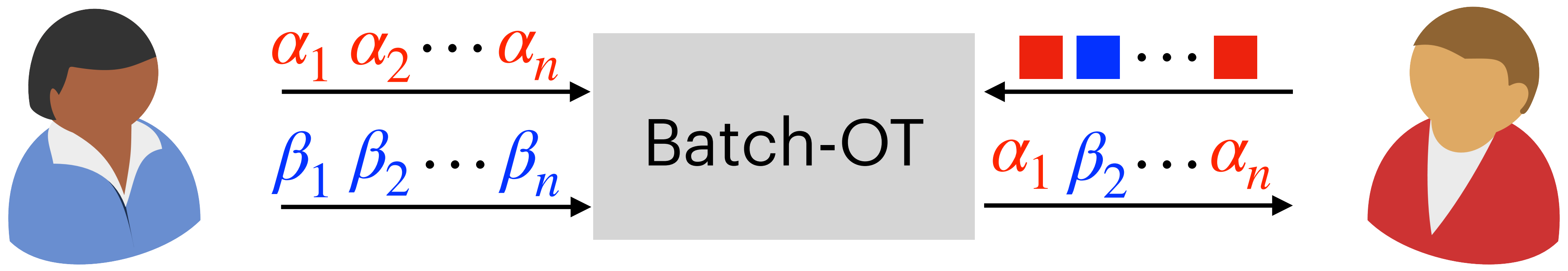
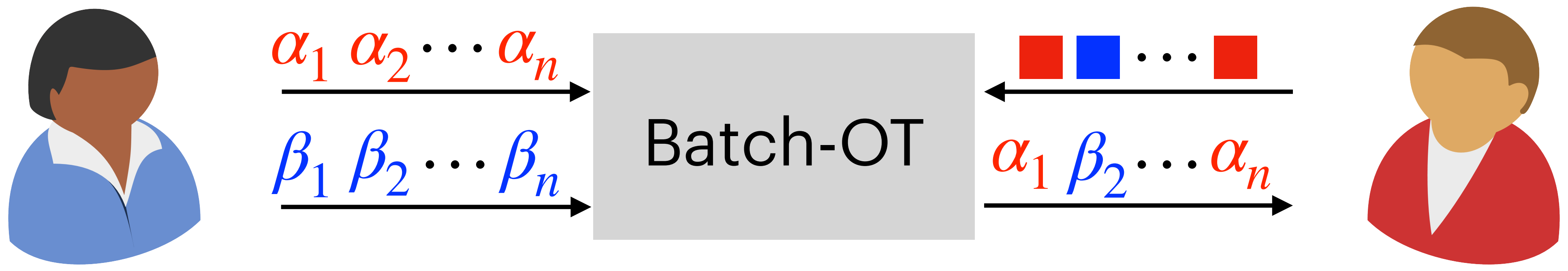


Applications 1: Compactness

Batch-OT with optimal rate from DDH



Ideal World Communication: $2n$ bits

Applications 1: Compactness

Batch-OT with optimal rate from DDH

Ideal World Communication: $2n$ bits

$X = \alpha_1,\ \ldots,\ \alpha_n,\ \beta_1,\ \ldots,\ \beta_n$

$y =$

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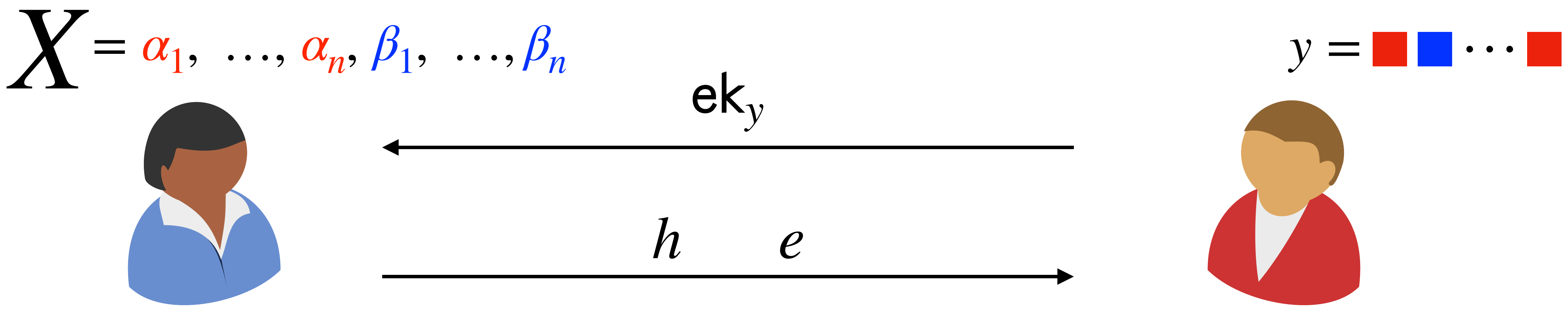
Ideal World Communication: $2n$ bits



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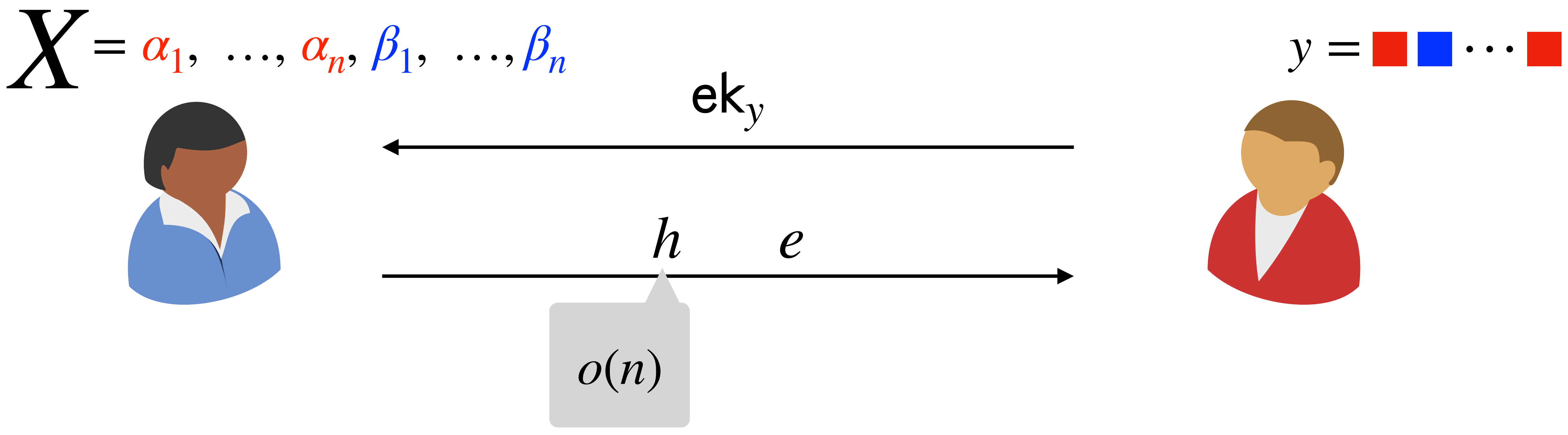

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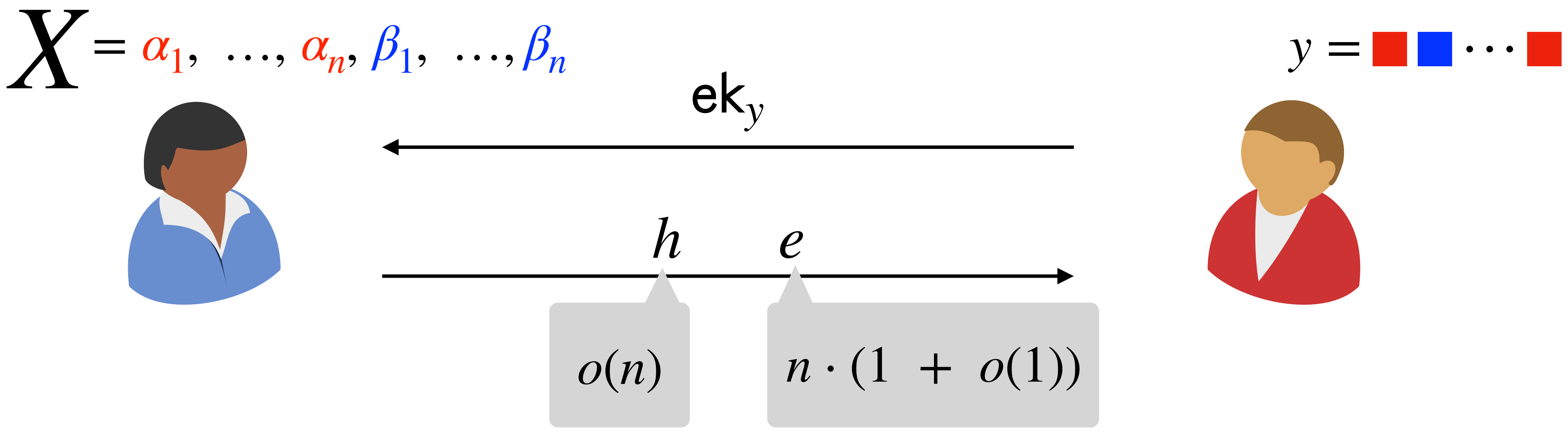

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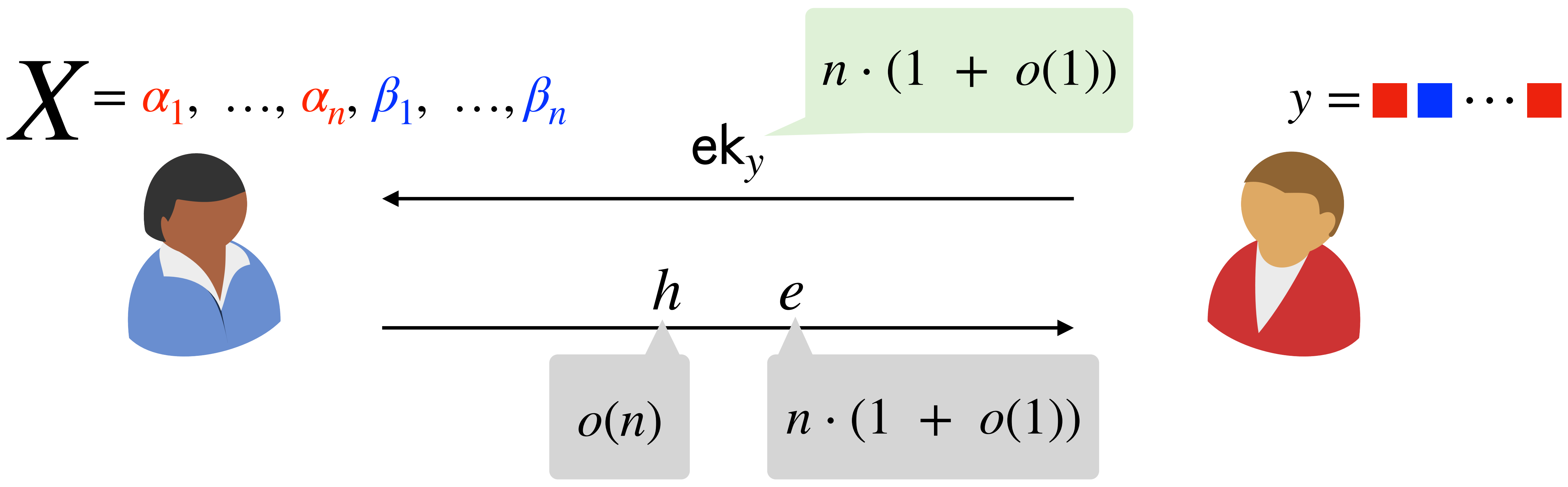
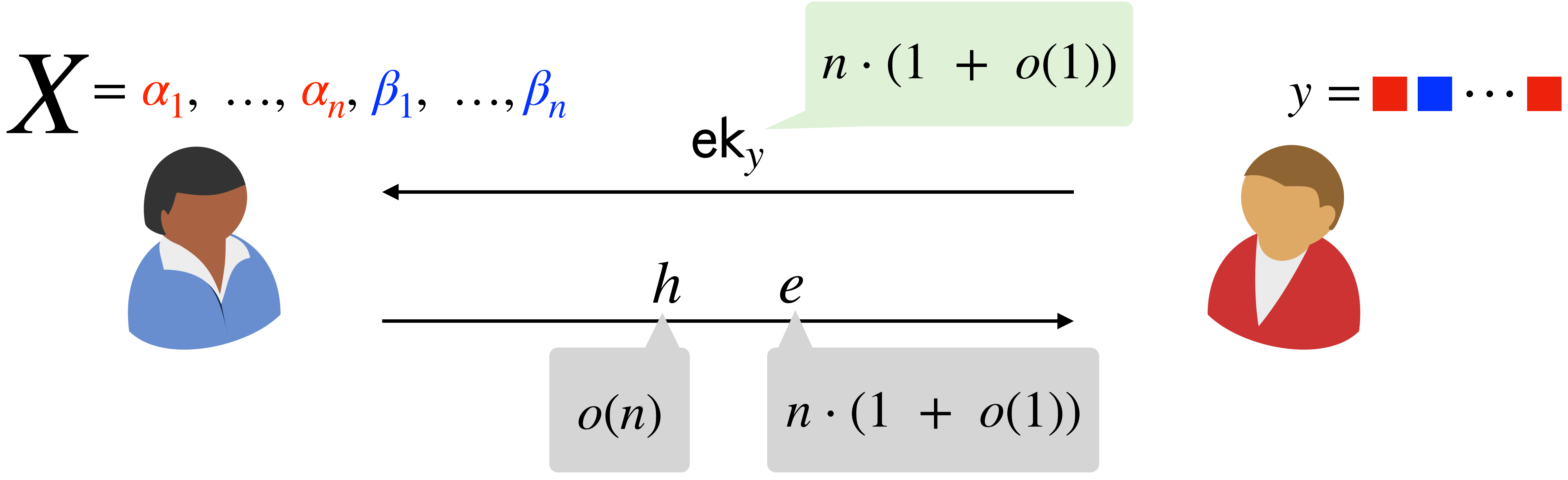


Applications 1: **Compactness**

Batch-OT with optimal rate from DDH

Ideal World Communication: $2n$ bits



Communication: $2 \cdot n \cdot (1 + o(1))$ bits

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This Work: $1 + o(1)$ rate

Semi-honest statistical sender privacy

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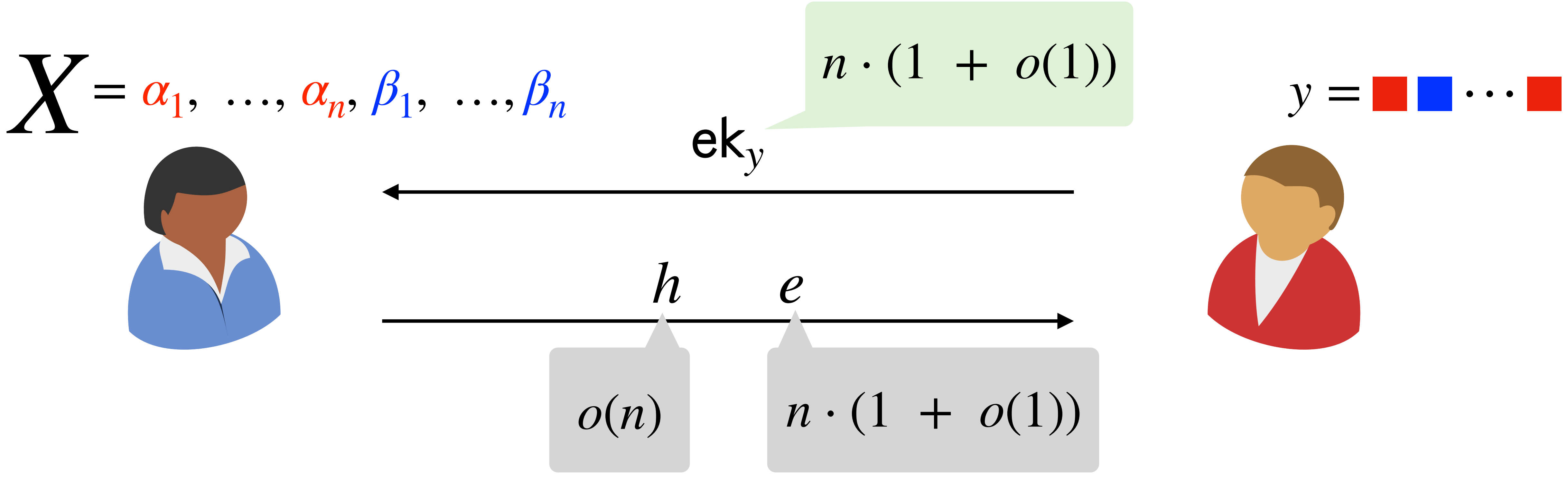

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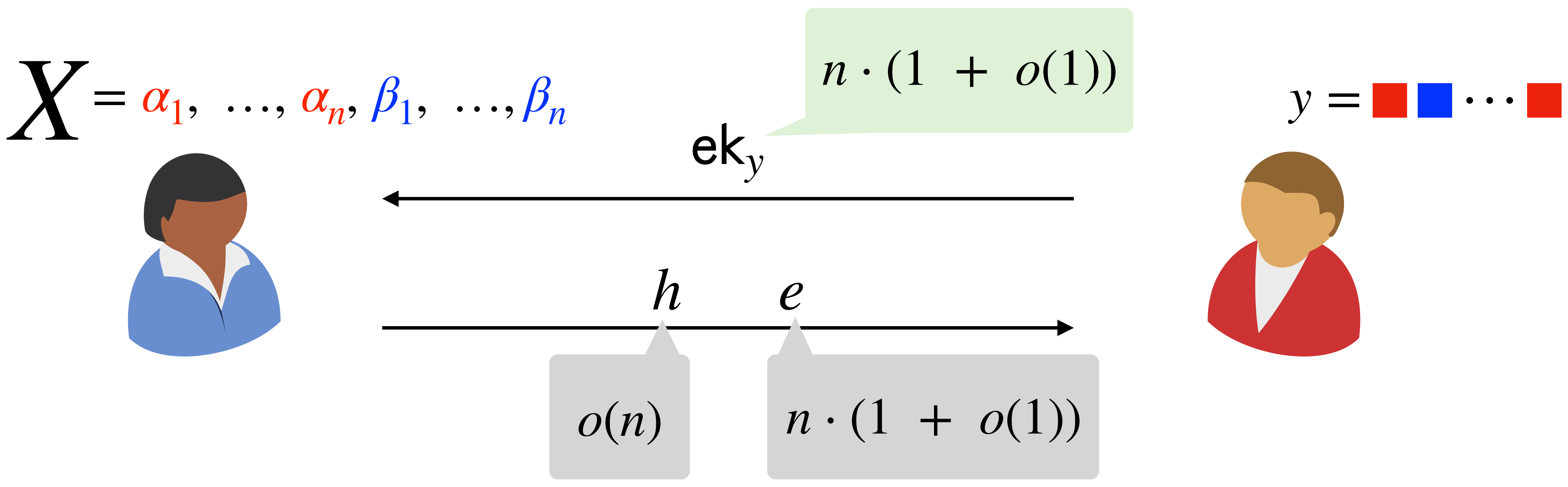

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DDH + LPN

[Boyle-Giboa-Ishai'17]:

DDH

$n \cdot (4 + o(1))$ bits communication

PKI setup

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Implications of Batch-OT with optimal rate

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String OT: $o(n)$ bits sender-to-receiver communication and $n \cdot (1 + o(1))$ bits receiver-to-sender communication

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Lossy Trapdoor Functions (LTDF): Rate-1 LTDF with public key size $n \cdot (1 + o(1))$ bits
Rate-1 LTDF with public key size $o(n)$ bits with CRS

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Private Information Retrieval: Client computation poly(n, λ)
Database size: 2^n
Upload communication $n +$ poly(λ) bits
Download communication $n \cdot$ poly(λ) bits

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Upload communication $n +$ poly(λ) bits
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Other Implications

Branching programs over encrypted data

Correlated symmetric PIR

Applications 2: Compactness + **Expressivity**

Sublinear 2PC from DCR, with one-sided statistical security for layered circuits

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Sublinear 2PC from DCR, with one-sided statistical security for layered circuits

Bilinear-$\mathsf{NC}^1 \supseteq \log\log$-depth circuits

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This Work: $|x| + (2 + o(1)) \cdot \dfrac{|C|}{\log\log|C|} + |y|^{2/3} \cdot \mathrm{poly}(\lambda)$ bits communication

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Linear communication in **computationally** secure input

Sublinear communication in **statistically** secure input

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Sublinear in size of circuit

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[Couteau-Meyer-Passelégue-Riahinia'23]: $|x| + |y| + \dfrac{|C|}{\log\log|C|} + \mathrm{poly}(\lambda)$ bits communication

Circular security of Paillier

Layered circuits over $\mathbb{Z}_N$

Applications 3: Compactness + Expressivity + **Reusability**

Improving Communication in the Amortized Setting

Public function F

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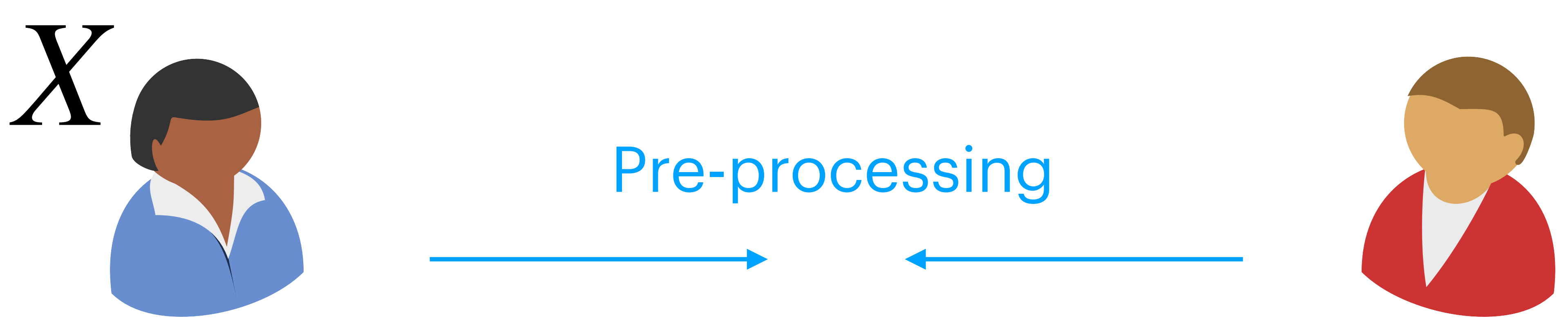
Improving Communication in the Amortized Setting

Public function F



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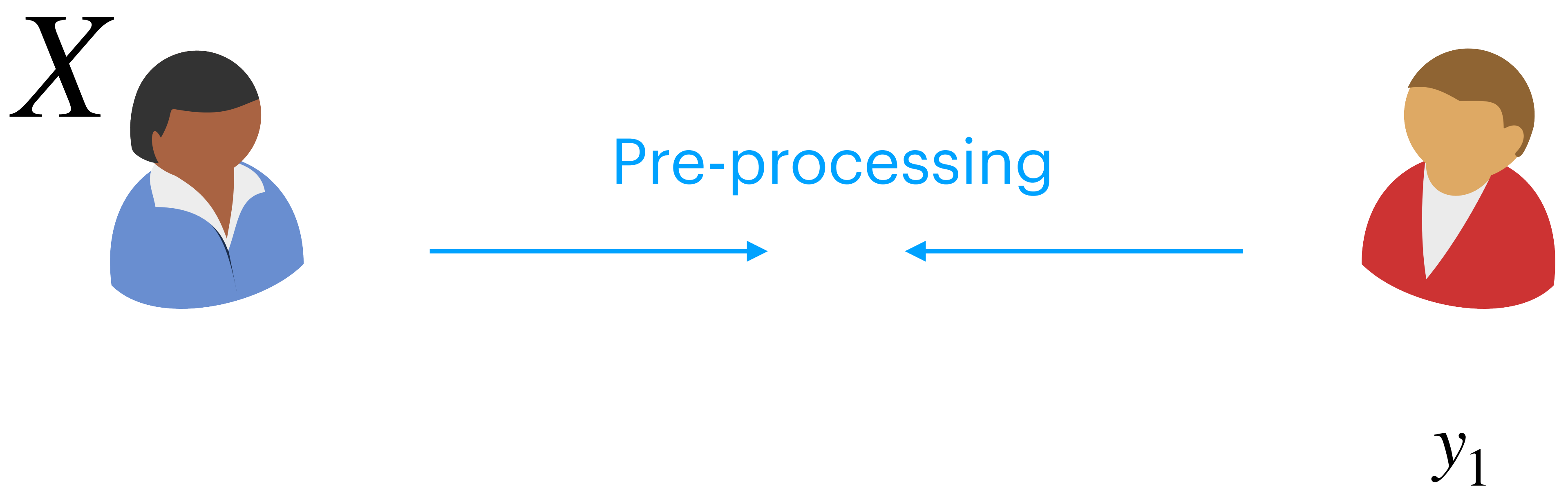

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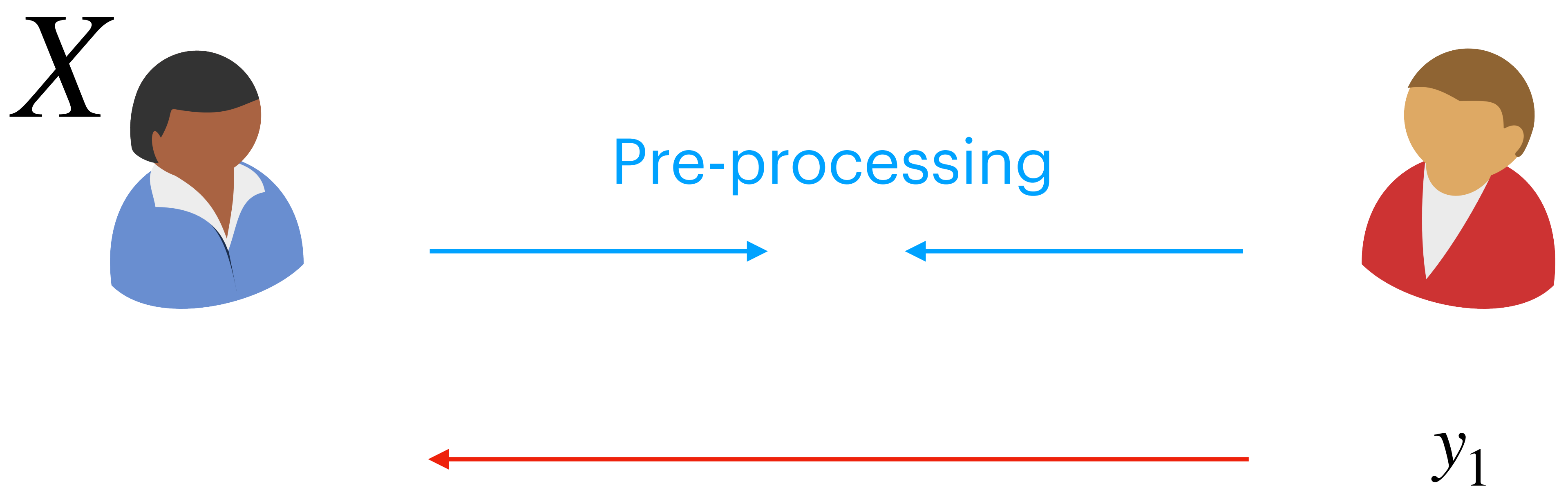

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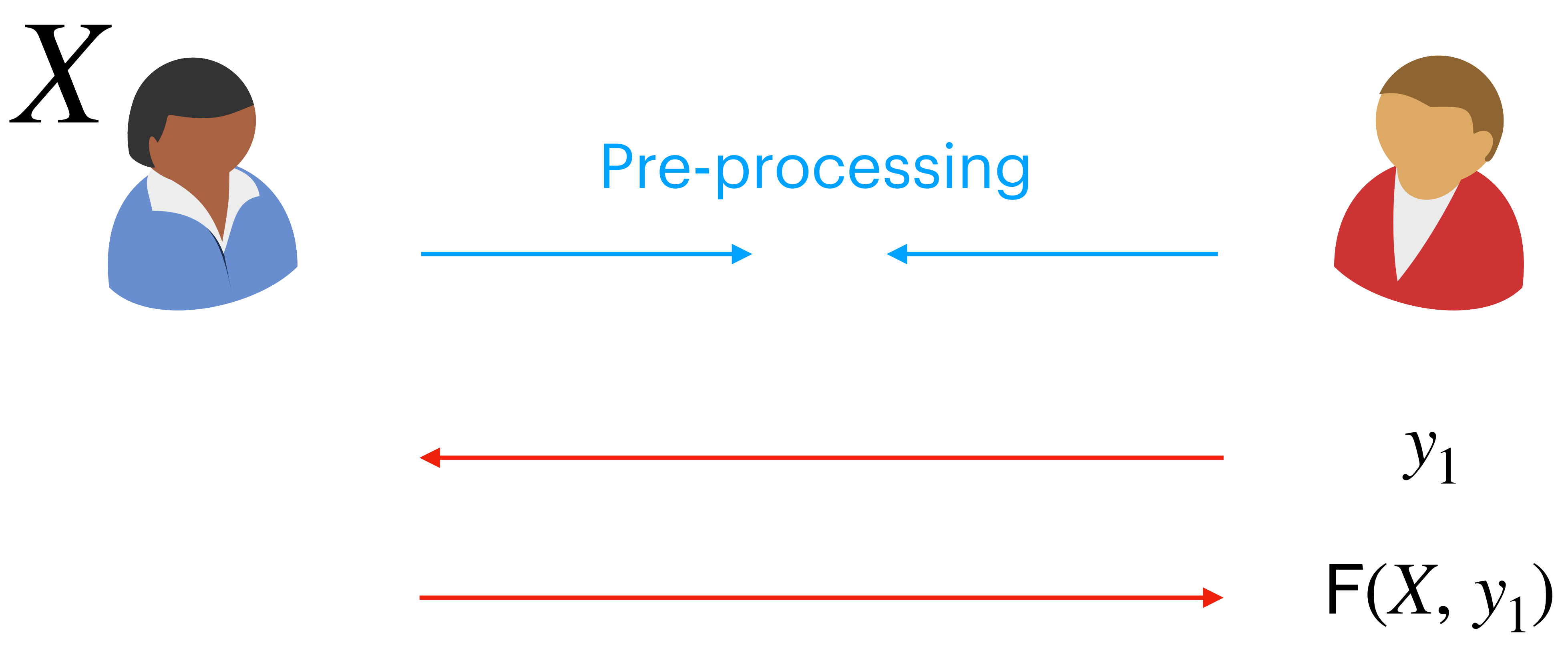

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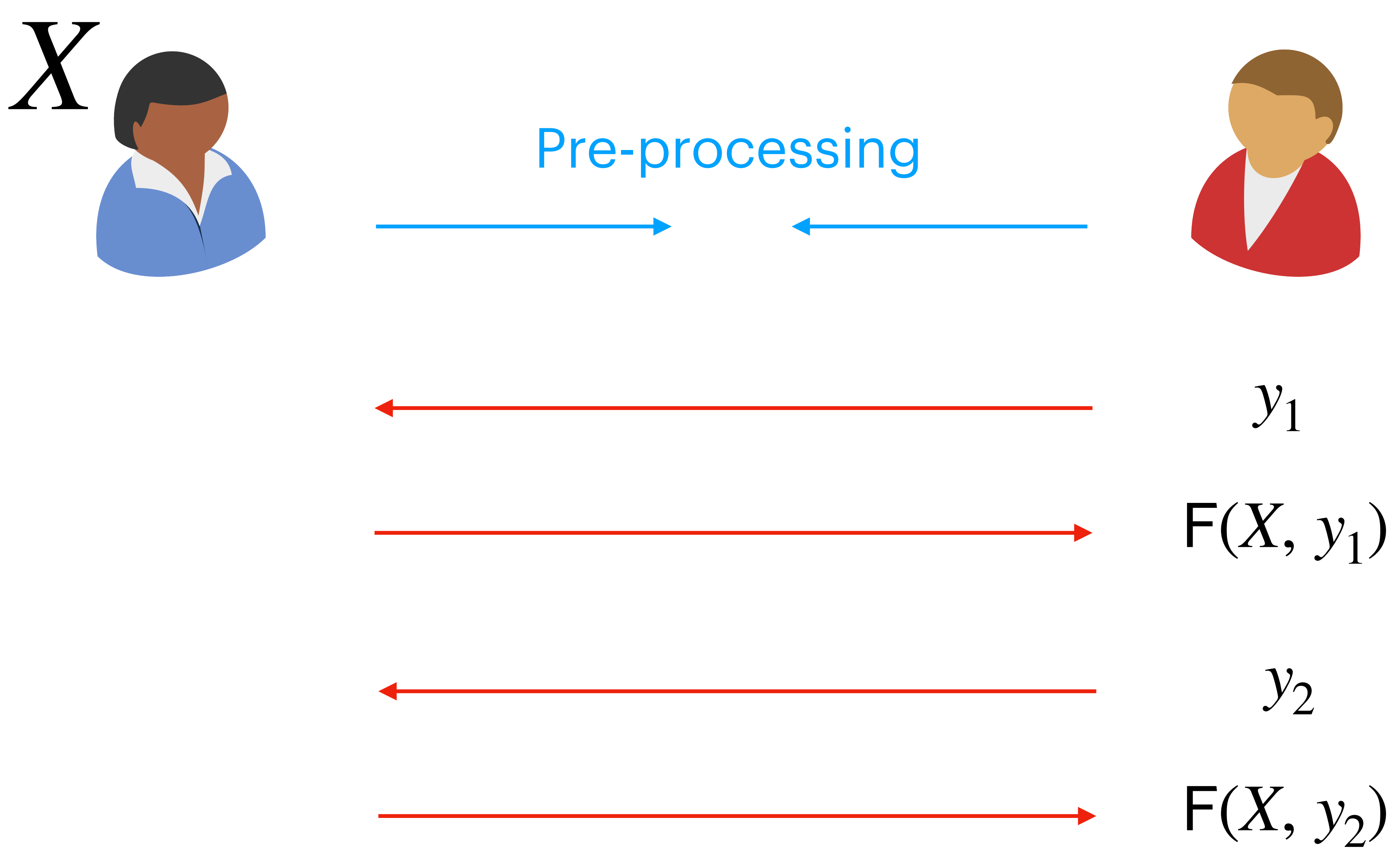

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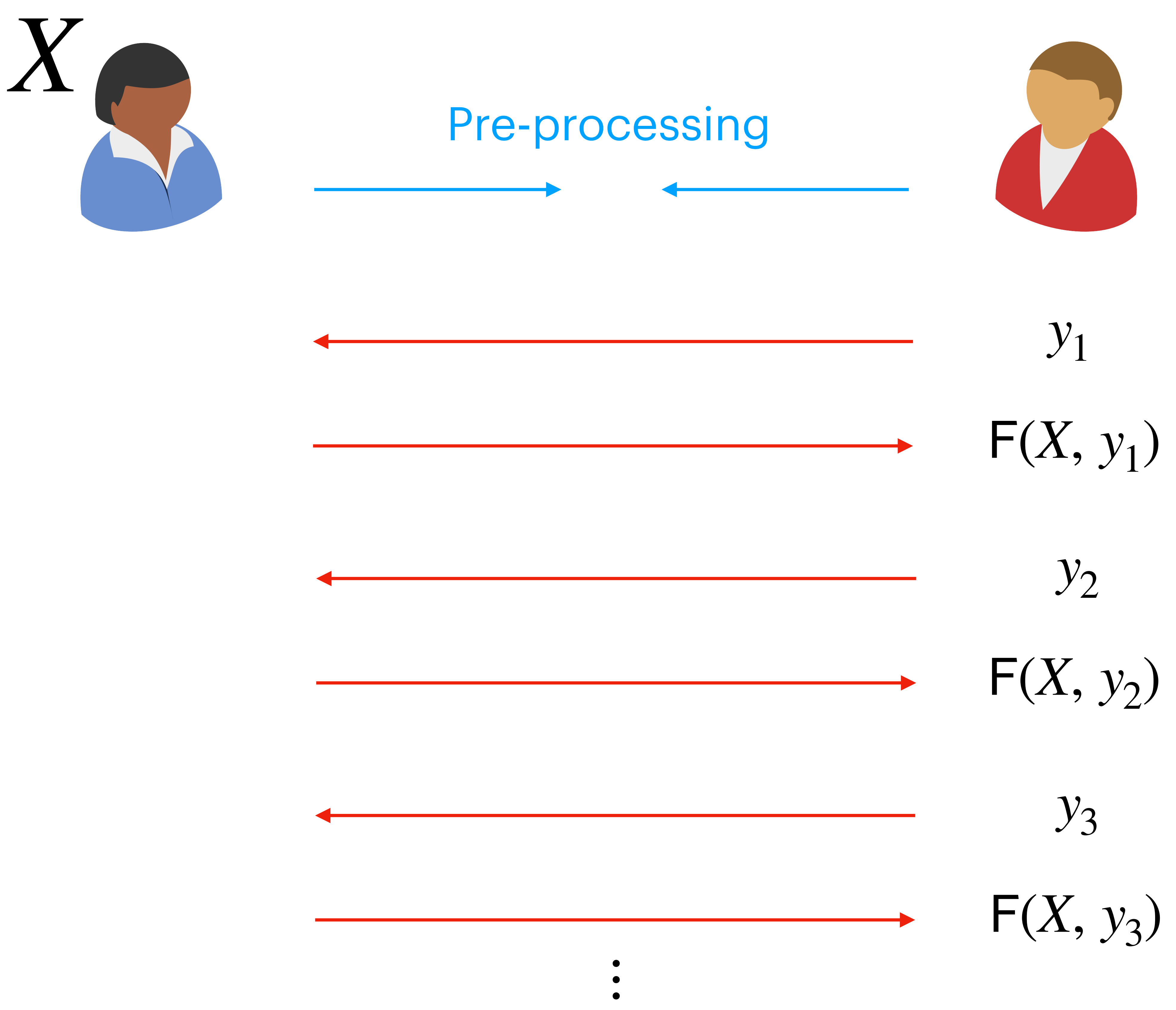

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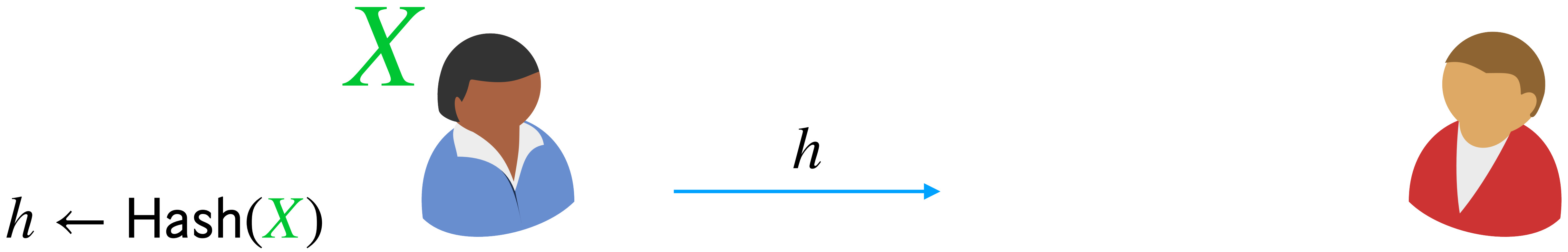

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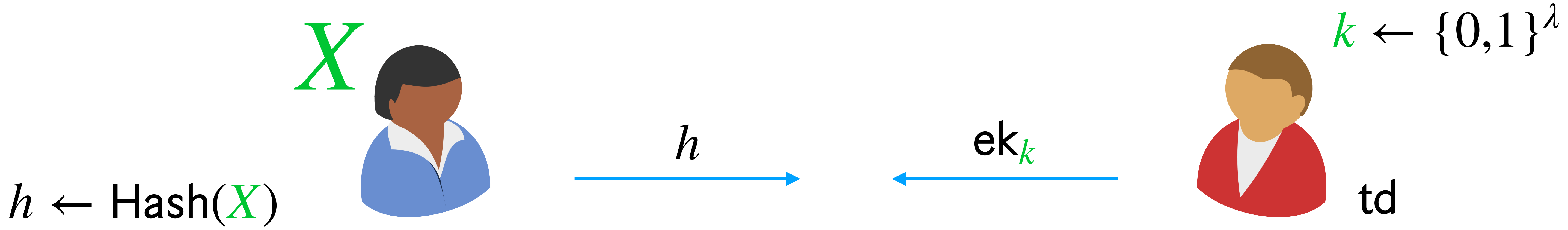

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Public function F

X

$h \leftarrow \text{Hash}(X)$

h →

← ek_k

$k \leftarrow \{0,1\}^\lambda$

td

y_1

$z_1 = y_1 \oplus \text{PRF}(k, 1)$

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Public function F

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$h \leftarrow \text{Hash}(X)$

$\xrightarrow{h}$

$\xleftarrow{\text{ek}_k}$

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td

y_1

$z_1 = y_1 \oplus \text{PRF}(k, 1)$

$\xleftarrow{z_1}$

z_1 ensures privacy of y_1

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$\text{F}_1(X, k) = F(X, z_1 \oplus \text{PRF}(k, 1))$

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Public function F

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$e_1 \leftarrow \mathsf{Encode}(\mathsf{F}_1, \mathsf{ek}_k, X)$ $\xrightarrow{e_1}$

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$\text{F}_2(X, k) = F(X, z_2 \oplus \text{PRF}(k, 2))$ $\xleftarrow{z_2}$ $z_2 = y_2 \oplus \text{PRF}(k, 2)$

$e_2 \leftarrow \text{Encode}(\text{F}_2, \text{ek}_k, X)$ $\xrightarrow{e_2}$ $F(X, y_2) \leftarrow \text{Decode}(\text{td}, e_2, h)$

$\vdots$

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Improving Communication in the Amortized Setting

Public function F

X  $h \leftarrow \mathsf{Hash}(X)$

$\xrightarrow{h}$ $\xleftarrow{\mathsf{ek}_k}$

 $k \leftarrow \{0,1\}^\lambda$ td

$\mathsf{F}_1(X, k) = F(X, z_1 \oplus \mathsf{PRF}(k, 1))$

$e_1 \leftarrow \mathsf{Encode}(\mathsf{F}_1, \mathsf{ek}_k, X)$

y_1

$\xleftarrow{z_1}$

$z_1 = y_1 \oplus \mathsf{PRF}(k, 1)$

(e_1, h)

Reusability $\Longrightarrow$ Encoding of k can be reused with functions chosen on-the-fly

$\mathsf{F}_2(X, k) = F(X, z_2 \oplus \mathsf{PRF}(k, 2))$

$e_2 \leftarrow \mathsf{Encode}(\mathsf{F}_2, \mathsf{ek}_k, X)$

$\xleftarrow{z_2}$

$z_2 = y_2 \oplus \mathsf{PRF}(k, 2)$

$\xrightarrow{e_2}$

$F(X, y_2) \leftarrow \mathsf{Decode}(\mathsf{td}, e_2, h)$

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$k \leftarrow \{0,1\}^\lambda$

$h \leftarrow \mathsf{Hash}(X)$

$\xrightarrow{h}$

$\xleftarrow{\mathsf{ek}_k}$

td

$|z_1| = |y_1|$

y_1

$\mathsf{F}_1(X, k) = F(X, z_1 \oplus \mathsf{PRF}(k, 1))$

$\xleftarrow{z_1}$

$z_1 = y_1 \oplus \mathsf{PRF}(k, 1)$

$e_1 \leftarrow \mathsf{Encode}(\mathsf{F}_1, \mathsf{ek}_k, X)$

$\xrightarrow{e_1}$

$F(X, y_1) \leftarrow \mathsf{Decode}(\mathsf{td}, e_1, h)$

DCR: $|e_1| = |F(X, k)|$

$\mathsf{F}_2(X, k) = F(X, z_2 \oplus \mathsf{PRF}(k, 2))$

$\xleftarrow{z_2}$

$z_2 = y_2 \oplus \mathsf{PRF}(k, 2)$

$e_2 \leftarrow \mathsf{Encode}(\mathsf{F}_2, \mathsf{ek}_k, X$

e_2

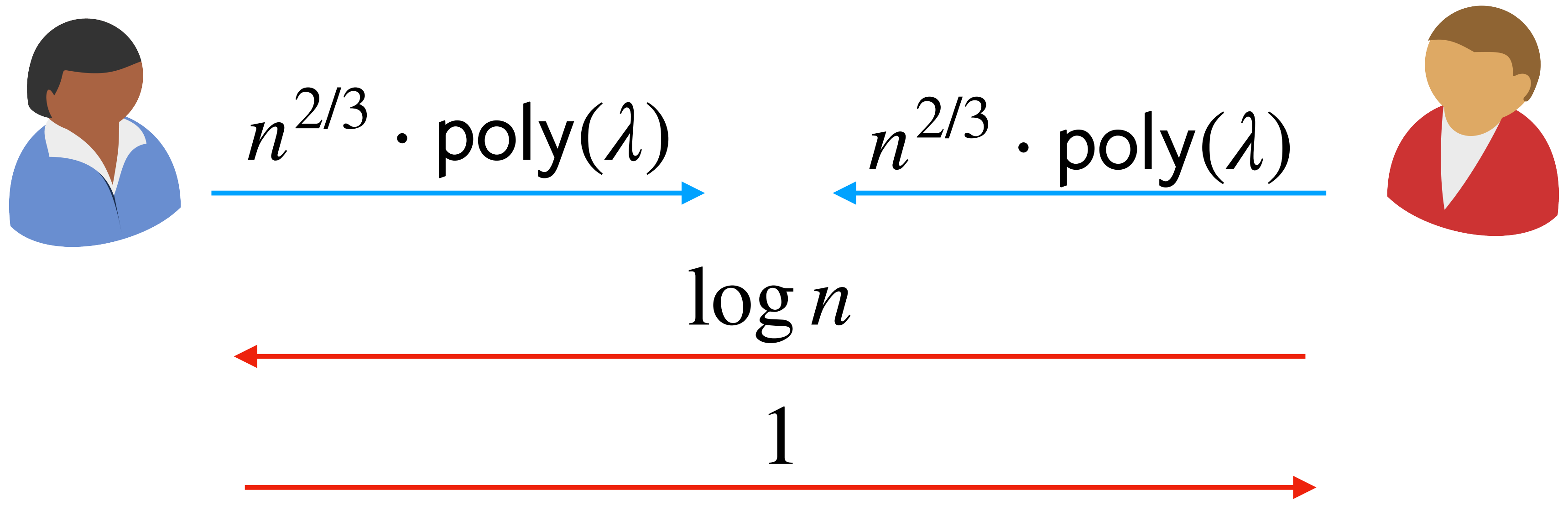
$y_2) \leftarrow \mathsf{Decode}(\mathsf{td}, e_2, h)$

Optimal communication per query from DCR

Applications 3: Compactness + Expressivity + **Reusability**

Optimal preprocessing symmetric Private Information Retrieval from DCR

$|X| = n$



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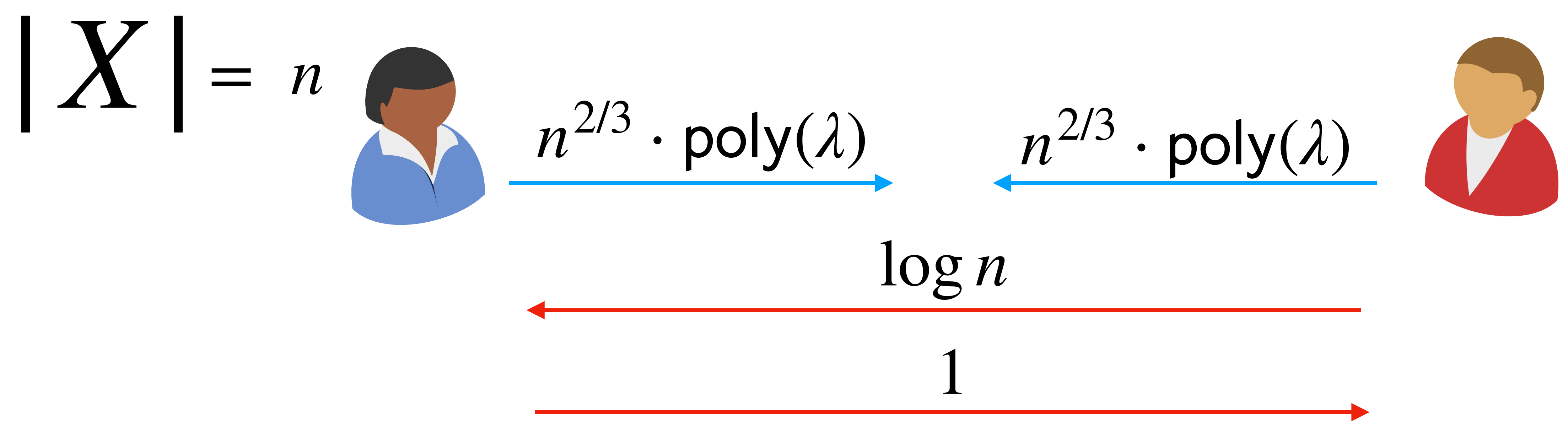
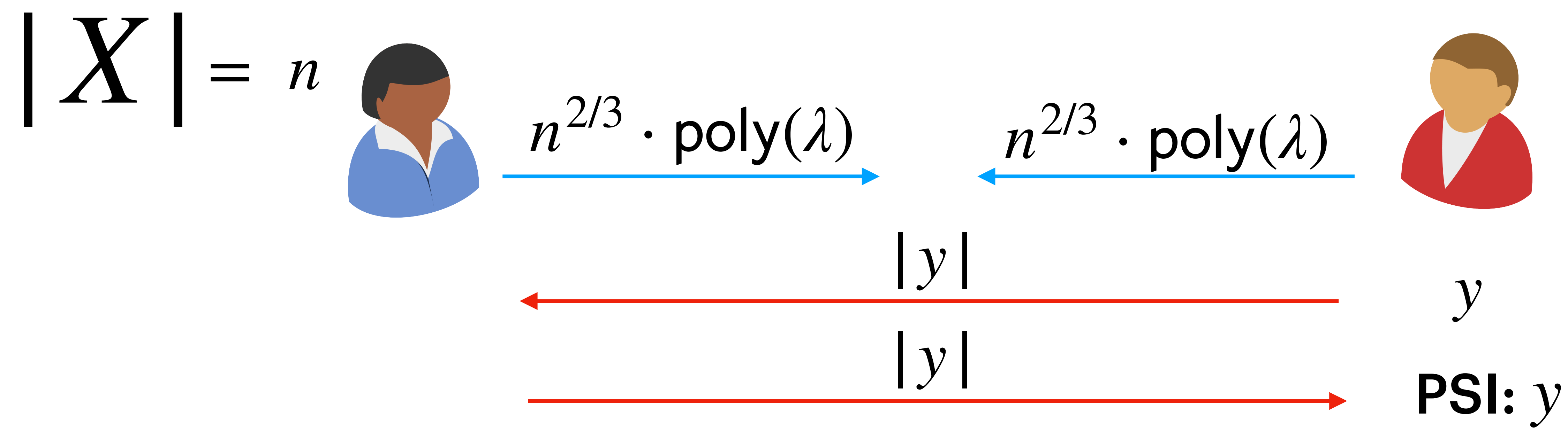


Rate-$\frac{1}{2}$ Private Set Intersection (PSI) and Fuzzy-PSI from DCR



PSI: $y \in X$

Fuzzy-PSI: Is y close to an element in X

Constructing Enhanced TDH

Constructing Enhanced TDH

Staged Homomorphic Secret Sharing

[Couteau-Meyer-Passelégue-Riahinia’23]

+

Trapdoor Hash Functions

[Döttling-Garg-Ishai-Malavolta-Mour-Ostrovsky'19]

Alternative view: Extending Succinct HSS [Abram-Roy-Scholl’24] using Staged HSS

Staged Homomorphic Secret Sharing

X

y

Staged Homomorphic Secret Sharing

X

y

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

Staged Homomorphic Secret Sharing

X

$\mathsf{ct}_y \leftarrow \mathsf{Encrypt}(\mathsf{pk}, y)$

y

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

Staged Homomorphic Secret Sharing

X

$\mathsf{ct}_y \leftarrow \mathsf{Encrypt}(\mathsf{pk}, y)$

$\mathsf{share}_A, \mathsf{share}_B \leftarrow \mathsf{Share}(\mathsf{pk}, X)$

y

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

Staged Homomorphic Secret Sharing

X

$\mathsf{ct}_y \leftarrow \mathsf{Encrypt}(\mathsf{pk}, y)$

$\mathsf{share}_A, \mathsf{share}_B \leftarrow \mathsf{Share}(\mathsf{pk}, X)$

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ct_y share_A

StagedEval

out_A

ct_y share_B

StagedEval

out_B

Staged Homomorphic Secret Sharing

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$\mathsf{ct}_y \leftarrow \mathsf{Encrypt}(\mathsf{pk}, y)$

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ct_y share_A

StagedEval

out_A

ct_y share_B

StagedEval

out_B

$\mathsf{out}_A + \mathsf{out}_A = F(X, y)$

Bilinear-NC^1 program

Staged Homomorphic Secret Sharing

X

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ct_y share_A

StagedEval

out_A

ct_y share_B

StagedEval

out_B

Key Ingredient: Secure and succinct protocol to distribute input shares

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ct_y share_A

StagedEval

out_A

ct_y share_B

StagedEval

out_B

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share_A

$\mathsf{ct}_y \quad \mathsf{share}_A$

StagedEval

out_A

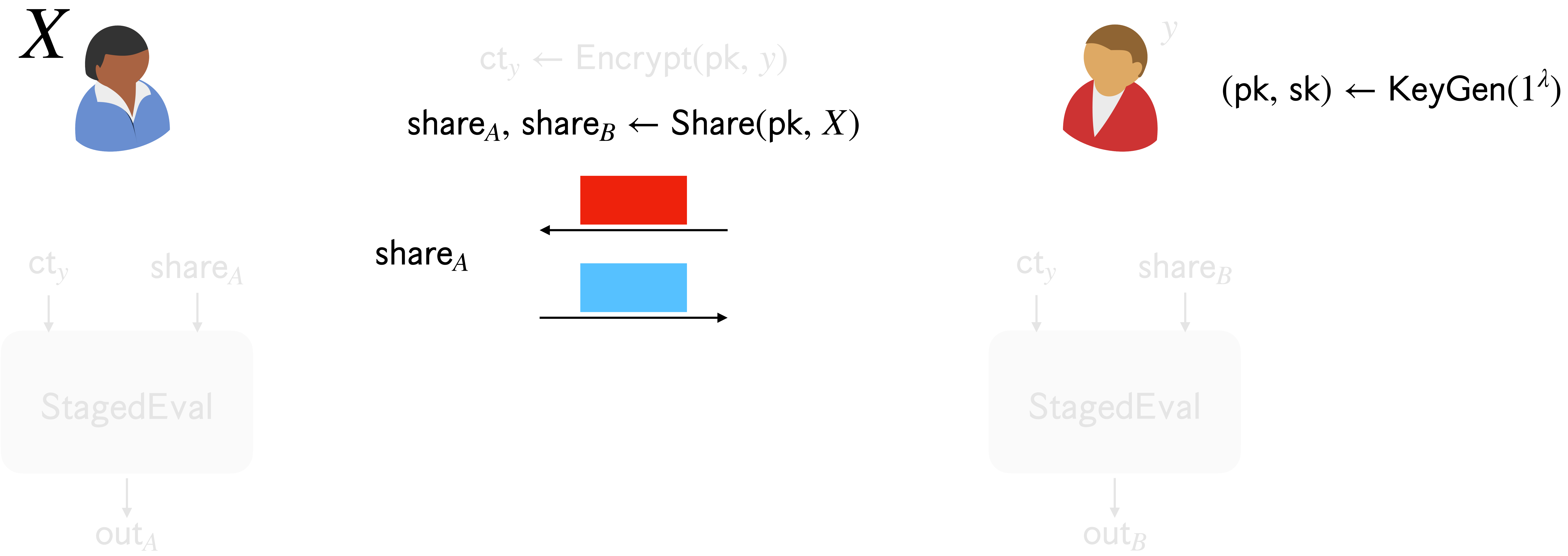
$\mathsf{ct}_y \quad \mathsf{share}_B$

StagedEval

out_B

Key Ingredient: Secure and succinct protocol to distribute input shares

Staged Homomorphic Secret Sharing



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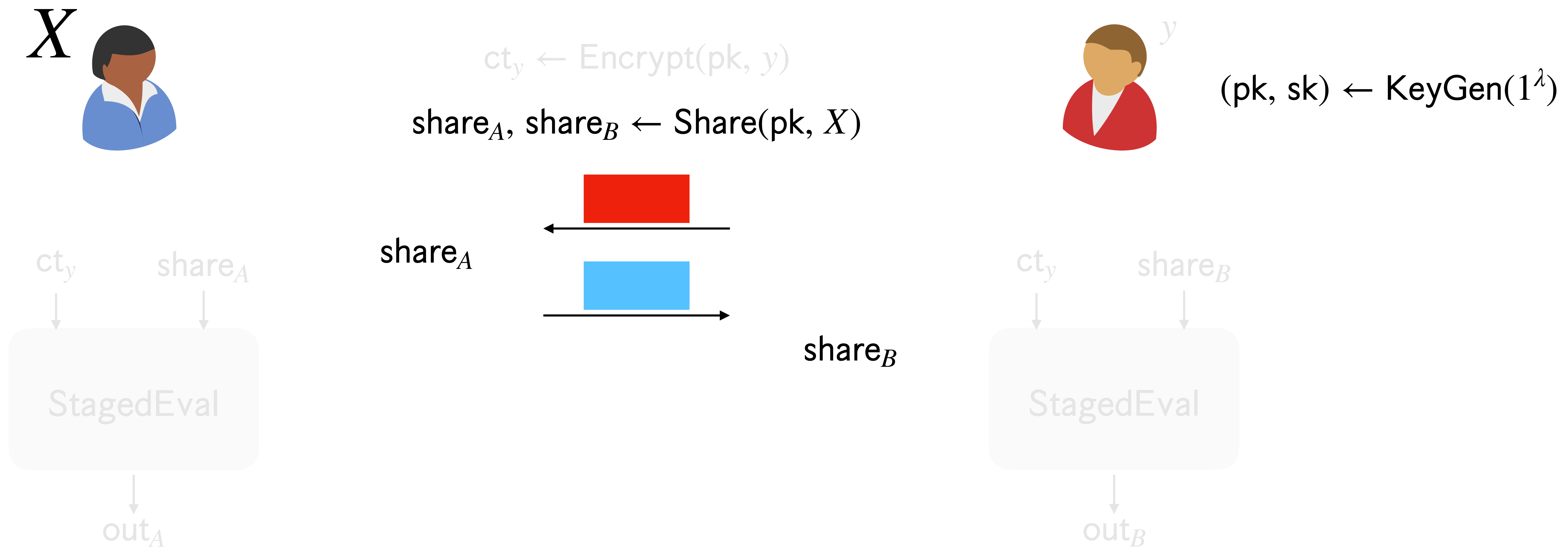

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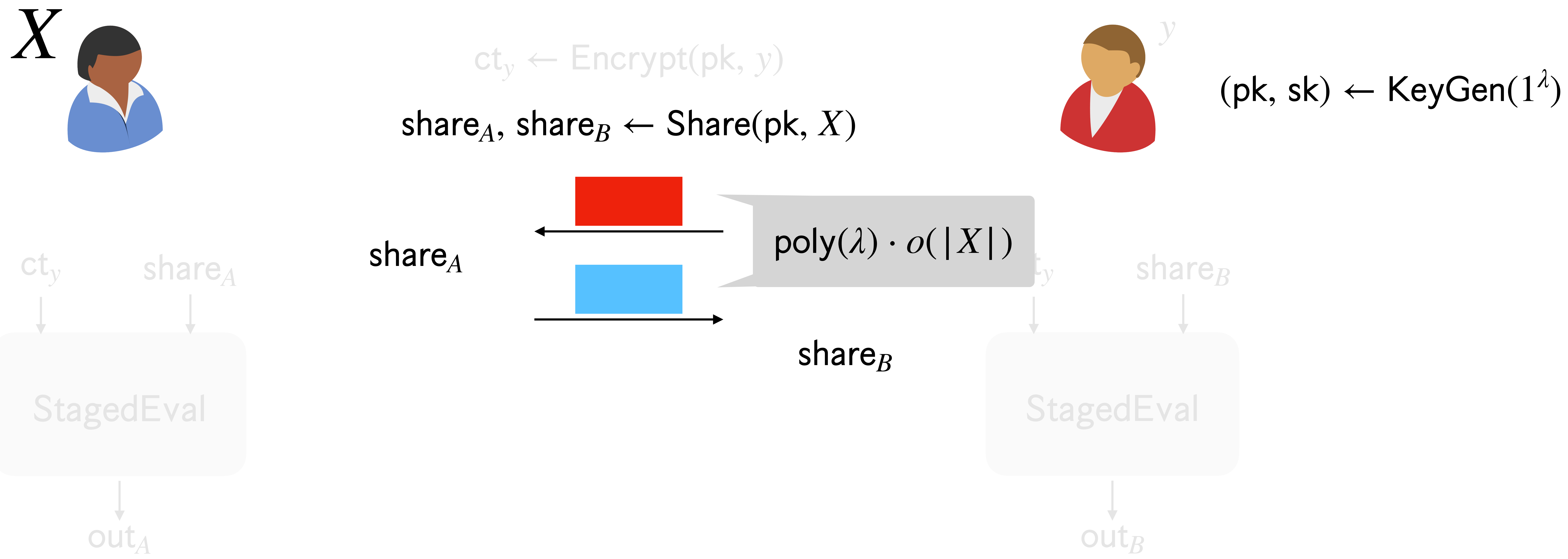

Key Ingredient: Secure and succinct protocol to distribute input shares

Enhanced TDH = Staged HSS + Succinct Distribution of Shares

Public function F

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Public function F

X

y

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Enhanced TDH = Staged HSS + Succinct Distribution of Shares

Public function F

X

y

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$k \leftarrow \{0, 1\}^\lambda$

$z = y \oplus \mathsf{PRG}(k)$

$\mathsf{ct}_k \leftarrow \mathsf{Encrypt}(\mathsf{pk}, k)$

Enhanced TDH = Staged HSS + Succinct Distribution of Shares

Public function F

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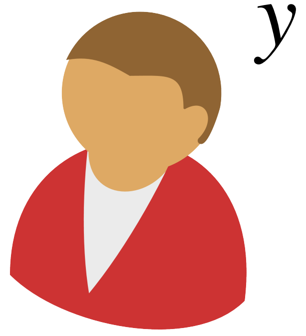

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$\mathsf{ct}_k, \quad z,$

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Compute share_A from

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Compute share_A from

$\mathsf{ct}_k, \quad z,$

$\mathsf{share}_A \rightarrow$ StagedEval $\rightarrow \mathsf{out}_A$

$\mathsf{ct}_k \rightarrow$

Evaluate: $\mathsf{F}'(X, k) = F(X, z \oplus \mathsf{PRG}(k))$

Enhanced TDH = Staged HSS + Succinct Distribution of Shares

Public function F

X

y

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

$k \leftarrow \{0, 1\}^\lambda$

$z = y \oplus \mathsf{PRG}(k)$

$\mathsf{ct}_k \leftarrow \mathsf{Encrypt}(\mathsf{pk}, k)$

Compute share_A from

ct_k, z,

$\mathsf{share}_A \rightarrow$ StagedEval $\rightarrow \mathsf{out}_A$

$\mathsf{ct}_k \rightarrow$

Evaluate: $\mathsf{F}'(X, k) = F(X, z \oplus \mathsf{PRG}(k))$

, out_A

Enhanced TDH = Staged HSS + Succinct Distribution of Shares

Public function F

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$k \leftarrow \{0, 1\}^\lambda$

$z = y \oplus \mathsf{PRG}(k)$

$\mathsf{ct}_k \leftarrow \mathsf{Encrypt}(\mathsf{pk}, k)$

Compute share_A from

ct_k, z,

$\mathsf{share}_A \rightarrow$ StagedEval $\rightarrow \mathsf{out}_A$

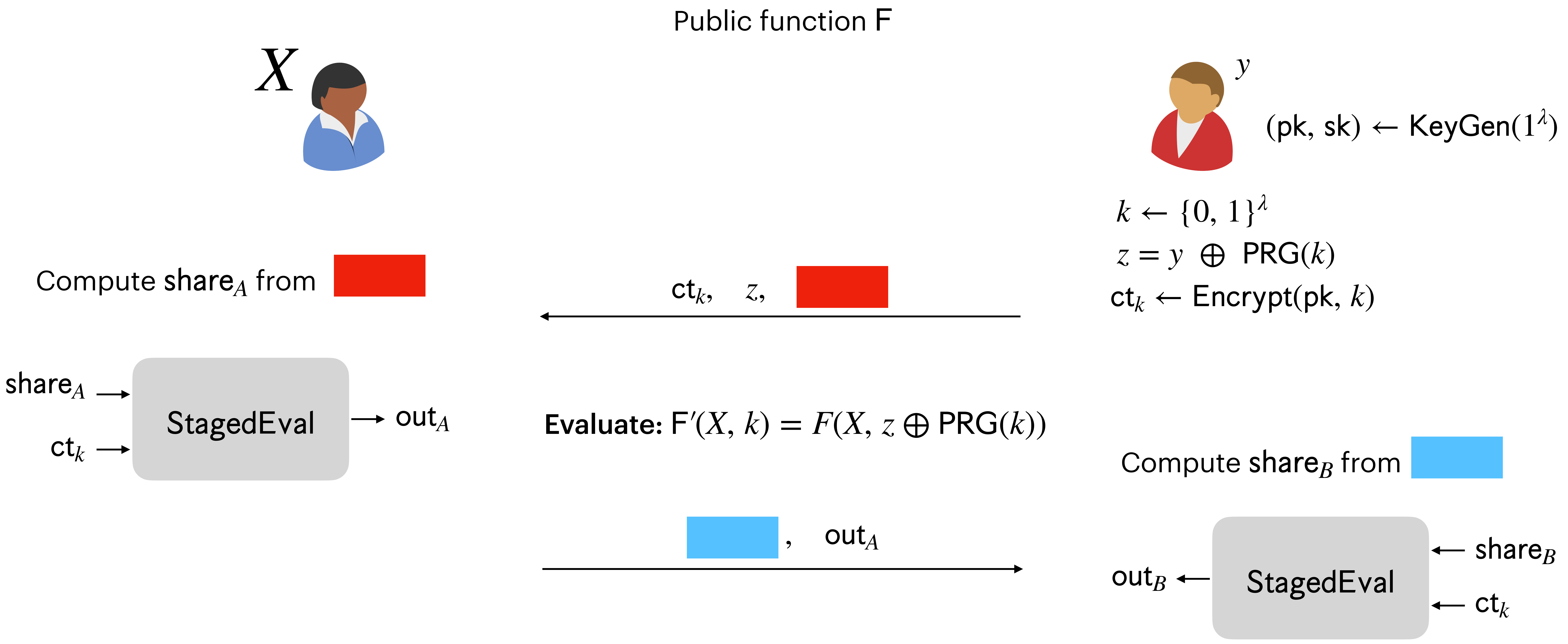
$\mathsf{ct}_k \rightarrow$ StagedEval

Evaluate: $\mathsf{F}'(X, k) = F(X, z \oplus \mathsf{PRG}(k))$

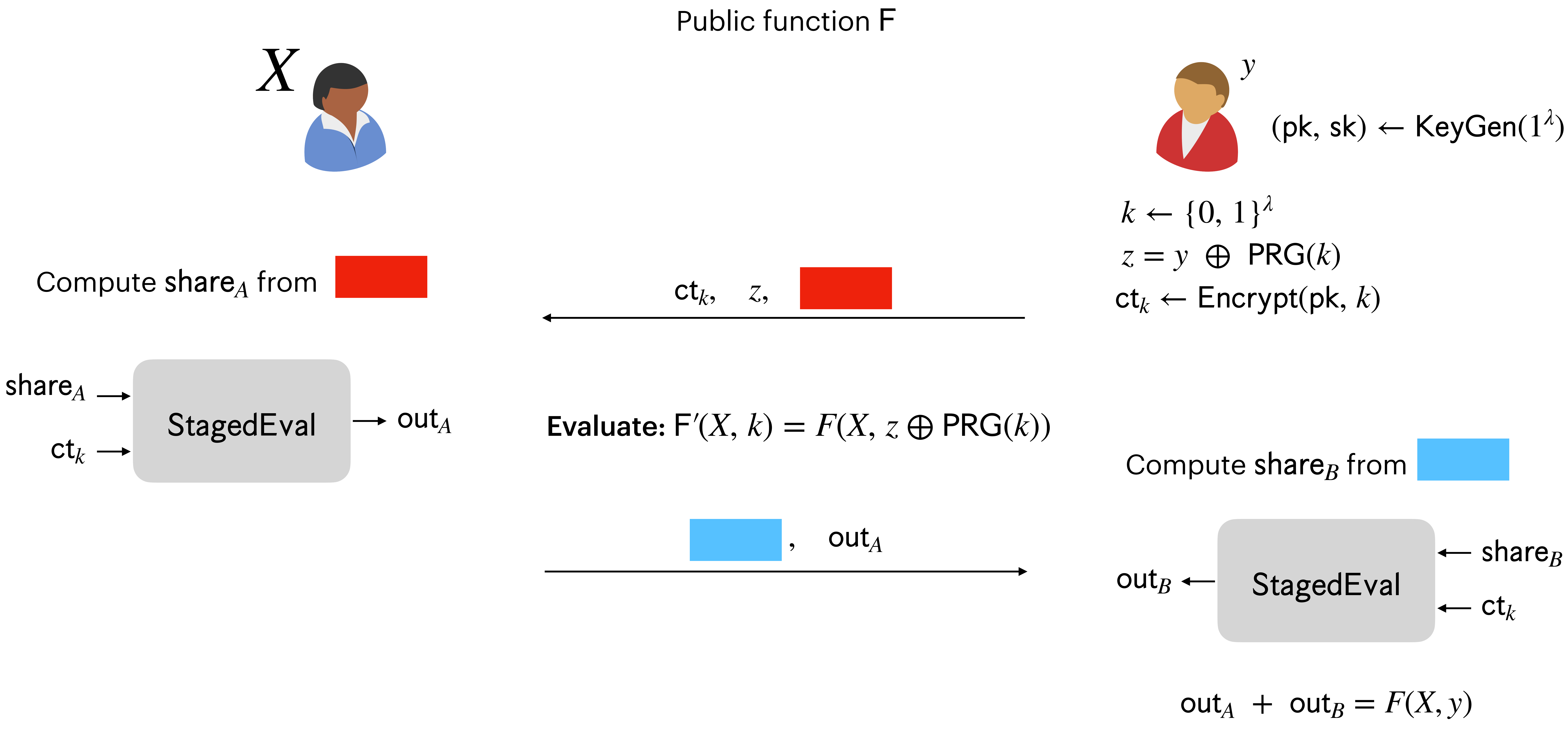
Compute share_B from

, out_A

Enhanced TDH = Staged HSS + Succinct Distribution of Shares



Enhanced TDH = Staged HSS + Succinct Distribution of Shares



Enhanced TDH = Staged HSS + Succinct Distribution of Shares

Public function F

X

y

$(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$

Encoding key ek_y

$|\mathsf{ek}_y| = |y|(1 + o(1))$

$k \leftarrow \{0, 1\}^\lambda$

$z = y \oplus \mathsf{PRG}(k)$

$\mathsf{ct}_k \leftarrow \mathsf{Encrypt}(\mathsf{pk}, k)$

Compute share_A from

ct_k, z,

$\mathsf{share}_A \rightarrow$ StagedEval $\rightarrow \mathsf{out}_A$

$\mathsf{ct}_k \rightarrow$ StagedEval

Evaluate: $\mathsf{F}'(X, k) = F(X, z \oplus \mathsf{PRG}(k))$

Compute share_B from

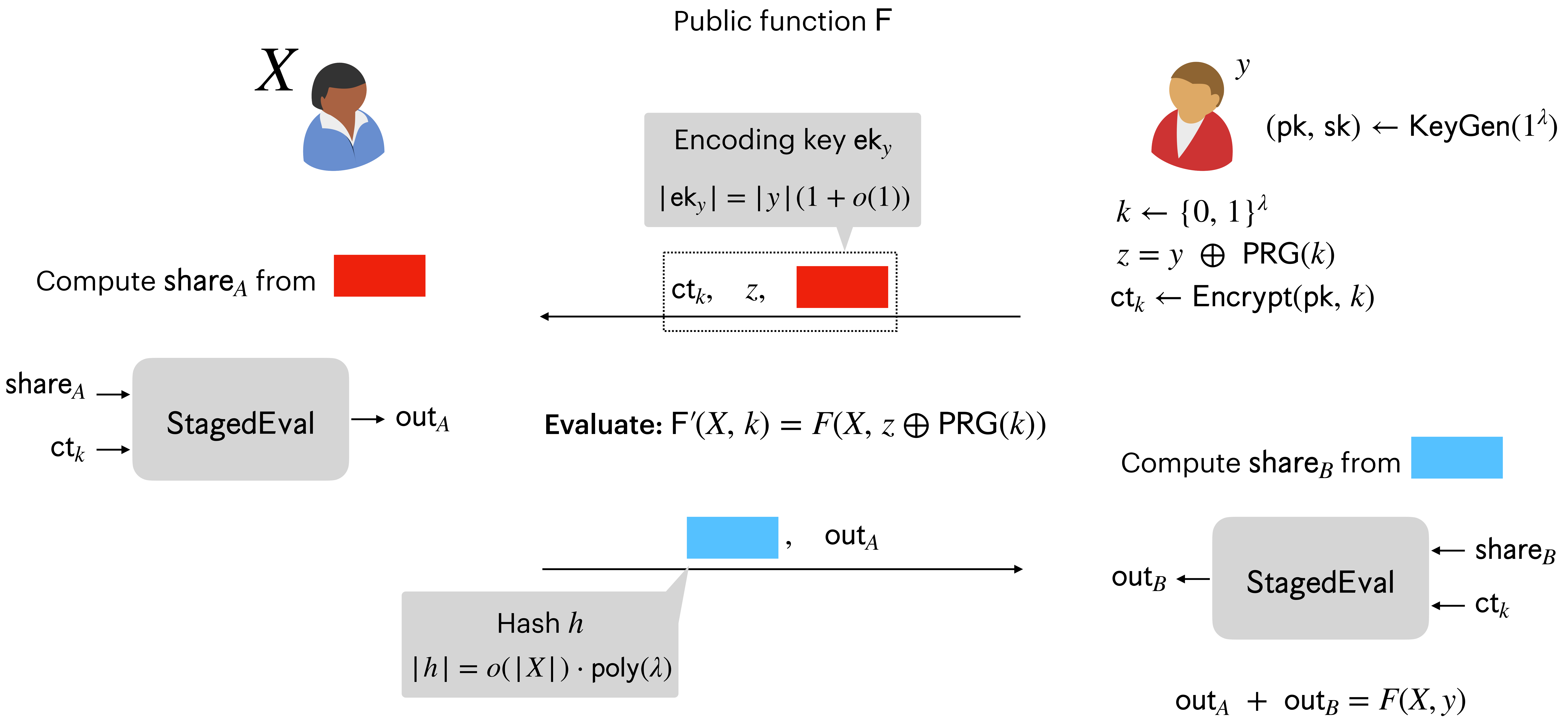
, out_A

$\mathsf{out}_B \leftarrow$ StagedEval $\leftarrow \mathsf{share}_B$

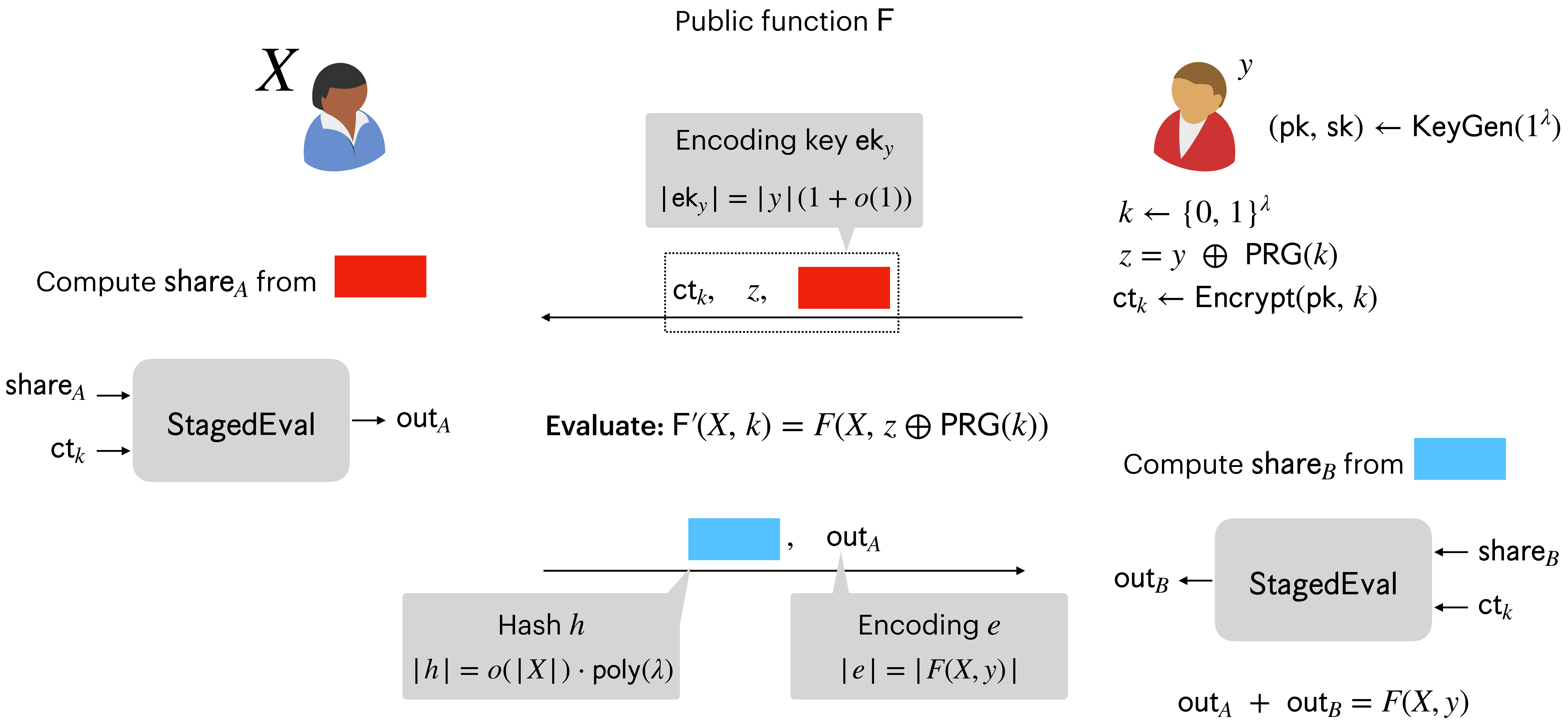
StagedEval $\leftarrow \mathsf{ct}_k$

$\mathsf{out}_A + \mathsf{out}_B = F(X, y)$

Enhanced TDH = Staged HSS + Succinct Distribution of Shares



Enhanced TDH = Staged HSS + Succinct Distribution of Shares



Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

$\text{Share}(\mathsf{pk}, X)$

share_A

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

For each bit b of sk

$\text{ElGamal}(\mathsf{pk}, b \cdot x_1) \quad \cdots \quad \text{ElGamal}(\mathsf{pk}, b \cdot x_n)$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

Share(pk, X)

share_A

For each bit b of sk

$$\mathsf{ElGamal}(\mathsf{pk}, b \cdot x_1) \quad \cdots \quad \mathsf{ElGamal}(\mathsf{pk}, b \cdot x_n)$$

$$=$$

$$(g^{r_1}, g^{\mathsf{sk} \cdot r_1} \cdot g^{b \cdot x_1}) \quad \cdots \quad (g^{r_1}, g^{\mathsf{sk} \cdot r_1} \cdot g^{b \cdot x_n})$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

Share(pk, X)

share_A $\qquad$ share_B

$\mathbb{G}, g$

$\text{sk} \leftarrow \mathbb{Z}_p$

$\text{pk} = g^{\text{sk}}$

For each bit b of sk

ElGamal(pk, $b \cdot x_1$) $\cdots$ ElGamal(pk, $b \cdot x_n$)

$=$

$(g^{r_1}, g^{\text{sk}\cdot r_1} \cdot g^{b \cdot x_1}) \quad \cdots \quad (g^{r_1}, g^{\text{sk}\cdot r_1} \cdot g^{b \cdot x_n})$

$g^{r_1} \ldots g^{r_n}$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

$\mathbb{G}, g$

sk $\leftarrow \mathbb{Z}_p$

pk $= g^{\mathsf{sk}}$

Share(pk, X)

share_A $\qquad$ share_B

For each bit b of sk

$$\mathsf{ElGamal}(\mathsf{pk}, b \cdot x_1) \quad \cdots \quad \mathsf{ElGamal}(\mathsf{pk}, b \cdot x_n)$$

$$=$$

$$(g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_1}) \quad \cdots \quad (g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_n})$$

$$g^{r_1} \ldots g^{r_n}$$

share_B only consists of the random component of the ciphertext

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$$\mathsf{share}_A \xleftarrow{} \mathsf{Share}(\mathsf{pk}, X) \xrightarrow{} \mathsf{share}_B$$

For each bit b of sk

$$\mathsf{ElGamal}(\mathsf{pk}, b \cdot x_1) \quad \cdots \quad \mathsf{ElGamal}(\mathsf{pk}, b \cdot x_n)$$

$$=$$

$$(g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_1}) \quad \cdots \quad (g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_n})$$

$$g^{r_1} \ldots g^{r_n}$$

share_B only consists of the random component of the ciphertext

Attempt:

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \dots, x_n)$

$\text{Share}(\text{pk}, X)$

share_A $\quad$ share_B

$\mathbb{G}, g$

$\text{sk} \leftarrow \mathbb{Z}_p$

$\text{pk} = g^{\text{sk}}$

For each bit b of sk

$\text{ElGamal}(\text{pk}, b \cdot x_1) \quad \cdots \quad \text{ElGamal}(\text{pk}, b \cdot x_n)$

$=$

$(g^{r_1}, g^{\text{sk}\cdot r_1} \cdot g^{b \cdot x_1}) \quad \cdots \quad (g^{r_1}, g^{\text{sk}\cdot r_1} \cdot g^{b \cdot x_n})$

$g^{r_1} \dots \; g^{r_n}$

share_B only consists of the random component of the ciphertext

Attempt:

$k \leftarrow \{0, 1\}^\lambda$

$r_1, \dots, r_n \leftarrow \text{PRG}(k)$

$g^{r_1} \dots \; g^{r_n}$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

$\mathsf{Share}(\mathsf{pk}, X)$

share_A $\quad$ share_B

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

For each bit b of sk

$$\mathsf{ElGamal}(\mathsf{pk}, b \cdot x_1) \quad \cdots \quad \mathsf{ElGamal}(\mathsf{pk}, b \cdot x_n)$$

$$=$$

$$(g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_1}) \quad \cdots \quad (g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_n})$$

$$g^{r_1} \ldots \; g^{r_n}$$

share_B only consists of the random component of the ciphertext

Attempt:

$$k \leftarrow \{0, 1\}^\lambda$$

$$\xleftarrow{\;k,\ \mathsf{pk}\;}$$

$r_1, \ldots, r_n \leftarrow \mathsf{PRG}(k)$

$g^{r_1} \ldots \; g^{r_n}$

$r_1, \ldots, r_n \leftarrow \mathsf{PRG}(k)$

$g^{r_1} \ldots \; g^{r_n}$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

Share(pk, X)

share_A $\quad$ share_B

$\mathbb{G}, g$

$\text{sk} \leftarrow \mathbb{Z}_p$

$\text{pk} = g^{\text{sk}}$

For each bit b of sk

ElGamal(pk, $b \cdot x_1$) $\quad \cdots \quad$ ElGamal(pk, $b \cdot x_n$)

$=$

$(g^{r_1}, g^{\text{sk}\cdot r_1} \cdot g^{b \cdot x_1}) \quad \cdots \quad (g^{r_1}, g^{\text{sk}\cdot r_1} \cdot g^{b \cdot x_n})$

$g^{r_1} \ldots \; g^{r_n}$

share_B only consists of the random component of the ciphertext

Attempt:

$r_1, \ldots, r_n \leftarrow \text{PRG}(k)$

$g^{r_1} \ldots \; g^{r_n}$

$g^{\text{sk}\cdot r_1} \; \ldots \; g^{\text{sk}\cdot r_n}$

$\xleftarrow{\;k,\ \text{pk}\;}$

$k \leftarrow \{0, 1\}^\lambda$

$r_1, \ldots, r_n \leftarrow \text{PRG}(k)$

$g^{r_1} \ldots \; g^{r_n}$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$$\mathsf{share}_A \xleftarrow{} \mathsf{Share}(\mathsf{pk}, X) \xrightarrow{} \mathsf{share}_B$$

For each bit b of sk

$\mathsf{ElGamal}(\mathsf{pk}, b \cdot x_1) \quad \cdots \quad \mathsf{ElGamal}(\mathsf{pk}, b \cdot x_n)$

$=$

$(g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1}) \quad \cdots \quad (g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$

$g^{r_1} \ldots \; g^{r_n}$

share$_B$ only consists of the random component of the ciphertext

Attempt:

$k \leftarrow \{0, 1\}^\lambda$

$r_1, \ldots, r_n \leftarrow \mathsf{PRG}(k)$

$g^{r_1} \ldots \; g^{r_n}$

$\xleftarrow{\;k,\ \mathsf{pk}\;}$

$r_1, \ldots, r_n \leftarrow \mathsf{PRG}(k)$

$g^{r_1} \ldots \; g^{r_n}$

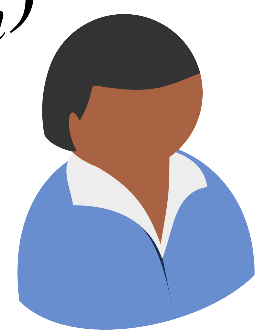
$g^{\mathsf{sk}\cdot r_1} \; \ldots \; g^{\mathsf{sk}\cdot r_n}$

Alice needs g^b to compute $g^{\mathsf{sk}\cdot r_i} \cdot g^{b\cdot x_i}$ but sending this would be insecure

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$



$\mathsf{Share}(\mathsf{pk}, X)$

share_A share_B

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

For each bit b of sk

$\mathsf{ElGamal}(\mathsf{pk}, b \cdot x_1) \quad \cdots \quad \mathsf{ElGamal}(\mathsf{pk}, b \cdot x_n)$

$=$

$(g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1}) \quad \cdots \quad (g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$

$g^{r_1} \ldots \; g^{r_n}$

share_B only consists of the random component of the ciphertext

Observation: Can be computed succinctly using techniques from Trapdoor Hashing [Döttling-Garg-Ishai-Malavolta-Mour-Ostrovsky'19]

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$X = (x_1, \ldots, x_n)$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \ldots, x_n)$

$$(g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$$
$$\vdots$$
$$(g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$$

$g^{r_1} \quad \ldots \quad g^{r_n}$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \ldots, x_n)$

$$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$$
$$\vdots$$
$$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$$

$g^{r_1} \quad \ldots \quad g^{r_n}$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$$g_1,\ \ldots,\ g_n \leftarrow \mathbb{G}$$
$$r_1 \leftarrow \mathbb{Z}_p$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \ldots, x_n)$

$(g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$

$\vdots$

$(g^{r_1}, g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$

$g^{r_1} \ \cdots \ g^{r_n}$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$(g_1, \ \ldots, \ g_n)$

$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b}, g_2^{\mathsf{sk}\cdot r_1}, \ \ldots, \ g_n^{\mathsf{sk}\cdot r_1})$

$g_1, \ \ldots, \ g_n \leftarrow \mathbb{G}$

$r_1 \leftarrow \mathbb{Z}_p$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \ldots, x_n)$

$$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$$
$$\vdots$$
$$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$$

$g^{r_1} \ \ldots \ g^{r_n}$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$$(g_1,\ \ldots,\ g_n)$$
$$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \ldots,\ g_n^{\mathsf{sk}\cdot r_1})$$

$g_1,\ \ldots,\ g_n \leftarrow \mathbb{G}$

$r_1 \leftarrow \mathbb{Z}_p$

$$h = (g_1,\ \ldots,\ g_n) \cdot X^T = \prod_{i=1}^{n} g_i^{x_i}$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \ldots, x_n)$

$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$

$\vdots$

$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$

$g^{r_1} \ \ldots \ g^{r_n}$

$\mathbb{G},\ g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$(g_1,\ \ldots,\ g_n)$

$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \ldots,\ g_n^{\mathsf{sk}\cdot r_1})$

$g_1,\ \ldots,\ g_n \leftarrow \mathbb{G}$

$r_1 \leftarrow \mathbb{Z}_p$

$$h = (g_1,\ \ldots,\ g_n) \cdot X^T = \prod_{i=1}^{n} g_i^{x_i}$$

$$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \ldots,\ g_n^{\mathsf{sk}\cdot r_1}) \cdot X^T$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \ldots, x_n)$

$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$

$\vdots$

$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$

$g^{r_1} \ \ldots \ g^{r_n}$

$\mathbb{G},\ g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$(g_1,\ \ldots,\ g_n)$

$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \ldots,\ g_n^{\mathsf{sk}\cdot r_1})$

$\longleftarrow$

$g_1,\ \ldots,\ g_n \leftarrow \mathbb{G}$

$r_1 \leftarrow \mathbb{Z}_p$

$$h = (g_1,\ \ldots,\ g_n) \cdot X^T = \prod_{i=1}^{n} g_i^{x_i}$$

$$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \ldots,\ g_n^{\mathsf{sk}\cdot r_1}) \cdot X^T$$

$$= \left(\prod_{i=1}^{n} g_i^{x_i}\right)^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1}$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \ldots, x_n)$

$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_1})$

$\vdots$

$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_n})$

$g^{r_1} \ \ldots \ g^{r_n}$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$$\xleftarrow[\qquad]{(g_1,\ \ldots,\ g_n) \quad (g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \ldots,\ g_n^{\mathsf{sk}\cdot r_1})}$$

$g_1,\ \ldots,\ g_n \leftarrow \mathbb{G}$

$r_1 \leftarrow \mathbb{Z}_p$

$$h = (g_1,\ \ldots,\ g_n) \cdot X^T = \prod_{i=1}^{n} g_i^{x_i}$$

$$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \ldots,\ g_n^{\mathsf{sk}\cdot r_1}) \cdot X^T$$

$$= \left(\prod_{i=1}^{n} g_i^{x_i}\right)^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1} = h^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_1}$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \dots, x_n)$

$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$

$\vdots$

$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$

$g^{r_1}\ \dots\ g^{r_n}$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$(g_1,\ \dots,\ g_n)$

$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \dots,\ g_n^{\mathsf{sk}\cdot r_1})$

$g_1,\ \dots,\ g_n \leftarrow \mathbb{G}$

$r_1 \leftarrow \mathbb{Z}_p$

$$h = (g_1,\ \dots,\ g_n) \cdot X^T = \prod_{i=1}^{n} g_i^{x_i}$$

$$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \dots,\ g_n^{\mathsf{sk}\cdot r_1}) \cdot X^T$$

$$= \left(\prod_{i=1}^{n} g_i^{x_i}\right)^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1} = h^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1}$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

$$(g^{r_1},\ g^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$$

A change in "base" from $g^{\mathsf{sk}\cdot r_1}$ to $h^{\mathsf{sk}\cdot r_1}$
does not affect staged HSS evaluation

$$= h^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1}$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \ldots, x_n)$

$$(h^{r_1},\ h^{\mathsf{sk}\cdot r_1}\cdot g^{b\cdot x_1})$$
$$\vdots$$
$$(h^{r_1},\ h^{\mathsf{sk}\cdot r_1}\cdot g^{b\cdot x_n})$$

$$h^{r_1}\ \ldots\ h^{r_n}$$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$(g_1, \ldots, g_n)$

$(g_1^{\mathsf{sk}\cdot r_1}\cdot g^{b}, g_2^{\mathsf{sk}\cdot r_1}, \ldots, g_n^{\mathsf{sk}\cdot r_1})$

$g_1, \ldots, g_n \leftarrow \mathbb{G}$

$r_1 \leftarrow \mathbb{Z}_p$

A change in "base" from $g^{\mathsf{sk}\cdot r_1}$ to $h^{\mathsf{sk}\cdot r_1}$
does not affect staged HSS evaluation

$h = (g_1, \ldots$ $\prod_{i=1}$

$$(g_1^{\mathsf{sk}\cdot r_1}\cdot g^{b}, g_2^{\mathsf{sk}\cdot r_1}, \ldots, g_n^{\mathsf{sk}\cdot r_1})\cdot X^T$$

$$= \left(\prod_{i=1}^{n} g_i^{x_i}\right)^{\mathsf{sk}\cdot r_1}\cdot g^{b\cdot x_1} = h^{\mathsf{sk}\cdot r_1}\cdot g^{b\cdot x_1}$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \dots, x_n)$

$(h^{r_1},\ h^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$

$\vdots$

$(h^{r_1},\ h^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$

$h^{r_1}\ \dots\ h^{r_n}$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$(g_1,\ \dots,\ g_n)$

$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \dots,\ g_n^{\mathsf{sk}\cdot r_1})$

$g_1,\ \dots,\ g_n \leftarrow \mathbb{G}$

$r_1 \leftarrow \mathbb{Z}_p$

$$h = (g_1,\ \dots,\ g_n) \cdot X^T = \prod_{i=1}^{n} g_i^{x_i}$$

$$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b},\ g_2^{\mathsf{sk}\cdot r_1},\ \dots,\ g_n^{\mathsf{sk}\cdot r_1}) \cdot X^T = h^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1}$$

Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

Goal

$X = (x_1, \ldots, x_n)$

$$(h^{r_1}, h^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1})$$
$$\vdots$$
$$(h^{r_1}, h^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_n})$$

$h^{r_1} \ \ldots \ h^{r_n}$

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$$(g_1, \ \ldots, g_n)$$

$$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b}, g_2^{\mathsf{sk}\cdot r_1}, \ \ldots, g_n^{\mathsf{sk}\cdot r_1})$$

$$(g_1^{\mathsf{sk}\cdot r_2}, g_2^{\mathsf{sk}\cdot r_2} \cdot g^{b}, \ \ldots, g_n^{\mathsf{sk}\cdot r_2})$$

$g_1, \ \ldots, g_n \leftarrow \mathbb{G}$

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$$h = (g_1, \ \ldots, g_n) \cdot X^T = \prod_{i=1}^{n} g_i^{x_i}$$

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Succinct Distribution of Staged Input Shares

Structure of Staged Input Shares

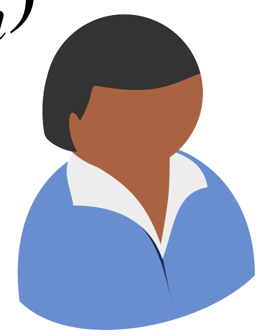
Goal

$\mathbb{G}, g$

$\mathsf{sk} \leftarrow \mathbb{Z}_p$

$\mathsf{pk} = g^{\mathsf{sk}}$

$X = (x_1, \ldots, x_n)$



$(h^{r_1}, h^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_1})$

$\vdots$

$(h^{r_1}, h^{\mathsf{sk}\cdot r_1} \cdot g^{b \cdot x_n})$

$h^{r_1} \ \ldots \ h^{r_n}$

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$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b}, g_2^{\mathsf{sk}\cdot r_1}, \ \ldots, g_n^{\mathsf{sk}\cdot r_1})$

$\vdots$

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$$\vdots$$

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$h^{r_1}, \ldots, h^{r_n}$ can be computed similarly

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Structure of Staged Input Shares

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Structure of Staged Input Shares

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$\xrightarrow{h}$

$h^{r_1}, \ldots, h^{r_n}$

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Hash needs to be randomized for security

$\xrightarrow{\quad h \quad}$

$h^{r_1}, \ldots, h^{r_n}$

$h^{r_1}, \ldots, h^{r_n}$ can be computed similarly

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$\xrightarrow{h}$

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$h = (g_1, \ \ldots, \ g_n) \cdot X^T$

$O(\lambda \cdot n^2)$

$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b}, g_2^{\mathsf{sk}\cdot r_1}, \ \ldots, \ g_n^{\mathsf{sk}\cdot r_1}) \cdot X^T = h^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1}$
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$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b}, g_2^{\mathsf{sk}\cdot r_1}, \ \ldots, g_n^{\mathsf{sk}\cdot r_1})$
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$(g_1^{\mathsf{sk}\cdot r_n}, g_2^{\mathsf{sk}\cdot r_n}, \ \ldots, g_n^{\mathsf{sk}\cdot r_n} \cdot g^{b})$

$O(\lambda \cdot n^2)$

Can be reused with multiple hashes $\Longrightarrow$ rebalancing gives $O(n^{2/3} \cdot \lambda)$ total communication

$h = (g_1, \ \ldots, g_n) \cdot X^T$

$(g_1^{\mathsf{sk}\cdot r_1} \cdot g^{b}, g_2^{\mathsf{sk}\cdot r_1}, \ \ldots, g_n^{\mathsf{sk}\cdot r_1}) \cdot X^T = h^{\mathsf{sk}\cdot r_1} \cdot g^{b\cdot x_1}$
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$O(\lambda)$

h

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Conclusion

- Discussed approach assumes circular security of ElGamal. Constructing from plain DDH and DCR requires extending to circular-secure variants [Boneh-Halevi-Hamburg-Ostrovsky’08] [Brakerski-Goldwasser’10]
- DDH-based Staged HSS evaluation has noticeable error probability which affects privacy. Requires developing new techniques to build trapdoor hash functions

Conclusion

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- DDH-based Staged HSS evaluation has noticeable error probability which affects privacy. Requires developing new techniques to build trapdoor hash functions

Thank You